Epistemic Games in Integration: Modeling Resource Choice

Katrina E. Black* and Michael C. Wittmann*¶†

*Department of Physics and Astronomy, University of Maine, Orono, ME 04469, USA
¶College of Education and Human Development, University of Maine, Orono, ME 04469, USA
†Center for Science and Mathematics Education Research, University of Maine, Orono, ME 04469, USA

Abstract. As part of an ongoing project to understand how mathematics is used in advanced physics to guide one's conceptual understanding of physics, we focus on students' interpretation and use of boundary and initial conditions when solving integrals. We discuss an interaction between two students working on a group quiz problem. After describing the interaction, we briefly discuss the procedural resources that we use to model the students' solutions. We then use the procedural resources introduced earlier to draw resources graphs describing the two epistemic game facets used by the students in our transcript.

Keywords: Physics education research, differential equations, air resistance.
PACS: 01.40Fk

INTRODUCTION

We find that students typically use one of two methods to apply boundary conditions to integrals that must be performed when solving first-order, separable differential equations (FOSDEs) in a physics setting. In one method, an undetermined integration constant ("+c") is added following integration and the boundary condition is used to find the value of the constant. In the other method ("limits"), the boundary or initial conditions are used to choose appropriate limits of integration. Although these methods lead to physically equivalent solutions, the choice of method, whether tacit or deliberate, is found to affect students' abilities to arrive at complete descriptions of the physics. We found that in physics-like scenarios, students who used the "+c" method did not use the boundary condition to find the value of the integration constant; rather, they left the constant unspecified. We present a detailed transcript of two students discussing these methods and model their reasoning in terms of conceptual and procedural resources[1,2] as well as a description of their framing[3] of the problem and the epistemic games[4,5] they use.

I. BACKGROUND AND TRANSCRIPT

In 2007, students in an intermediate mechanics course using small-group learning materials (the *Intermediate Mechanics Tutorials*[6,7]) took part in group quizzes. In one quiz, students were asked to find the horizontal velocity with respect to time of a beach ball thrown horizontally, assuming a v^2 air resistance and no appreciable vertical motion during the motion. In one group, two students, Max and Phil (*aliases*), having set up the equation (Eqn. 1) and separated variables to create

$$m\frac{dv}{dt} = -bv^2 \qquad (1)$$

two integrals (Eqn. 2), debate whether to use the "+c" or the "limits" method to integrate the *dt* side of the separated differential equation.

$$\frac{-m}{b}\int\frac{dv}{v^2} = \int dt \qquad (2)$$

(Earlier, they indefinitely integrated the *dv* side without adding a constant of integration.)

Max, advocating for "limits," and Phil, advocating for "+c," are explicit about their reasons for choosing each method. Phil's reasoning provides a possible explanation for the failure to find the value of the integration constant observed in earlier studies of students using the "+c" method.

At this point in the discussion, Phil, who has led the group throughout, asserts that the next step in the solution is to indefinitely integrate the right hand (*t*) side and add an integration constant.

Phil: so we get negative m over b times negative v to the negative one equals t plus c.

Max: No, this is where you should be going just plain t, or t-naught to t, or zero to t, because initial time is when you throw it to some time, so just go to t instead of t plus c. <pause> Because then your integration of time would probably be going from zero, when you throw it, to t, some time later. So you don't need c there

In an earlier part of the problem (transcript not given), Max had not objected that limits were not used when performing the v-integral. In the t-integral, he objects to the addition of a constant. His use of language (line 3: "you should be going" and lines 5 and 6: "just go to t") implies that he would prefer using limits of integration based on the physical situation. Phil, however, is not convinced. Although he offers to change the name of the constant to indicate that it represents a time, he is unwilling to assign that constant any particular value.

Phil: Hmmmmm… or you could do t plus t-naught. You need a constant though. I mean yeah, if you did it

When Max asks why a constant is needed (line 13), Phil reveals his reasoning: without a constant, you don't have a "function" (lines 22–24):

Max: Why would you need a constant in time? You just go from 0 to t.
Phil: Because it's <sigh> --
Max: -- t-naught to t
Phil: -- that's not a function, well
Max: well, you could go from t-naught to t, but t-naught is just initially zero when you start throwing the ball.
Phil: If you, ssssss, no, you can't do that, because then you're not setting up a function. You're setting up a function that's only ok in certain cases, you see what I mean?

From lines 22 to 24, we infer that what Phil is actually referring to is a family of functions. A function "that's only OK in certain cases" is not general enough. It seems that, to Phil, the integration constant must remain as general as possible. In contrast, Max is trying to use information specific to this problem in working through the mathematics.

Later, Phil and Max are still discussing the limits versus integration constant issue. Now, Phil does not object to using limits if they are arbitrary (for example, from t_0 to t) but still does not want to apply a specific boundary condition to the equation. The problem statement does not specifically say that you throw the ball at t = 0, and Phil seems uncomfortable using that common physics convention. Again, he argues that the final equation must be one that "anybody can use" (a family of functions) and not "one that you know" (an equation with a specific boundary condition applied):

Phil: I just can't imagine integrating without having an extra constant. I guess if you said t-naught is zero, so, I mean, yeah
Max: but t-naught is the initial time
Phil: But you don't know that's zero.
Max: You can just mark it as zero anyway. <pause> for consistency. Why, why do you need some extra time there?
Phil: Yeah, but then you're not creating an objective function that anyone can use. You're making a function that **you** know, and that's great, but it's not really, I mean,
Max: Well, no.
Phil: the math has to cover everything. We'll compromise, we'll say t minus t-naught. Write that. That work?
Max: That's fine
Phil: OK

In lines 33 to 35, we again see Phil trying to be as general as possible. Max, though, is looking for consistency between the physics and the math (lines 30 to 32). As we describe below, their compromise (lines 39 to 41) satisfies both their agendas in solving this problem, and they are able to move forward without actually resolving the situation.

II. CONCEPTUAL AND PROCEDURAL RESOURCES

Resources are a model to describe small chunks of student knowledge[1,2]. Although most resources discussed in the literature are conceptual, the definition of resources as pieces of knowledge also leaves room for procedural, factual, and analytical resources.

We describe several procedural resources that we observe students using during the integration described above. In particular, these resources are used after the students have separated variables but before the final solution has been reached. These resources are part of larger activities, *Definite Integration* and *Indefinite Integration*, which describe the algorithmic computation of an integral. We have found, in several years of observation, that a student's past exposure to integration in math and physics classes often has a strong influence on which of these resources is activated. We state explicitly that we use the term "resources" here to describe constructs and procedures that often require and contain sub-steps that we do not make explicit. Instead, our goal is to clarify the many different kinds of procedures that students must carry out when solving integrals in a physics class. Analyses at different grain sizes (*e.g.*, smaller procedural steps) are possible.

Two procedural resources that students might use when integrating a physically meaningful equation are *Find Value* and *Choose Limits*. Each connects the physical meaning of the equation to the mathematical formalism and involves several steps when applied. These resources are frequently used in conjunction with the resource *Extract Boundary Condition*, in which the student uses the problem statement to develop the mathematical form of the boundary conditions.

We have found (though the transcript above does not show it) that the presentation statement of the boundary condition can influence how strongly the *Extract Boundary Conditions* resource is primed and what other resources it gets linked to. In a math-like problem, the boundary condition, if it exists, is already stated in its mathematical form; it is pre-extracted in the form $v(0)=0$ (note the lack of units in this math-like formalism). On the other end of the continuum, a very physics-like problem has no boundary conditions stated, either mathematically or linguistically. In this case the student must reason about the described situation to determine the boundary conditions. An intermediate type problem might state the boundary conditions explicitly in words, leaving the student to do the translation to mathematics.

In this language, we observe Phil and Max using different procedural resources and arguing about the implementation of *Extract Boundary Conditions* and the subsequent of activation of *Find Value* for Phil and *Choose Limits* for Max. From this, we can construct a more complete description of their work, combining the representation of resource graphs [2] with a description of epistemic games in physics [8].

III. DESCRIBING EPISTEMIC GAMES WITH GRAPHS OF PROCEDURAL RESOURCES

Much as resources are a kind of schema, epistemic games are one kind of script [8]. (The former describes conceptually-linked ideas, the latter time-ordered actions.) Epistemic games are sets of rules that are followed when creating new knowledge. They have starting conditions, moves, and an ending condition, known as the epistemic form. When solving a problem, a student takes a particular path through the primed resources of his or her frame. If the links between these resources are robust and often activated in the same order in varying situations, we can call this path an epistemic game. Because the resources primed in a setting depend on other resources activated in that setting, the epistemic games available for play are also heavily personally and contextually dependent; more than one game may be available in a particular situation. Similarly, since resources can be applied in a variety of situations, the same game may be available in different situations. In physics, epistemic games have been described at a fairly large grain size [5].

In our case, both Max and Phil can be seen as playing variations of the game "Mapping Meaning to Mathematics," described by Tuminaro [5]. This game has the moves *Develop story* about physical situation, *Translate quantities* in physical story to mathematical entities, *Relate mathematical identities* in accordance with physical story, *Manipulate symbols*, and *Evaluate story*. Specific details differ for Max and Phil, though.

Each specific move in the "Mapping Meaning to Mathematics" game can be described as a procedural resource. To describe the game as a whole, we build a resource graph out of these procedural resources. In general, at this large grain size, the network is relatively linear and strongly unidirectional. In practice, we find that certain loops exist and are commonly activated as a whole. This phenomenon is even more evident at a smaller grain size, where is it is often impossible to determine the order in which resources are activated. A resource graph allows us to represent this situation.

IV. FACETS OF EPISTEMIC GAMES

Although Phil and Max have identical end conditions at a large grain size (an equation describing the speed of the ball), in a finer grain analysis, they differ. Phil requires an equation that "can cover anything" (line 38), while Max questions the need for an unspecified constant. Thus, Phil and Max can be described as playing different versions of one epistemic game.

We call these games "Finding a Family of Functions" and "Fitting the Physical Situation." Both have similar moves and are facets of the "Making Meaning of Mathematics" epistemic game. The early moves in each game are so similar that it is not until near the end of the problem solution that Phil and Max have a difference of opinion on how to proceed.

Phil is playing the game "Finding a Family of Functions." Several procedural mathematical resources are part of this game, including *Compute Antiderivative* and *Add Constant*. The entrance conditions for this game are a differential equation without boundary conditions. The problem statement is very physics-like; the boundary condition is not stated in words or math. Thus, he frames the activity as one of finding the most general solution to a physics problem. Without an explicit boundary condition statement, Phil does not activate *Extract Boundary Conditions*. He begins to solve the problem using "+c," but through argument with Max does not arrive at an ending point.

In Max's case, *Extract Boundary Conditions* is activated by the physical scenario and by his intention (line 30-32) for "consistency" (seemingly between the mathematics and the physics). He makes explicit use of physics conventions in the broader game of "Mapping Meaning to Mathematics." Although the problem statement does not explicitly state that the initial time is equal to zero, Phil feels that an initial time must be defined, and that zero is a conventional initial time for this type of problem. As a result, Phil can more easily be described as playing the "Mapping Meaning to Mathematics" game, even as his variation is one of "Fitting the Physical Situation."

To better represent the "Making Meaning of Mathematics" game and its two facets of "Finding a Family of Functions" and "Fitting the Physical Situation," we represent the individual procedural resources in a resource graph with directed arrows to indicate the possible allowable moves (see Fig. 1). We note that our resource graph is situation and student dependent. In this case, it is drawn to show the moves of two different students

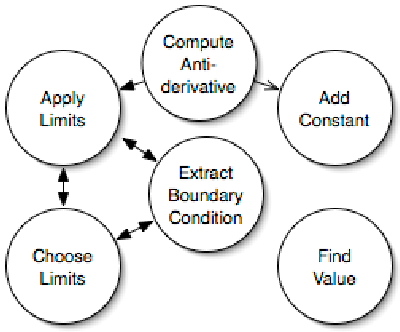

Figure 1: A partial resource graph of two epistemic games. Phil's game is represented by open arrows; Max's by solid

Both Phil and Max begin at *Compute Antiderivative*. Phil's game is, from here, very simple. He needs only *Add Constant* to make a family of functions from the antiderivative. It is clear that these resources are very strongly linked. In lines 25-26, Phil states that he "just can't imagine integrating without having an extra constant". However, although undoubtedly Phil has the resources *Extract Boundary Conditions* and *Find Value*, they are not activated here. In line 29, he states that boundary conditions cannot be used because "you don't know that's zero". To Phil, only the problem writer can determine boundary conditions, and we can infer that if the boundary condition were given in the problem statement, his framing of the problem would be such that *Extract Boundary Condition* and *Find Value* would be activated.

Max's game is rather more complex, and the exact order in which the resources are applied is difficult to determine from the transcript. This is represented in the resource graph by the use of double-ended arrows. Evidence exists for the activation of three resources -- *Apply Limits, Choose Limits, and Extract Boundary Conditions*. Additionally, since Phil does not object when Max integrates both sides of the equation, we can infer that *Compute Antiderivative* is also active.

In several places (lines 3-9, 14, 18), Max uses the language "go" or "going." In these cases,*Apply Limits* is the resource in action. In lines 3-6, Phil gives several possibilities for limits; it is clear that he needs *Choose Limits* to narrow down the options. At the end of that statement (lines 7-9) he uses just one set of limits, zero to t, and describes the reason for his choice.

In lines 18-20, we see a good example of *Extract Boundary Conditions*. Phil states that although you could integrate from t_0 to t, it reasonable to use the common physics convention of calling t_0 "zero," even though this information is not specifically given in the problem. It is worth noting that *Choose Limits* and *Extract Boundary Conditions* are not necessarily linked; a student's procedure for choosing limits may not involve the boundary conditions.

SUMMARY

Epistemic games can be described as a particular pathway through activated procedural resources. As an example, we look at an interaction between two students as they attempt to solve a first order, separable differential equation. One student prefers the "+c" method, the other the "limits" methods. The openness of this discussion allows a clear look into the reasoning that each student uses.

Using the transcript, we define several procedural resources, show how they can be organized into two facets of a previous described epistemic game, and produce a resource graph which allows visualization of this portion of the epistemic games. By representing two correct mathematical procedures in terms of shared resources, we help clarify the types of thinking in which students engage when learning to apply mathematical reasoning to physics.

ACKNOWLEDGMENTS

We thank Kate McCann for her assistance in gathering the video data.

This material is based upon work supported by the National Science Foundation under grants DUE-0441426, DUE-0442388, and REC-0633951.

REFERENCES

1. D. Hammer, American Journal of Physics 67 (Physics Education Research Supplement), S45 (2000).
2. M.C. Wittmann, Physical Review Special Topics - Physics Education Research 2 (2), 020105 (2006).
3. D. Tannen and C. Wallat, in The Discourse Reader, edited by A. Jaworski and N. Coupland (Routledge, New York, 1987), pp. 346.
4. A. Collins and W. Ferguson, Educational Psychologist 28 (1), 25 (1993).
5. J. Tuminaro and E.F. Redish, Physical Review Special Topics - Physics Education Research, submitted (2006).
6. B.S. Ambrose, American Journal of Physics 72, 453 (2004).
7. M.C. Wittmann and B.S. Ambrose, "Intermediate Mechanics Tutorials," available at http://perlnet.umaine.edu/imt/.
8. J.P. Byrnes, Cognitive Development and Learning in Instructional Contexts, 2nd ed. (Pearson Allyn & Bacon, 2000)