USING THE CONTEXT OF PHYSICS PROBLEM SOLVING TO EVALUATE THE COHERENCE OF STUDENT KNOWLEDGE.

Mel Stephan Sabella, Doctor of Philosophy, 1999

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ABSTRACT

We use the context of problem solving to show that students exhibit a local coherence but not a global coherence in their physics knowledge. When presented with a problem-solving task, students often activate a coherent set of knowledge called a schema to solve the problem. This schema consists of strongly related knowledge and procedures. Although the schemas students develop in the physics course are usually sufficient for success in the class, they are often insufficient for solving complex problems. Complex problems require that students have a deep understanding where they have integrated their qualitative knowledge with their quantitative knowledge and have integrated related physics topics. We show that our students activate schemas consisting of small amounts of knowledge and these schemas are often isolated from other schemas.

Physics Education Research (PER) has shown that students in introductory physics lack a deep understanding of physics principles and concepts. Through research-based curricula, conceptual understanding can be improved. In addition PER has shown that students can be taught problem solving skills through a modified curriculum. Despite these improvements, students still have difficulty developing a coherent knowledge of physics. In particular, students often have difficulty connecting related physics concepts. In addition, they view quantitative problems and qualitative questions as distinct types of tasks, possessing different types of knowledge and different sets of rules for responding.

We discuss some possible methods that physics instructors and physics education researchers can use to examine coherence in student knowledge. Using these methods, we provide evidence for the local coherence in student physics knowledge by identifying distinct schemas for different physics topics and concepts, as well as distinct schemas for qualitative and quantitative knowledge. After identifying some of these difficulties in student understanding, we look at how students are connecting qualitative knowledge to quantitative knowledge after going through concept-based curriculum. The research identifies benefits as well as shortcomings in the concept-based curriculum and talk about possible modifications that may foster coherence. In addition, we compare performance on quantitative questions between a physics class using the traditional problem-solving recitation and a class using *Tutorials in Introductory Physics* on quantitative problems.

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by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
1999

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Chapter 1: Introduction and Overview

What do we mean by coherent physics knowledge?

Many physics education researchers have stated that students possess a fragmented knowledge of physics consisting of isolated facts and formulas.[1] In this dissertation we use the context of problem solving to show that calculus-based physics students as well as advanced physics majors exhibit a local coherence but not a global coherence in their physics knowledge. When a student is presented a problem-solving task the student often activates a coherent set of knowledge called a *schema* to solve the problem. This schema consists of strongly related knowledge and procedures that are used to accomplish the goal of the problem.

The schemas students develop in the physics course for solving problems are usually sufficient for success in the class. Unfortunately these schemas are often insufficient to solve complex problems. *Complex problems* require that students have a deep conceptual understanding; *deep conceptual understanding* means that students have integrated their qualitative knowledge with their quantitative knowledge and have integrated related physics topics. We will show that the knowledge our students possess is often incorrect and is often inconsistent with their other knowledge. In addition, activated schemas consist of a small amount of knowledge and these schemas are often isolated from other schemas.

As an example, in this dissertation we analyze student responses to a problem involving the topics of dynamics and work-energy. Many students employed a schema consisting only of knowledge from dynamics, which was insufficient to solve the problem. Because students' schemas are isolated from one another it was difficult for them to activate the relevant schema. For these students, the concepts of dynamics and work-energy were only weakly linked.

This problem also illustrates that the responses students gave to the qualitative questions sometimes contradicted their answers to the quantitative questions. This suggests that students possess schemas for qualitative knowledge and schemas for quantitative knowledge. Again we see that the task activates a particular schema (either qualitative or quantitative) which is isolated from the other. For expert problem-solvers, a novel problem would tend to activate both qualitative and quantitative knowledge that they would use to solve the problem. Schema theory can therefore be used to help us make sense of the following issues about physics problem solving:

(1) Success on traditional quantitative problems tells us little about qualitative understanding and
(2) Success on qualitative questions tells us little about quantitative problem solving skills.

Physics Education Research (PER) has provided many useful results about students understanding of physics. One particular research result is that students often lack a deep understanding of the underlying principles and concepts even though they can succeed in the course.[2] PER has also shown that the problem solving skills many of our students possess are very different than those of expert problem solvers.[3]

Evaluating students in the introductory physics course usually involves performance on traditional quantitative problems, like the ones found at the end of the chapter in a physics textbook. In the introductory physics course it is often the case that students are rewarded with good grades by applying formulas and facts without a deep understanding. Acquiring a fragmented knowledge consisting of unrelated facts and formulas becomes an efficient way for students to succeed in the course. But when students are presented with novel problems we often see them attempt to apply formulas and patterns of solution that are inappropriate.

David Hammer examined the role coherence plays in student understanding.[4] He interviewed two students who, at the beginning of an introductory physics course for pre-medical students, had very different ideas about how to approach the course. Liza relied heavily on memorization and pattern matching, while Ellen tried to build a deep understanding of the material and reconcile her in-class experience with her intuition. Despite having expert-like qualities, Ellen was forced to abandon these qualities in favor of a more fragmented knowledge focusing on formulas and facts in order to succeed in the course.[5]

Results showing that students have a poor qualitative understanding of the material has led many physics instructors and physics education researchers to address the quality of conceptual knowledge. Physics education researchers have shown that through research based curriculum, students can develop better qualitative understanding.[6] Results from this dissertation will show that improving qualitative understanding does not always transfer to quantitative problem solving.

Motivation

If students are to solve novel problems such as problems they may encounter in research, or problems they may encounter later in their careers, in school or in work, they will need to be able to apply an integrated set of knowledge. Different principles and concepts must be linked together and the qualitative aspects of a problem must be linked to the quantitative aspects of a problem. This integration is an important characteristic of physics understanding. To accomplish this, the traditional introductory course requires a new set of paradigms. We believe that expert-like analytic reasoning requires that students integrate their qualitative knowledge and their quantitative knowledge. Reif and Heller state that teaching problem solving skills and concepts are not sufficient; the two must be integrated.[7]

We believe that students leaving our physics courses must possess an integrated set of knowledge that can be used in solving research problems. Without coherent knowledge students will be unable to adapt to new situations. Therefore, integrating physics knowledge should be a major goal of physics instruction. In addition, the evaluation of coherence is an important component in the assessment of student understanding. Traditional exams and homework assignments give us a very limited understanding of our students' knowledge and how our students are integrating aspects the course. For this reason, the University of Maryland Physics Education Research Group has begun to develop and implement measures that can evaluate whether our students are developing coherent physics knowledge. The context of physics problem-solving provides the physics community with effective ways to

evaluate coherence. This dissertation will provide the physics education researcher, as well as the physics instructor, with a number of tools that can be used to evaluate coherence.

Dissertation overview

Schema Theory and Previous Research in Student Problem-Solving

Ideas from education and cognitive science play an important role in our discussion of the coherence in student knowledge and problem-solving ability. In particular, schema theory will serve as the theoretical framework for the results of this study. We therefore provide the reader with a detailed background on schema theory and how it relates to problem solving.

There is currently a substantial body of work dedicated to student problem solving, covering many diverse topics. We will discuss the research done in how students solve problems in physics. Much of this research focuses on the differences between expert and novice problem-solvers.

Physics Education Research: Methods and Context

After providing a background on schema theory and the existing research in student problem-solving we will discuss the background for the research presented in this dissertation. For the benefit of those unfamiliar with PER we provide a description of some of the goals of PER and some of the methods researchers use to accomplish these goals.

The methods used in this dissertation to evaluate the coherence of student knowledge in the context of problem-solving involves specific applications of the general methods of PER that include one-on-one interviews, open-ended exam questions, and multiple-choice questions. Most of the research in this dissertation focuses on written responses to open-ended exam questions.

The research is conducted with two different populations of students. The first population is the undergraduate engineering majors, enrolled in the introductory physics sequence (Physics 161, 262, 263) at the University of Maryland. The second population is advanced physics majors, all of whom are enrolled in graduate level courses at the University of Maryland.

Results: Students use locally coherent knowledge in problem-solving contexts.

We first show that the schemas students activate to answer questions and solve problems strongly depends on the context of the problem. Exam problems and multiple-choice questions that test the same physics concepts were given to undergraduate engineering majors. Even though the corresponding exam questions and multiple-choice questions would be considered essentially identical by physicists, students often answered inconsistently. This is a clear example of the context triggering a particular way to respond to a question.

In the next section, we present results showing that specific principles and concepts that are closely integrated in an experts' knowledge are distinct in our students' knowledge. In a problem involving dynamics and work-energy we observed

that students had difficulty applying both concepts to the problem. Most students focused on using ideas from dynamics to solve the problem. When they found that their knowledge from dynamics was not enough many were unable to activate the relevant schemas for solving the problem. In another example, we show that many students view the work done by a piston in a thermodynamic process as either the area under the PV curve or $\int Pdv$. These two procedures for calculating work are seen as distinct methods.

We also observe that students possess schemas for qualitative knowledge and quantitative knowledge, which tend to be isolated from one another. We analyze responses to different problem-solving tasks that were given by students in each part of the three semester, calculus-based, introductory physics course, at the University of Maryland. We demonstrate that our students can have qualitative knowledge and quantitative knowledge that sometimes contradict one another. Our results show that students can solve traditional questions correctly without a basic conceptual understanding of the material. In addition, we see that students can solve qualitative questions correctly, yet many are unable to apply this qualitative understanding to quantitative problems.

Results: How does the Tutorial curriculum affect student coherence and problem-solving ability?

Between 1993 and 1999 The University of Maryland used conceptual worksheets called *Tutorials*[8] in the discussion sections of some of its introductory calculus-based physics classes. Tutorials come from a research and curriculum development program at the University of Washington led by Professor Lillian C. McDermott. The tutorial section replaces the traditional one-hour problem-solving recitation with worksheets the students go through in groups of three or four. The tutorials used at UMd include both tutorials that have been developed by the Physics Education Group at the University of Washington (PEG) and tutorials that have been developed by the Physics Education Research Group at the University of Maryland (PERG). Research conducted with students at both institutions and a number of pilot sites has shown that the tutorial curriculum can aid students in gaining a qualitative understanding of the material in the course. But there has been little research on whether student problem-solving skills benefit from the tutorial curriculum.

Our results show that in certain contexts a conceptual curriculum such as *Tutorials* can improve problem-solving performance for some students. In one study, we examine how student coherence develops as they progress through the introductory mechanics course. Analyzing the dynamics of a class shows that, although some of the students are making the connection between qualitative and quantitative knowledge, many are not. This illustrates some of the limitations of the Tutorial curriculum. In another study we compare student performance on quantitative problems in two instructional modes. One population went through the tutorial curriculum and the other went through traditional problem-solving recitations. In this study we observed that students in the tutorial section performed much better than students in a problem-solving recitation.

Curriculum Designed to address Problem-Solving Ability

There are a number of innovative curricula designed to foster problem-solving ability. These will be discussed in the final section of this dissertation. In addition we will discuss some modest efforts conducted by the PERG at the University of Maryland to help students make the connection between qualitative knowledge and quantitative problem solving. Curricula materials designed by the PERG are included in the Appendix.

Summary and Speculations for the Future

In the conclusion we briefly present some of the major results from this work. We also summarize the methods we used in this dissertation to evaluate coherence in the students' understanding of physics. These methods can be used by instructors and physics education researchers to understand the type of connections students are making in the course. The data we present is dependent on the context of the problems we have studied. Future work might therefore focus on the general characteristics of different types of schemas; characteristics that cut across many contexts.

[1] See A. A. diSessa, "Knowledge in Pieces," In *Constructivism in the Computer Age,* G. Forman and P. Pufall (Eds.) (Lawrence Erlbaum, NJ, 1988), and F. Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," Am. J. Phys. **63**, 17-32 (1995).

[2] See L. C. McDermott, and the Physics Education Group, A perspective on physics education research as a guide to the improvement of instruction, unpublished collection, (1998); I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. **53** (11) 1056-1064 (1985);

[3] See chapter 2 and the references contained therein.

[4] D. Hammer, "Two approaches to learning physics," Phys. Teach. **27** (9) 664-670 (1989).

[5] See Ref. 4.

[6] See L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned – closing the gap," Am. J. Phys. **59** (4) 301-315 (1991), and R. Hake, "Interactive engagement versus traditional methods: a six-thousand-student survey of mechanics test data for introductory physics courses,: Am. J. Phys., 65 (5) 418-428 (1997).

[7] F. Reif and J. I. Heller, "Knowledge structures and problem solving in physics," Educational Psychologist, **17** (2), 102-127 (1982).

[8] Tutorials come from a research and development project from the University of Washington Physics Education Group. See L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).

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Chapter 2: Schema Theory and Previous Research on Student Problem-Solving

Introduction

In this dissertation we use the context of physics problem-solving to evaluate the coherence of physics knowledge. We can infer, from analyzing student responses, that students activate sets of coherent knowledge called schemas to solve physics problems. We therefore discuss schema theory and some of the previous research done on student problem-solving in this chapter. Schemas and schema theory will serve as the theoretical framework for our analysis. Schemas allow us to go beyond the basic elements of knowledge and understand the connections between the basic elements. After describing the theoretical framework we discuss some of the previous work into how experts and novices solve problems.

Beyond the Basic Elements

We begin our discussion of schemas by describing a different type of knowledge element, called a *phenomenological primitive*, and showing how it fits into the schema theory framework.

diSessa states that intuitive physics "consists of a rather large number of fragments rather than one or even any small number of integrated structures one might call theories."[1] diSessa refers to many of these basic elements of reasoning as *phenomenological primitives* (*p-prims*), because they are based on experience and because "they simply happen."[2] These phenomenological primitives (p-prims) are basic reasoning elements that cut across topics.[3] diSessa therefore believes that in order to cause conceptual change we cannot simply address specific misconceptions; we must address more global attributes.

Examples of p-prims include *Ohm's P-prim, Force as Mover, Continuous Force (or maintaining agent), Dying Away, Dynamic Balance*, and *Overcoming*.[4] Because p-prims cut across contexts, developing expert understanding involves modifying their activation conditions.[5] Particular p-prims may be appropriate or inappropriate, depending on when and where they are applied. Unlike misconceptions p-prims are not "correct" or "incorrect", according to Hammer.[6]

Two p-prims that will come up in this dissertation are the *maintaining agent p-prim* and the *Ohm's law p-prim*. Maintaining agent is a term used by Hammer to describe the continuous push (force) p-prim.[7] The maintaining agent p-prim is used to explain why students often (incorrectly) believe that a force is needed to maintain motion. Like other p-prims the maintaining agent p-prim can be applied across contexts. Another example of this p-prim is that a supply of energy is needed to keep a bulb lit.[8] In this case the p-prim is appropriate. diSessa describes the Ohm's law p-prim as "one of the most fundamental and pervasive p-prims."[9] It is related to the interpretation of the voltage, resistance, and current in Ohm's law but, like the other p-prims, it can be applied in many contexts. The Ohm's p-prim is increased effort (V) leads to increased result (I) and increased resistance (R) leads to decreased result (I). For example if I increase the force (effort) I exert on a block, I increase the

acceleration (result). Also, if I increase the friction between the block and the surface and apply the same push I decrease the acceleration.

In some of the examples included in this dissertation, we observe that students use these p-prims in their problem solutions. Therefore the schema a student uses to solve a particular problem may consist of p-prims. A schema may also be composed of elements of knowledge that get applied in specific situations. Minstrell refers to these elements as *facets*.[10] Minstrell states that facets may relate to content, they may be strategic, or they may represent a generic piece of reasoning.

While the work of diSessa and Minstrell focus more on these basic elements, in this dissertation we look at the structure of connections between knowledge elements. We have observed that students use strongly associated knowledge sets (schemas) to solve physics problems. We have also observed that although many students posses locally coherent knowledge (or schemas), they only sometimes possess coherence between these sets of knowledge (characterizing a global coherence). The context of physics problem-solving allows us to infer the characteristics of the schemas our students employ in different situations.

Definition of a Problem

"Problem-solving" has many different meanings, so it is important to specify the particular definition used in this dissertation. To define problem-solving it is first necessary to define the characteristics of a problem. Newell and Simon define~~d~~ a *problem* as a situation in which an individual "wants something and does not know immediately what series of actions he can perform to get it."[11] For many physics instructors, physics problems are the kind of task that are usually found at the end of the chapters in introductory college physics textbooks.[12] These problems present a situation where information is given and some or all of the information is required to obtain a numeric or symbolic answer. We use this description of a physics problem for this dissertation. *Problem-solving* is the process one goes through to construct the answer to a problem.

Schema Theory

In this section we present the background on schema theory and argue that schema theory is well suited for the theoretical framework for our data. A schema is a set of coherent knowledge that gets brought up in a set of similar contexts or situations. Schemas contain facts, rules, p-prims, and other spontaneous responses that are used to accomplish a certain goal. In this chapter we build from chains of responses and items called patterns of association to schemas and discuss schema theory in some detail. Throughout the discussion we make comparisons between expert problem-solving schemas and novice problem-solving schemas. After discussing the theoretical aspect of schemas we explain how we use the term schema in this dissertation. An example of the schema(s) a student might develop after going through the introductory physics course is presented. Those readers less interested in the details of the theoretical framework may choose to skip these sections and go straight to the summary for the chapter.

From Patterns of Association to Schemas

The important concept in understanding cognitive responses are *patterns of association*. This phrase summarizes the observation that an individual's response to a particular context or situation is a chain of related responses or items. These items are not rigidly controlled or predictable, but depend on both the external context and the individual's internal mental state. Since the latter is not directly observable we treat the associations as probabilistic. A pattern of association may be strong if a link is activated in a large variety of situations or weak if a link is rarely activated. Patterns of association may contain any type of knowledge (i.e. facts, formulas, concepts, rules, etc.)

Michael Wittmann describes patterns of association for mechanical waves in his doctoral dissertation.[13] His data show that students often possess weak patterns of association, which they apply to physics tasks. These weak associations often imply fragmented sets of knowledge containing pieces of information that are inconsistent with one another. A strong pattern of association is characterized by pieces of knowledge that are frequently elicited together in a wide variety of situations. We refer to robust patterns of association as *schemas*. Although schemas are not necessarily "correct" (as evaluated by an expert) and may contain inconsistencies, the relations among the pieces of knowledge are important for us to identify if we are to understand how students reason. Note that a schema is constructed by the interaction of an individual's cognitive structure with a cueing context. It is not simply a structure in an individual. This is of considerable importance and will be discussed in more detail below.

Success in understanding physics requires the development of expert-like schemas as well as the building of connections between different schemas. We will show that it is not enough to get our students to the stage where they are employing coherent schemas to solve problems; they also need to build connections between different schemas.

Introduction to Schemas and Schema Theory

The following sections will provide the reader with a detailed description of schemas and discuss some extensions we make to the existing models. The two main sources for this background on schema theory come from the work of Rumelhart[14] and Marshall.[15] As described by Rumelhart, schemas are

> the fundamental elements upon which all information processing depends. Schema[s] are employed in the process of interpreting sensory data, . . . in retrieving information from memory, in organizing actions, in determining goals, . . . in allocating resources, and generally in guiding the flow of processing in the system. . . . A schema . . . is a data structure for representing the generic concepts stored in memory. . . . [Schemas represent knowledge] about . . . objects, situations, events, sequences of events, actions, and sequences of actions.[16]

According to Marshall a particular schema will have the characteristics shown in Table 2 - 1.

There are three types of knowledge that govern how an individual will perform on a problem-solving task:

- *Declarative* – knowledge that is composed of concepts and facts, and is static,
- *Procedural* – knowledge that is composed of rules, that consists of skills and techniques, and
- *Schematic* – knowledge that combines procedural and declarative knowledge.[17]

• A schema is a basic storage device.
• A schema has a network structure.
• The degree of connectivity among the schema's components determines its strength and accessibility.
• A schema is a flexible structure, accessible through many channels.
• Schemas have no fixed size; they may be large or small.
• Schemas may embed and overlap.

Table 2 - 1

The characteristics of a schema.

Important characteristics of schemas for problem-solving

One of the main activities associated with a schema is determining whether it provides the appropriate knowledge for dealing with a presented context.[18] Once a schema is activated an individual must decide whether the declarative facts in the activated schema correspond to the problem. Then the problem solver must decide whether he or she can use the procedural rules in the schema to obtain the goal of the task.

The particular schema that is activated depends on the cues that are presented to the individual. In this study, these cues come from the problem-solving task and are based on some idea or concept in the given situation. A desirable trait for a problem solver is to be able to use the relevant characteristics of a problem to link to a schema that will help solve the problem. Depending on how information is encoded by the individual, cueing an appropriate schema may be easy or difficult. There are a number of studies which provide evidence that experts and novices encode information differently.[19] This may cause the expert and novice to use different schemas even though they may be given identical cues.

In order for schemas to be useful to a problem solver he or she must be able to map from a new situation or problem to an existing schema. This flexibility allows schemas to adjust to the problem-solving task. When instructors solve problems for students at the board, they want their students to develop schemas for solving a range of similar problems. It is often the case that when our students see a problem solved at the board they develop a very narrow and specific schema that can only be applied to a particular problem and not a class of problems. We refer to this as *pattern matching*.

We sometimes see in the classroom that the schemas students possess are often not flexible enough to adapt to different problem-solving situations. They attempt to solve a new problem based on how a sample problem has been solved, even though it may be inappropriate. Although pattern matching is a type of schema, since it consists of knowledge and procedures for applying the knowledge, it is not characterized by the dynamic nature of an effective schema. The pattern-matching schema is static and can only be applied to very specific situations. In order to succeed in the course many students attempt to memorize a large set of pattern-matching schema that they can apply to many different problems.

For a schema to be useful in problem-solving its components must be linked together and not just exist as isolated facts and pieces of knowledge. Marshall represents schemas with nodes and links from node to node.[20] The nodes represent declarative facts and procedural rules. Lines connecting nodes represent links or associations among facts and rules. Marshall refers to these representations as *schema graphs*. Implicit in the representation is a particular triggering context and probability weightings on each of the links. Figure 2 - 1 shows two sample schemas, where the first is completely linked and the second is only partially linked.

Some links are stronger than others in an individual's schema. Marshall describes this extension to schema theory when she talks about the relation of neural networks to schema theory.[21] With weights placed on the links, when one node is activated, that node will link to nodes with stronger connections more often. In a particular schema the internal links would be stronger than the links to other schemas,

Completely linked schema graph *Partially linked schema graph*

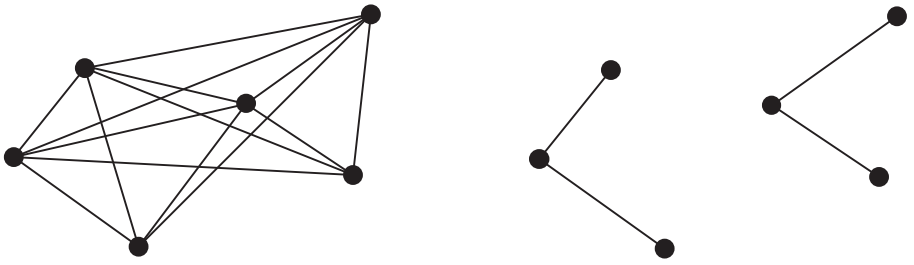

Figure 2 - 1

Representations of schemas. The nodes represent declarative knowledge and procedural rules. The lines represent relations between different nodes.

although there may be exceptions. This will make it possible for the individual to select appropriate associations from a particular node, and not simply activate every thing associated with it. The weightings are important for organization. One idea that comes from Polya is that in some cases the expert may not have more knowledge than the novice but will simply use it better.[22] The organizational aspect of schema theory comes from the network of connected knowledge.[23]

It is important to note that schemas are a classification that we apply to describe an individual's thought processes, not a rigid structure in the brain. Individuals make many associations; when we have a set that have a reasonably high probability of triggering to each other (i.e. are coherent with each other) we identify it as a schema.

The final characteristic for individual's schema to be useful for problem-solving is that it must be reasonably complete and accurate. Besides experts and novices having knowledge encoded differently, experts also have more correct knowledge.[24] Expert schemas are therefore more often composed of bundles of knowledge about the physical world that are both internally consistent and also externally consistent. We refer to such schemas as *physical models*. *Internal consistency* requires that the nodes of knowledge in the model be consistent with each other while *external consistency* requires that different models be consistent with each other. In contrast, the novice may have his or her schema, composed of pieces of inconsistent knowledge.

Schema use in this dissertation

In this dissertation we use a general definition of schemas that characterizes them as a set of knowledge that gets brought up in a problem-solving situation. The definition we use comes from the ideas from Marshall and Rumelhart, presented earlier in the chapter.[25] Our analysis of student understanding focuses on their responses to different physics problems. By examining these responses we can infer certain characteristics about the types of schemas our students use.

Let us now turn to an example of schemas in the context of dynamics and work-energy. Introductory physics texts often treat dynamics and work-energy as separate and weakly connected topics. Bagno and Eylon state that "although some of the textbooks attempt to locally organize the information (e. g. within a single chapter) by giving a summary or table, there are no comprehensive attempts to organize the information at a global level."[26] Most textbooks place dynamics and work-energy in different chapters and, although some make a formal connection between the two, most rarely build substantial links between the topics.

Experts and novices differ in the way they integrate knowledge from different topics. One example, presented in this dissertation, is in the context of dynamics and work-energy. We show that for the expert, the topics of dynamics and work-energy are strongly related. If the task requires them to do so, the expert can go from dynamics knowledge to work-energy knowledge easily. The novice finds it very difficult to go from the knowledge in one topic to the knowledge in another topic. We show that even when presented with a complex problem requiring multiple topics (for example: dynamics and work-energy) novices tend to activate schemas containing

knowledge on a single topic. Figure 2 - 2 shows a schematic of an expert's and a novice's possible schema structures for dynamics and work-energy knowledge.[27]

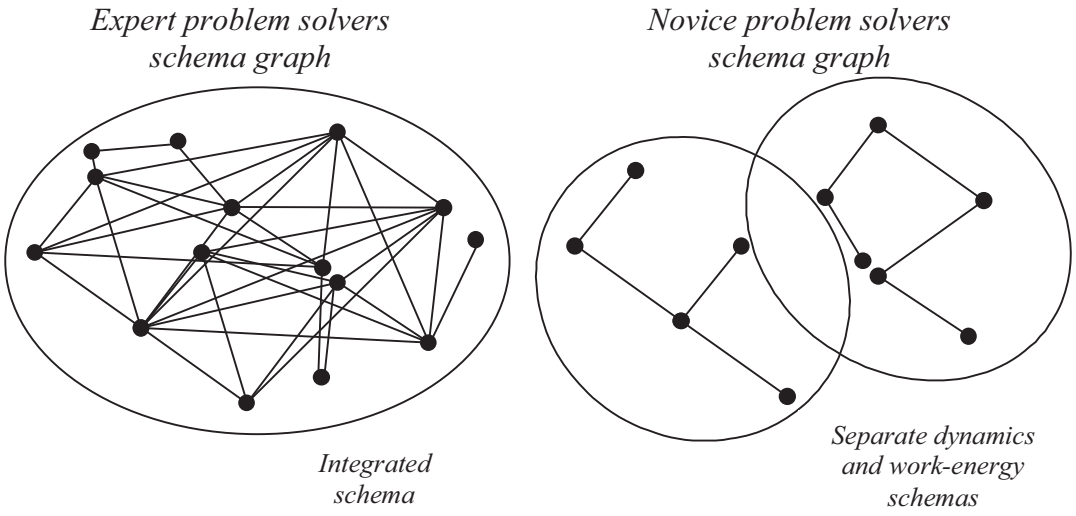

Figure 2 - 2

Graphs representing two schema. The graph on the left represents an expert's well-linked structure and the graph on the right represents a novice's partially linked structure. The pattern of connections are the largest difference between the two schema graphs.

Existing Research in Problem-solving

Introduction

Previous research on student problem-solving has looked at three major areas. David Maloney describes these different areas in his extensive review published in the *Handbook of Research on Science Teaching and Learning*.[28] The first area examines how individuals solve problems. The second examines how pedagogical methods can be employed to improve student problem-solving. The third consists of research into issues of transfer, what students learn from solving problems, and other topics.[29]

In this dissertation I use the context of problem-solving to look at coherence in student knowledge of physics. When our physics students are presented with problems they bring a set of knowledge that they use to accomplish the goal of the problem. By inferring the schemas they bring to a task we are able to make claims about the coherence of their knowledge. Since this research focuses on how students solve problems I concentrate on the particular aspect of the problem-solving literature describing how students solve problems. The most relevant topics from the existing

literature are: the difference between expert and novice problem-solvers and the hierarchical organization of knowledge.

In order to understand why novices have so much difficulty solving complex problems it is necessary to elicit and describe the differences between expert problem-solvers and novice problem-solvers. Characterizing these differences is a common thread through each paper we discuss in this section. Of particular importance to this work is the research on knowledge organization since these topics play a large role in understanding why and how certain sets of knowledge are brought to a problem-solving situation.

Differences between expert and novice problem-solvers

In this section we discuss five classic papers that describe the differences between novice problem solvers and expert problem solvers. The studies focus on the strategies and representations experts and novices use when solving problems. The papers by Larkin and Chi et al. describe the types of schemas experts and novices have and are particularly important to this dissertation. The five papers are summarized in the following list.

- Bashkar and Simon compared the strategies experts and novices used to solve problems (1977).
- Larkin and Reif compared the use of qualitative and quantitative knowledge in expert's and novice's solutions to problems (1979).
- Larkin focuses on the representational processes that differentiate experts and novices (1983).
- Anzai and Yokoyama investigated the way experts and novices use models to solve problems and how cues presented in problems affect student solutions (1984).
- Chi et al. looked at the way experts and novices categorize problems and the types of schemas novices and experts possess (1981).

In early research on problem-solving, experts and novices were characterized by the different types of strategies they used to solve problems. Bhaskar and Simon looked at how a teaching assistant in a chemical engineering thermodynamics course solved six problems. To analyze the think-aloud protocols they used a computer program called SAPA (Semi-Automatic Protocol Analysis) along with a human coder. SAPA encoded the basic processes: "producing a relevant equation, evaluating a variable, solving an equation, and so on" while the human coder transcribed the semantic information.[30] SAPA provided the researchers with a framework that they could use to identify whether the participant followed the scheme and where he deviated from the scheme. The participant was observed to follow a method where he identifies the goal and identifies the current state and then performs operations to try to get from the current state to the goal of the problem. Bhaskar and Simon refer to this strategy as *means-ends analysis*.[31]

Larkin and Reif examined how an expert (a physics professor) and a novice (a student who just completed his first course in mechanics) solved five mechanics problems in a think-aloud setting.[32] They found that after an initial description of the problem the novice began to construct a mathematical description of the problem. The

novice then proceeded to combine equations to eliminate the undesirable quantities. After the expert's initial description of the problem, he constructed a qualitative description of the problem and then a mathematical description.

Larkin and Reif stated that experts tend to think more about the underlying principles and concepts when answering physics problems. They describe how using concepts allows students to go from the more global aspects of a problem to the more specific aspects of a problem.[33] But even when underlying concepts are understood, students may have difficulty solving problems. In order to use conceptual understanding in problem-solving it is not enough for students to just have a deep conceptual understanding about a topic; they must also organize their knowledge so that the knowledge can be accessed readily.[34]

In the 1980's there were an increasing number of studies focusing on the representational processes that novices and experts use to solve problems.[35] One such study done by Larkin identified two types of representations used in solving problems, the naïve representation and the physical representation.[36] The *naïve representations* involve real world objects and evolve in real time.[37] The term naïve has negative connotations associated with it so we believe that a better word to describe these representations are *concrete representations*. In contrast, what Larkin refers to as *physical representations*, or what we prefer to call *abstract representations* contain conceptual entities, such as forces. (We will continue to use Larkin's nomenclature for this section.) Novices were seen to limit themselves to naïve representations, while the expert problem solvers had access to both the naïve and the physical models. Larkin also states that there is a close relationship between the physical representations and the application of quantitative physics principles. The construction of the physical representation is the step before an equation or formula can be applied. Therefore the connection between the conceptual physical representation is linked by experts to the quantitative methods. If students possess only a naïve representation, the quantitative formulation will be harder to construct. In addition, the formulation may not be related to a physical representation, although it may be related to a naïve representation.

Larkin states that certain schemas are used for producing physical representations. Two examples used by Larkin are a force schema and a work-energy schema. The force schema would be used to construct a physical representation for the principle that the total force on a system is equal to the mass of the system times its acceleration. The work-energy schema would be used to construct a physical representation for the idea that the total energy of a system depends on the amount of work done to it.[38] She refers to place holders for information needed to solve problems as *slots*. A particular slot gets filled when a particular piece of information is brought up by the problem-solver.

In one study Larkin gave eleven experts and eleven novices an inclined plane problem that could be solved using the ideas of force or the ideas of work and energy. A table was constructed by Larkin that contains the necessary slots that would have to be filled to solve the problem. For instance, if one decides to employ a force schema he or she might state that the component of the gravitational force along the incline is $mg \sin\theta$. Once the individual uses this information its slot is filled. Larkin analyzed the order in which experts and novices filled the slots. She found that experts begin

filling slots corresponding to known quantities, while novices did not show an order for filling the slots. The result that "experts tend to work forward, to 'develop knowledge' is reinterpreted here by saying that experts fill slots in a schema to make a physical representation, starting with slots related to known quantities."

The study by Anzai and Yokoyama investigated the way experts and novices use internal models to solve problems. They also examined the "ability of a problem-solver to generate, or make a shift to, a new internal model that would lead to the correct answer by attending to [a] set of clues."[39] They refer to this as *semantic sensitivity*. In their paper they describe three types of internal models. An *experiential model* is a set of knowledge generated from experience, a *correct scientific model* is a set of scientific concepts and relations that are correct, and a *false scientific model* is a set of scientific knowledge that is incorrect.

To give an idea of the research Anzai and Yokoyama conducted we present two of their studies in some detail. Both the studies we discuss involve the yoyo problem shown in Figure 2 - 3. In the problem a yoyo sitting on a surface is pulled by a string to the left and the students are asked which direction the yoyo rolls. The correct answer to the yoyo question is that it will move to the left since the string produces a torque about the point of contact out of the page. If a student uses an experiential model, where the yoyo rotates about its axle (i.e. when it is dropped) to answer this question, the student will most likely state that the yoyo moves to the right. Students might also use a false momentum model where they treat the center of the yoyo's axle as the center of rotation.

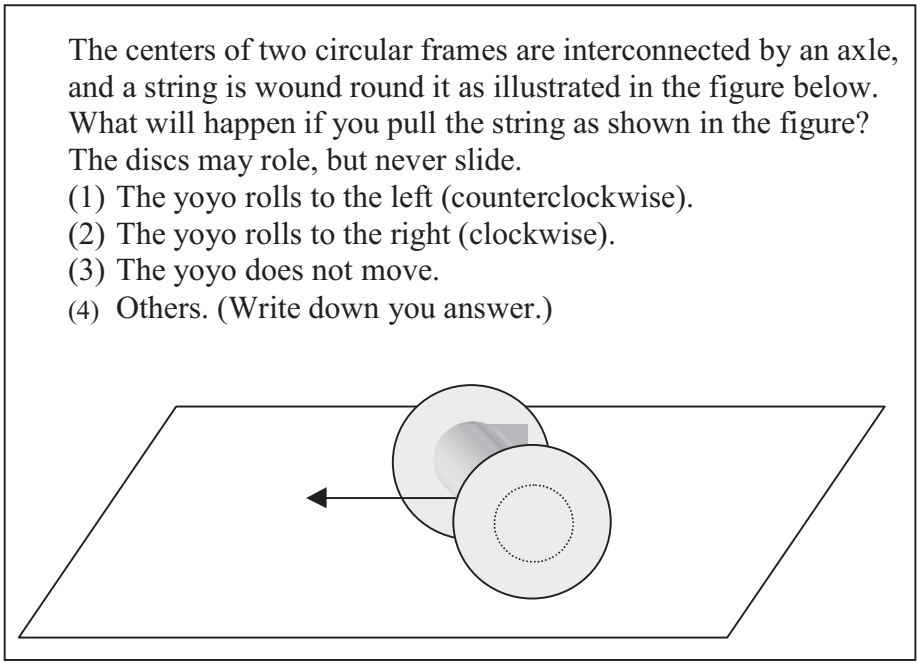

Figure 2 - 3

Yoyo problem asked by Anzai and Yokoyama.

Anzai and Yokoyama looked at the think-aloud protocols of two experts and one novice on the yoyo problem.[40] The two experts (E1, E2) were physics professors and the novice (N1) was a freshman in science and engineering. The first participant, E1, generated a single model that was then used to solve the problem. E2 first generated a false scientific model, but his model was changed to the correct scientific model as he solved the problem. The novice student, N1, first used an experiential model and then made the shift to a false scientific model. When presented with cues N1 was able to generate the correct scientific model but also brought up the experiential model. This shows that N1 was able to have multiple competing models for a single problem.[41] The results on the yoyo problem and the other two problems asked by Anzai and Yokoyama showed that the two experts shifted toward scientific models while the novice seemed to go back and forth between the correct scientific model and the other models.

In a large-scale study with 216 students, Anzai and Yokoyama gave five variations of the yoyo question. The differences from one question to another were modifications in the figure of the yoyo. Each variation included a different physical cue. By doing this, Anzai and Yokoyama were able to determine what types of physical cues were necessary for the students to apply the correct scientific model. They found that including the physical cues of either the direction of rotational momentum, the location of the fulcrum, or the frictional force did not produce significant improvement in the performance. Because the false models were so robust it was necessary to provide the students with both the position of the fulcrum and with the direction of the rotational motion to get them to perform better.[42]

The conclusion they drew from their set of studies is that "semantic sensitivity to cues may depend on principles that the cues are related to and also the knowledge the presently evoked model is based on."[43] This study is similar in some ways to the type of studies we will be considering. Instead of providing cues that are modifications to the problem's figure, we will examine how students respond to cues in the form of qualitative questions. We demonstrate that cues that would tend to help an expert solve a problem can hurt the performance of students, due to the fact that a novice's schemas are often isolated from one another.

Chi et al. performed a study where eight undergraduates who had just completed a semester of mechanics (novices) and eight graduate students in physics (experts) were asked to categorize a set of twenty-four physics problems.[44] Chi et al. contrasted the application of surface features by novices to the application of principles and concepts by experts in grouping these problems. They conducted a number of studies with these sixteen students.

We will discuss one of their studies in detail. In it, they selected 24 problems, from chapters 5 through 12 (3 from each chapter), from Halliday and Resnick's *Fundamentals of Physics* textbook.[45] The participants were presented with the problems on cue cards and were asked to group the problems based on the similarities of their solutions. Participants were not given the opportunity to solve the problems.

Novices categorized the problems in terms of the surface features (or surface structure). Surface features include the objects referred to in the problem or the literal physics terms in the problem. For example, novices might group problems together if they involve springs, or inclined planes (i.e. by the objects). They might also group a

set of problems together if they involve friction or center of mass (i.e. by the physics terms).[46] Experts tended to categorize the problems in terms of the underlying principles and concepts (deep structure). For example, experts might group a set of problems together if they can be solved using energy conservation.

Chi et al. postulate that a "problem can be tentatively categorized" by the problem solver after a first look at the problem's features. Therefore, a problem's representation is not fully constructed until after the first categorization has been completed. Chi et al. also state that a category, and the knowledge that comes with the category, constitute a schema. This categorization activates a set of knowledge (schema) that the problem solver will use to accomplish a goal. Chi et al. state that the content of the schema then determines the problem representation. By showing that novices and experts will categorize problems according to different features (surface vs. deep), they were able to conclude that experts and novices possess schemas that contain different types of knowledge.

In one study, Chi et al. looked at the basic solution methods that the participants applied to problems, the identification of features in the problem statement that led them to the basic approach, and the process of constructing a problem representation. The study was conducted with two physicists, who had frequently taught the introductory physics course and two novices, who had just received A's in the college level mechanics course. Participants were asked to outline the basic approaches they would take to solve specific problems. They were also asked to state which features of the problem helped them decide on the basic approach. Think-aloud protocols were done with these four participants on twenty problems.

In analyzing the protocols, Chi et al. found that both experts and novices construct problem representations based on their category knowledge or schema. The researchers could identify triggers from the problem statement and examine how these triggers activated different schemas. Experts used problem statements to trigger principles, while the novices used problem statements to trigger equations and isolated facts. The problem representation is therefore based on the initial categorization process, which comes from the cues in the problem, and the completion of the solution is based on the knowledge available in the schema. The initial process is a bottom up process; the individual will activate schema based on the specifics of the problem. Once the schema is activated the process proceeds in a top down manner.[47] Principles and concepts guide an expert's representation while surface features guide the novice's representation. They conclude their study by stating that

> experts' schema[s] contain a great deal of procedural knowledge, with explicit conditions for applicability. Novices' schema[s] may be characterized as containing sufficiently elaborate declarative knowledge about the physical configurations of a potential problem, but lacking abstracted solution methods.[48]

This lack of procedural knowledge may be due to the novices' schemas being small and isolated from their other schemas. When presented with a problem a schema may be activated with a set of inappropriate procedural rules. If this schema is

isolated from another schema containing the appropriate procedural rule the student will not be able to solve the problem.

Hierarchical Organization

The three papers discussed in this section compare the organization of the knowledge structures of experts and novices. It is suggested that experts exhibit a hierarchical organization allowing them to retrieve information more easily than novices. The research shows that the development of hierarchical structures is effective in helping novices become better problem solvers. These papers discuss how experts may have their knowledge linked and are therefore important to our discussion of schemas and the types of knowledge structures individuals use to solve problems. The following list summarizes the papers included in this section.

- Reif and Heller characterize expert problem solvers as possessing hierarchically organized knowledge while novices are categorized by poorly organized knowledge (1982).
- Eylon and Reif followed up on the idea of a hierarchy by proposing a model for effective problem-solving that in addition to being hierarchical, contained information about the implementation of the tasks (1984).
- Bagno and Eylon found that explicitly asking students to identify links between different topics helped them obtain a hierarchical structure that could then be applied to problem-solving tasks (1997).

In their 1982 paper Reif and Heller presented a theoretical perspective on a model of expert problem-solving.[49] They suggest that expert problem-solvers possess knowledge that is organized hierarchically.[50] This structure of knowledge allows the expert to retrieve relevant information much more easily than the novice. Reif and Heller describe the novice's knowledge as "fragmented, consisting of separate knowledge elements that can often not be inferred from each other or from other knowledge."[51,52] Reif and Eylon provided experimental evidence that teaching a hierarchical structure can help students solve physics problems.

A *hierarchical organization* schematically resembles a family tree. Eylon and Reif describe the hierarchical organization as a structure with general knowledge elements at the top level and specific elements placed at lower levels. They state that this type of structure allows the individual to efficiently search for information and that in an expert physics student, the top levels are composed of basic definitions and principles, while the bottom levels are composed of equations and formulas. Figure 2 - 4 shows a schematic of the hierarchical knowledge organization from Eylon and Reif's 1984 paper.[53]

Eylon and Reif proposed a model for effective problem-solving which has the characteristics of being hierarchical as well as containing information for the implementation of tasks. They refer to this structure as hierarchical and *task oriented*. (This is similar to our description of schemas, in that schemas contain declarative knowledge and procedural rules.) They state that an expert uses this knowledge structure by making gross decisions at an early stage and then making more specific decisions. The hierarchical structure is therefore employed in a top down manner.

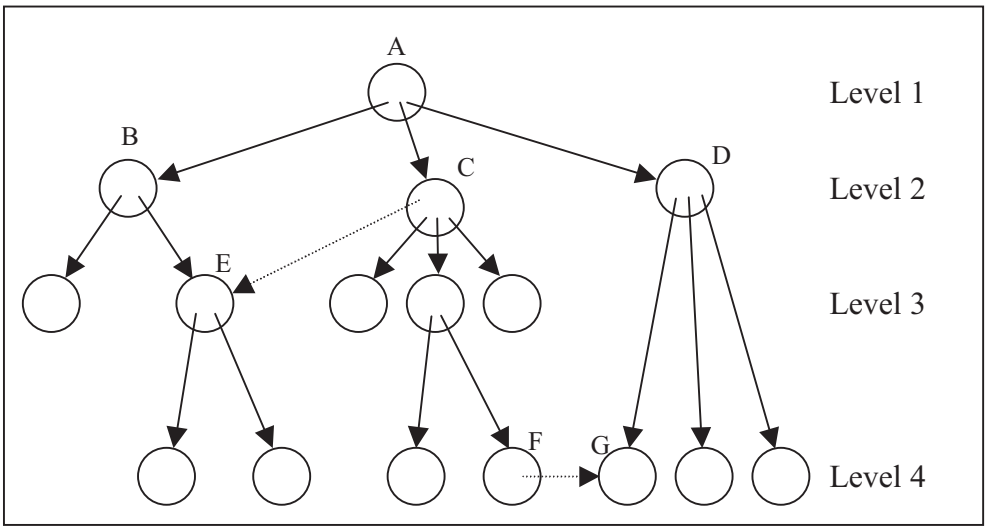

Figure 2 - 4

Schematic diagram of a hierarchical knowledge organization from Eylon and Reif.

Although Reif and Eylon do not provide evidence that experts actually possess this structure they do look at whether teaching introductory students this structure helps them on different types of tasks.[54] They used 36 paid volunteers who were enrolled in the introductory mechanics course. The participants were divided into three groups and each group went through a different treatment. Each treatment group was presented with written texts about a problem argument. The first group (*H-treatment*) was presented a hierarchical two-level organization of a particular physics argument. The second group (*S1*) was presented with a single level organization which consisted of the lower level in the hierarchical organization, and the third group (*S2*) was presented the single-level organization twice. The only difference in the H-version and the S-version was that the S-version omitted all the titles connecting the argument to the overview.[55] These groups were then evaluated on their performance on different types of tasks.

Students were first given *acquisition tasks* that were designed to assure that a student has acquired the given organization. They were then given four performance tasks. Students were asked to:

- reproduce an argument unaided in the *free recall tasks*,
- give the next step in an argument in the *cued recall task*,
- diagnose a mistake in a similar argument to the one in the text in the *debugging task*,
- and carry out similar arguments with changed premises in the *modification tasks*.

They found that students in the H-treatment group performed better than the students in S-treatment group. Tasks asking students to summarize the argument in

about five statements and tasks asking students to order a scrambled list of summary statements were designed to probe the nature of students' internal knowledge organization. Performance on these tasks indicated that students in the S-treatment groups acquired knowledge that was merely locally connected. Other studies discussed in their paper provide additional insight into knowledge organization and task performance.[56]

In another study on the importance of knowledge organization, Bagno and Eylon use two-dimensional structures that show particular concepts and how different concepts are related to each other to aid their students in obtaining a hierarchical knowledge structure in the domain of electromagnetism.[57] These two-dimensional structures come from the work of Novak et al. and are referred to as *concept maps*. Figure 2 - 5 shows a sample concept map from their paper. This map relates the concepts of electric charge (q), electric current (I), electric field (E), magnetic field (B), and force (F).

In their study concept maps were used as instructional tools in that students would actively construct concepts maps using a problem-solving approach. The learning sequence consists of the following.

- *Solve* – The student solves a problem.
- *Reflect* – The student identifies a relationship and compares it to other relationships.
- *Conceptualize* – The student develops and elaborates the concepts.
- *Apply* – The students apply the knowledge to novel problems and situations and use the concept map to describe different processes.
- *Link* – The student links the new part of the concept map to the existing concept map.

In doing this they hoped their students would form an explicit relationship between problem-solving and knowledge structure as well as treat conceptual difficulties in relation to knowledge structure.[58] They used three treatment groups. *Treatment E* used the five-step approach outlined above, *treatment* C_1 included dealing with conceptual difficulties but did not include the active construction of the concept map, and *treatment* C_2 received only regular instruction.

Bagno and Eylon compared performance among the different groups using a total of 190 students. They looked at four aspects:

- *Content and form of knowledge representations* – Students were asked to summarize the main ideas in electromagnetism in the order of their importance.
- *Conceptual understanding* – Students were to comment on the correctness of different statements about electromagnetism and explain.
- *Application* – Students were asked to solve a standard problem and a more complicated, unfamiliar problem.
- *Transfer* – Students were asked to read an unfamiliar passage and write down the main concepts and relations in it.

Bagno and Eylon found that group E performed better on each of these tasks leading them to conclude that actively constructing the concept maps created a link between the concepts and how to apply the concepts in problem-solving.

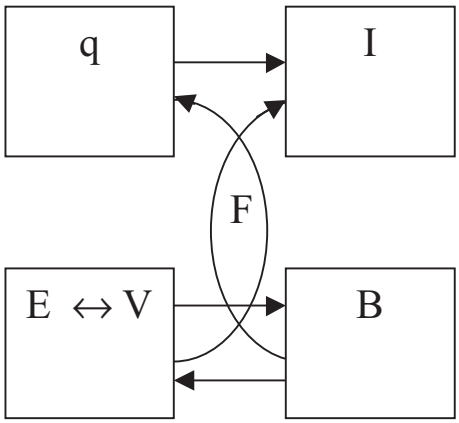

Figure 2 - 5

Concept map for electromagnetism from paper by Bagno and Eylon.

Summary

Schema theory is used to place our data into a theoretical framework. Simply stated, schemas are robust patterns of association that get activated when an individual is presented with a set of similar problems to solve. These bundles of knowledge contain both declarative knowledge and procedural rules. For instance, when presented with a problem involving a block on an inclined plane, an individual may activate a schema for dynamics, which would contain information about Newton's Laws and how to apply them in a problem. Knowledge elements in a schema are linked, and therefore associated with other knowledge elements in the schema; therefore they are locally coherent. In addition, schemas may contain both correct as well as incorrect information.

In this dissertation we use this cognitive model in the context of solving physics problems. Individuals go through a number of steps when given a problem. They first use their set of declarative knowledge in order to determine which schema to activate in a given situation. They must then go through the conditions necessary for the particular schema to hold true. If the conditions are not met, they must find the relevant schema. They should also ask themselves how a particular schema will help them obtain their goals for the problem. The next step is attempting to solve the problem by implementing the set of procedural rules in the schema.

Much of the previous research in understanding problem-solving describes the difference between expert and novice problem solvers. Experts are found to have more conceptual knowledge and are more inclined to use this conceptual knowledge in answering quantitative problems. Experts are also found to work forward when

solving problems, while novices tend to use formulas and equations in a careless manner. These differences are related to the differences in the expert's and the novice's schemas. Besides having more knowledge in the expert's schemas, the schemas that experts possess are composed of different types of knowledge, which are organized differently than in the novice's. An expert's schemas are organized in terms of the underlying principles and concepts, while the novice's schemas are organized according to more superficial features. Expert's schemas are also observed to contain more procedural rules than the novice's schema. These procedural rules determine how the declarative knowledge in the schemas are to be used in different situations.

In chapter 5 we show that an expert problem-solver activates a large well-structured schema when presented with a physics problem-solving task. In addition, the expert is able to activate a set of integrated knowledge much more easily than a novice. Novice's individual schemas tend to contain much less information and the individual schemas tend to exist as isolated sets. We provide evidence for the existence of topic based schemas and schemas that are qualitative and schemas that are qualitative in the novice student.

[1] A. A. diSessa, "Knowledge in Pieces," In *Constructivism in the Computer Age,* G. Forman and P. Pufall (Eds.) (Lawrence Erlbaum, NJ, 1988).
[2] See Ref. 1.
[3] See Ref. 1.
[4] For more information see Ref. 1.
[5] D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).
[6] See Ref. 5.
[7] See Ref. 5.
[8] This is an example from Ref. 5.
[9] see Ref. 1.
[10] J. Minstrell, "Facets of students' knowledge and relevant instruction," In Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop, R. Duit, F. Goldberg, and H. Neidderer, (Eds.) (IPN, Kiel Germany, 1992), pp. 110-128.
[11] A. Newell and H. A. Simon, *Human problem solving*. (Prentice Hall, NJ, 1972).
[12] Here we are describing quantitative problems. A more detailed description is provided in chapter 6 where we differentiate qualitative questions and quantitative questions.
[13] M. C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).
[14] D. E. Rumelhart, "Schemata: The building blocks of cognition," In J. T. Gurthrie, (Ed.), Comprehension and Teaching: Research reviews. International Reading Association, Inc. 3-27 (1981).
[15] S. P. Marshall, "Assessing Problem Solving: A short term remedy and a long term solution," In *The teaching and assessing of mathematical problem solving*, R. I Charles and E. A. Silver (Eds.) (L. Erlbaum Associates and the National Council of Teachers of Mathematics, VA, 1988) pp. 159-177.
[16] See Ref. 14.
[17] See Ref. 15.
[18] See Ref. 14.
[19] See B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984) and M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).
[20] See Ref. 15.
[21] S. P. Marshall, *Schemas in Problem-solving*, (Cambridge University Press, NY, 1995).
[22] G. Polya, *How to solve it*, (Doubleday, NY, 1945).
[23] see Ref. 21.
[24] See M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).

[25] See Ref. 21 and Ref. 14.
[26] E. Bagno and B. Eylon, "From Problem Solving to a knowledge structure: An example from the domain of electromagnetism," Am. J. Phys. **65** (8) 726-736 (1997).
[27] This example is discussed in detail in chapter 5.
[28] D. P. Maloney, "Research on Problem Solving: Physics. In *Handbook of Research on Science Teaching and Learning*, D. L. Gabel (Ed.), (Macmillan Publishing Co, NY, 1994), pp. 327-354.
[29] See Ref. 28.
[30] R. Bhaskar and H. A. Simon, "Problem Solving in Semantically Rich Domains: An example from engineering thermodynamics," Cog. Sci., **1** 193-215 (1977).
[31] Bhaskar and Simon 1977
[32] J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979).
[33] For a discussion on local and global attributes see J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979). See also L. C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993).
[34] Discussions on the organization of knowledge are found in Ref. 24, and in J. H. Larkin, J. McDermott, D. P. Simon, and H.A. Simon, "Expert and Novice Performance in Solving Physics Problems," Science, **208** (20) 1335-1342 (1980), and B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984).
[35] See Ref. 28.
[36] J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A.L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.
[37] Larkin adopts the term naïve representation from the process of envisionment described by de Kleer.
[38] See Ref. 36.
[39] Y. Anzai and T. Yokoyama, "Internal Models in Physics Problem Solving," Cognition and Instruction, **1** (4) (1984).
[40] See Ref. 39.
[41] See Ref. 39.
[42] See Ref. 39
[43] For more information see Ref. 39.
[44] See Ref. 24.
[45] As cited by Chi et al., D. Halliday and R. Resnick, *Fundamentals of physics*, (Wiley, NY, 1974).
[46] These examples come from Ref. 24.
[47] See Ref. 24.
[48] See Ref. 24.
[49] F. Reif and J. I. Heller, "Knowledge structures and problem solving in physics," Educational Psychologist, **17** (2), 102-127 (1982).
[50] See Ref. 49.

[51] F. Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," Am. J. Phys. **63**, 17-32 (1995).
[52] For an example see F. Reif and S. Allen, "Cognition for Interpreting Scientific Concepts: A study of acceleration," Cognition and Instruction, **9** (1), 1-44 (1992).
[53] See B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984).
[54] See Ref. 53.
[55] For more info see Ref. 53.
[56] See Ref. 53.
[57] See Ref. 26
[58] See Ref. 26.

Chapter 3: Background, Methods, and Context

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Chapter 3: Background, Methods, and Context

Physics Education Research Background

This chapter will provide the reader with a summary of some of the general methods used by physics education researchers to evaluate student understanding. In this dissertation we apply these methods in specific ways to develop a better understanding of the coherence of student content knowledge. This chapter also describes the context of the research.

Methods of Physics Education Research

Physics education researchers use a number of different methods to probe student understanding. These methods include interviews, multiple-choice questions, and open-ended exam problems. This dissertation uses all of these methods, but most of our data comes from written responses on open-ended exam problems. In this section we provide a brief summary of these general methods followed by a more detailed description of the specific ways the methods were employed in this dissertation.

Interviews

Interviews provide the most in-depth probe into student understanding. In an interview one or more students will be present with a researcher. The researcher will often have a protocol that may include a specific physics problem, or may include a series of questions concerning a demonstration. The type of interview will depend on the type of study being conducted. During the interview the researcher will try to ask questions that probe existing student understanding, rather than questions that guide students to an improved understanding.

The interview method gives the researcher the ability to probe deeply into student understanding. The researcher can follow up student responses that are unclear with questions designed to elicit clearer explanations. In addition, questions can be constructed on the spot based on interactions with the students. Statements and questions made by the interviewer are also a limitation of the interview method because the question (or measurement) affects the student answer (or observation). Even carefully constructed questions, which try not to guide the students, will affect the student response.

The interviews that were conducted for this work involved students from both the introductory engineering physics course (6 students) as well as advanced students (6 students) studying physics at the University of Maryland. All interviews discussed in this work were done on a one-on-one basis with volunteers and involved students solving problems. Much of the previous work done in understanding student problem solving involves this type of interview. The students were first given a brief introduction to the interview method where they were informed about the anonymous nature of the interview and were told that they should state everything they were thinking out loud. The situation is a bit unnatural for the students. Other researchers

have done interviews with two or more students, creating a more natural setting.[1] In the one-on-one interview it was occasionally necessary to remind the students to state what they were thinking and writing out loud. The interviews I conducted typically lasted forty minutes to one hour. After the interview, students were given the opportunity to discuss any aspect of the physics course they wanted. Except for the interviews conducted with the advanced students, all interview questions were based on the material students were currently learning in the course. Because of this, the post-interview discussion often focused on understanding the correct solution to the problem given in the interview. Asking questions that were relevant to the course material also provided motivation for students to participate in the interviews. After the interviews were conducted, most were transcribed and analyzed. Complete transcripts are provided in Appendix C and D for the interviews that are used in this dissertation.

Interviews are usually conducted with a small sample of students because of the amount of time involved in conducting and transcribing interviews. The sample of students that were interviewed was obtained by asking for volunteers in the physics course. A list of the students who were willing to participate in the research was created. A number of students were contacted, at random, from the list, and a time for each interview was arranged. Because participation in this type of study is voluntary, the students who volunteer are not a random sample. Most of the students whose interviews are presented in this work had grades ranging from C's to A's.

Multiple-Choice Questions and Diagnostics

Multiple-choice diagnostics have become an increasingly popular tool to evaluate student performance. They are the easiest type of measurement device to implement. Because of this, researchers are able to obtain large sample sizes and are also able to conduct statistical analyses on the results. Despite their ease in implementation, data from multiple choice questions are difficult to interpret. There is still debate about what these tests actually measure and a danger that instructors will view the results from these questions in a casual and simplistic manner. [2]

One of the most popular diagnostics used is the Force Concept Inventory (FCI), which was designed by D. Hestenes, M. Wells, and G. Swackhammer and was published in the *Physics Teacher* in 1992.[3] The success of the FCI in drawing out student misconceptions is due to its careful construction and validation. Before the FCI was constructed, the authors performed in-depth studies on the nature of student difficulties in mechanics and collected the results of a large body of published work. The FCI is one of the products of this work. The precursor to the FCI was the Mechanics Diagnostic Test (1985), which was validated through the use of interviews and statistical methods. Because of the similarity between the two instruments, Hestenes et al. did not follow the same procedures to evaluate the validity of the FCI, although they did perform interviews with 20 high school students and 16 graduate students about their answers on the FCI.[4]

According to the Hestenes et al, the FCI "assesses a student's overall grasp of the Newtonian concept of force … [and it] can be used for both instructional and

research purposes."[5] The questions on the FCI all deal with mechanics concepts and they are phrased in terms of real world contexts. Because of its construction the FCI can be used both before and after physics instruction. After administering the FCI to their students, most instructors are surprised at their students' poor performance. Because of this, the FCI provided a wake-up call for many instructors. Despite satisfactory performance on exam problems and homework sets, the FCI showed that many students exhibited basic conceptual difficulties in mechanics.

The success of the FCI has prompted the development of other diagnostic tools to evaluate student understanding. Thornton and Sokoloff have developed another mechanics diagnostic called the Force Motion Concept Evaluation (FMCE).[6] Because most of the work conducted in PER is conducted in mechanics, The FCI and FMCE are the most popular diagnostic tools used by instructors and researchers. But there are a number of newer tools being developed to evaluate student understanding of other topics in physics. Diagnostics now exist to evaluate student understanding of waves[7] and student understanding of concepts in electricity and magnetism.[8]

Despite the growing popularity of these instruments there are few studies documenting how student performance on diagnostics correspond with other measures of student understanding. We find that, because student knowledge is often only locally coherent, the context of a problem becomes an important factor in the way students will respond to a question. In this dissertation we will use multiple-choice and corresponding open-ended questions to probe for coherence in student understanding of physics.

Open Ended Questions and Problems

Questions that ask for detailed written responses are the primary research tool used in this dissertation. These open-ended questions include quantitative questions, qualitative questions, and questions that combine the two. The *quantitative physics problem* is the type of problem at the end of the chapter in introductory physics texts. These problems usually require that the students find the appropriate formula or formulas, manipulate the formulas to fit a given situation, identify the givens and unknowns, and solve for one or more numerical values. Often, little conceptual understanding is required for these problems. These traditional problems are often included on homework assignments and exams in the introductory physics course. A second type of open-ended question is the qualitative question. The *qualitative question* is not as common in the calculus-based course, although it has become a major tool for physics education researchers to use in evaluating student understanding and many texts now include them at the end of the chapters. Qualitative questions usually involve little or no symbolic manipulation or formulas. They require the students to draw upon their qualitative resources and reasoning ability. A third type of question is a *hybrid* or *bridging problem*. These questions require the students to draw upon both their qualitative knowledge and their quantitative knowledge. This may be done by explicitly breaking the problem into qualitative and quantitative parts. It may also be done by designing questions for students to solve that require conceptual understanding. These questions will often require the students to either show all their

work or to explain their reasoning. By doing this we are able to get a deeper understanding of what our students are doing. Each type of question serves different purposes for the researcher and instructor.

In this dissertation, quantitative problems that require students to use their conceptual understanding will be used. Sometimes the students are given a problem and simply asked for a certain unknown while other times they are asked a series of qualitative questions before the final quantitative question. These types of questions allow us to look explicitly at student qualitative knowledge, student quantitative knowledge, and the links between them. Comparisons are sometimes made between students who solved identical quantitative questions with and without qualitative parts. Students' solutions can then be compared and the use of qualitative knowledge in solving problems can be evaluated.

Summary of Research Methods

Each of these three methods has advantages and disadvantages. The main limitation of the interview is the small sample size. To show the distribution of responses, physics education researchers can turn to multiple choice questions and open-ended questions.

Multiple-choice tests are the easiest tool to implement but they are difficult to interpret. This does not imply that multiple-choice tests are not worthy of attention. They provide us with many interesting results about student understanding; but these results may be very different than results on other measures.

Open-ended exam problems are a middle ground between multiple-choice questions and interviews. They can provide the researcher with fairly large sample sizes, and provide the researcher with more information about student reasoning than a multiple-choice question. Due to the nature of this study, open-ended questions were used as the basis of most of the work. Interviews were conducted to probe more deeply into student understanding after open-ended questions were administered.

Error Analysis

Estimates of the standard error are included in some of the studies in this dissertation. Occasionally we present analyses that compare two quantities. For instance we compare the performance of two different populations in chapter 5 and chapter 8. In order to make claims about our results it is therefore necessary to estimate the error in our measurements.

Error is calculated by making the crude assumption that student responses approximate a binomial distribution. Using this simplified model gives a rough approximation of the uncertainty in our measurements. We use $\sigma^2 = E(x^2) - [E(x)]^2$ and the fact that the expectation value for a binomial distribution is $E(X) = \sum_{r=0}^{N} \frac{N!}{(r-1)!(N-r)!} p^r q^{N-r}$ where p is the probability of the response we are interested in, q $(= 1 - p)$ is the probability of a different response, and N is the total number of students in the study. The standard deviation is therefore given by

$\sigma = \sqrt{Npq}$ and the deviation in the percentage of a particular response is $\sigma_P = \sqrt{pq/N}$. Since most of the data in this dissertation is reported in terms of the percentages, σ_P will be used to calculate most of the error in our measurements.

Context of the Research

Undergraduate engineering students

Most of the work presented in this dissertation was conducted in the engineering physics sequence at the University of Maryland. The course is a three-semester calculus-based course. The first semester of the sequence (Physics 161) covers mechanics. The second semester (Physics 262) covers the topics of heat and temperature, thermodynamics, mechanical waves, electric fields and electric circuits. The third semester (Physics 263) covers the topics of magnetism, electromagnetic waves, geometric and physical optics, and modern physics. In the second and third semesters students must also enroll in Physics 262A and Physics 263A, the respective laboratory courses. Students do not receive a separate grade for the laboratory. Instead the grade is incorporated into the overall grade for the course. Approximately 200 - 300 students are enrolled in each part of the sequence and these students are divided among two to three lecture classes, each taught by a different instructor.

Each part of the three part sequence consists of three hours a week of lecture and fifty minutes of either a traditional TA-led problem solving recitation or a tutorial section. In the tutorial section the students participate in research-based curriculum emphasizing qualitative understanding.

Students are usually not told whether the class has tutorial sections or traditional recitation sections when they register for the course. Because of this, the populations enrolled in the class with recitations and the population enrolled in a class using tutorials are nearly identical. The PERG at the University of Maryland runs the tutorial part of the class. Because of limited resources, only one or two courses in the sequence will be engaged in tutorials each semester. For instance, in one semester there may be tutorials in both the physics 161 course and the physics 263 course, but the 262 course may have traditional recitations. During most semesters, all the day sections of a particular course will be engaged exclusively in a tutorial curriculum or exclusively a traditional recitation curriculum. Because of logistic reasons the night class at UMd usually has recitations. Students who start in a tutorial class in Physics 161 will usually have tutorials in the physics 262 and 263 courses as well (provided they take them in sequence with no breaks.) Likewise, students who begin with the recitation class will usually have traditional recitations for the remainder of the sequence.

We will use the following terminology to differentiate between the different components of the engineering sequence consisting of Physics 161, 262, and 263. A particular part of the sequence will be referred to as a *course*. Within the course, different lectures are referred to as *classes* and within the classes discussion (tutorial or recitation) will be referred to as *sections*. Therefore a student may be in the

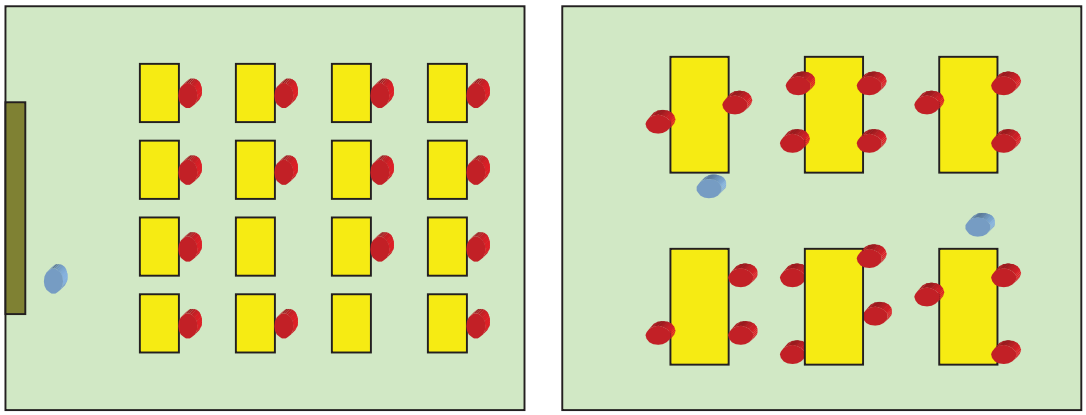
engineering sequence, in the Physics 161 course, in the class taught by Dr. X, in the discussion section that meets Wednesday morning.

In a traditional recitation section students typically watch a TA demonstrate how to solve problems at the board. Students often do not actively participate in recitation. Occasionally a TA will introduce innovative instruction in the recitation such as group work, but this not common.

In the tutorial section, students work through researched-based curriculum materials designed by the University of Washington[9] and the University of Maryland physics education groups. Their design is based on extensive research into the student understanding of various topics. Tutorials use conceptual worksheets that promote active learning. Students work in groups of three or four and mostly interact with the other members of their groups. Two facilitators (TAs) go around and ask the students guided questions to help the students construct the understanding for themselves. A schematic of the tutorial section and the recitation section is shown in Figure 3 - 1.

There are a number of important implementation details associated with the tutorial curriculum. These details will be discussed in chronological order to give the reader a clear picture of how the tutorials run in the introductory course. The major components of the tutorial curriculum are pretests, TA training sessions, tutorial sections, tutorial homeworks, and tutorial exam questions. Pretests, TA training sessions, tutorial sections, and tutorial homework assignments all occur once a week. Exam questions occur three to four times during the semester. The following paragraph will describe an ideal implementation of the curriculum. Because of different circumstances, implementation of the tutorials is not always ideal.

The first aspect of the tutorial curriculum are pretests. These one to two page activities are given in the first 10 minutes of the lecture after the material has been lectured on, but before students have gone through tutorial instruction on the material. Students receive credit for participating in the pretests but do not get graded on their responses or get them back. No solutions are posted to the pretests, although the same



(a) (b)

Figure 3 - 1

Schematic showing a traditional recitation class (a) and a tutorial class (b).

issues are dealt with in the tutorial section where the students are given feedback. Pretests serve two main purposes; they help the students determine what aspects of the material they are having difficulty with after lecture on the material and give them a chance to commit to an answer on their own before the tutorial; they also give the TAs for the class a chance to become familiar with the student difficulties. In addition, the pretests help students identify certain topics as important.

During the TA training sessions the TAs are put in the same role as the students. They first take the same pretest the students took earlier in the week and go over their responses. They then look through and discuss their students' pretests. TAs are therefore made aware of the difficulties some of the students are having with the material that is to be covered in the upcoming tutorial section. By understanding the student errors and difficulties, the TAs are in a better position to address difficulties and questions that will come up in the tutorial sections.

After looking over the pretests and discussing the student responses with the other TAs, the TAs start the actual tutorial. Going through the tutorial serves two purposes. TAs must become familiar with the material in the tutorial if they are to ask the students guided questions on the material. The skill of asking questions that guide the students and allow them to construct the knowledge for themselves is often very difficult to acquire since most of the TAs have learned the material by having explanations given to them.[10] Many TAs therefore want to teach the students in the same way they have been taught. Unfortunately this method is not effective for the vast majority of our introductory physics students.[11] The second purpose of the meeting is to give the TA's the opportunity to confront and resolve any difficulties they, themselves, may have with the material.

Tutorial sections last fifty minutes and are given once a week, just like the TA-led problem solving recitations. Students enter the class and begin working on the conceptual worksheets once their group is assembled. The tutorial worksheets either come from the *Tutorials in Introductory Physics*[12] book or are given to the students as handouts. Students mainly interact with their group members. Two TAs are in the tutorial room at once, asking students guided questions about the tutorial material. Students keep the tutorial worksheets to help them with the tutorial homework assignments and to help them study for the exams. Attendance is not required in the tutorial sections, although attendance is typically quite good. Attendance in the tutorial section during the period of this study was approximately 80%. Besides getting a better understanding of the material, incentive for coming to tutorial comes from the graded tutorial homework assignments and the graded exam question based on one of the tutorials.

Tutorial homework is assigned each week in addition to the textbook homework assignments given by the instructor for the class. Because of the additional tutorial homework, there are fewer problems assigned from the textbook. Tutorial homework is based on the tutorial the students have just completed and is due the following week. The tutorial homework assignments are graded and returned to the students.

Most of the incentive for attending the tutorial session probably comes from the tutorial exam question. Students are told at the beginning of the semester that one

question on each exam is based on the tutorial. If students do not attend the tutorial sections the exam question will be very difficult for them; if they attend and work diligently through the tutorials they usually perform well on the exam question.

In order to help students form coherence in their qualitative and quantitative knowledge the UMd PERG has begun designing instructional material to help students make connections between the concepts they develop in tutorial and quantitative problem solving. *Problem Solving Tutorials* and *Bridging problems* are implemented on a small scale to help the students make this bridge.

Problem-solving tutorials occur either once or twice a semester. They are implemented in the same way other tutorials are implemented, but instead of having students construct an understanding of a particular topic or topics, they include applications of the concepts developed in earlier tutorials in the context of three to four carefully constructed problems. These problems consist of both qualitative parts and quantitative parts to help the students develop more coherence between the concepts and quantitative problem-solving.

Bridging problems are sometimes given as part of the tutorial homework assignments. They are supplements to the more conceptual part of the tutorial homework assignments and are based on the tutorials the students have just completed. Like the problems given in the problem-solving tutorials, the bridging problems contain both qualitative and quantitative questions designed to help the students make the link between these two types of knowledge.

Advanced Students

In addition to the undergraduate engineering majors a small number of studies were conducted with advanced physics students. These students volunteered for interviews on topics in mechanics and physical optics. Two students were second-year graduate students, three were first-year graduate students, and one student was an upper-level undergraduate student who was enrolled in a number of graduate courses. All students were physics majors at the University of Maryland.

Summary of the Data

The data that is presented in this dissertation comes from the many different physics topics that span the three semesters of the engineering sequence at the University of Maryland (UMd). One reason for the wide range of topics is to show that the difficulties students have in making the connections between their qualitative and quantitative knowledge and making connections across physics topics and concepts is not topic dependant. It also shows the pervasiveness of this fragmentation in physics knowledge.

Table 3 - 1 summarizes the data that is presented in the dissertation. It contains a brief description (name) of the question, the format of the class, the format of the question, the date of the study, and the location in the dissertation where the data are first presented.

Description of Problem (name)	Format of Class	Format of question	Date Administered	Location in dissertation
NI, NII, and motion diag.	Phys 161, w/ tutorial	Open-ended exam prob.	Spring 1996	Chapter 4
NIII-carts	Phys 161, w/ tutorial	Open-ended exam prob.	Fall 1995	Chapter 4
Force Concept Inventory	Phys 161, w/ tutorial	MC questions.	Fall 1995, Spring 1996	Chapter 4
Hand-block	Graduate Students	Prob. Solving Interview	Fall 98	Chapter 5
Hand-block	Phys 161, w/ recitation	Open-ended quiz prob.	Fall 97	Chapter 5
Tension-block	Phys 161, w/ tutorial	Open-ended bridging prob.	Spring 97	Chapter 5
Tension-block	Phys 161, w/ tutorial	Prob. Solving Interview	Spring 97	Chapter 5
Thermo-piston	Phys 262, w/ tutorial	Open-ended pretest	Fall 97	Chapter 5
Inductive Circuits	Phys 263, w/ tutorial	Open-ended exam prob.	Spring 99	Chapter 6
Momentum Question	Phys 161, w/ recitation	Open-ended exam prob.	Fall 97	Chapter 6
Resistive Circuit	Phys 262	Open-ended exam prob.	Fall 97	Chapter 6
Electric Fields and Potential	Phys 262, w/ tutorial	Open-ended bridging prob.	Fall 97	Chapter 6
Physical Optics I	Phys 263, w/ tutorial	Open-ended exam prob.	Spring 96	Chapter 6
Tension-Two Blocks	Phys 161, w/ tutorial	Open-ended exam prob.	Fall 98	Chapter 7
Heat Transfer	Phys 262	Open-ended exam prob.	Spring 96	Chapter 8
Physical Optics II	Phys 262	Open-ended exam prob.	Spring 96	Chapter 8

Table 3 - 1

Table showing a summary of each question(s) discussed in the dissertation.

[1] This method is used in B. Sherin, "The Symbolic Basis of Physical Intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, School of Education, University of California, Berkley, (1996).
[2] For a discussion on how to interpret multiple-choice tests see R. S. Steinberg and M. S. Sabella, "Performance on multiple-choice diagnostics and complimentary exam problems," Phys. Teach. **35** (3), 150-155 (1997) and D. Huffman and P. Heller, "What does the Force Concept Inventory actually measure?" Phys. Teach. **33**, 138-143 (1995), D. Hestenes and I. Halloun, "Interpreting the force concept inventory, a response to the March 1995 critique by Huffman and Heller," Phys. Teach. **33**, 502 (1995), and P. Heller and D. Huffman, "Interpreting the force concept inventory, a reply to Hestenes and Halloun," Phys. Teach. **33**, 503 (1995).
[3] D. Hestenes, M. Wells and, G. Swackhammer, "Force Concept Inventory," Phys. Teach. **30** (3) 141-153 (1992).
[4] For more information on the FCI see ref. 3 and J. M. Saul, "Beyond Problem Solving: Evaluating introductory physics courses through the hidden curriculum," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).
[5] See Ref. 3.
[6] R. K. Thornton and D. R. Sokoloff, "Assessing student learning of Newton's laws: The Force and motion concept evaluation and the evaluation of active learning laboratory and lecture," A. J. Phys. **66** (4) 338-351 (1998).
[7] M. C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).
[8] Private communications with R. Thornton, D. Sokoloff and D. Maloney, T. Okuma, C. Heiggelke
[9] L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).
[10] Most physics majors will spend time outside of class to reason out the information themselves. Unfortunately, most of our service course students do not do this.
[11] Redish states that physicists as a group seem to be selected for the character of being able to learn on their own in E. F. Redish, "Millikan Award Lecture (1998): Building a Science of Teaching Physics," Am. J. Phys. **67** (7), 562-573 (1999).
[12] See Ref. 9.

Chapter 4: Using Context to Probe for Coherence within Physics Topics

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Chapter 4: Using context to probe for coherence within physics topics

Introduction

In this chapter we examine how students respond to questions about a particular physics topic that are posed in different contexts. The data clearly shows that when students are given a question or problem they often activate schema partially based on the context of the question. This leads to many of our students answering inconsistently on questions that deal with identical physics concepts. The knowledge that is brought to the task is often characterized by a local coherence, which indicates that our students activate schemas of physics knowledge that are isolated from one another.

In addition, the knowledge that is contained in the two schemas may even be contradictory. This causes certain contexts to trigger different types of physics knowledge, which include different procedural rules, and different declarative knowledge. An expert answering the two problems would tend to bring up schemas that depend more on the underlying principles. Even though the context of the question is different the expert will usually solve the two questions in the same way, using a consistent set of knowledge.

In this study we have changed the contexts by changing the format of the questions (i.e. multiple choice vs. open-ended) and by changing the type of questions (i.e. real world vs. physics class.) Multiple choice diagnostics are often used to assess student understanding of physics, but there have been few studies documenting how student responses in a multiple-choice context compares with their responses in other contexts such as interviews or open-ended exam questions.

Question Number (old version)	Topic
18	Newton's 1st and 2nd Laws
9,22	Impetus Force
7	Motion Diagrams
2,11,13,14	Newton's 3rd Law

Table 4 - 1

List of four topics covered on the FCI and their corresponding questions.

In this study the same group of students were asked questions on a multiple-choice diagnostic and on open-ended problems. The multiple-choice questions we

used were chosen from the Force Concept Inventory (FCI).[1] The FCI, as well as other multiple-choice instruments have become increasingly popular. Because of their popularity and ease of use, they play a large role in evaluating curriculum and also in developing curriculum and instructional strategies. Despite its wide acceptance and use there have been few studies published on how student performance on the FCI correlates with their understanding of the subject matter.[2,3,4]

Students were given the FCI as an ungraded diagnostic, near the end of the physics 161 course at UMd. In order to compare how students answer questions on the FCI to how they answer questions on an open-ended exam problem we developed and administered the two final-exam problems shown in Figure 4 - 1 and Figure 4 - 2. The exam problems dealt with a number of concepts that are tested for on the FCI. In particular, we examined the concepts of Newton's 1st Law (NI), the Impetus Force, motion diagrams, and Newton's 3rd Law (NIII). Table 4 - 1, shown on the previous page, shows the concepts and the corresponding questions on the FCI. Students had instruction on all the material presented in this chapter prior to the administering of the FCI and final exam.

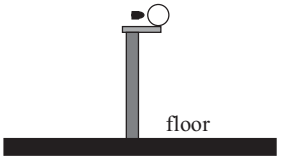
The Physics Education Research Group at UMd has been administering the FCI for several years both before instruction and after instruction. In this study the

Ignore all friction and air resistance in this problem.

A. A steel ball resting on a small platform mounted to a hydraulic lift is being lowered at a constant speed, as shown in the figure at right.

i. Draw a free body diagram of the ball. Describe each type of force.

ii. Compare the magnitudes of the forces you have drawn.
Explain your reasoning.



B. As the ball is moving down, a bullet moving horizontally hits the exact center of the ball (see figure at right) and then ricochets straight back. This causes the ball to immediately fall off the platform

i. Draw a free body diagram of the ball after it is no longer in contact with the bullet or the platform. Describe each type of force.

ii. A vector that represents the velocity of the ball just before the bullet hits is shown below. Draw vectors that could represent the velocity at each of the 2 other times indicated. The scales of the 3 vectors should be consistent with each other. Explain your reasoning.

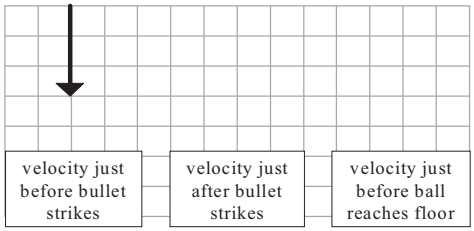


Figure 4 - 1

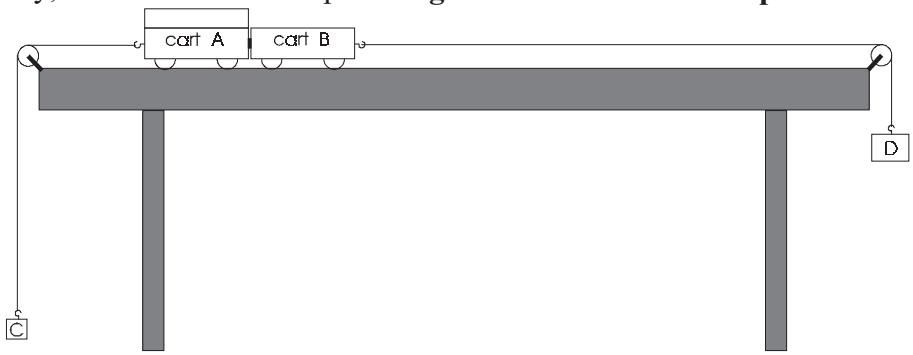
Exam problem testing student understanding of Newton's 1st and 2nd laws, and motion diagrams.

original version of the FCI has been used. Although some questions have been changed on the new version, it does not affect the conclusions we can draw from the results. A full copy of the FCI is included in Appendix E.

The work was conducted by Richard Steinberg and myself and has since been published in *The Physics Teacher*.[5]

The problem shown in Figure 4 - 1 was asked in spring 1996, in a single lecture class. Twenty-eight students also completed the FCI at the end of the semester. The problem shown in Figure 4 - 2 was given to three different lecture classes in the fall of 1995. In these classes, 128 students also completed the FCI. The data we discuss are broken down according to concept. Richard Steinberg wrote the problem shown in Figure 4 - 1 and Jeff Saul wrote the problem shown in Figure 4 – 2.

Two carts, A and B ($Mass_A > Mass_B$), are placed on a table then stuck together with Velcro. Using pulleys, two small blocks, C and D ($mass_C < mass_D$), are connected by light strings to the carts as shown below. Initially, the carts are held in place. **Ignore all friction in this problem.**



At t = 0, the carts are released. At t = 3 seconds, the Velcro pulls apart and the two carts separate. At some later time, cart A returns to its starting point.

a. Draw and label two separate free-body diagrams, **one for each cart**, for a time after the carts start moving but before the Velcro pulls apart.

b. Rank all the horizontal forces from both your diagrams by magnitude, from largest to smallest. **Explain the reasoning that you used to rank the forces**.

Figure 4 - 2

Exam problem testing student understanding of Newton's 2nd and 3rd Laws.

Newton's 1st Law

The first concept we will discuss is NI. Previous research in physics education has shown that students have many profound difficulties with the concept of NI and NII.[6] The FCI uses specific misconceptions that students have as distractors. These distractors as well as the correct response are included as choices for each FCI question. The distractors are based on research into student understanding of the various concepts. Halloun and Hestenes also conducted many interviews with students to validate the FCI.[7]

The FCI question that tests the concept of NI, on the FCI is question 18. The question involves an elevator moving upward at constant speed. Students are asked to compare the force exerted on the elevator by the cable to the force of gravity exerted on the elevator. The UMd PERG has consistently seen about 50% of the students answering this question correctly after instruction, by stating that the two forces are equal. The most common error, given by approximately 40% of the students was that the upward force on the elevator by the cables was larger than the downward force due to gravity. This response is inconsistent with NI and NII but is consistent with the naïve belief that a force is needed to maintain a constant velocity.

The first part of the problem shown in Figure 4 - 1 is very similar to question 18 on the FCI. If student models were consistent we would expect them to answer this question the same as they answered question 18 on the FCI. In the exam problem, instead of an elevator being raised at constant speed, a steel ball is being lowered at constant speed. The students are asked to draw the free-body diagram for the ball and then compare the magnitudes of the forces acting on the ball. The correct answer to the problem would include the students identifying the normal force from the platform on the ball directed upward and the force of gravity on the ball acting downward. The magnitudes of the two forces must be equal since the ball is moving at constant velocity.

A comparison of how the students responded in the two contexts shows that these two questions are not equivalent for our students. Of the twenty-eight students who answered both the FCI question and the exam question we see 54% of the students answering the FCI question correctly and 90% of the students answering the open-ended question correctly. Ten of the thirteen students who answered the FCI question incorrectly selected choice A, stating that the force directed up from the cable was greater than the downward force of gravity. Two of the three students who answered the exam question incorrectly gave responses that were consistent with choice A on the FCI.

Since the students in the study are matched, meaning that they took both the FCI and the open-ended question, we can examine how they answered on each instrument. These correlations are displayed in Table 4 - 2. It is interesting to note the large difference in the way students performed on the two questions, despite the similarity in the items.

One difference between the multiple-choice question and the corresponding open-ended question is that in one situation an object is moving up (elevator) and in the other situation an object is moving down (ball). A physicist will view this difference to be irrelevant in answering the question. More importantly many instructors will see this as irrelevant to the way the students solve the problem. We have investigated whether the direction of the object's motion has an effect on how students answer the questions.

Upon the recommendation of a colleague we undertook an investigation in the Physics 161 class in spring 1999. During the last week of lecture students in one class were given an open-ended diagnostic and students in the other class were given a multiple-choice diagnostic. It was announced to the students that the diagnostics would not count toward their grades but that they should take them seriously.

N=28

Exam Question	FCI	
	Correct:	Incorrect
Correct	14	11
Incorrect	1	2

Table 4 - 2

How students answered the FCI question on Newton's 1st Law and how they answered on the corresponding open-ended problem.

Students in one class, taking the open-ended diagnostic, were given three open-ended questions in addition to two multiple-choice questions. Two versions of the *ball-platform question*, shown in Figure 4 - 1, were included on the diagnostic. Both versions were given in each tutorial section. In one version the ball was moving down at constant velocity, as in Figure 4 - 1, and in the other version the ball was moving up at a constant velocity.

Students in the other class were given the FCI, which included the *elevator-tension question*. Two versions of this question were given in different tutorial sections. One version was identical to the FCI question described above. On the other version the elevator was moving down, instead of up, at constant speed. All other questions on the FCI were identical. It should also be noted that for this study the new version of the FCI was used. The elevator-tension question is essentially the same on both the old and the new version of the FCI.

The student responses indicate that the direction of the motion of the object does have some effect on how the students answered the multiple-choice question, but not the open-ended question. The results are shown in Figure 4 - 3. On the open-ended question, 84% of the students are answering the question correctly on the version with the platform moving down and 76% answered correctly on the version with the ball going up. On the multiple choice question 40% answered correctly with the elevator moving down, while 60% answered correctly with the elevator moving up. The results suggest that on the multiple-choice question some students are answering differently on the two versions. Differences on the open-ended question are not significant. Error bars are provided to show whether the results are indeed different based on an estimate of the error.[8]

The data shows that on the multiple-choice question the direction of the elevator's motion made a difference in the way students answered. Students tended to answer correctly more often when the elevator was moving up instead of down.

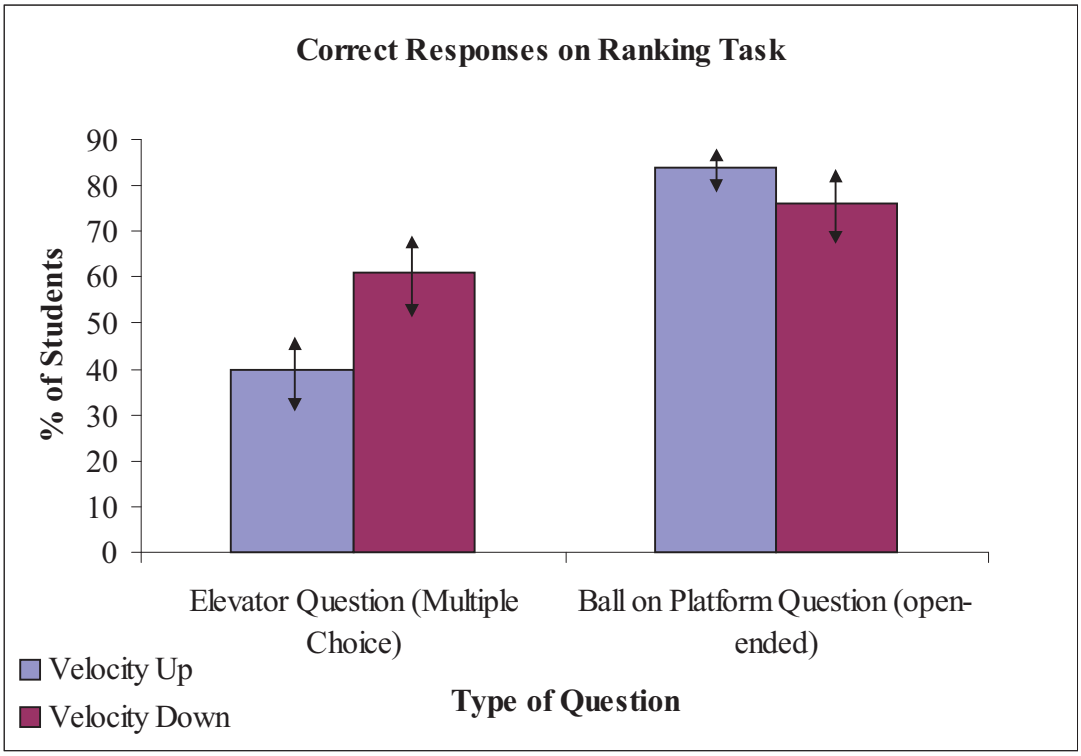


Figure 4 - 3

Comparison of how students answered the questions depending on whether the object was moving up or down.

Impetus Force

Students often have the naïve belief that an object that receives a brief impulse will experience the force from the impulse even after the impulse is over. For instance, if a ball is thrown up in the air, students will often include the force of the hand on the ball even though the ball is no longer in contact with the hand. This is a common error and is well documented in the literature.[9] This topic is addressed on the FCI by questions 9 and 22. The first question involves a sliding hockey puck getting kicked and the second question involves a golf ball after it has been hit. Students are asked to identify the forces that are acting on the puck and ball after the initial strike. The correct response was given by 46% of the students for the question about the hockey puck. Students answered the question about the golf ball correctly 43% of the time. Although the percentage of students answering correctly is about equal for these two similar FCI questions the students who answered one question correctly did not necessarily answer the other question correctly. Only 29% of the students answered both questions correctly. Therefore 71% of the students stated that there was either a force in the direction of motion or a force of the hit, on at least one of the two questions. These responses are consistent with the idea of an impetus force.[10] The results also indicate that even on the same type of measurement instrument students

can answer differently to very similar questions.

Part B of the problem in Figure 4 - 1 is very similar to these two FCI questions. The steel ball on the platform is hit by a bullet and is knocked off the platform. The students are asked to draw a free-body diagram for the ball when the ball is no longer in contact with the platform and the bullet. A correct answer would state that only the force due to gravity would be acting on the ball.

The relationship between how students answered the three questions is shown in Table 4 - 3. The table indicates that all students who answered both FCI questions correctly answered the open-ended question correctly. It also shows that students who did not answer any of the FCI questions correctly were unable to answer the exam question correctly. Despite this consistency, the table also shows that over one third of the students did not answer all the questions the same way. We expect an expert to answer all three questions in the same way.

N=28

	FCI		
Exam Question	0 FCI's correct:	1 FCI correct	2 FCI's correct
Correct	1	7	8
Incorrect	10	2	0

Table 4 - 3

How students answered the FCI questions on the impetus force and how they answered the corresponding open-ended problem.

Motion Diagrams and the Velocity Vector

The last part of the problem shown in Figure 4 - 1 asks the students to sketch a velocity vector for the ball that is consistent with the velocity vector just before the bullet strikes the ball. FCI question 7 asks a similar question about the speed of a hockey puck right after being kicked in a direction perpendicular to its motion. Although in one case the students are asked to draw a velocity vector and in the other case students are asked to pick the correct description of the velocity vector, the questions are similar in the concepts they are testing. These results will provide us with additional insight into student coherence and point out some of the limitations of multiple-choice questions. Table 4 - 4 shows how students performed on the two parts of Bii and FCI question 7.

The correct choice on the FCI was choice E, which states that the speed after the kick is greater than the speed before the kick and greater than the speed it "acquires" from the kick, but smaller than the arithmetic sum of these two. Choice E was selected by 61% of the class. The class performed almost as well on the first part

of Bii. About half the class drew the velocity vector correctly by drawing a vector with a vertical component that was the same as the vertical component just before the kick and a non-zero horizontal component.

The results also showed that only 12 of the 28 students who answered both the FCI and the exam had answers where the FCI response matched the exam response. In addition, four of the students gave answers on the exam that did not correspond to any of the choices on the FCI. The remaining 12 students had answers on the exam that did not correspond to the answers that were given on the FCI. One observation that is particularly interesting is that three of the students drew velocity vectors on the exam with vertical components that were shorter than the velocity before the ball was struck by the bullet. If the students answered correspondingly on the FCI they would answer the FCI question correctly, despite having this particular error. All three of these students stated that the speed would be independent of the initial speed of the puck on the FCI.

N=28

Exam Question	**FCI**	
	Correct	Incorrect
2 Correct	8	0
1 Correct	4	6
0 Correct	5	5

Table 4 - 4

Performance on the two parts of exam problem Bii (on motion diagrams) and the corresponding FCI question.

Students also struggled with the second part of Bii, where they were asked to draw the velocity vector just before the steel ball hits the ground. The nature of the difficulties on the exam made comparisons with similar FCI questions difficult.

Newton's 3rd Law

Newton's 3rd Law is a very difficult concept for our students to grasp and these difficulties are well documented in the literature.[11] Students often make the error that big objects exert larger forces or that the object that is accelerating is providing the bigger force. The concept of NIII has the most questions devoted to it on the FCI. Questions 2, 11, 13, and 14 all involve two objects interacting in some way and students are asked to choose from five different descriptions about the forces the objects are exerting on one another.

The problem shown in Figure 4 - 2 addresses NIII, although it is couched in terms of a larger problem. In the problem two carts are attached together and are

accelerating on a tabletop. The problem asks the students to first draw free-body diagrams for each of the carts and then rank the relative magnitudes of the horizontal forces. Students must recognize the existence and the magnitudes of the forces while describing the behavior of a multifaceted problem. The reason the open-ended question was posed this way was to prevent the knee-jerk response "equal and opposite." A superficial understanding of the relationship between the force that cart A exerts on cart B and the force that cart B exerts on cart A is unlikely to result in a correct response.

Because of the large number of students answering both the FCI and the exam question and the relatively large amount of questions devoted to the concept of NIII on the FCI we were able to perform additional studies. By having four questions on the FCI devoted to the concept of NIII we can get a better idea of the consistency in students' knowledge of NIII. Because 128 students answered both the exam question and the FCI, as opposed to 28 students, we can perform some other types of analysis. In particular, we will compare student responses on the exam to their responses on question 13 of the FCI. Question 13 is the most similar to the exam question.

Table 4 - 5 shows how students performed on the exam question and the FCI questions. The five categories correspond to the number of students who answered 0, 1, 2, 3, or 4 of the FCI questions correctly. The table indicates that 67% of the students who answered all four FCI questions correctly answered the exam question correctly. Similarly 65% answered the exam question correctly with three FCI questions correct, 53% for two FCI's correct, 29% for one FCI correct, and 14% for no FCI questions answered correctly. This data shows that performance on the FCI is correlated with performance on the exam; the more questions students answer correctly on the FCI the better they do on the corresponding exam question. It is a little surprising that even if students answered all four of the FCI questions correctly, which many instructors would say indicates a good conceptual understanding of NIII, only 67% of these students answered correctly on the exam.

N=128

	FCI				
Exam Question	0 FCI's correct:	1 FCI correct	2 FCI's correct	3 FCI's correct:	4 FCI's correct
Correct	3	8	11	21	24
Incorrect	11	20	8	10	12

Table 4 - 5

How students answered the FCI questions on Newton's 3rd Law and how they answered the corresponding open-ended problem.

The most interesting comparison to make is the way students answered question 13 on the FCI and the exam question. Question 13 on the FCI is shown in Figure 4 - 4. In it, a car pushes a truck and the entire system accelerates. On the exam question the two carts interact and the system accelerates. Therefore, in both cases two objects remain in contact and accelerate uniformly for the entire motion considered. Table 4 - 6 shows how students responded on the two similar questions. If the students' knowledge about NIII is consistent the off-diagonal elements of the table would be close to zero. The chart shows that only about half of the students fell on the diagonal indicating that these similar questions were answered inconsistently by about half the class. The open-ended question on the exam gives the students the opportunity to explain their reasoning. There were 21 students on the exam that clearly stated their reasoning for their incorrect responses. When we look at how these students answered question 13 on the FCI, we see that only six of them answered FCI question 13 in a way that was consistent with their reasoning used on the exam.

A large truck breaks down out on the road and receives a push back into town by a small compact car as shown in the figure below.

While the car, still pushing the truck, is speeding up to get up to cruising speed

(A) the amount of force with which the car pushes on the truck is equal to that with which the truck pushes back on the car.

(B) the amount of force with which the car pushes on the truck is smaller than that with which the truck pushes back on the car.

(C) the amount of force with which the car pushes on the truck is greater than that with which the truck pushes back on the car.

(D) the car's engine is running so the car pushes against the truck, but the truck's engine is not running so the truck cannot push back against the car. The truck is pushed forward simply because it is in the way of the car.

(E) neither the car nor the truck exert any force on the other. The truck is pushed forward simply because it is in the way of the car.

Figure 4 - 4

FCI question 13. This question is the most similar to the exam problem on NIII. In both cases the system of interacting objects is accelerating.

N=128

Exam Problem Response	FCI Question 13 response: Correct	Forces not equal	No contact forces shown	Other
Correct	33%	16%	0%	1%
Forces not equal	12%	20%	0%	0%
No contact forces shown	2%	2%	0%	0%
Other	5%	9%	0%	1%

Table 4 - 6

Comparison of how students responded on FCI question 13 and on the open-ended problem. Approximately half of the students answered inconsistently on the two instruments.

Summary

Our results show that students often answered differently on questions from the FCI and our open-ended problems, even though the questions were essentially the same. This provides us with evidence that, even within a particular topic, student content knowledge is fragmented. This study also suggests that student understanding cannot be measured by only one type of evaluation tool.

There are a number of possibilities that might account for some of the differences we have observed in how the students answered the corresponding questions. These possibilities include the fact that student models for physical systems can be ill formed and inconsistent, the FCI looks colloquial while the exam question looks formal, and the nature of the measurement instruments are different.

The different responses indicate that for many of our students the schemas they activate strongly depend on the context of the question. In contrast, an experienced physicist will tend to activate schemas that are more dependent on the concepts and principles involved and not so much on the context of the questions.

One possible reason different schemas get activated is because the exam question looks like a physics question, with carts and strings and free-body diagrams, while the FCI looks more colloquial. Instead of carts colliding on a table the FCI asks questions with objects from the real world. An anecdote given by Eric Mazur epitomizes this distinction. Upon looking at the questions on the FCI, one student asked, "Professor Mazur, how should I answer these questions? According to what

you taught us, or by the way I think about these things?"[12,13] Different questions might cue schemas for physics knowledge or for experiential knowledge, therefore causing students to answer differently. These ideas are related to the way students view physics in that many students do not relate the concepts and principles they learn in their physics course to the real world.[14]

The fact that the exam is open-ended and the diagnostic is multiple choice may also have an effect on how students answer the two types of questions. We found that most of the incorrect responses on the exam question were choices on the FCI but there were a handful that were not. We also noticed that there were a number of answers given on the exam that were incorrect but would be measured as correct on the FCI. In addition we saw students give correct answers on the exam for the wrong reasons. Another important feature that separates the multiple-choice questions from the open-ended questions is that the FCI includes distractors that are very appealing to the student. The distractors may have triggered responses, either right or wrong, that would not have been produced by the students on their own.[15] These choices on the FCI can trigger particular schemas. One of the goals in the physics class is to have students recognize physics principles in a wide variety of real-world contexts in addition to physics-class contexts. The distractors on the FCI measure the robustness of the physics schemas students have learned.

It is important to note that these results do not imply that one type of measurement instrument is better than the other. They both provide the instructor and the physics education researcher with valid, yet different, information about student understanding. When used in conjunction with one another they provide a useful tool in evaluating students coherence about different topics and concepts in physics. The danger is when instructors or physics education researchers use only one type of measurement tool to evaluate student understanding of physics.[16]

Because of their ease of use, many physics educators are solely using multiple-choice diagnostics to evaluate their students' understanding of the subject. They are therefore assuming the correlation between measurement items to be much greater than it is. It also may lead to instructors viewing student learning in a simplified way. This data shows that student understanding is very complex and depends on a number of factors.

This chapter demonstrates that students may use particular sets of knowledge (schemas) to answer a question or solve a problem. We have shown that different schemas often get activated when the context of question changes, even though the underlying physics is the same. In addition the knowledge in the schemas, used in the corresponding exam and FCI questions, often does not overlap.

[1] D. Hestenes, M. Wells and, G. Swackhammer, “Force Concept Inventory,” Phys. Teach. **30** (3) 141-153 (1992).
[2] For an account of the research that preceded the FCI, see I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. **53** (11) 1043-1055 (1985).
[3] For a discussion of a factor analysis on the FCI, see D. Huffman and P. Heller, "What does the Force Concept Inventory actually measure?" Phys. Teach. 33, 138-143 (1995); D. Hestenes and I. Halloun, "Interpreting the force concept inventory, a response to the March 1995 critique by Huffman and Heller," Phys. Teach. 33, 502 (1995); P. Heller and D. Huffman, "Interpreting the force concept inventory, a reply to Hestenes and Halloun," Phys. Teach. 33, 503 (1995).
[4] For an example that includes thorough research on reliability of multiple-choice physics diagnostics, see R.J Beichner, "Testing student interpretation of kinematic graphs," Am. J. Phys. 62, 750-762 (1994); for further discussion see references therein.
[5] R. S. Steinberg and M. S. Sabella, "Performance on multiple-choice diagnostics and complimentary exam problems,” Phys. Teach. **35** (3), 150-155 (1997).
[6] L. C. McDermott, P. S. Shaffer, and M. D. Somers, “Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood’s machine,” Am. J. Phys. **62** (1) 46-55 (1994) and J. Clement, "Students' preconceptions in introductory mechanics," Am. J. Phys. **50** (1) 66-71 (1982); I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. **53** (11) 1056-1064 (1985); I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. **53** (11) 1043-1055 (1985).
[7] See Ref. 2.
[8] A brief discussion of the estimate in the error is presented in chapter 3.
[9] See Ref. 6.
[10] For a discussion on the impetus force see Ref. 2.
[11] See D. P. Maloney, "Rule-governed approaches to physics: Newton's Third Law," Phys. Educ. **19**, 37-42 (1984). and E.F. Redish, J.M. Saul, and R.N. Steinberg, “On the effectiveness of active-engagement microcomputer-based laboratories,” Am. J. Phys. **66** (3), 212-224 (1998).
[12] E. Mazur, *Peer Instruction*, (Prentice Hall, NJ, 1997).
[13] This student recognized that she had two different schemas for a particular question.
[14] The connection of physics and reality are discussed in E. F. Redish, J. M. Saul, and R. N. Steinberg, “Student expectations in introductory physics,” Am. J. Phys. **66** (3), 212-224 (1998).
[15] M.C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).
[16] Redish cautions that it rarely pays to build universal theories come from a few good insights, see E.F. Redish, "Millikan Award Lecture (1998): Building a Science of Teaching Physics," Am. J. Phys. **67** (7), 562-573 (1999).

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Chapter 5: Using Complex Problems to Evaluate Coherence in Physics Understanding

Introduction

In this chapter we examine how students approach complex problems that require the integration of multiple schemas. In order for students to solve complex problems, it is important that our students integrate the knowledge from different physics topics into a coherent package. When given a problem-solving task students will activate a schema that they will use to solve the problem. Unfortunately, the schema that is activated is often not sufficient to solve the problem and it is difficult for many of our students to activate the relevant schema. We also begin discussing isolated schemas in students' qualitative and quantitative knowledge.

The first study we present involves one-on-one interviews with advanced students studying physics. These students were given a problem involving the concepts of dynamics and the work-energy theorem. The data shows a clear distinction in students who are able to go back and forth between knowledge for related topics and students who do not exhibit coherence between different physics topics. We present interview transcripts along with *reasoning maps* that represent the students' solutions to the problem. The transcripts and maps can help researchers and instructors look for coherence between different physics topics.

After looking at the case studies with advanced students we look at the way classes of undergraduate students in the Physics 161 course perform on two variations of the *dynamics-work energy problem*. We present responses to an ungraded quiz, a bridging problem, and a one-on-one interview protocol. We find that many of the undergraduate students exhibit schemas that are characterized by local coherence, but not by the global coherence that would characterize an expert problem solver. The data also shows that qualitative force-based questions cause the students to activate a force schema, which was isolated from the concepts of work and energy. Students who were not presented with the qualitative questions were more likely to apply their knowledge of work and energy to solve the problem.

To show the pervasiveness of these difficulties, we also present data from the physics 262 class. Student responses to two versions of a problem involving the work done by a piston in a thermodynamic process are analyzed. The data shows that qualitative questions, asked on one version of the problem, caused students to perform worse on the question. When students were not presented with the qualitative questions they were more likely to activate a quantitative schema they could use to solve the problem.

Dynamics and Work-Energy Problem

Introduction

Two versions of a dynamics-work energy problem were administered to advanced physics students and undergraduate engineering students. The original

version was given to undergraduate students in a bridging problem format and in a one-on-one interview format in the Spring '97 semester. A revised version was written in the Fall '97 semester. It has been given as a one-on-one problem-solving interview with advanced students and as an ungraded quiz with undergraduate engineering majors. All students involved in these studies had completed instruction on the material.

Graduate Student Interviews

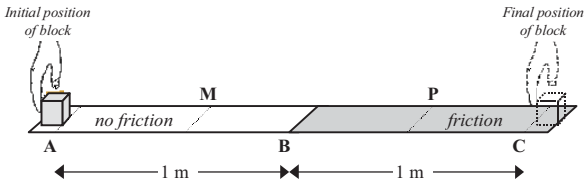
We presented the dynamics-work energy problem to six students enrolled in graduate classes at the University of Maryland, as a problem-solving interview.[1] The problem was written by the author and is shown in Figure 5 - 1 along with a model solution. The graduate students were first given a short version of the question, consisting of only parts a and d. The long version, shown in the figure, was given to one of the students (after he was having difficulty solving the short version) to help him solve the problem.

In a problem-solving interview the researcher provides the student with a problem and asks the student to solve the problem explaining what he or she is thinking and writing.[2] It was usually not difficult to get the students to talk about the physics. In addition to getting the students to explain their work out loud, the researcher must be careful not to guide the students. Instead, the researcher must ask questions to get the students to provide a clear record of their understanding and reasoning. The transcripts presented in the dissertation include the code name of the student and the gender of the student. The transcripts contain the following short-hand notation: [ ] indicate comments about the interview added after the fact, {—} is a short pause, [pause] is a long pause, {…} indicates that unimportant words were purposely omitted from the transcript, and (IA) indicates that the words were inaudible.

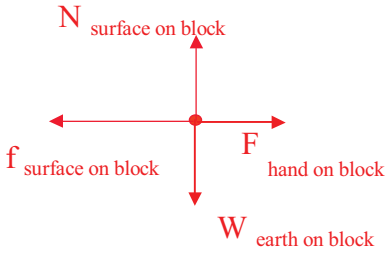
The pool of volunteers contained one upper-level undergraduate student who was enrolled in graduate level classes, three first-year graduate students, and two second-year graduate students. The complete transcriptions of the advanced student interviews presented in this dissertation are included in Appendix C.

The three first-year graduate students had many conceptual difficulties with the problem. The two-second year and the upper-level undergraduate student answered the question correctly, with little or no prompting. The undergraduate student seemed to exhibit the most coherent knowledge. He would continuously go back and forth between the concepts of work-energy and forces. These excerpts of the interview transcripts provide insight into how advanced students solve this problem. (The names presented are code names chosen by the students.)

A hand applies a force to a small 1 kg block from "A" to "C." The block starts at rest at point "A" and then comes to rest at point "C." The block moves along a frictionless surface from "A" to "B" and then travels an **equal** distance along a surface with friction from "B" to "C" with the force of the hand **remaining constant**. The force of the hand is 2 N to the right and the distance from "A" to "C" is 2 m. (See figure above.)



a) **Draw a free body diagram for the block when it is at "P."**



b) Is the magnitude of the net force acting on the block at "M" greater than, less than, or equal to the magnitude of the net force acting at "P"? Explain your reasoning.

Since the change in kinetic energy from A to B and from B to C are equal, the magnitudes of the net works are equal therefore the magnitudes of the net forces are equal.

c) i. Draw a vector representing the acceleration of the block at "P." If the acceleration is zero state that explicitly.

Since the block is coming to rest at C and the force of the hand and the force from the friction are constant the acceleration must be toward the left.

ii. Does the magnitude of the acceleration increase, decrease, or remain the same as the block moves from "B" to "C"? Explain.

The magnitude of the acceleration vector remains the same since the two forces acting on the block are constant.

d) **Calculate the coefficient of kinetic friction μ.**

From (b) we know that $T - f = -T$ and $f = \mu mg$ therefore $\mu = .41$

Figure 5 - 1

Revised version of the original bridging problem with a model solution. This problem was asked as an ungraded quiz in the Physics 161 class.

Eagle

A model solution to this problem would involve the application of multiple physics principles and concepts. In particular we would like to see the students tying the ideas of the work-energy theorem to the ideas of force. This particular section of a transcript is from the interview with Eagle, the upper level undergraduate student. Eagle integrates different physics concepts into a schema that he uses to solve the problem. In this section of the interview, Eagle is looking for the coefficient of kinetic friction. He has already drawn a correct free-body diagram.

> *E: Let's see— the block travels an equal distance (IA) with the force remaining constant — Let's see — (IA) — let me think — does it say anything about the speed — it doesn't — oh okay I see — I suppose the force is being applied until the end of the trajectory and the block stops due to the friction and not that the hand stops.*
>
> *I: The hand keeps applying from A to C.*
>
> *E: Okay — So I'm going to calculate the kinetic energy that the block has until point B —*
>
> *I: How come you're doing that?*
>
> *E: To find out what the total — what the energy it loses on the friction surface is — which should tell me —yes of course — what the force acting against it was. So that is going to be 2 Newtons times 1 meter, which is 1 Joule and that is equal to ½ m v squared — ... v being the velocity of the block — and that is exactly what it is going to lose which means the force — the friction force should be equal to 4 Newtons in the other direction — meaning ... in the direction C to A, or to the left — So that the net force being applied on the block is 2 Newtons in the other direction so that the loss of energy is equal to the gain of energy in the first half of the trajectory.*

At this point Eagle is connecting the ideas of force and work-energy and using the two concepts to solve the problem. He makes these connections throughout.

> *I: How did you know the loss in energy was the same as the gain in energy?*
>
> *E: Because it started at rest and it ends at rest — I suppose. I assume that is what it means when it travels an equal distance — …* [Rereads part of the question.] *So the force is 4 Newtons, which is equal to the magnitude of the normal force times μ — the kinetic friction coefficient— ... so the normal force is equal to the weight of the block which is 1 kilogram times 9.8 so — μ is equal to 4 over 9.8 which is about .4.*

I: … Can you compare the magnitude of the net force at M to the net force at P — how would they compare?

E: The magnitude of the net forces? — well they should be equal and opposite —

I: And how did you know that?

E: By the same argument — because I assume that the force due to friction — which is constant along the whole surface since the weight of the block doesn't change — I assumed that it was equal and opposite to the force — I'm sorry — I mean the sum of the force being applied by the hand and the friction should be totally equal and opposite to just the force applied by the hand on the block so that the loss of energy is equal along the same distance traveled — so they will be equal and opposite.

The question concerning the magnitude of the net forces on the two regions was particularly difficult. Eagle answers correctly without hesitation by applying the integrated knowledge in his schema for this problem. Even some of the graduate students who solved for μ correctly answered the question about the net force incorrectly (at least at first.)

Peter

Peter is a 2nd year graduate student. Like Eagle, he solved the problem correctly in a short amount of time. Unlike Eagle, he used a dynamics schema to solve the problem, instead of a single schema containing knowledge of dynamics and work-energy. It is interesting that even though Peter has correctly solved the problem he first states that the magnitude of the net force at P is smaller than the magnitude of the net force at M. He does correct himself soon after. The excerpt below is taken from the interview.

P: … so it will be minus 4 over 2 — which is the same acceleration — but opposite sign … and now we can plug it back into this equation (points to F - $\mu_k N$ = —ma) for the force and the coefficient of friction and so we have 2 Newtons — we are going to put numbers immediately — 2 minus μ_k will be equal to F plus ma over N — μ_k is 2 Newtons plus … I'm using here the absolute values of the acceleration over the 10 … so we are going to have 0.4.

I: How does the net force at M compare to the net force at P?

P: Umm — the net force at P is smaller than at M — by the amount of the kinetic friction. The net force is smaller by this amount because — the y components of the two forces are canceled out. So the only difference — they will be cancelled out at point A too — and the only difference comes at point P

because of the introduction of the force of friction, which is directed opposite to the applied force.

I: So can you draw me a vector for the net force at point P — how would that look?

P: It would look — almost caught me there — yeah the force of friction is bigger than the force — this is net force — since the acceleration is negative — so negative y direction — net force — according to the famous Newton's second Law should … I'll put it here to support my statement — the net force should be in the same direction as acceleration —

Granola

Granola, a 1st year graduate student, exhibited many conceptual difficulties while solving this problem. In addition we see a lack of coherence in his knowledge, evident from his inability to go back and forth between the two topics and his contradictory statements in the interview.

Granola identifies all the forces in the free-body diagram correctly. He later incorrectly describes the force of the hand to be greater at point P because the block is still moving toward the right. This indicates that Granola has the common misconception that force is proportional to velocity. In addition, Granola states that at point C the force of the hand would be equal to the force of friction. (Although this is true after the block has stopped, it seems that Granola makes the statement because the velocity is zero at C and not because it remains at zero.) When solving for the coefficient of friction Granola sets the two forces equal and solves for μ. But this solution does not feel right to him. In the interview Granola tries to draw from different physics principles when his analysis using dynamics does not seem correct, but these alternate principles lead to dead ends and Granola goes back to thinking about the forces.

G: Yeah. So wait maybe that should be right. [pause] *That's not right at all — … it doesn't seem right to me — just give me a second — I just started to solve it assuming it would be easy — then realized maybe it wasn't as easy as I thought. Could also do the work — the work from here* [points to A] *to there* [points to B] *— no that's got nothing to do with it — no — friction — 2 Newtons — the force of the hand remains constant — because according to this — this wouldn't be stopped there — it would stop there if this were the case — … if this were the case if it was 9.8 Newtons and 2 Newtons because that is the same circumstance that we have at B — so it would stop at B the way I have it set up — why would it stop at C? There is something about the 1 meter that I'm not getting — I'm not thinking very well. There has to be something to do with … the velocity — with the hand force I'm thinking — so I think — there is something to do with friction apparently —*

This excerpt shows that Granola is using pieces of his knowledge to try to trigger some procedure that he can use to solve the problem. In this small excerpt he brings up work, the force of the hand, the force of friction, the 1 meter, and the velocity. Unfortunately, these items lead to dead ends even though these ideas are all closely related to the work energy theorem and the definition of work, stating that $\Delta KE = W = \sum F \cdot d$. Granola is therefore unable to activate a schema containing knowledge about work and energy. Without links between these pieces of knowledge Granola cannot get to the work-energy theorem. Eventually Granola obtains the correct solution to the problem after some assistance from the interviewer.

Erica

Another student who had serious conceptual difficulties with the material was Erica. Like Granola, Erica drew a correct free-body diagram for the block but seemed to be confusing velocity with acceleration throughout the interview. One way to look at this type of error is in terms of diSessa's p-prims. P-prims are small logical building blocks that can be applied in many different situations.[3] The particular p-prim that seems to come up here is the maintaining agent p-prim, which states that a force is required to keep an object moving.[4] A section of the transcript follows.

I: How does this force [friction] *compare to that force* [hand]*?*

E: Well if it's still moving forward then this [points to force of hand] *is bigger then this* [points to friction force] *— it's not enough to stop it.*

This statement is particularly interesting because Erica states NII correctly during the interview. Erica also believes that once the block is stopped the net force has to be zero, despite the fact that she states NII correctly. The excerpt below shows that Erica is very unwilling to give this up, even though she has already written down the algebraic form of NII. Here we see that her qualitative responses and her quantitative responses are contradicting each other even though they are being presented very closely in time. Erica seems to have a conceptual form of NII (force implies motion), which directly contradicts the quantitative form of NII ($\sum F = ma$).

I: So what happens at point C?

E: It stops — zero — velocity equals zero — ...Oh — I see what your saying — but that is because the force is not acting anymore — the hand is not pushing anymore —

I: The hand acts all the way to point C.

E: Oh — so it just stops at point C — for no reason? — ...

I: The force of the hand is remaining constant from point A to point C

E: Then if there is no impediment there then friction is greater than the hand pushing it.

I: How did you know that? How did you know friction was greater?

E: How? Because the force of the hand is the same — so friction must have dominated that.

I: How did you know it dominated?

E: Because it came to a stop —

I: Before, you said this force [points to force of hand] *was greater than the force of friction.*

E: Cause it was moving that way … — The coefficient must have been — No — Not the coefficient — I don't know — if the force of the hand is the same then the friction must have been different.

Advanced students' reasoning maps

These interviews show that even advanced students exhibit fragmentation between related physics topics. Three of the six students had many contradictory remarks in their interviews. These contradictions were often hard for the students to resolve. This was partially due to a weak conceptual understanding of the material and not attaching conceptual meaning to the equations. These students were also unable to go back and forth between the ideas of force and work and energy.

We present *reasoning maps* to represent some of the interview data. Each main statement from the interview is presented in the map, with a link to the next statement and each statement is coded. Statements are shaded lightly (yellow in the color version) if they are based on ideas that come from dynamics knowledge, dark gray (blue in the color version) if they come from work and energy, speckled if it is unclear where they come from, and shaded darkly (red in the color version) with words written in italics if the statement was made by the interviewer.

The maps show that the three students who had difficulty with the problem had distinct force and work-energy schemas and had difficulty going between them. The maps of two of the three students who had difficulty with this problem are shown in Figure 5 - 2 and Figure 5 - 3. Figure 5 - 2 is a map of Granola's interview. We can see that he primarily uses the ideas of force and dynamics in the interview. Although he mentions statements about work and energy they lead to dead ends and he goes back to thinking about the forces and the motion of the block.[5] The map also shows that Granola makes many contradictory remarks in his interview. Some of these inconsistencies would not exist if the appropriate links were made between different knowledge elements.

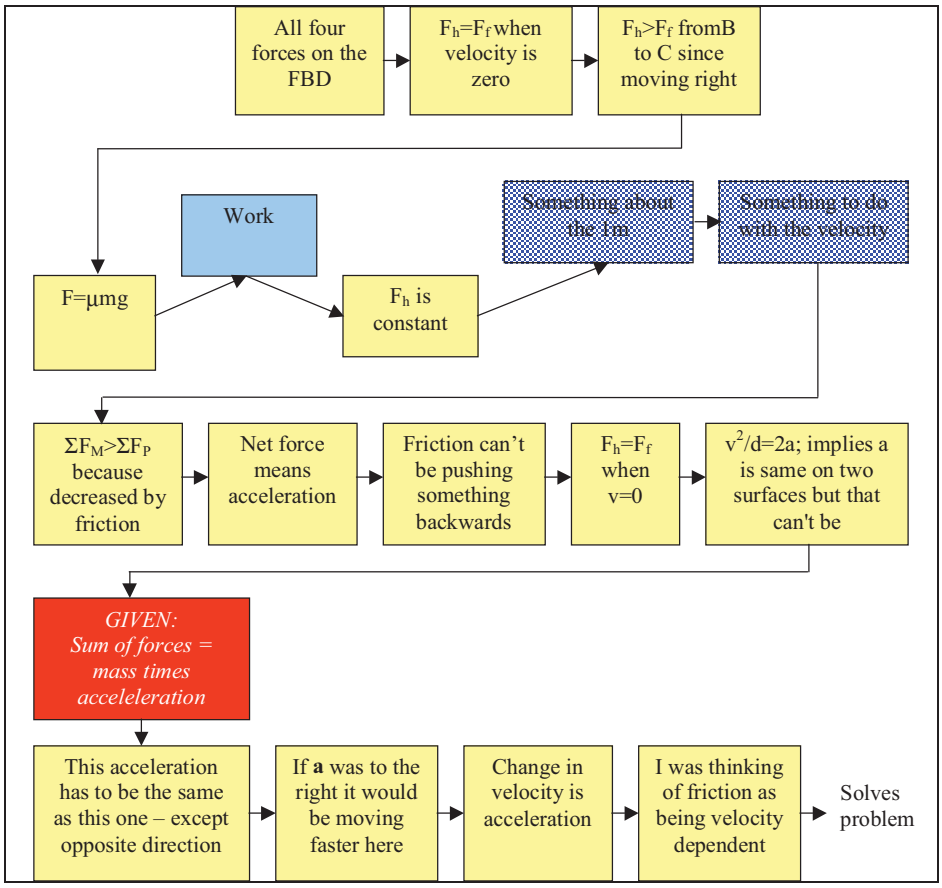


Figure 5 - 2

Reasoning map showing the main statements from Granola's interview. The map shows that although he tried to bring up the ideas of work and energy they lead to dead ends.

Erica's map is shown in Figure 5 - 3. It shows that she also tries to solve the problems by thinking about the forces involved in the situation through most of the interview. I asked her three questions relating to the work-energy theorem but she had difficulty tying these ideas into her analysis using forces.

In contrast, the other three students performed very well on the problem. These three students provide us with a picture of how experts might solve the problem. Connections between content knowledge are made more easily for these students. In addition the experts had more correct conceptual knowledge that they attached to the equations.

Two maps are shown in Figure 5 - 4 and Figure 5 - 5 for the students who solved the problem correctly. Even though both students did not have difficulty with the problem and each students' statements were consistent and correct, the two solutions were quite different. Eagle went back and forth between his knowledge of dynamics and his knowledge for work and energy. Peter primarily used the ideas that would be associated with a dynamics schema. When Peter needed some additional information he used a formula from kinematics. (This formula could also be interpreted as coming from the work-energy theorem but Peter did not explicitly make this connection.) We have shaded the statement lightly (yellow) because he uses the form of the equation usually introduced in dynamics. Peter's reasoning map shows that he has a coherent force schema but we cannot say anything about his work-energy schema or how well these topics are integrated. Eagle's map is shown in Figure 5 - 4 and Peter's map is shown in Figure 5 - 5.

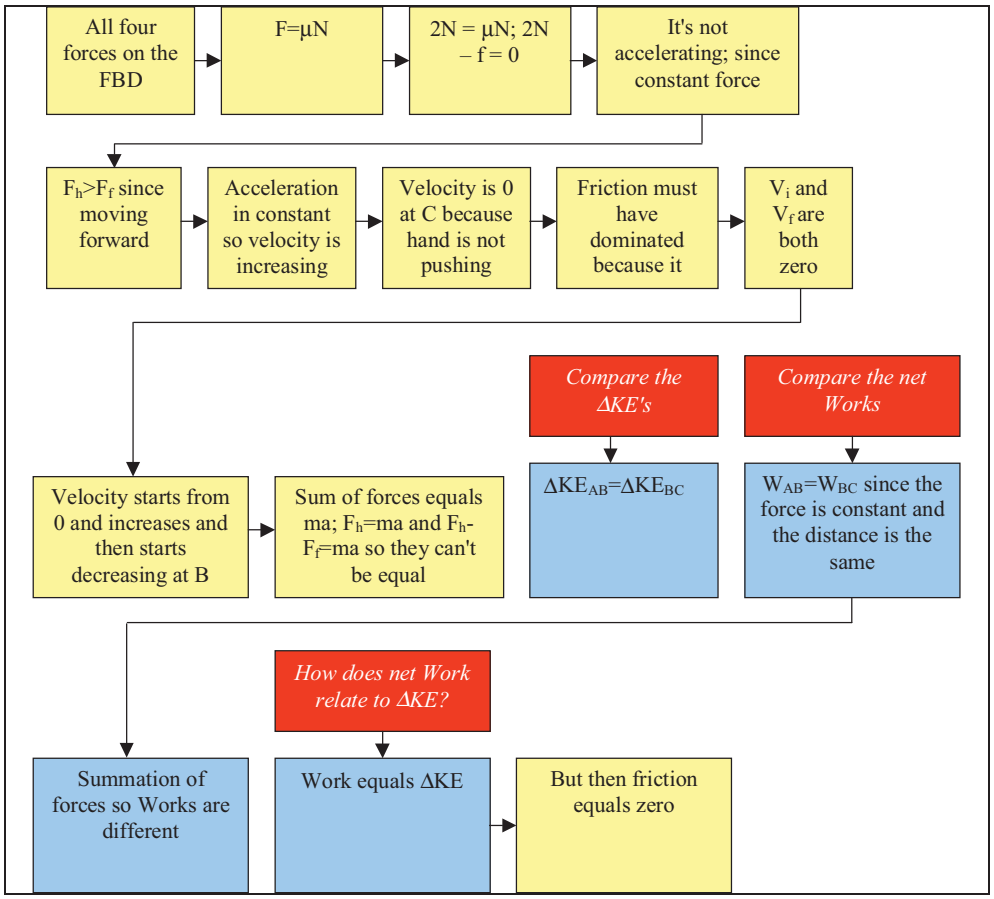


Figure 5 - 3

Erica's reasoning map showing that even with hints from the interviewer it was difficult for her to reconcile the dynamics information with the work and energy information.

The interviews and the reasoning maps show what we mean by a schema. They are strong patterns of association between particular knowledge elements in response to a given context. They show that for different individuals the same knowledge can be connected in different ways and for different individuals connections can be strong or weak.

In addition, we observed that Granola had a similar cognitive style (i. e. he could also be described as a lateral thinker.) In his interview he attempted to trigger other types of knowledge, from different physics topics; recall the excerpt where he brings up work, velocity, and the distance the block travels. In that section of the interview he is thinking laterally by attempting to activate knowledge from different topics. But because his schemas are isolated, these attempts lead to dead ends.

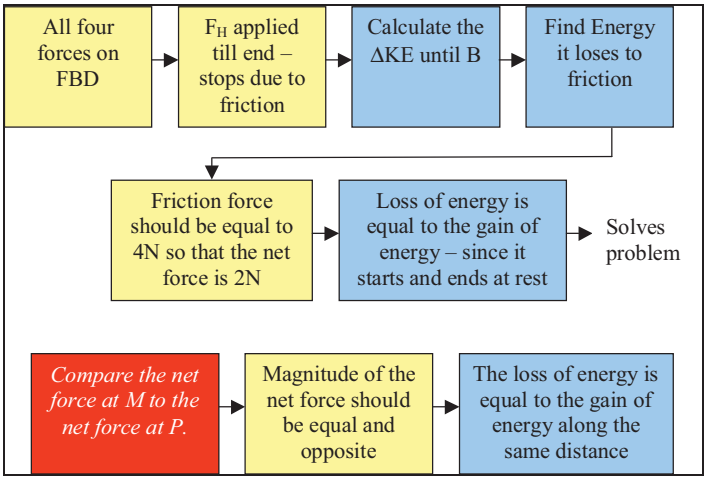


Figure 5 - 5

Eagle's reasoning map showing that he was able to go back and forth between the schemas for force and work and energy.

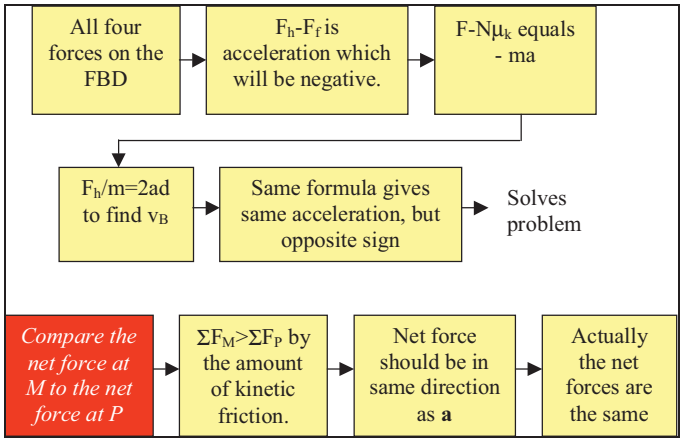


Figure 5 - 4

Peter's reasoning map showing that even though he used the knowledge from the dynamics schema almost exclusively he was able to solve the problem in very few steps.

Analysis of Ungraded Quiz

A short version and a long version of the question shown in Figure 5 - 1 was given to students in the physics 161 course without tutorials. Only the questions in boldface appeared on both the short and the long version of the problem.

Physics 161 was divided into three classes and each class was taught by a different instructor. The question was asked in the recitation sections of the class in the format of an ungraded quiz. The short version of the question was administered to 40 students and the long version was administered to 69 students. The short version of the question was asked in the two smaller classes and the long version was asked in the large class. Because the quiz was asked in the recitation sections, and not all students attend recitations, not all students in the class participated in the study. The quiz was allotted 15 minutes of the recitation.

The results on the ungraded quiz indicate that, although many instructors would like to believe that conceptual questions help the students trigger the correct answer, it can actually hurt their performance. These results are both surprising and disturbing. Many instructors believe that our students think, learn, and organize their knowledge the way we (physicists) do, except that they have less physics content knowledge.[6] This leads some instructors to a number of assumptions as they teach their course. They often ignore student epistemologies and ignore the way students organize their knowledge. We, as instructors, often assume that if we can get students to learn a set of items, they will organize their knowledge of this set of items the same way we have them organized. In reality our students often do not make the same links between different items that we do. Cues that help us activate a set of different interrelated schemas do not necessarily help our students activate those schemas. In addition, some of these cues, or triggers, can cause students to activate a particular schema that may be isolated from the relevant schema for a given task.

We will first discuss the student responses on the long version of the question and then show the results of a comparison on the responses to the last part of the question, which was answered by students on both versions of the problem.

Part a of the question asked students to draw a free-body diagram for the block when it was on the surface with friction. Almost all students correctly identified all four forces on the block. In most cases it was difficult to check the relative magnitudes of the forces, and in practice students are usually not expected to make their forces consistent with one another unless the forces are equal.

On part b of the problem, only 12% of the students answered correctly. The results are shown in Table 5 - 1. The most common error, given by 56% of the students, was that the magnitude of the net force on the non-friction surface was greater than the magnitude of the net force on the friction surface. We can explain this error in terms of diSessa's Ohm's P-prim.[7] The Ohm's primitive comes from the compensating type of reasoning that is associated with Ohm's Law.[8] A part of the Ohm's primitive states that an "increased resistance leads to less result."[9] Because the block first travels over a non-friction surface and then over a surface with friction the resistance on the block increases thereby decreasing the result, which in this case can be interpreted as the net force. This type of response is common with both the undergraduate engineering students and the advanced physics students.

N=69

Compare magnitude of net force at **M** to **P**	Correct: The net force is equal	Incorrect: F_{net} greater on non-friction surface	Incorrect: F_{net} greater on friction surface
	12% ± 4%	56% ± 6%	26% ± 5%

Table 5 - 1

Performance on the question asking students to compare the magnitudes of the net forces in the two regions. This question proved to be extremely difficult for the students.

There were also a large number of students stating that the net force on the frictionless surface would be less. One way to explain this result is that students were not considering the vector nature of the forces and were just thinking of the net force as the number of forces acting on the block. This is evident from some of the students' responses. Two examples of student responses showing each type of incorrect response are shown in Figure 5 - 6. They have been typed from the student papers.

In part c the students are asked two conceptual questions about the acceleration vector on the surface with friction. The results, shown in Table 5 - 2, indicate that 41% of the students answered correctly, that the direction of the acceleration vector was to the left. Only 32% of the students stated that the magnitude of the acceleration vector does not change as the block moves from point B to point C, where it comes to rest. These results are consistent with previous work indicating that students often treat acceleration as if it were proportional to velocity.[10] There were also a significant percentage of students stating that the acceleration of the block at point P was zero.

Two sample student responses	Case 1: "It is greater at M because there is no frictional force working against the F_{hand}."	Case 2: "[It is] less than [at M] because friction is being invoked at point P in addition to the 2N."

Figure 5 - 6

Sample student responses comparing the net force on the friction surface to the non-friction surface.

The results on the qualitative questions indicate that students still have many conceptual difficulties with NII, even though instruction on Newton's laws were completed a few weeks before this study was conducted. We are now in a position to examine the student responses on the final part of the problem, where students are asked to calculate the coefficient of kinetic friction.

Figure 5 - 7 shows the results on the quantitative aspect of this problem for the students taking both the long and the short version. It shows the percentage of correct responses as well as the percentage of students setting the net force on the block equal to zero on the region with friction. We should also note that only students who had enough time to attempt the final part were included in the analysis. Therefore, all the percentages listed include only the students who answered the final question. Even though students had covered this material in lecture and had homework assignments on the material fewer than 30% of the students answered this question correctly.

Student performance was significantly different on the two versions of the problem. The results show that students performed better on the short version of the problem.

N=69

Acceleration vector at point **P**	Correct: Vector directed to the left	Incorrect: Zero	Incorrect: Vector in the direction of motion
	41% ± 6%	20% ± 5%	19% ± 5%

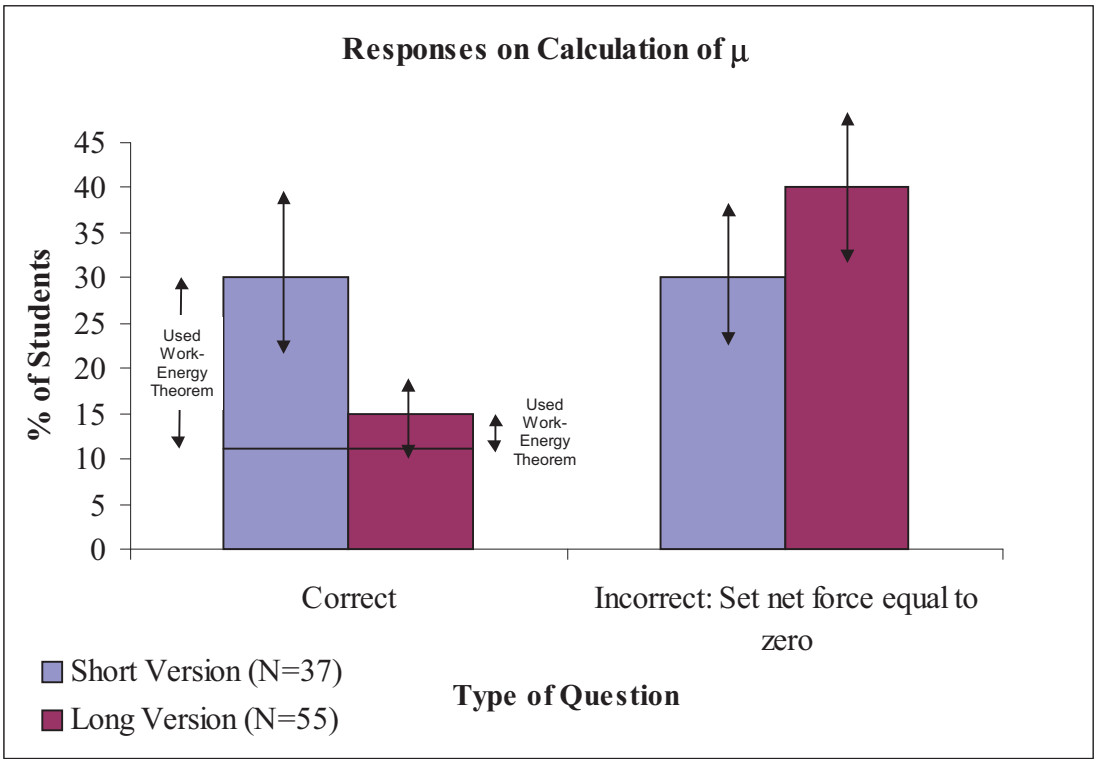
How does the vector change?	Correct: Acceleration is constant	Incorrect: Acceleration is decreasing
	32% ± 6%	55% ± 6%

Table 5 - 2

Performance on the questions concerning the acceleration vector of the block on the surface with friction.

We would also like to examine the methods of solution used by the students who answered correctly. We can see from Figure 5 - 7 that more students used the ideas of work-energy to solve the question on the short version of the problem. Most of the students who did not solve the problem using work-energy used the ideas from kinematics to try to solve for the acceleration of the block from point B to point C.

Larkin talks specifically about schemas for force and work-energy, but does not talk about how these schemas are linked.[11] If the conceptual questions lead students into a dynamics or force schema and that schema is not linked to the work or work and energy schema, the students may try to solve the problem using only the force schema. Previous research has also shown that novice problem solvers tend to focus on the surface features of a problem.[12] Since all the conceptual questions focus on force it is possible that our students responded by triggering a dynamics schema. This was actually the intent of providing the conceptual questions. But, we hoped that our students would use the dynamics schema to activate the work-energy schema. Based on our results this is not the case. In contrast, experts will tend to bring a larger



Data does not include students that did not complete the problem.
Unfinished: 8% on short version, 20% on long version

Figure 5 - 7

Performance on the quantitative part of the problem showing that students performed better on the short version. The graph also indicates that the methods of solution on the long and short version are different.

set of knowledge that is integrated to the problem-solving task. Experts will also be more able to activate additional schema if needed.

In addition to schemas for organizing specific physics content, the data also suggests that our students often have isolated schemas for their qualitative knowledge and their quantitative knowledge within a given physics topic. The qualitative and quantitative questions provide information on the coherence between qualitative and quantitative knowledge. There are two main types of errors students can make. Students may be able to solve for the coefficient of kinetic friction correctly yet have serious conceptual errors, or they may have qualitative ideas that they do not apply when answering the final quantitative question.

Table 5 - 3 shows examples of student inconsistencies on this problem. Even though some of the students could solve the quantitative question correctly many of those answering correctly had serious conceptual difficulties. Answering the quantitative question correctly requires that the student use the fact that the

magnitudes of the accelerations are equal in the two regions. (Note that only 8 students answered part d correctly on the long version.) We observe that five of the eight students, who answered the final part correctly, stated incorrectly that the magnitude of the net force was different in the two regions. In addition, three of the students who solved for the coefficient of friction correctly stated that the acceleration vector was decreasing from point B to point C. Perhaps an even more surprising result is that seven of the students who drew a non-zero acceleration vector in part c set the net force equal to zero when solving the quantitative question.

Of the students who could calculate μ correctly many made serious conceptual errors.	Incorrect: Stated that the magnitude of the net force in the two regions were different on part B	Incorrect: Stated that the acceleration vector decreases as the block moves from B to C	
	5 students	3 students	**N=8**

Of the students who had a non-zero acceleration vector some answered inconsistently on the quantitative part.	Incorrect: Stated that the net force was equal to zero when solving for μ.	
	7 students	**N=41**

Table 5 - 3

Inconsistencies in the student responses on the hand-block problem.

In Figure 5 - 8 we represent the possible paths of solution to the two versions with *solution maps*. The solution maps indicate strong paths and weak paths by the thickness of the lines that link the different responses. This allows us to map the number of students going from one response to the next response and allows us to see what types of responses to a certain part of the question tend to trigger certain responses on the next part. To make the maps easier to read, links that are sufficiently weak are excluded. It is important to note that the solution maps show the paths of solution for the entire class; they do not tell us about individual students. Therefore we cannot say that most of the students followed the thick lines to the final part of the question. Among other things, the maps do show that the conceptual questions seem to scatter the responses away from a correct analysis using the work-energy theorem.

The data from the ungraded quiz indicates that, for our students, physics content knowledge may be organized by schemas that are only weakly linked. If our students were developing coherence in their content knowledge, qualitative questions would tend to help in solving these problems, instead of hurt. In the problem discussed in this chapter, qualitative questions that lead students to a force schema tended to isolate them from other pieces of knowledge that could have been helpful in

solving the problem. We have also begun to see that many of our students form isolated schema for their qualitative and quantitative knowledge.

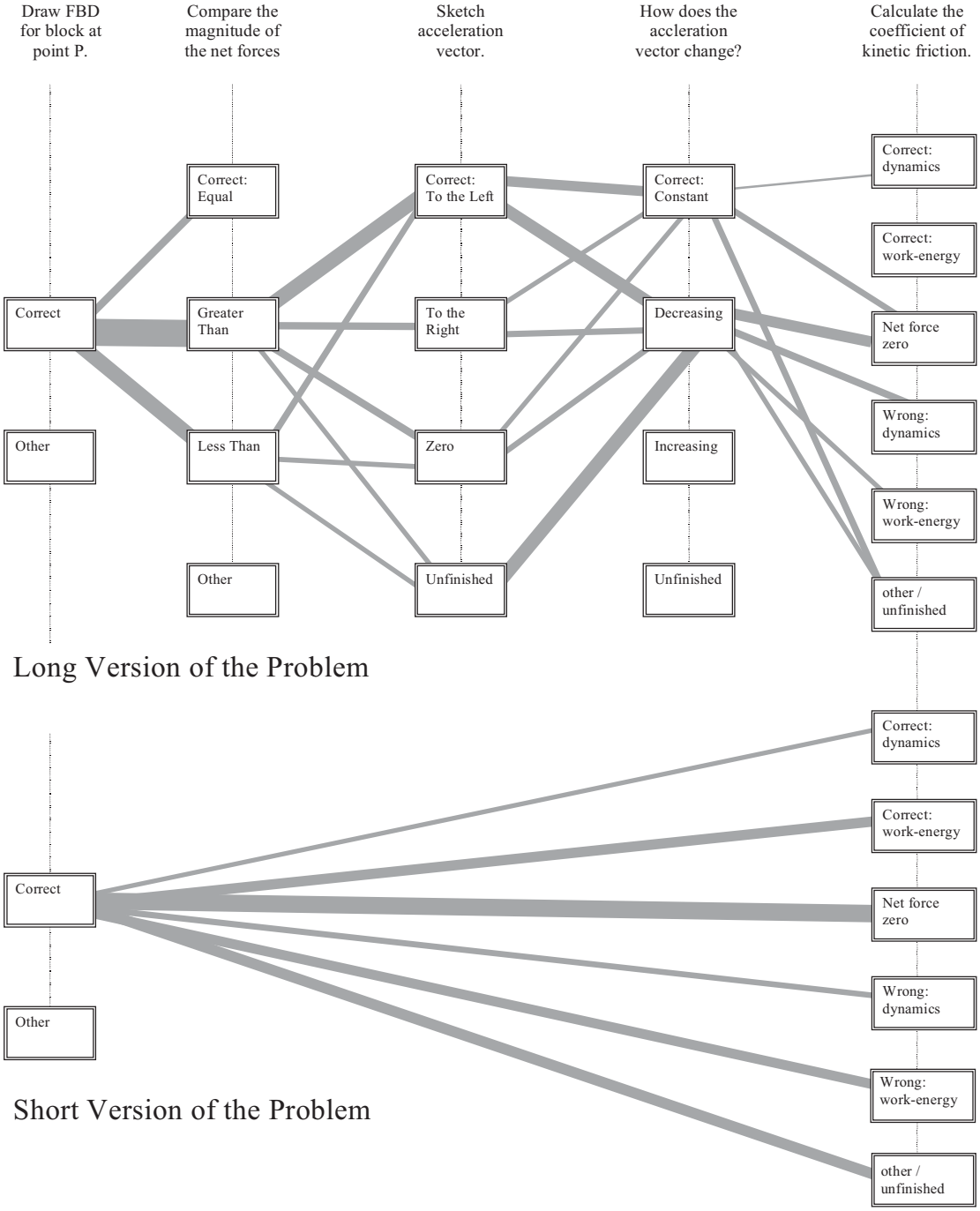


Figure 5 - 8

Paths of solutions on the long version and the short version of the problem. The intermediate qualitative questions seem to lead students away from using the ideas of work and energy.

Bridging Problem Analysis

The results presented so far in this chapter have described isolated student schemas for different physics topics. The data also indicates that many of our students have isolated schema for their qualitative and their quantitative knowledge in Newtonian dynamics. We have observed that many of our students answer inconsistently on the qualitative and quantitative parts of a single question. The following results concentrate on the qualitative and the quantitative schemas our students use in answering a similar question involving dynamics and work and energy.

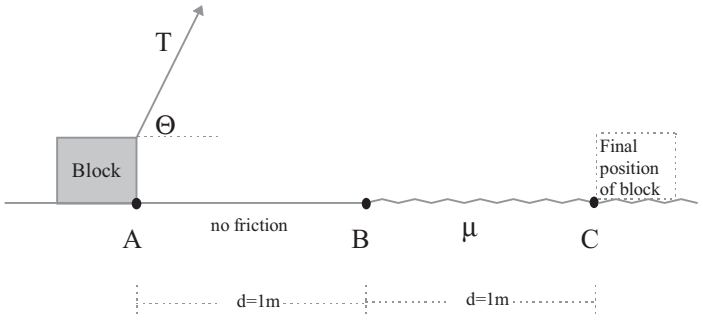
An older version of the problem shown in Figure 5 - 1 was asked as a bridging problem[13] in Spring '97. We observe that in their solutions many of the students directly contradicted one of the statements in the exposition of the problem. This result prompted us to perform more studies in order to understand whether students were making a careless error or a more profound error.

Bridging problems attempt to help students incorporate the qualitative knowledge they were developing in tutorial with the quantitative problems they were solving on exams and textbook homework assignments. The bridging problem, asked in the physics 161 class at the University of Maryland with tutorials, is shown in Figure 5 - 9 with a model solution. The students are asked two qualitative questions and then a final quantitative question about the coefficient of kinetic friction. They are first asked to draw a free-body diagram and then asked to compare the magnitudes of the net forces in the two regions. The question about the net forces acting on the two blocks was intended to help the students make the connection between work, forces, and changes in kinetic energy.

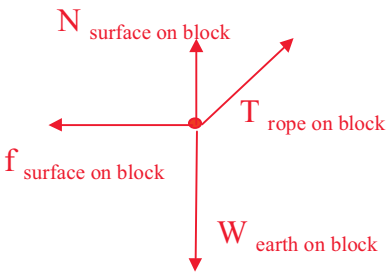
We will concentrate on the responses to part (c) of this question. The most common error involved the students setting the net force equal to zero on the friction surface in order to calculate the coefficient of friction. This response seemed odd since the problem explicitly states that the block slows down from point B to point C. Despite this statement many students set the net force equal to zero implying that the acceleration of the block in the region from B to C was zero.

The results, along with a sample student response, are shown in Figure 5 - 10. The figure shows that only about 35% of the students answered this question correctly after lecture and tutorial instruction on the work-energy theorem. The response shown in Figure 5 - 10 is an incorrect response in which the student sets the sum of the forces in the x direction equal to zero. Unfortunately the student does not explain his or her reasoning so it is difficult to extract much more information from this response. One of the difficulties with quantitative questions is that it is often hard to probe deeply into the students reasoning based on a collection of formulas. Open-ended conceptual questions are sometimes more helpful but can be limited in the same way.

A block is pulled from point A to point C by a tension force, T, as shown. The block starts off at rest at point A and speeds up to point B. The block then slows down finally stopping at point C (as shown). The magnitude and direction of the tension are constant throughout the motion. Note that the block is first pulled across a surface with no friction and is then pulled an equal distance across a surface with friction.



a) Draw a free-body diagram for the block when it's between B and C.



b) Is the magnitude of the net force acting on the block from A to B greater than, less than, or equal to the magnitude of the net force acting on the block from B to C? Explain your reasoning.

Since the change in kinetic energy is equal the magnitudes of the net works are equal therefore the magnitudes of the net forces are equal.

c) Calculate the coefficient of kinetic friction between the surface and the block between B and C if $\Theta=60^{o}$, $m_{block}=1.5kg$, and T=5N.

From (b) we know that $T\cos\theta - f = -T\cos\theta$ and $f = \mu\,(mg - T\sin\theta)$ therefore $\mu=.41$

d) Calculate the work done by friction to move the block from A to C. Show all work.

Since $f = 2T\cos\theta = 5N$ the work is $W = f\,d = 5\,J$

Figure 5 - 9

Bridging problem asked as part of the tutorial homework assignment and as a one-on-one interview with undergraduate students.

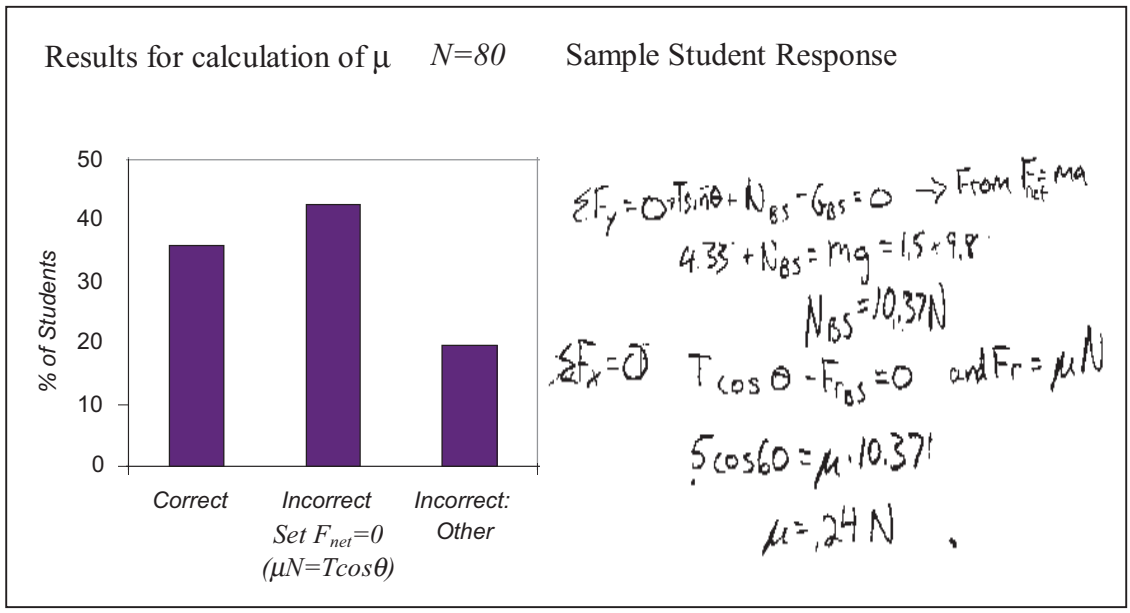


Figure 5 - 10

Performance on the quantitative part of the bridging problem and a sample student response. The sample shows the most common error that students made on this problem.

Interview Analysis

We followed the bridging problem analysis with one-on-one interviews, allowing us to further probe student understanding. Results of the interview analysis showed that the most common mistake, of setting the net force equal to zero, could be explained by the students not attaching the correct qualitative knowledge to the quantitative statement of NII. In particular, one student (out of the six interviewed) applied NII over a time interval rather than at an instant. Although these ideas are not consistent with NII they do show that our students are capable of rather sophisticated reasoning. Without using interviews as a research tool it is difficult to show the types of sophisticated reasoning that we, as instructors, can build upon to help students achieve the correct understanding.

We conducted one-on-one problem-solving interviews with six students from the Physics 161 class. The six students were chosen at random from a pool of eighteen volunteers.[14] Most students in the class were given the opportunity to volunteer for the interviews.

Interviews were conducted with students who solved the bridging problem discussed earlier in the chapter. Because of this, all students being interviewed were seeing the problem for the second time. Complete transcripts for the interviews presented in this section are found in Appendix D.

Pink

We will first look at Pink's[15] response in answering the final part of the problem about the coefficient of kinetic friction. On the bridging problem homework assignment Pink made the error of setting the net force equal to zero on the friction surface.

In this interview, Pink began the final part of the question by examining the forces on the block. Eventually she set the net force in the *x* direction equal to *ma*, unlike what she did on the homework assignment. She did however state, in the interview, that the acceleration is zero when the block reaches point C.[16] As Pink starts to get stuck when trying to calculate the acceleration, I suggested that she think about the work-energy theorem, which states that the net work equals the change in kinetic energy. The following is an excerpt of Pink's interview with some commentary.

> *P: Force times the distance is 1/2 m v squared.* [pause] *... force equals mass times acceleration times distance which is the work and that equals 1/2 times the mass times the final velocity squared. — so between A and B the final velocity is* —[pause]
>
> *I: How does the change in kinetic energy from A to B compare to the change in kinetic energy from B to C?*
>
> *P: ... The change in kinetic energy from A to B is going to be 1/2 times the mass times the velocity and from B to C — its going to be — well you're staring out with a velocity so — They will be the same because you are starting from rest and you get to a final v and that is the initial velocity from here to here and you're stopping. So the velocity is the same and the mass is the same so the change in kinetic energy is the same.*

At this point Pink seemed to be having difficulty applying the work-energy theorem to this situation. The reason that the hint to use the change in kinetic energy was given was to provide a cue that might help her make the connection, and activate a schema for work and energy. Although she was able to correctly identify the changes in kinetic energy the work-energy theorem was not brought up, which indicates a very weak link between these two topics. At a later stage in the interview Pink begins to focus on the acceleration of the block.

> *P: So the acceleration is zero.*
>
> *I: So how do you get that?*
>
> *P: Because it is not acceleration ... because it is not accelerating because the velocity is not changing.*
>
> *I: What do you mean the velocity is not —*

P: Well, it is starting and ending at the same velocity so the change in the velocity is zero. And the acceleration is change in velocity over change in time so its zero.

I: So you said acceleration is zero because it starts off at rest and it ends at rest ... so the change in velocity from A to ...

P: A to C is zero —

[Pink then proceeds to solve the problem, setting the sum of forces equal to zero. She therefore has $T\cos\theta - \mu_k n = 0$ and gets $\mu_k = .24$.]

In this section Pink states that the acceleration of the block is zero because it starts at rest and ends at rest. One explanation for this type of response is that Pink is not breaking up the problem into local and global parts.[17] She correctly identifies the average acceleration from the beginning to the end of the motion but then uses NII, which is valid at a single instant in time. The time issue associated with equations is an interesting subject that warrants more research.[18] A more general way to classify this error is to notice that Pink does not seem to be attaching appropriate conceptual meaning to the equation for NII. This is an error we often see when students solve quantitative questions. To get a better idea of Pink's misinterpretation of NII, I ask her about the equation she has just written.

I: This equation here — this F cosine 60 plus the force of friction equals ma — when does this equation apply?

P: Between B and C.

I: Between B and C— so between B and C —

P: And between A and B it is just T cosine theta.

I: T cosine equals —

P: Mass times acceleration.

I: ... Can you write the sum of the forces in the x direction between A and B?

P: ... It's just T cosine theta cause that is the only force in the x direction.

I: And that's equal to ma?

P: Yeah —

I: And the acceleration?

P: Well, it's going to be ... cause it starts from rest and its got a final velocity at B so its just change in velocity over change in time.

I: So it is accelerating? —

P: Between A and B — and its decelerating between B and C.

I: ... so when you wrote this equation here [points to $T\cos\theta - F_r = ma$] — *this is for — you said this is for the part of the motion between B and C?*

P: Yeah.

Here Pink sees a contradiction in her reasoning after I intervene. Pink states explicitly that the equation she wrote down applies in the region from B to C and she also stated the correct definition of acceleration and that the block is decelerating from B to C. Earlier, she wrote down that the net force (from B to C) is equal to zero. To justify it she stated that the acceleration from A to C is zero. These statements can be attributed in part to a lack of coherence in her schema for dynamics. Some of her statements directly contradict each other. By the end of the previous exchange she starts to believe that the net force from B to C is non-zero. Different pieces of knowledge get brought out depending on the cues presented. In the section above, the questions I present cue a different set of knowledge. In the next section Pink resolves the discrepancy.

P: So if you have ... I guess if you add T cosine theta then it would be for the whole thing

I: Say that again.

P: If you add T cosine theta again that would add all the forces in the x direction from here to that [points to A and C]. *So this would be* [writes a 2 in front of the tension force — she has therefore changed $T\cos\theta - \mu_k n = ma$ into $2T\cos\theta - \mu_k n = ma$]

I: So you would get 2 T ... so what is this equation again?

P: If we add T cosine theta — that's the forces between A and B plus the forces between B and C it would be the whole thing.

I: So what do you mean the whole thing?

P: I mean the sum of the forces ... from A to C.

I: The sum of the forces from A through C is this guy [points to $2T\cos\theta - \mu_k = ma$.]

P: Right.

I: Now why do you say that?

P: Its just the sum of the force from A to B plus the forces from B to C.

I: So when you say this [points to $2T\cos\theta - \mu_k = ma$] *what's the acceleration?*

P: Well, there it is zero.

I: There its zero because ...

P: Because the change in velocity over the change in time is zero —

I: ... So, is it zero here? [Points to old sum of forces from B to C which is $T\cos\theta - F_r = ma$.]

P: Umm — That's between B and C — well it's decelerating so it can't be zero.

In this section Pink resolves her dilemma but instead of switching from global (or average acceleration) to the instantaneous acceleration which would be correct, she reinterprets NII as a global quantity. Pink applies NII across time summing the forces throughout the entire motion. She is therefore exhibiting some profound conceptual difficulties with Newton's Laws.

These difficulties seem to come from Pink not being able to see the role of time in the equation for NII. Redish discusses Newton's 0th (N0) Law in his Millikan paper. N0 states that "at a time t, an object responds only to forces that are exerted on itself at time t."[19] This is not trivial for our students. Each equation a student uses has information about time that is not explicit in the equation. For instance, when using NII the quantities on either side of the equal sign are evaluated at the same instant. In contrast, when using conservation laws, such as conservation of momentum the quantities on either side of the equal sign are evaluated before and after a collision. Bruce Sherin discusses ways to put extra information into equations to help students attach more conceptual information to equations.[20] Attaching time information to different formulas may help students make this connection. The time issue is an interesting area of research. There are very few studies examining this difficulty.

It is also interesting that Pink's last statement would yield the correct answer to the problem. Because the accelerations are equal and opposite on the frictionless surface and on the surface with friction, summing the forces on each region and adding the two NII equations would give you zero. In the next chapter we present results showing that students can get the correct answer to a quantitative problem even though they may have serious conceptual difficulties. Again, we see why physics education researchers must support analysis on written responses with the analysis of interviews.

It is also important to note that Pink is using some very sophisticated reasoning to solve the problem and resolve her inconsistency. She constructs her own theoretical interpretation for the algebraic equation of NII and this interpretation does not come from a book or an instructor. Even though it is incorrect it allows her statements to be consistent with each other. The interview allows us to observe the expert-like reasoning skills that our students may possess, such as Pink's interpretation of NII. It is easy to view wrong answers (and correct answers) in a simplified way when we are simply presented with written responses to homework and exam questions because we are unable to see the types of reasoning our students are capable of. Interviews can provide us with information on the resources students have, which we can build upon, to help students come to a correct solution to the problem. Although Pink's time averaged use of NII is a sophisticated piece of reasoning and it allows Pink to obtain the correct answer for the coefficient of kinetic friction it may lead to more difficulties in the future.

Michelle

In another interview we saw the same mathematical error (i.e. solving for μ by setting the force of the hand equal to the force of friction), although in this interview it is done for a different reason. This excerpt comes from an interview with Michelle, who was one of the top students in the class. In this excerpt Michelle begins solving for the coefficient of kinetic friction.

> *M: Friction — I usually start with any definition or formula I can think of — so that is μ N and in this case it is not moving off the table or surface so you are going to have the normal plus the Tension times sine theta will equal m of the block times gravity. And we can solve for N so we can put in this equation* [points to μ N = F_r] *— They give us mass of block ...1.5 times 9.8 meters per second squared minus the tension which is 5 N times sine sixty degrees. ... I'd get a number for this and plug that in here* [points to μ N = F_r] *— Friction — because it is going to come to a stop at point C you are going to take friction at rest will be equal to tension times cosine of theta. ... Because at the end it is going to be at rest so it is not going to be moving in the horizontal direction — I took that at point C.*
>
> *I: So you are saying that at point C the force of friction has to be equal to that* [points to T cos 60] *because it is at rest?*
>
> *M: So they are equal — 5 N times cosine sixty — that will give me another number which can go into there* [points to μN = F_r] *and I can solve for μ*
>
> *I: ... So at point C you said it was at rest so these two forces are equal?*
>
> *M: Yes — I said it's got a velocity coming this way and the friction is eventually going to slow it to a stop and the μ isn't changing at all so —*

I: So the acceleration at point C is what?

M: The acceleration at point C is zero because it is not moving.

Michelle is making a different type of error. She is summing the forces in the region from B to C but only using point C to evaluate the acceleration of the block. This error is similar since we again see that the quantity on the left side of the equation is being evaluated at a different time than the quantity on the right side of the equation in NII. Again the time issue seems to be a factor in the way the students respond to the question. Later in the interview, after some intervention, Michelle resolves some of these issues.

These interviews show why it is important to probe student understanding in a number of different ways. The interview provides a detailed look into students' reasoning. It is impossible to say that each student who made the error of setting the horizontal component of tension equal to the force of friction went through the same type of reasoning Pink and Michelle went through. But these results do give us an idea of some of the difficulties we must look out for. They also tell us about some of the resources students have that we can build upon.

One possible explanation for the incorrect response is a lack of conceptual information attached to the quantitative form of NII. In order for our students to apply equations correctly at appropriate times it is necessary that we attach conceptual meaning to the equations they use. As stated earlier, when our students view an equation, they often observe it as simply a tool to obtain an unknown variable. This seemed to be the case with Michelle and Pink. Although they had strong reasoning skills and often had the correct concepts, they did not attach the concepts to the equations and they did not check that there was consistency between the concepts and the equations.

Thermodynamics Question

Introduction

The question discussed in this section was posed as a pretest for the physics 262 course with tutorials at UMd. The pretest preceded a problem-solving tutorial that was developed by the UMd PERG.[21] Problem-solving tutorials are given after the students hear lectures on the material and after they go through tutorials based on the subject. The students taking this pretest have therefore had lecture and tutorial instruction on the material.

Pretest Analysis

Two versions of the thermodynamics problem were given to the students. Figure 5 - 11 shows the problem with a model solution. Each version was asked in a different lecture class with a different instructor. The first version had many qualitative parts before a final quantitative part, while the second version had only the final quantitative part. The problem involved a piston that undergoes an isothermal expansion and then an isovolumetric process. Only the bold faced question appeared

on both the short version and the long version of the question. This problem probes a schema for work. Some nodes that an individual may activate in a work schema are:

- the work is equal to the force dotted with the displacement,
- the work is the integral of Pdv,
- the work is equal to the area under a PV diagram, etc.

A thermodynamics problem involving a piston would most likely activate the last two nodes in order to solve the problem.

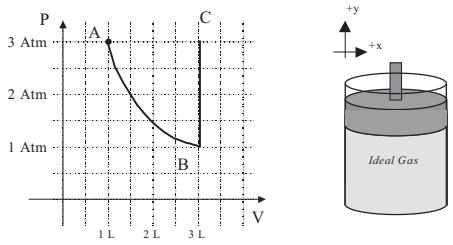
The most interesting data comes from looking at how the students answer the final question where they are asked to calculate the work done by the gas in the process A⇒B⇒C. First we will concern ourselves with looking at the general categories of correct and incorrect. Table 5 - 4 shows the results on the quantitative part for the two classes. (Only the students who answered the final part are included in the analysis.)

The data shows that the students performed better on the short version where they are simply asked to calculate the work done in the process. This result was surprising because one would think that the conceptual questions on the long version would help the students solve the problem. In particular, questions a and b, where the students are asked to draw a PV diagram, should help them calculate the work done by the piston.

In order to determine why the performance on the short version is better, we break down the responses into more specific categories. Table 5 - 5 shows that on the long version of the question a large percentage of the students incorrectly calculated the area under a triangle and a square. Most of these students (16 out of 18) drew the isotherm as a straight line. The students who drew the PV diagram have therefore cued into the node that work is the area under the curve, whereas the students who were given the short version cued into the node that $\int Pdv = Work$. Because of a lack of conceptual understanding in the isothermal processes the piston goes through (only 61 out of 114 graphed the isotherm correctly), the students on the long version actually perform worse. We expect the students taking the short version of the pretest to have the same conceptual difficulties with graphing the isothermal process we observed with the students on the long version.[22] Sketching the PV curve seems to have triggered a geometrical solution for the problem, while the students who were not given the qualitative questions solved the problem using calculus. The two versions of the questions have therefore cued different schemas with different procedural rules.

An expert problem solver would tend to relate the geometrical and calculus based knowledge so that performance on the long and short versions would be similar. When experts attempt to solve problems, they will tend to draw upon both their qualitative and their quantitative knowledge in most situations. Occasionally experts are presented with problems that they may have solved many times, in which case they may not need to use both their qualitative and quantitative knowledge. For the most part this will only happen when the expert is reasonably sure of the answer. Any doubt will usually cause experts to check for consistency between their qualitative and quantitative knowledge.

An ideal gas in a container with a piston starts at a pressure of 3 Atm and a volume of 1 L. The gas first goes through an isothermal process ending up with a pressure of 1 Atm and a volume of 3 L. The gas then undergoes a process at constant volume ending up with a final pressure of 3 Atm.



a) Sketch the isothermal process on the PV diagram and label the resulting state "B."

b) Sketch the constant volume process on the PV diagram and label the final state "C."

c) What is the direction of the force exerted on the piston by the gas in the container in the process A⇒B? Is the work done by the gas positive, negative, or zero? Explain.

The direction of the force from the gas on the piston is in the +y direction. Since the volume of the gas is increasing the displacement of the piston is in the +y direction. The work done be the gas is positive since the force and the displacement are in the same direction.

d) What is the direction of the force exerted on the piston by the gas in the container in the process B⇒C? Is the work done by the gas positive, negative, or zero? Explain.

Again the force of the gas on the piston is in the +y direction. Since the displacement of the piston is zero the work done by the gas on the piston is also zero.

e) Calculate the work done by the gas in the process A⇒B⇒C. Show all work.

The work done by the gas is equal to the area under the isotherm. With the temperature remaining constant we have

$$\int_A^B PdV = \int_A^B nRT/V\, dV = 3.3Atm \cdot L$$

Figure 5 - 11

Two versions of a thermodynamics question asked as a pretest with a model solution. The first version contained all five parts and the second version consisted of only the last part.

Calculate the work done.	Correct: $\int Pdv = Work$	Correct: Area under the curve (approx)	Incorrect	
Long version	20% ± 5%	7% ± 3%	73% ± 5%	**N=74**
Short version	44% ± 7%	0%	56% ± 7%	**N=55**

Table 5 - 4

Performance on the final part of the question.

Calculate the work done.	Incorrect: area of Δ + □	Incorrect: area of Δ	Incorrect: PΔV, ΔPΔV, ΔPV	Incorrect: other	
Long version	24% ± 5%	5% ± 3%	23% ± 5%	21% ± 5%	**N=74**
Short version	2% ± 2%	11% ± 4%	33% ± 6%	10% ± 4%	**N=55**

Table 5 - 5

A more detailed description of how students answered the final part of the problem.

Summary

This weak link between qualitative knowledge and quantitative knowledge can actually cause our students to perform worse when they are presented with qualitative questions before the final quantitative part. This may be due to students activating a particular schema because of the qualitative questions and then getting trapped in that particular schema. If the schema that is activated does not contain all the information needed for the problem, the student will not be able to solve the problem. Evidence from the hand-block ungraded quiz and the thermodynamics pretest supports these conclusions. In the hand-block problem we observed that some students tended to get cued into a dynamics schema where they could not make the link to the work and energy schema. In the thermodynamics problem we saw that students activated a schema for work which involved the area under the *PV* curve and could not make the link to the integral of *PdV* when conceptual questions were presented.

This chapter provides researchers and instructors with methods that can be used to evaluate student coherence. The methods we used involved problem-solving interviews, and open-ended questions. By asking questions with parts that are qualitative and parts that are quantitative we were able to identify contradictions and

inconsistencies in student knowledge and reasoning. We are also able to probe for coherence between the schemas students have for different physics topics.

We have introduced two new representations that can help instructors and researchers look for coherence: reasoning maps and solution maps. We can also see how students are using concepts in answering the quantitative questions by examining performance on different versions of a problem. We believe that forming coherence and evaluating coherence should be an explicit goal of the physics course.

The responses students gave on the qualitative and the quantitative parts of a problem also indicate that students have isolated schemas for qualitative and quantitative knowledge. The conceptual responses on the hand-block problem were not always consistent with the quantitative expression for NII. On the thermodynamics problem, the quantitative expression for the work done by the piston involving the integral of PdV was used less often for students who answered qualitative questions first. In the following chapters we provide more detail on these issues.

[1] Problem-solving interviews are discussed in chapter 3.
[2] Another option for a problem-solving interview would involve two or more students. Bruce Sherin discusses the advantages of using two students. One particularly relevant benefit is that when two students are present they must talk to each other and explain what they are doing. In contrast, the one-on-one interview is a more artificial setting for the student. The researcher often has to remind the student to explain their thoughts and what they are writing out loud. Because we do not want any external cues or aids for the student, we decided to use the one-on-one interview setting.
[3] For more information see chapter 2 pages 10-12 and A. A. diSessa, "Knowledge in Pieces," In *Constructivism in the Computer Age,* G. Forman and P. Pufall (Eds.) (Lawrence Erlbaum, NJ, 1988), pp. 1-24; and D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).
[4] Maintaining agent is a term used by Hammer to describe diSessa's continuous force p-prim. See D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).
[5] We believe that the graduate students who could not use work-energy in this problem could have done a straightforward work-energy problem.
[6] E.F. Redish, "Millikan Award Lecture (1998): Building a Science of Teaching Physics," Am. J. Phys. **67** (7), 562-573 (1999).
[7] For more information on P-prims see Ref. 3.
[8] P-prims were discussed in chapter 2 on pages 10-12.
[9] See Ref. 3.
[10] D. E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," Am. J. Phys. **49** (3), 242-253 (1981).
[11] J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.
[12] M.T.H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).
[13] Bridging problems are discussed in detail in chapter 3.
[14] Problem-solving interviews are discussed in detail in chapter 3.
[15] Students use code names they choose in the interviews.
[16] Pink may be making the common error that the acceleration at point C is zero because the velocity is zero at point C. She may also be answering correctly by recognizing that the block remains at rest at C. It is difficult to tell from the transcript.
[17] L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993).
[18] Bruce Sherin discusses some of these issues in B. Sherin, "The Symbolic Basis of Physical Intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, School of Education, University of California, Berkley, (1996).
[19] See Ref. 6.

[20] For more info see Ref. 18.
[21] Problem-solving tutorials are described in chapter 3.
[22] We believe that the two student populations we investigated are similar. Both populations went through the same curriculum and both instructors for the course were well liked by their students.

Chapter 6: Using Complex Problems to Evaluate Coherence in Qualitative and Quantitaitve Knowledge

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Chapter 6: Using Complex Problems to Evaluate Coherence between Qualitative and Quantitative Knowledge

Introduction

In addition to schemas centered around physics topics, the last chapter provided evidence that many of our students have isolated schemas for qualitative and quantitative knowledge. In this chapter we present results showing how students perform on qualitative questions and quantitative questions about various physics topics. The results we present in this chapter will show that the qualitative knowledge and quantitative knowledge our students possess about a particular topic often do not overlap, and may even be contradictory. For example, we have already seen how sometimes students write down the equation for Newton's 2nd law (NII) yet still believe that a net force is needed to keep an object moving at constant velocity. We will show that students often do not attach qualitative meaning to their quantitative knowledge.

Research on physics problem-solving has clearly demonstrated that experts tend to use their qualitative knowledge, more than novices, in solving quantitative problems.[1] In addition, Larkin has found that experts will use their qualitative understanding in formulating physical representations.[2] In this chapter we specifically focus on the coherence between qualitative knowledge and quantitative knowledge for novice problem-solvers. Our results indicate that students' qualitative and quantitative schemas for answering questions and problems are only weakly linked. We show that students often use either a qualitative schema to solve a problem or a quantitative schema, but rarely use integrated schemas.

This implies that giving our students either qualitative questions or quantitative questions on exams to evaluate their performance will only evaluate the quality of a particular schema; it will tell us little about the quality of the coherence in their physics knowledge. For instance, in an introductory physics course for pre-medical and engineering students, the students are usually evaluated by their performance on quantitative problems. Students who are good at formula manipulation are often rewarded by receiving good grades in the course.[3] To get a deeper understanding of our students' knowledge, physics education researchers try to incorporate different types of evaluation techniques to probe deeper into students' understanding.[4]

One common assumption instructors make is that the ability to solve traditional homework and exam problems implies qualitative understanding. There is by now an extensive literature that documents that quantitative skills do not always imply qualitative understanding and our results confirm this. This supports the claims of physics education researchers who have shown that students often pass the introductory physics course without a basic conceptual understanding of the material.[5] Students leave our courses with many of the same misconceptions with which they entered; but now they can solve traditional problems.

We also provide results that show that improving students' qualitative understanding does not imply that students will automatically use their qualitative understanding in solving quantitative problems. Therefore, innovative curricula that

have been shown to improve our students' qualitative understanding are not sufficient to produce effective problem-solving in some contexts.

By applying the theoretical framework of schema theory to qualitative knowledge and quantitative knowledge we can begin to make sense of these issues. Schema theory helps us understand why success on quantitative problems tell us little about the quality of qualitative understanding, and why success on qualitative questions tell us little about the quality of problem-solving skills. Unlike expert problem-solvers who tend to integrate qualitative and quantitative knowledge, our students find it difficult and consider it unnecessary to go back and forth between these two types of knowledge.

At this point it will be useful to define what we mean by quantitative questions and qualitative questions. For experienced physicists this distinction is not necessarily an obvious one. An experienced physicist will usually see a problem as containing both qualitative and quantitative aspects. For instance, when presented with an equation such as $x - x_o = \frac{1}{2}at^2$ experienced physicists will integrate a story of the motion with the equation. They will picture an object starting at some position, from rest, moving along a straight line with increasing velocity. Novice physics students will often just see the equation as a way to solve for a particular variable; they do not consider the qualitative picture of what is happening important. For this dissertation we will define *quantitative questions* as questions that either require the students to solve for a given variable in terms of symbols or to actually calculate a specific value for a particular variable. Often these problems can be solved without requiring a qualitative understanding of the underlying physics; students often solve these problems by selecting a formula from a list (by identifying the presence of particular variables) and then manipulating equations. These types of problems are typically found at the end of the chapter in a physics textbook.[6] *Qualitative questions* are questions where the student reasons conceptually instead of algebraically about a given situation. Again, for some questions the distinction will be difficult. Although these definitions are simple, they will allow us to classify many questions as being either qualitative or quantitative.

In the paragraph above we have classified qualitative and quantitative questions simply by the questions themselves, but this is a simplification. A more accurate definition of a qualitative and a quantitative question would depend on the characteristics of the problem and the individual solving the problem. Simply stated, a question which activates qualitative knowledge is a qualitative question and a question that activates quantitative knowledge is a quantitative question. The particular knowledge an individual has will play a large role in which schema gets activated, or whether a schema with both qualitative and quantitative knowledge is activated.

To make the distinction clearer let's look at an example from Serway[7] shown in Figure 6 - 1. Based on our simple definition, using only the characteristics of the question, we would tentatively classify this problem as quantitative because the students are asked to calculate the value of the potential differences for a set of circuit elements. Most experts would see this question as more of a qualitative question. Rather than applying Kirchoff's voltage law on two different loops, they would reason conceptually about the current and the properties of the different circuit elements.

After a conceptual analysis they would then apply some simple formulas, instead of solving two simultaneous differential equations, to find an answer. Some students treat this question as quantitative, applying little conceptual reasoning, while others treat this question as qualitative. Two student responses are shown in

Figure 6 - 2 (a, b) depicting the different ways different students can approach this problem. One students clearly answers the questions using qualitative reasoning, while the other student clearly begins with a quantitative analysis.

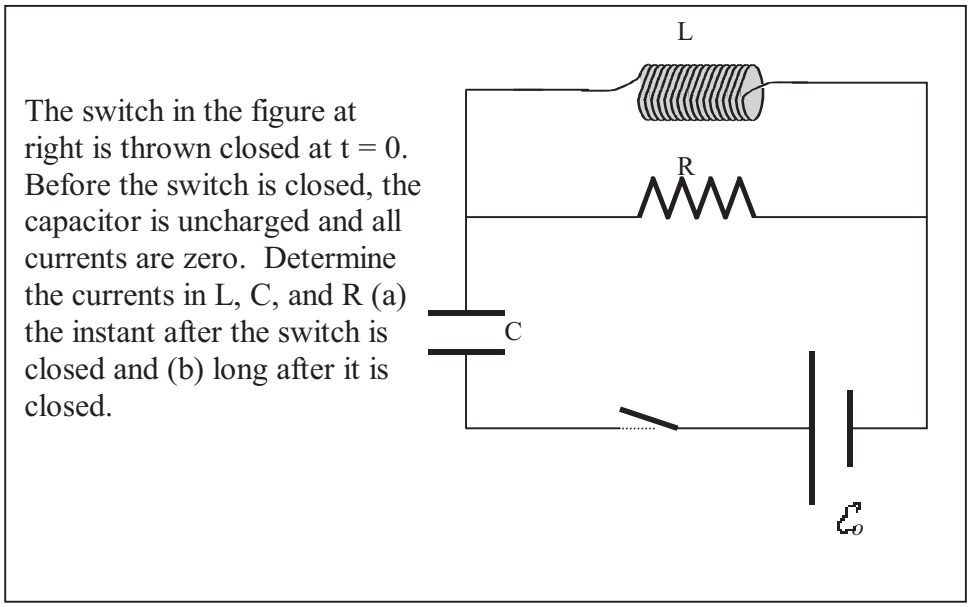


Figure 6 - 1

Inductive circuits problem from Serway. This problem can be interpreted as either a qualitative problem or a quantitative problem depending on the individual solving the problem.

We provide evidence for the presence of largely distinct qualitative and quantitative schemas with data from the three semesters of the engineering physics sequence at the University of Maryland (Physics 161, 262, and 263.)[8] This chapter examines responses on qualitative and quantitative parts to open-ended questions. We do not differentiate the students in the tutorial sections and the recitation sections for most of the studies in this chapter.

We demonstrate the fragmentation between the qualitative knowledge and quantitative knowledge by identifying two types of student responses. In the first case we observed that students can often solve quantitative questions correctly without a conceptual understanding of the physics involved in the question. In the second case we observe that students may have the correct conceptual understanding but not apply this conceptual understanding in solving a quantitative problem.

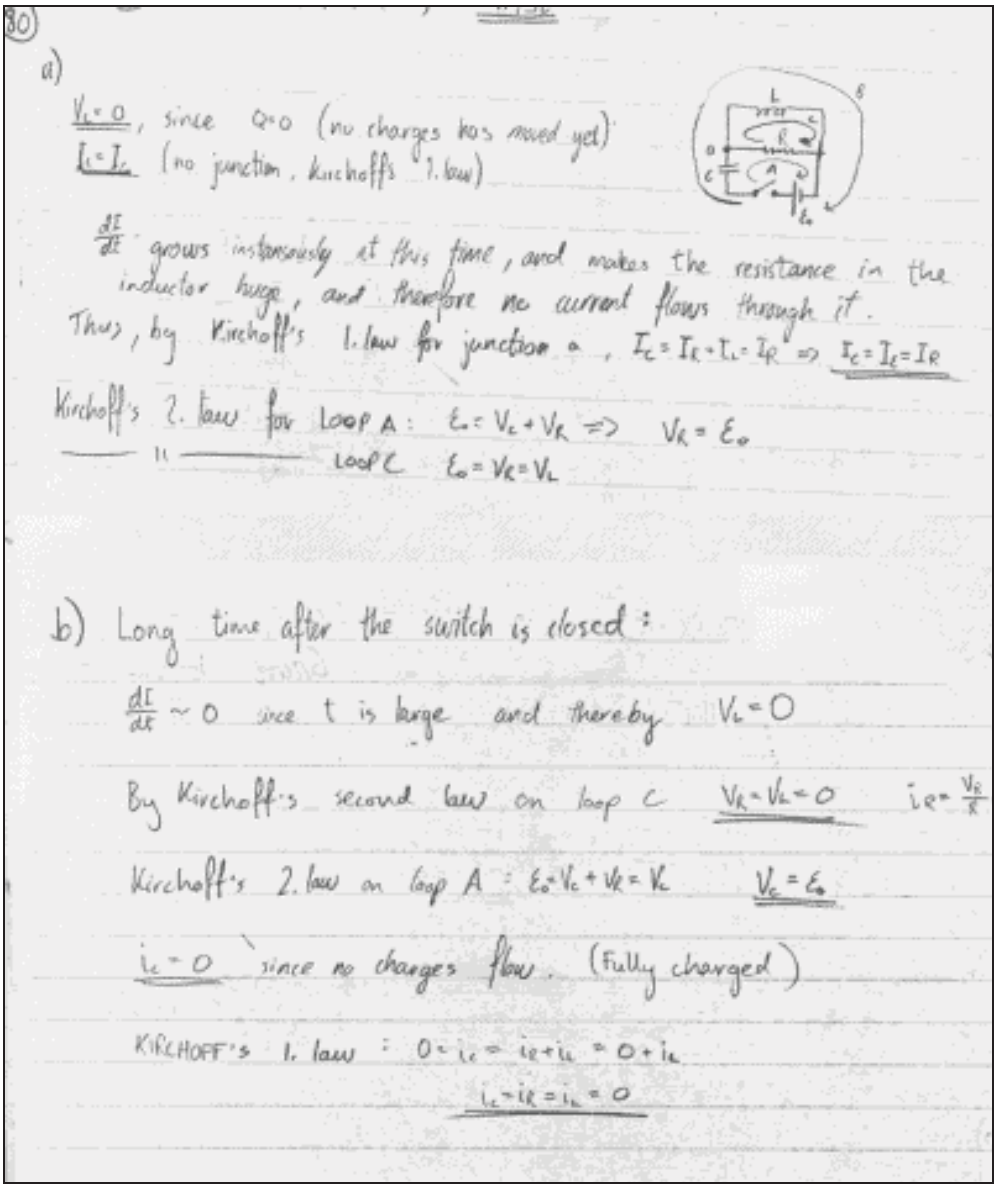


Figure 6 - 2

(a)

Sample student response showing how students can treat one question either qualitatively or quantitatively. In this solution the student uses his qualitative understanding.

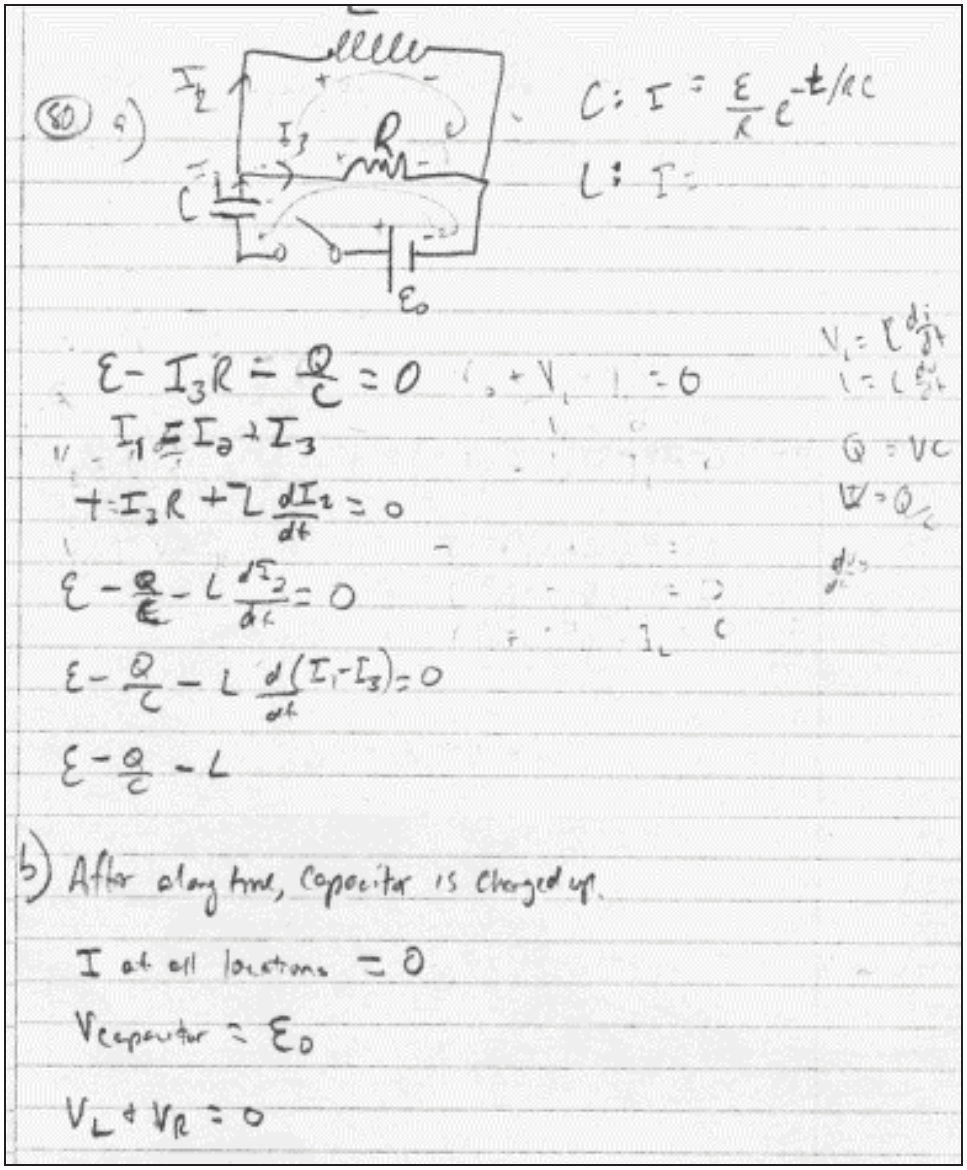


Figure 6 - 2

(b)

Sample student responses showing how students can treat one question either qualitatively or quantitatively. In this solution the student uses a quantitative analysis.

Sample Analysis

Before attempting to describe our large-scale studies we will first look in detail at a sample student response to an exam problem.[9] This allows the reader to become acquainted with some of the issues of interest. This analysis also clarifies what we mean by qualitative and quantitative questions and responses.

The question shown in Figure 6 - 3 was written by the author and was asked in the Physics 263 class at the University of Maryland. The question was asked on an exam and tests the students' knowledge of inductive circuits with a DC voltage source.

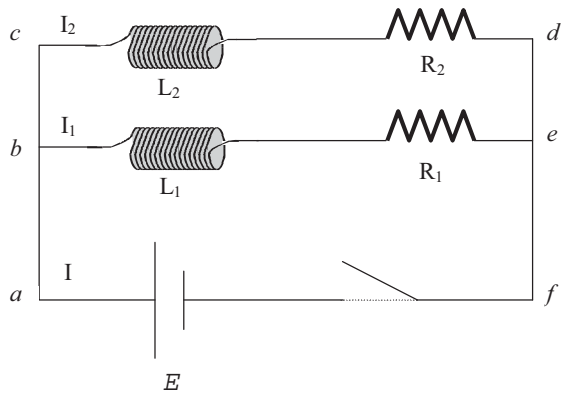
The problem begins with a quantitative question asking the students to write down the circuit equations using Kirchoff's Laws. The next three parts are qualitative questions where students are first asked to reason about the potential differences across each circuit element and then asked to sketch the graph of the current I_2, given the current I_1. The last part of the problem is a quantitative question asking about the energy stored in the inductors a long time after the switch has been closed.

For an expert problem solver none of the parts of this problem are exclusively qualitative or quantitative. The expert problem solver will use schemas containing both types of knowledge to solve each of the parts (if the responses are not obvious). We also expect the expert problem solver to apply qualitative and quantitative reasoning more consistently then our students, since these two types of content knowledge are more integrated in the expert.

One student's response to the exam questions is given in Figure 6 - 4. This student's responses on the quantitative and the qualitative questions are often contradictory. This student is not representative of the whole class or even most of the class, but the response provides us with an example of the type of incoherence we often observe when students are solving complex physics problems requiring both quantitative and qualitative skills.

The student's response in Figure 6 - 4 has been typed in order to make it easier to read. This student shows a clear distinction between his qualitative and quantitative knowledge. For the quantitative questions, parts a and e, the student correctly applies his quantitative knowledge. He uses Kirchoff's voltage law (KVL) over the two loops correctly, identifying the voltage drops across the inductors and the resistors. He also has the correct signs for each of the voltage drops. For the final part the student correctly uses the fact that the current will be a maximum a long time after the switch is closed. One may then conclude that this student has a good understanding of inductive circuits. Without solving the circuit equations, the student realized that the current will become constant after a long time, by either applying a piece of qualitative knowledge or remembering a fact that the student associates with this type of problem.

Consider the circuit shown at right consisting of a battery, two different inductors, and two identical resistors, where $\mathcal{E}$ = 20 V, $R_1 = R_2 = 10\ \Omega$, $L_1 = 10\ \mu H$, and $L_2 = 5\ \mu H$. Before the switch is closed there is no current in the circuit. Assume that the battery, the wires, and the inductors have no resistance.

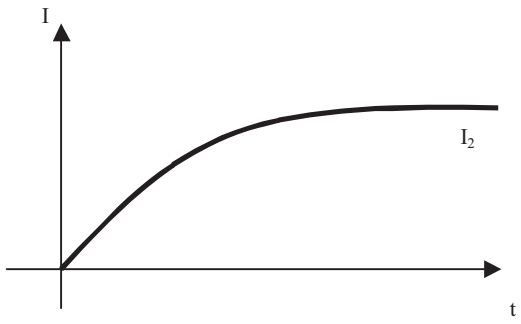


a. Use Kirchoff's Law to write the circuit equation for loop *acdf* and loop *abef*.

b. Rank the magnitudes of the potential differences over the battery, L_1, L_2, R_1, and R_2 **a very short time after** the switch is closed. If any of the potential differences are zero state that explicitly. Explain your reasoning.

c. Rank the magnitudes of the potential differences over the battery, L_1, L_2, R_1, and R_2 **a long time after** the switch is closed. If any of the potential differences are zero state that explicitly. Explain how you know.

d. The figure at right shows the current I_2 as a function of time. Redraw this figure in your exam booklet. On the same figure sketch the graph of I_1. Be sure to make your sketch consistent with the sketch of the current I_2 and be sure to label each of the currents. Explain your reasoning on qualitative grounds using your answer to part a. (You do not need to solve the circuit equations.)



e. Calculate the energy stored in each inductor **a long time** after the switch has been closed. Show all work.

Figure 6 - 3

Inductive circuits problem asked as an exam question in the physics 263 class. This question is an example of a tutorial question with quantitative parts.

When we analyze the qualitative questions we see that the student's qualitative understanding has serious deficiencies, despite his correct answers to the quantitative questions. In part b, a short time after the switch is closed, the student states that the change in current is zero, therefore the voltage drop over the inductors is zero. Here, the student may be confusing current with change in current. The confusion of a quantity and its rate of change is a common error, and appears in the student confusion of acceleration and velocity.[10] The next part of the problem asks about the voltage drops a long time after the switch has been closed. The student states that the voltage drops over the inductors are the largest because of the current in the inductors, even though he states that $V_L = -L\,dI/dt$ and in part d he states that the current will reach a maximum value of $\mathcal{E}/R$.

a. For loop acdf, $\mathcal{E} - L_2\,dI_2/dt - I_2R_2 = 0$

For loop abef, $\mathcal{E} - L_1\,dI_1/dt - I_1R_1 = 0$

(since R_1=R_2 we can replace both with R_2 but since the diagram has both R_1 and R_2 I'll leave it in.)

b. $|V_{R1}|=|V_{R2}|>|V_{L1}|=|V_{L2}|=0$
A short time after the switch is closed the change in current across the inductor is zero and V_L=0. V_{R1}=V_{R2} because R_1 = R_2.

c. $|V_{L1}|>|V_{L2}|>V_{R1}=V_{R2}=0$
$V_{L2} > V_{L1}$ because I_1=I_2 so the only difference is their L and L_1>L_2. The inductor produces a magnetic field thanks to the current and the induced current. The V_{R1} and V_{R2} are zero because all the current is going through the inductors.

d. $\mathcal{E} - L\,dI/dt - IR = 0$, $\mathcal{E}/R - L/R\,dI/dt - I = 0$

e. $U_f = \frac{1}{2}LI_f^2$

A long time after the switch has been closed $I = Imax$

$Imax = Q\omega = \mathcal{E}/R$

Therefore

$U_{f1} = \frac{1}{2}(10X10^{-6})(20/10)^2 I_f^2$, $U_{f2} = \frac{1}{2}(10X10^{-6})(20/10)^2 I_f^2$

$= 2\ X\ 10^{-5}$ *Joules*

Figure 6 - 4

Student solution to the question in Figure 6 - 3.

When using Kirchoff's voltage law (KVL) in part a, the student presumably uses the fact that the currents are the same in series. He uses I_1 to describe the current across the leg of the circuit *be* and I_2 to describe the current across leg *cd*. This correct application of KVL is contradicted in part c when the student states that the voltage drops over the resistors are zero because all the current is going through the inductors. This provides us with another example of how a question may cue a qualitative schema activating a different response than a question that may cue a quantitative schema. Besides having contradicting qualitative and quantitative knowledge, a student might also possess contradicting qualitative knowledge that can be activated by different cues.

It is important to note that we are not simply looking at whether students score better on quantitative questions or qualitative questions testing the same ideas. We would like to understand the structure of their qualitative and quantitative knowledge and identify consistency and inconsistency in the students' reasoning. The above example shows that the student directly contradicts his qualitative statements and quantitative statements even when the statements deal with the same physical phenomena. The following examples are done on a larger scale to show the pervasiveness of these difficulties.

Momentum Question

The question shown in Figure 6 - 5 was asked in the non-tutorial Physics 161 class at the University of Maryland.[11] This question was posed on the final exam for the class. It consists of two qualitative questions and two quantitative questions.

> Two blocks collide on a frictionless surface. After the collision the two blocks stick together. Block A has a mass M and is initially moving at speed v in the $+x$ direction. Block B has a mass $2M$ and is initially at rest.
>
> a) Draw a free body diagram for each block at an instant ***during*** the collision. Rank the magnitudes of the horizontal forces in your free body diagram.
> b) What is the final velocity of the blocks after the collision? Show all work.
> c) Calculate the change in momentum of block A. Calculate the change in momentum of block B. Show all work.
> d) Can a system whose momentum is conserved be made up of smaller systems whose individual momenta are not conserved? Explain your reasoning.

Figure 6 - 5

Question on momentum conservation asked on the Physics 161 final exam.

The students are first asked to draw a free-body diagram for the blocks during the collision. They are then asked about the final velocity of the two blocks after the collision. Since the collision is inelastic, students can determine the velocity of the blocks using the principle of conservation of momentum. In the next part, the students are asked about the changes in momentum of both blocks, A and B. The final part is a

qualitative question testing the students on the concept of conservation of momentum.[12] Parts b and c are quantitative questions requiring the application of conservation of momentum. The last three questions require the application of the principle of conservation of momentum.

In part a, students are asked to draw a free-body diagram for both blocks during the collision. Only about 38% of the students had correct free-body diagrams for the blocks during the collision and 15% of the students did not include a free-body diagram. The two most common errors were to include a force on block A in the direction of A's initial motion (9%) or to omit the horizontal forces during the collision (7%), accounting for 16% of the errors. Each of the remaining errors accounted for fewer than 5% of the responses.

In the ranking task, many students (65%) left the question unanswered, possibly because students in a traditional physics class often have little practice with ranking tasks. Only 18% of the entire class correctly ranked the NIII force pairs. The most common incorrect answer, given by 9% of the class was that the force exerted on block B by block A was larger. This is consistent with previous studies on student understanding of the third law.[13]

Students performed better on the quantitative questions in parts b and c. In part b, 81% of the students calculated the final velocity of the blocks correctly. The most common error, given by 11% of the students, was using conservation of energy, instead of conservation of momentum. These results would indicate to most instructors that students have a good understanding of the conservation of momentum. It shows that most of the students were capable of deciding when conservation of energy can and cannot be applied. In part c, 37% of the students calculated the change in momentum of each block correctly. Of the students who answered incorrectly, only 2% of the students stated that the change in momentum of each block was zero. Therefore, about 90% of the students in the class obtained non-zero responses for the change in momentum of each block.

In part d, students are given a general qualitative question about the conservation of momentum. Recall that about 81% of the students used the principle of conservation of momentum in part b and about 90% of the students calculated non-zero answers for the change in momentum of each individual block. We would therefore expect that most of the students would answer “Yes” to the question in part d, possibly citing their answer to part c as evidence. We find that only 53% of the students answered correctly, while 41% of the students answered “No”. This result shows a clear distinction between students’ qualitative and quantitative knowledge. It shows that having contradictory statements for these two questions is acceptable for many of the students. Either the students are not aware of the contradiction or they recognize the contradiction but do not attempt to resolve it. Table 6 - 1 summarizes student performance on the momentum problem for parts b, c, and d. Responses are divided into correct and incorrect with special cases of incorrect responses included in parenthesis.

N=115

	b) Calculate the final velocities.	c) Calculate Δp for blocks A and B	d) If $\Delta p_{sys} = 0$ can Δp_A and Δp_B be non zero.
Correct	81%	37%	53%
Incorrect	19%	63% (49% had non zero)	47% (41% had No)

Table 6 - 1

Summary of student performance on the momentum question (parts b, c, and d.)

Figure 6 - 6 shows a sample student response to the problem. Although this student answered the quantitative questions correctly using the principle of conservation of momentum, his response on the qualitative questions is incorrect. The free-body diagrams for both blocks include vectors labeled *v*, which most likely represent the final velocity, and not the forces on the blocks. On part d this student directly contradicts his responses to parts b and c. His response to part d also shows that the confusion does not come from a misconception about the term "system." He believes that if one part of the system loses momentum the entire system must also lose momentum. He is therefore not considering that if one part of the system loses momentum another part of the system can gain that momentum. There were a number of students who made this error.

To get a clearer picture of how students are responding to the quantitative question in part c and the qualitative question in part d we concatenated their responses on these two questions. The results of this comparison are shown in Table 6 - 2. (The questions for part c and d are paraphrased in the table.) The darkly shaded cells of the table represent the percentage of students who were consistent in their responses to the two questions. The lightly shaded cells represent the percentage of students who answered the two questions inconsistently. As an example, we see from the table that 37% of the students obtained the correct answer for the quantitative question (part c). Of those students 19% of the class answered the qualitative question consistently, stating that a system whose momentum is conserved can be made up of smaller systems whose momenta are not conserved. We also see 16% of the class answered inconsistently, stating that a system who's momentum is conserved must be made of smaller systems whose momenta are conserved, even though they had shown in part c that the change in momentum of block A and block B were non-zero for this situation.

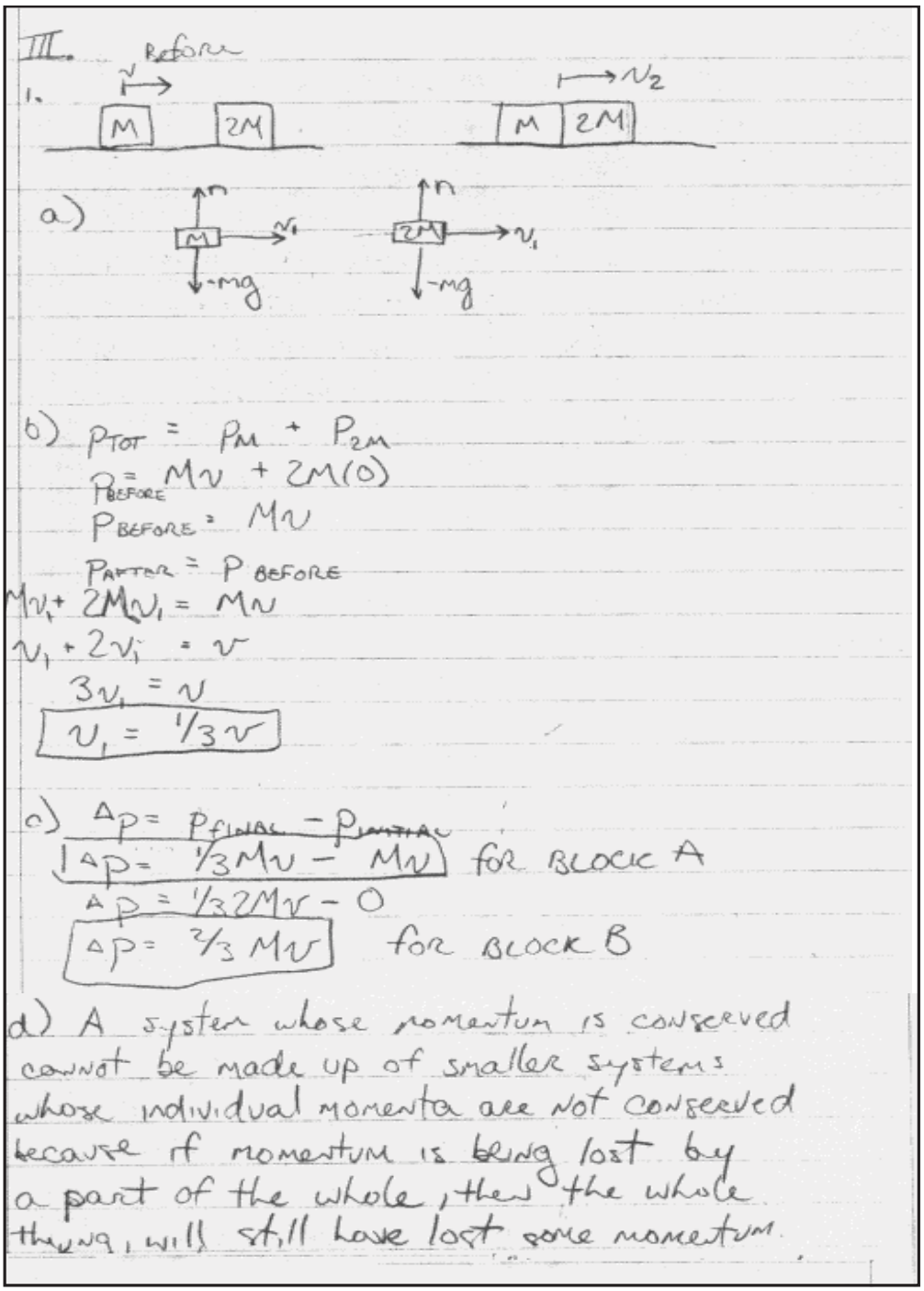


Figure 6 - 6

Sample solution to the momentum exam question shown in Figure 6 - 5.

Quantitative Question Response (c) Calculate ΔPA, ΔPB.	**Qualitative Question Response (d)** If $\Delta P_{sys} = 0$ can ΔP_A and ΔP_B be non zero?			**N=115**
	Correct: Yes	Incorrect: No	Incorrect: Other	TOTAL
Correct	19%	16%	2%	37%
Incorrect: Non Zero	27%	21%	1%	49%
Incorrect: Zero	1%	1%	0%	2%
Other	8%	3%	2%	13%
TOTAL	55%	41%	4%	**100%**

Table 6 - 2

Table showing how students answered different parts of the question in Figure 6 - 5. The results show that many students answer inconsistently on the quantitative and qualitative parts of the question.

Circuits Question

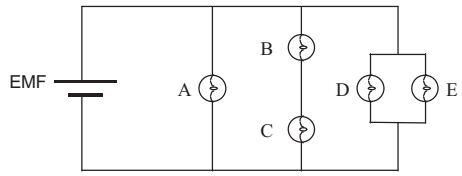
The mixed qualitative and quantitative question, shown in Figure 6 - 7, was asked in the physics 262 class at the University of Maryland. The results show that students can perform reasonably well on a quantitative question yet they can have serious conceptual difficulties.[14] This question is similar to the question presented by Eric Mazur[15] at Harvard University and a pair of questions given by Steve Kanim[16] at the University of Washington.[17]

The question shown in Figure 6 - 7 was given on the final exam in two Physics 262 classes. Each class consisted of three hours of lecture, two hours of lab, and either one hour of tutorial or one hour of traditional recitation each week. Students in the tutorial section had no specific tutorial instruction on circuits.

Part A of the problem is a qualitative question that comes from the University of Washington PEG.[18] Part B is a standard quantitative question. The correct solution to part A is that bulbs A, D, and E all have equal brightness because they are all connected in parallel and therefore each bulb will have the same voltage drop as the battery. Bulbs B and C would be dimmer since the resistance in that leg of the circuit is greater and the voltage drop over both bulbs is the same as the battery. One possible solution to part B would involve a straightforward application of Kirchoff's voltage rules. Another correct method used by students was to calculate the

In this problem, assume that the battery is ideal and that all wires have zero resistance.

A. Consider the circuit shown at right containing 5 identical bulbs labeled A-E. Rank the brightness of the bulbs from most bright to least bright. If any bulbs are equally bright, state that explicitly. Explain your reasoning.



B. For the circuit shown at right, find the current through R_1, the 10 ohm resistor. Show how you arrived at your answer.

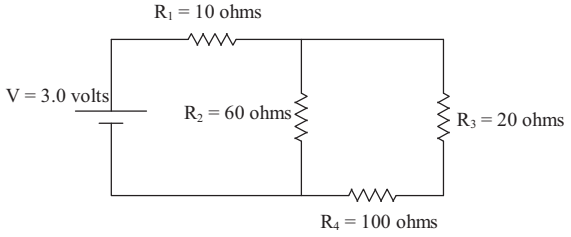


Figure 6 - 7

A final exam question with a qualitative question (A) and a quantitative question (B). Students performed better on the quantitative question despite it being more difficult than the qualitative question.

equivalent resistance in the circuit in order to find the current through R_1. Most experienced physicists would rate the quantitative question (part B) to be more difficult than the qualitative question (part A.)

The results from the two classes are combined in Table 6 - 3. The results indicate that students had significantly more difficulty with the qualitative question. Only 17% of the students answered this question correctly. These results are consistent with the results from the University of Washington PEG where 15% of their population answered this question correctly.[19] Their results also show that students have some fundamental misconceptions with the topics of current and voltage.

The work done by the University of Washington Physics Education Group on the topic of electric circuits gives us an idea of why some of the errors on the conceptual questions were made. Some incorrect models used by students include a model where current gets used up in a bulb and a model where the battery is a constant current source.

Only 44% of the students at Maryland answered the quantitative question correctly. Although performance was better on the quantitative question (part B), the

N=131

Part A: Qualitative	Correct: A=D=E>B=C	Incorrect: D=E>A>B=C	Incorrect: A>B=C>D=E	Incorrect: Other
	17%	21%	19%	43%

Part B: Quantitative	Correct	Incorrect: V_{batt}/R_1	Incorrect: R_{eq} wrong	Incorrect: Other
	44%	21%	12%	23%

Table 6 - 3

The types of responses given on the two circuits questions and the percentages of students giving each type of response.

percentage correct is far from where we would like it to be. The most common error involved the students' incorrect application of Ohm's Law. Students who made this error divided the battery voltage by the resistance, R_1. One possible explanation for this is that students are applying Ohm's Law without attaching conceptual meaning to it. Another explanation is that the students were applying the model where the battery acts as a constant current source, independent of the circuit.

The most relevant result to the issue of coherence between qualitative and quantitative schema is that most of the students who answered this question applied different models when solving parts A and B. Applications of Kirchoff's Rules and calculations of equivalent resistances tell us little about student reasoning. We see from this example that students can have incorrect models for current and voltage yet still correctly apply algorithmic methods, such as Kirchoff's Voltage Rules to solve standard problems.

N=131

Quantitative Question Response (c) Find the current through R_1.	**Qualitative Question Response (d)** Rank the brightness of the five bulbs.		
	Correct	Incorrect	TOTAL
Correct	11%	34%	45%
Incorrect	6%	50%	56%
TOTAL	17%	84%	**100%**

Table 6 - 4

Performance on the qualitative and the quantitative parts of the circuits question.

Table 6 - 4, on the previous page, shows how students performed on the qualitative part and the quantitative part of the circuits problem. We also see that students who have a good qualitative understanding of the material but do not necessarily apply their qualitative understanding to quantitative problems. We see that 35% of the students who answered the qualitative question correctly answered the quantitative question incorrectly. It is also interesting to note that only 11% of the students answered correctly on both the qualitative and the quantitative questions.

Electric Potential Problem

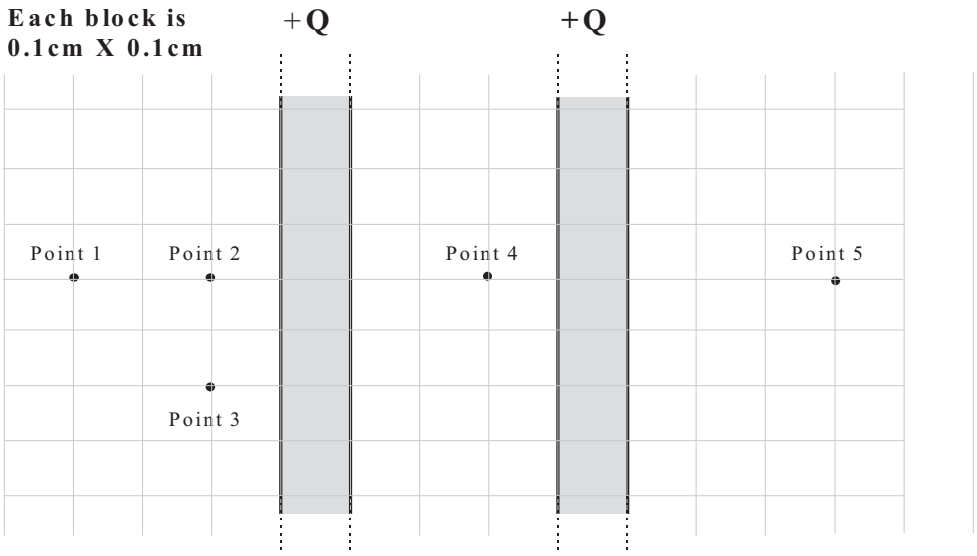
The three examples given above concentrated on students who could solve quantitative problems without a deep qualitative understanding of the material. These results support claims that have previously been reported in the literature. [20] A more surprising result that is usually not discussed, is that students can have the correct qualitative understanding, but do not transfer this qualitative knowledge to quantitative questions.

These results are important because some instructors view the introductory physics course as simply a place for students to develop analytic skills to solve problems. This study shows that good qualitative understanding, although necessary for effectively problem-solving, is not sufficient; a students' qualitative and quantitative schema must be connected. We believe that good analytic skills consist of not merely being able to perform algebraic manipulations; good analytical skills involve applying conceptual understanding and reasoning to solve quantitative problems.

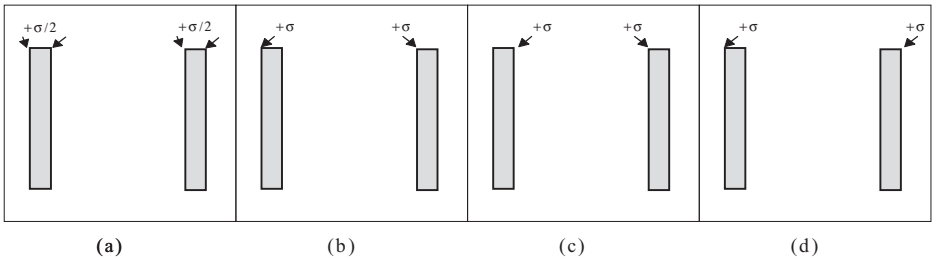
The following example was written by the author and was asked in the physics 262 class at UMd with tutorials as a tutorial homework bridging problem.[21]. The question is shown in Figure 6 - 8. We concentrate on the student responses to parts b and d although the responses on the other parts provide us with additional information about student understanding of this topic. We analyzed 56 student solutions.

The qualitative parts of this question have the students sketching electric field vectors at different positions and identifying the charge distribution on the metal slabs. Students performed well on the qualitative questions. Field vectors were, for the most part, drawn in the correct direction and had the correct magnitude. The most common error was drawing field vectors that changed in magnitude as you looked at points farther and farther away from the plates. About 80% of the students correctly indicated that the field would be zero at point 4, between the slabs, even though the point was not halfway between the slabs. This result will be particularly interesting when we look at how the students answered the final quantitative part of the problem. Students also performed well on the question about the charge distribution. We saw 88% of the students answering part a correctly.

Consider two very large conducting plates a distance *D* apart. Both plates have a charge of +Q on them. See figure at right.



a. Which diagram below shows the correct charge distribution on the plates? Explain your reasoning.



b. Draw vectors on the diagram above representing the electric field at points 1, 2, 3, 4, and 5. If the field is zero at any point indicate that explicitly.

c. A small charge q, initially at rest at point 1, is moved by a hand so that it comes to rest at point 3. Write an equation for the work done on the charge.

d. If the charge density, σ, is 2 X 10^{-9} C/m^2, determine the potential difference between points 1 and 5. Show all work.

Figure 6 - 8

Electric potential bridging problem asked as part of the tutorial homework assignment.

Students did not perform as well on the quantitative questions about the work done on a point charge to move it from point 1 to point 3 and the question about the potential difference from point 1 to point 5. Many students gave responses that contradicted their qualitative answers. In part c, 20% of the students wrote down the work done on a point charge to move it in the field of another point charge, instead of in the field due to parallel plates. Only about 32% of the students answered part c correctly, by treating the field as constant and taking into account the dot product for the force and displacement. The results from part d are even more striking. Even though about 80% of the students indicated that the field was zero between the two plates and was pointing in opposite directions on either side of the two plates, many of these students considered **E** to be constant when calculating the potential in part d. Our results show that 27% of the students treated the field as constant in the equation, $\int_1^5 E \cdot dl = -V$. Most of these students (21% of the entire class) stated, in part b, that the field between the two plates was zero, which indicates that they are not using their qualitative knowledge of the electric field in the equation. Table 6 - 5 summarizes the student performance on this problem. The table also lists some of the specific errors made by the students.

N=56

	a) Describe the charge distribution.	b) Draw field vectors at the points indicated.		c) Calculate the work from 1 to 3.	d) Calculate the potential from 1 to 5.
		Point 4	General		
Correct	88%	82%	63%	32%	41%
Incorrect	12%	18%	37%	68%	59%

(21% had vectors with different magnitudes, 7% drew correct field lines)

(20% used work done by a point charge, 11% had no dot product)

(27% treated the **E** field as constant in potential equation.)

Table 6 - 5

Summary of the results on the electric potential problem.

Instructors rarely attach conceptual meaning to equations in the physics course, and when they do, it is usually stated and not written on the board. This can lead to students creating separated schema for the qualitative and quantitative aspects of a concept or principle. In this situation we see that even though we can improve student qualitative knowledge, through the use of modified curriculum, there is no guarantee that these concepts will transfer to quantitative problems. In order for students to become expert problem solvers, not only do students have to develop better qualitative knowledge, they also have to be able to transfer this qualitative knowledge to new situations such as traditional problems.

Interference Problem

The next example shows that students in the top of the class (measured by total overall score) made the error of not connecting the mathematics with the concepts more often than the other students. This is surprising because instructors usually expect the students in the top of the class to have more coherence between the equations and the concepts and principles. In addition this particular error can be classified as a sophisticated type of error because of the level of reasoning involved in the solution. These results show that coherent qualitative and quantitative knowledge can benefit the entire student population, not just the students in the middle and bottom of the class.

The interference problem, shown in Figure 6 - 9, was posed on an exam in the physics 263 class at UMd, with tutorials. Students received instruction on double slit interference both in lecture and in the tutorial class. The physics 263 class at the University of Maryland uses four tutorials written by the PEG at the University of Washington. These tutorials are dedicated to the wave properties of light and physical optics. They have since been revised by the PEG and are included in *the Tutorials in Introductory Physics* book.[22] We will concentrate on part b ii of the problem.

The correct answer to part b ii can be obtained qualitatively by noticing that as the wavelength increases, the path difference, ΔD, required for the first minimum also increases. A larger value of ΔD means that the angle between the vertical and the path from the slits to the first minimum is greater. Therefore the point P_1 would be between the first minimum and the central maximum. A solution to the problem is shown in Figure 6 - 10. The particular error we will be concerned with shows the lack of coherence between the equation $\left(n + \frac{1}{2}\right)\lambda = d\sin\theta$ and the conceptual aspects of the double slit interference problem. Since we are given the distance to the first minimum, substituting n = 0 into the equation gives the relevant equation; we have $\frac{\lambda}{2} = d\sin\theta$ for this situation. Approximately 17% of the students made the mistake of substituting 2λ for λ since they were told that the wavelength of light was increased to 1000 nm. This left the students with the following equation, $\lambda = d\sin\theta$ which they stated was the condition for a maximum. These students therefore concluded incorrectly that point P_1 became the position for maximum constructive interference. The students who made this error were no longer considering the point P_1, since they changed the path difference, ΔD. We refer to this error as "equation-concept." Again, this is a sophisticated mistake, but a fairly common and profound error. Students who

answered this way were obviously using the concepts; they were using the fact that $\Delta D = \lambda/2$ for destructive interference and $\Delta D = \lambda$ for constructive interference. Students were therefore applying some concepts but the concepts were not correctly associated with the equation. Two sample student responses are shown in Figure 6 - 11 depicting this error.

Consider a plane wave of monochromatic green light, $\lambda = 500$ nm, that is incident normally upon two identical narrow slits (the widths of the individual slits are much less than λ). The slits are separated by a distance $d = 30$ μm. An interference pattern is observed on a screen located a distance L away from the slits. On the screen, the location nearest the central maximum where the intensity is zero (i.e., the first dark fringe) is found to be 1.5 cm from this central point. Let this particular position on the screen be referred to as P_1.

a. Calculate the distance, L, to the screen. Show all work.

b. In each of the parts below, one change has been made to the problem above (in each case, all parameters not explicitly mentioned have the value or characteristics stated above). For each case, explain briefly whether the light intensity at location P_1 would remain zero or not. If not, will P_1 become the location of a maximum constructive interference (bright) fringe? In each case, explain your reasoning.

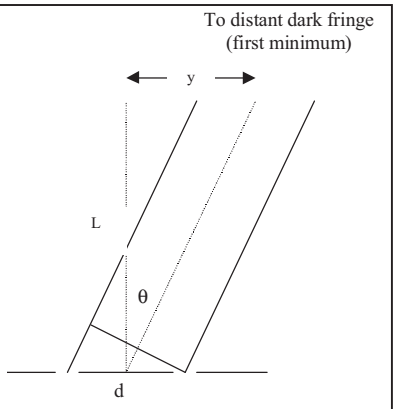
    i. One of the two slits is made slightly narrower, so that the amount of light passing through it is less than that through the other.
    ii. The wavelength is doubled so that $\lambda = 1000$ nm.
    iii. The two slits are replaced by a single slit whose width is exactly 60 mm.

Figure 6 - 9

Exam question on physical optics. We will focus on the student responses to part B ii.

The graph in Figure 6 - 12 shows the percentage of students answering correctly, and answering incorrectly vs. total score in the class. The total score in the class is based on homeworks, exams, labs, and a quiz. The graph also shows the percentage of students making the "equation-concept" error we discussed earlier. The graph is constructed in the following way. We first take the student with the highest score in the class and see the type of response he or she gave. The second point represents the top two students and the percentage of correct, incorrect, and "equation-concept" responses given. The third data point represents the top three student's responses and so forth. The final point represents the percentage of correct, incorrect, and "equation-concept" responses for the entire class. We can therefore look for trends in the responses for the students based on their overall score in the class.

a. The distance L to the screen can be obtained from $\left(\frac{1}{2}\right)\lambda = d\sin\theta$ since we know the distance to the first dark fringe. Since θ is small we have $L = \dfrac{dy_1}{\frac{1}{2}\lambda} = 1.8\ m$



b. i. If one slit is made narrower we will no longer have complete destructive interference although the point will still be a minimum.

ii. If l is doubled the ΔD would have to increase for the first minimum therefore P_1 would be in-between a minimum and a maximum.

iii. The 1st minimum will still be in the same place since the ΔD between the path beginning at the end of the slit and the path beginning at the middle of the slit must be λ/2 and the d for both cases is 30 mm.

Figure 6 - 10

Solution to the problem in Figure 6 - 9.

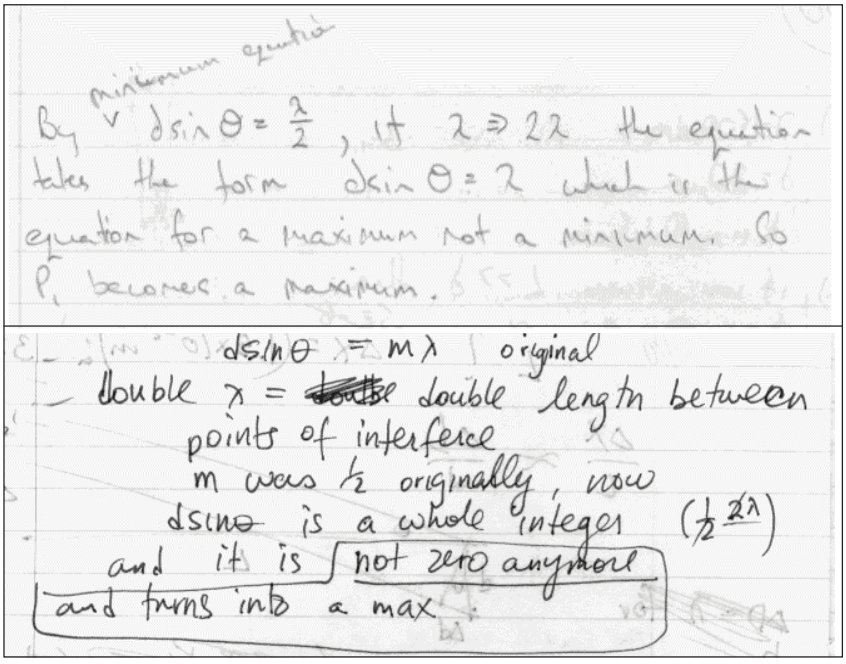


Figure 6 - 11

Two sample student responses showing the "equation-concept" error.

The data series representing the correct responses and incorrect responses behaves as we would expect. The percentage of correct responses increases as we move to the top of the class and the percentage of incorrect responses decreases as we move to the top of the class. The percentage of "equation-concept" type responses is surprising because this particular error actually increases as we move to the top of the class. This result shows that even the more successful students fail to link the equations to concepts. The data also indicates that the top students in the class may rely on equations more than the other students in the class, possibly because of the way they view physics knowledge and the way they view the applications of physics knowledge.[23]

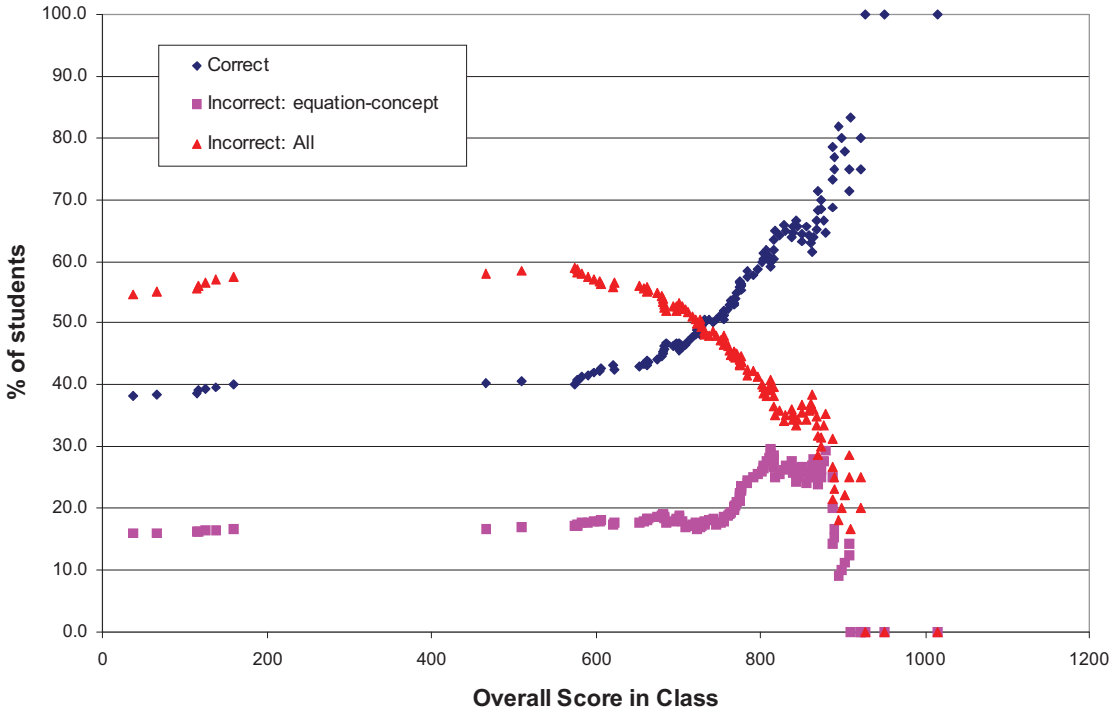


Figure 6 - 12

Graph showing the types of responses students gave versus their total score in the class. The percentage of students making the "equation-concept" error increases as the student score increases.

It would be too simple to say that these students are applying these equations blindly. Their performance on other measures and even their responses on this question show that these students do think qualitatively about equations. One explanation for these results is that the equations cue incorrect conceptual knowledge,

which may not be linked to the correct conceptual knowledge. The two sample student responses shown in Figure 6 - 11 indicate that some students are using the concepts but not using the concepts in conjunction with the equation. Physics education researchers have demonstrated that students can have many models at one time and that these models may even be contradictory.[24]

Summary

This chapter has shown that students have difficulty developing coherence between qualitative knowledge and quantitative knowledge in many areas of physics. Although many instructors assume that proficiency in solving textbook problems means that students understand the underlying principles and concepts, physics education research has demonstrated that this is not the case. Similarly many instructors feel that success on conceptual questions implies that students will be able to solve quantitative problems. Again, we see that this is not the case. In order for students to develop coherence in their content knowledge it is necessary for them to integrate their qualitative schema and their quantitative schema.

The examples presented in this chapter show that introductory physics courses often fail to accomplish the goal of developing coherence between qualitative and quantitative schemas. This incoherence persists from the beginning of the introductory engineering sequence until the end.[25] In addition, the data shows that students at the top of the class might also benefit from instruction aimed at developing coherence.

One might argue that genuine quantitative knowledge is constructed from qualitative knowledge and the quantitative problems are drawing out the qualitative difficulties the students are having. Therefore the students who are responding correctly on the qualitative questions, but not on the quantitative questions, actually do not have a good qualitative understanding. Although we agree that the students do not have a deep conceptual understanding of the material, where they are tying together qualitative and quantitative ideas, we believe that their correct qualitative responses can be used for the quantitative questions if the connection between these two schemas is made. So although these students do not possess a deep conceptual understanding of the material, their correct responses on the qualitative questions can serve as a resource for the quantitative questions.

[1] See F. Reif, "Teaching problem solving – A scientific approach," The Physics Teacher, 310-316 (1981); D. P. Simon and H. A. Simon, "Individual differences in solving physics problems," In *Children's Thinking: What develops?* R. S. Siegler (Ed.) (Lawrence Erlbaum, NJ, 1978), pp. 325-348; M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981); J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979).

[2] See chapter 2 and J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.

[3] D. Hammer, "Two approaches to learning physics," Phys. Teach. 27 (9) 664-670 (1989).

[4] These techniques are described in chapter 3.

[5] E. Mazur, *Peer Instruction*, (Prentice Hall, NJ, 1997).

[6] More complex questions combining algebraic manipulation with qualitative reasoning are present in most introductory textbooks. In our experience these are rarely assigned and when they are, are successfully solved by few of the students.

[7] R. A. Serway, *Physics for Scientists and Engineers*, (Saunders College, Philadelphia, 1996).

[8] The engineering physics course at the University of Maryland is described in chapter 3 of this dissertation.

[9] It is often beneficial for physics education researchers to first try to characterize particular students' responses before attempting studies, which characterize the class. This allows us to get a deeper understanding of what our students are doing and why they are doing it.

[10] J. Torney-Purta, "Schema Theory and Cognitive Psychology: Implications for Social Studies," Theory and Research in Social Education, **19** (2), 189-210 (1991), I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. 53 (11) 1056-1064 (1985).

[11] See chapter 3 for more information on the Physics 161 class at UMd.

[12] A similar question is asked in the Conservation of Momentum Tutorial from the University of Washington referenced elsewhere. The question was chosen, in part, because it proved to be a sticking point for the students in the tutorial class. Note that this particular class did not use the tutorial curriculum.

[13] See D. P. Maloney, "Rule-governed approaches to physics: Newton's Third Law," Phys. Educ. **19**, 37-42 (1984); and E. F. Redish, J. M. Saul, and R. N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **66** (3), 212-224 (1998).

[14] See P.S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, (1993).

[15] See Ref. 5.

[16] S. Kanim, "An investigation of Student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999).
[17] Steve Kanim has done extensive work on student responses to paired, qualitative and quantitative questions, in the contexts of electric circuits and electrostatics.
[18] See L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993) and P.S. Shaffer and L.C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity, Part II: Design of instructional strategies." Am. J. Phys. 60 (11), 1003-1013 (1992).
[19] For a more detailed account of this research see reference 18.
[20] See I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. **53** (11) 1056-1064 (1985); L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned – closing the gap," Am. J. Phys. **59** (4) 301-315 (1991).
[21] Bridging problems are discussed in detail in chapter 3 of this dissertation.
[22] L.C. McDermott, P.S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).
[23] These issues are discussed in D. Hammer, "Epistemological beliefs in introductory physics," Cognition and Instruction, **12** (2) 151-183 (1994).
[24] A number of researchers have discussed how different knowledge can be cued. See M. C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998); and B. Sherin, "The Symbolic Basis of Physical Intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, School of Education, University of California, Berkley, (1996). Wittmann describes how different types of questions cause students to bring up different models. He also shows that multiple models can brought up at the same time for some students. Bruce Sherin discusses how equations may cue certain p-prims and how certain p-prims may cue certain equations.
[25] Our results have also shown that these difficulties exist in physics graduate students also.

Chapter 7: How does Conceptual Instruction Affect Coherence Between Qualitative and Quantitaitve Knowledge?

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Chapter 7: How does Conceptual Instruction Affect Coherence between Qualitative and Quantitative Knowledge?

Introduction

This chapter documents the evolution of coherence between qualitative and quantitative knowledge as students go through a modified curriculum. By tracking one group of students through the physics 161 course we can look at the dynamics of how coherence develops between students' qualitative and quantitative schemas. Redish et al. have examined how students view coherence before students begin the physics course and after students have completed the course. They report that for traditional classes, students' views of coherence move away from an expert's views on coherence.[1] In particular students tended to shift toward believing that physics can be treated as unrelated facts or pieces.

In this study we track a class of engineering students as they go through *Tutorials in Introductory Physics*.[2,3] The previous three chapters have demonstrated that student content knowledge is often fragmented. We have seen that students often exhibit local coherence and use schemas consisting of either qualitative or quantitative knowledge, but they rarely use schemas containing integrated knowledge. This chapter will discuss some of our results about how a course which includes conceptual instruction can lead to some students forming links between the conceptual knowledge they develop and quantitative problem-solving. Even though we see some improvements in student coherence, the results are by no means dramatic and there is definite room for improvement. Our data will show some of the limitations in relying on the tutorial curriculum for building coherence between qualitative and quantitative knowledge.

We had the opportunity to give a similar exam problem to a class of Physics 161 students twice during the semester. The problem focused on Newton's 2nd law (NII) and tension. Students were given a problem involving two blocks attached by a massless string on a frictionless surface with force applied to the first block. This problem was included on the first exam in the Physics 161 course, after the students had traditional instruction on the material, but before they had tutorial instruction. The problem has been repeated on the third exam after the students went through the tutorial curriculum. The question tested the students' qualitative and quantitative understanding of NII and tension. In addition it tested whether students could apply their qualitative understanding of NII (i.e. the FBD) to a quantitative application of NII.

Context

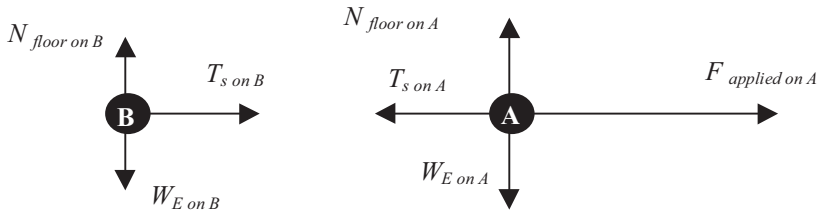
In the spring '98 semester there were three 161 classes, all using the tutorial curriculum. The same professor taught two of the classes (Classes A, B). The NII-tension problem was asked on the first exam and the third exam in two of the three classes (Class A, B) of the Physics 161 class. The third class (Class C) was asked the problem on the third exam only. The wording of the question changed slightly from

version 1 (Exam 1) to version 2 (Exam 3) and the labels on the blocks changed from version 1 to version 2. Version 2, along with a model solution is shown in Figure 7 - 1.[4] Version 1 was written by the instructor for the course, while version 2 was written by the University of Maryland PERG.

Because we are looking at the dynamics of student coherence it is important to identify when each type of instruction occurred in the class. A timeline for the course is shown in Table 7 - 1. The type of instruction is listed in the far-left column and the two columns on the right indicate when the specific type of instruction began and ended. (For items that only lasted one day, such as homework due dates and exams, there is no ending date.) In addition, although tutorials occurred over two days, each student only went to one (1-hour) section each week. There were three (1-hour) lectures a week for the students in Classes A and B, and two (1.5-hour) lectures a week for students in Class C.

Two blocks of masses m_A and m_B, connected by a massless string, are sliding to the right on a horizontal surface with no friction. Block *A* is pulled by a hand with a force $\mathbf{F}_{\text{hand}}$.

A. Draw separate free-body diagrams for each of the blocks.



B. Find the tension in the string that connects the two blocks.

$$\sum F_A = F_{applied} - T_{s\ on\ A} = m_A a \text{ and } \sum F_B = T_{s\ on B} = m_B a$$

$$\text{and } T_{s\ on\ A} = T_{s\ on\ B} = T \text{ therefore}$$

$$F_{applied} = (m_A + m_B)a \text{ so } T = {m_B\ a}\big/{(m_A + m_B)}$$

Figure 7 - 1

The NII-Tension problem, given on the first and third exams, with a model solution. The wording and formatting of the problem changed slightly from the first exam to the third exam.

All textbook homework for the class came from the Serway text.[5] For the homework entries, the chapter from which the problems came is listed as well as the number of problems from that chapter. For instance, *HW4 due, Chap 4(2), 5(6)* means that homework assignment 4 included two questions from chapter four and six questions from chapter five. Newton's laws are covered in Chapter 5 of Serway.

The topics for the tutorial entries are listed in parentheses. Because the different classes went through slightly different tutorial curricula some entries have multiple topics. For instance *TUT (NIII,Tension)* means that the students in some classes had gone through a NIII tutorial and the other students had gone through the tension tutorial. It should also be noted that the names of the tutorials provide the reader with a very limited idea about the content of the tutorial. The tension tutorial, for example, is designed to also help students with the related concepts of NII and NIII. Other tutorials try to link related concepts in a similar way.[6]

The first exam was administered following all lecture instruction and after the first homework assignment *(HW 4)* on Newton's Laws was due. In particular, one question on the homework was very similar to this exam problem. The question involved two blocks being pushed by a hand on a frictionless surface. It involved the same physical concepts except that the normal force between the blocks played the role of the tension force on the homework question. The second homework *(HW 5)* on

	Class A,B	**Class C**
Type of instruction	Date Begin - End	Date: Begin - End
Lecture (Newton's Laws)	9/18-9/30	9/22-9/29
Tutorial (Forces, FBD's)	9/29-9/30	9/29-9/30
HW4 due, Chap 4(2), 5(6)	9/30	9/29
Exam I	10/2	10/1
HW5 due, Chap 5(6), 6(2)	10/6	10/5
Tutorial (NII, NIII)	10/6-10/7	10/6-10/7
Tutorial (NIII, Tension)	10/13-10/14	10/13-10/14
Tutorial (Air Resistance)	10/20-10/21	10/20-10/21
Tutorial (Problem Solving)	10/27-10/28	10/27-10/28
Exam III	12/4	12/3

Table 7 - 1

Table showing a timeline for the section of the course covering Newton's Laws and Tension.

Newton's Laws was due after Exam 1. *HW 4* had one problem dealing explicitly with tension while *HW 5* had three problems explicitly dealing with tension. Tutorial instruction on Newton's Laws prior to the examination consisted of a single tutorial dealing with forces.

By the third exam the students had gone through a number of tutorials dealing with Newton's Laws. In particular, class A had a tutorial dealing with the subject of tension and the application of NII and NIII. The University of Washington Physics Education Group (PEG) created the bulk of the tutorials on Newton's Laws that were used in the three classes. The University of Maryland PERG created the remaining tutorials. Tutorials from the UW PEG, done by Class A, included tutorials on forces, NII and NIII, and Tension. Tutorials from the UMd PERG, done by Class A, included tutorials on air resistance and mechanics problem-solving.

The question we posed is similar to a question posed by the UW PEG. They describe some of their research on student understanding of Newton's Laws and Tension in their 1994 paper, "Research as a guide for teaching introductory mechanics: An illustration in the context of Atwood's machine."[7] They state that student difficulties with tension come from general difficulties with Newton's laws and acceleration and more specific difficulties about tension. The question shown in Figure 7 - 2 was asked by the PEG to elicit the difficulties students were having. The students who participated in this study were students at the University of Washington, who were enrolled in the calculus based physics course. The question involves two blocks connected by a string being pulled across a table by another string. The students were told that the mass of block A was less than the mass of block B and that the strings were massless. General errors documented by the PEG included "failing to (1) isolate an appropriate system, (2) identify correctly all the forces present, (3) discriminate properly between third law force pairs, and (4) recognize that it is the net force on a system that determines the acceleration."[8,9] Specific errors with the concept of tension included the idea that a string transmits a force. Students who stated that

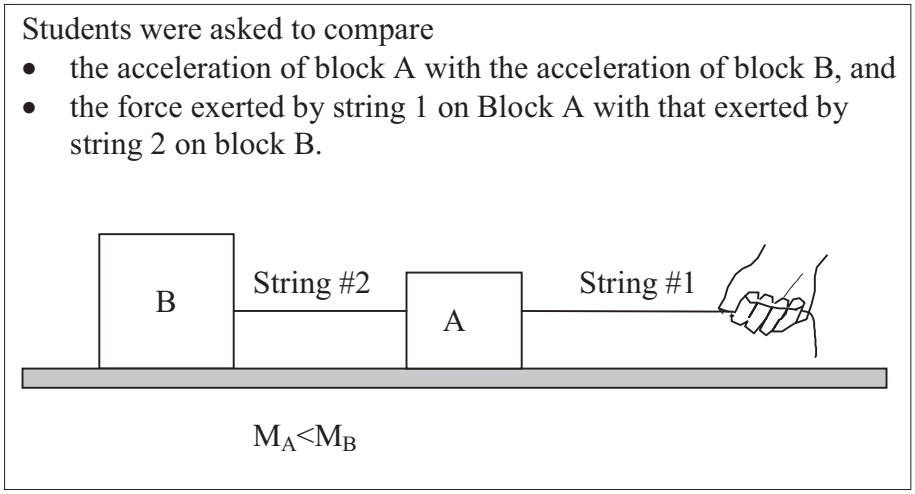


Figure 7 - 2

Question asked by the University of Washington PEG. The question is similar to the question posed in Figure 7 - 1.

the two tensions were equal (20%) seemed to have this particular belief.[10] Our data from the University of Maryland supports these results.

Pre-Tutorial responses

Student responses on version 1 of the problem, given at Maryland, showed that the students had many of the same difficulties that were reported by the Washington PEG. To get an idea of some of the difficulties the students were having we will concentrate on the responses given by Class A (N = 92). We concentrated on Class A because those students took the exam before students in class B and there were no absolute measures taken to ensure that students in class B had no prior knowledge about the exam. The data shows that these students did not connect their conceptual understanding with the quantitative calculation of the tension in the string. Performance on the qualitative part (free-body diagram) was good, but very few students answered the quantitative question correctly.

On the free-body diagrams, students had the most difficulty with the tension forces on each block. The tension force on the first block was drawn correctly by 70% of the students, while 12% of the students were missing the tension force on the first block. An additional 10% drew a line connecting the two blocks but did not include any direction for the force. (It is also possible that this line did not represent a force. It could have been included as simply part of the picture.) On the second block, 80% of the students drew the tension force correctly and 9% of the students drew a line with no direction. In addition to these errors, 4% of the students drew an extra force on their free-body diagrams. The extra forces were usually added to the 2nd blocks free-body diagram but this extra force was not labeled consistently from one student to another. The other forces that were acting on the blocks, including the weight, normal, and the applied force, were drawn correctly by over 90% of the class. Figure 7 - 3 summarizes how the students performed on the free-body diagrams. (Figure 7 - 4 shows how students performed post tutorial on the free-body diagrams. Solid vectors represent correct forces and dotted vectors represent incorrect forces.

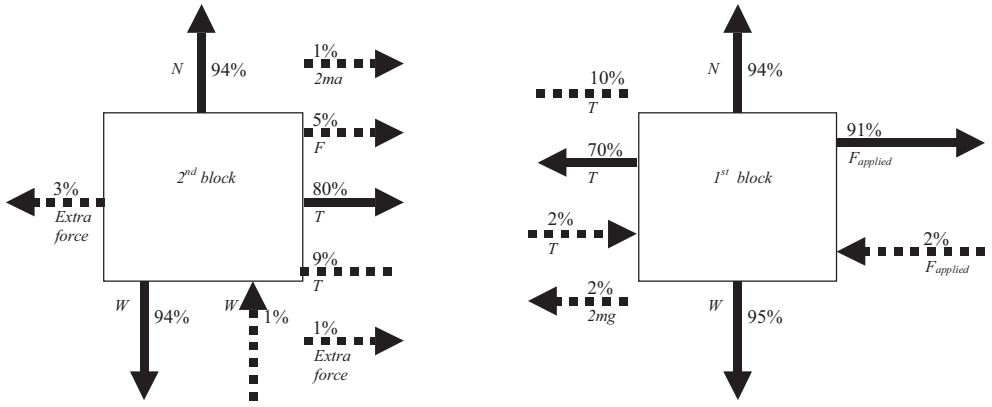


Figure 7 - 3

Student performance on the free-body diagrams for each block on Exam 1 (pre-T). Solid vectors represent correct forces and dotted vectors represent incorrect forces.

Only 7% of the students answered correctly on the quantitative question, where they are asked to calculate the tension in the string. These results show that a correct free-body diagram had little effect on how students answered the quantitative question. Possibly because of the wording in the problem, some students obtained an answer for the tension that did not include all the given quantities. We therefore created a category called correct (general) which included students who solved for the tension correctly and students who summed the forces correctly but did not solve for the tension in terms of just the given quantities (These responses were classified as "correct-unfinished.") The correct (general) category accounted for 29% of the students.

There were two profound errors that indicated that students were not attaching a correct conceptual meaning to the algebraic form of NII. The first error, "eq," involved the students setting the applied force equal to the tension force. The second error, "s," involved the students summing the forces on a particular system and setting it equal to the mass of a different system times the acceleration of that system. A student might sum the forces on the first block yet set that equal to the mass of both blocks times the acceleration.

Figure 7 - 5 shows examples of four types of responses that students gave on this problem. Many of the students answering incorrectly could not be categorized due to the nature of their responses. The first part of Table 7 - 2 shows how the students answered the quantitative question on exam 1.

Post-Tutorial instruction

Student performance on version 2 of the question, which was asked on exam 3, was better than their performance on version 1. Although performance improved on both the qualitative part, where they are asked to draw free-body diagrams for each block, and on the quantitative part, where they were asked to calculate the tension, the improvement was not dramatic. Again students performed well on the free-body diagrams. On version 2 all forces were drawn correctly by more than 90% of the students, although 4% of the students still drew an extra horizontal force. Figure 7 - 4 shows the results on the free-body diagrams. There was also improvement on the quantitative question. The second row of Table 7 - 2 shows the results for the calculation of the tension.

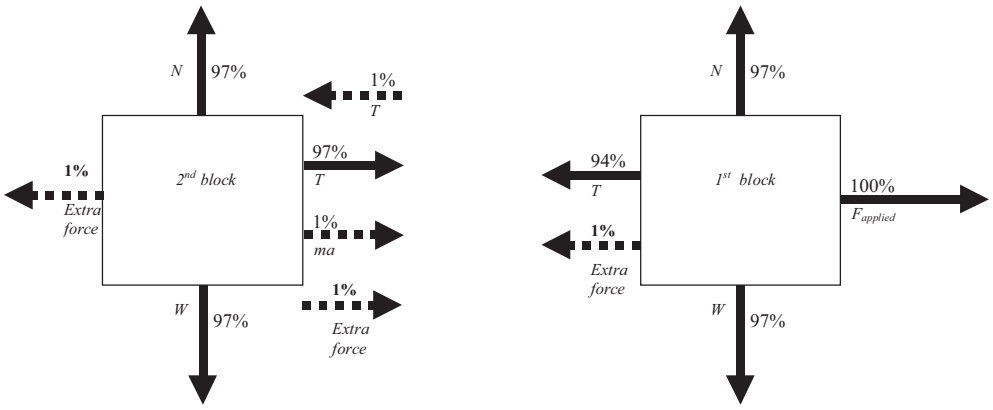


Figure 7 - 4

Student performance on the free-body diagrams for each block on exam 3 (post-T). Solid vectors represent correct forces and dotted vectors represent incorrect forces.

It is difficult to say why the students are performing better on the third exam. Besides having additional tutorials on NII the students had an additional homework assignment involving NII problems. In addition, the students were seeing the exam problem for the second time and the instructor in the course went over the first exam in Classes A and B. Each of these components most likely had some effect on how the students performed on the third exam.

We believe that seeing the problem a second time did not benefit the bulk of the class. Class C was presented this question only once, on the third exam. Their performance on this question was slightly worse than the performance from class A and Class B, but Class C performed worse on many types of measures throughout the

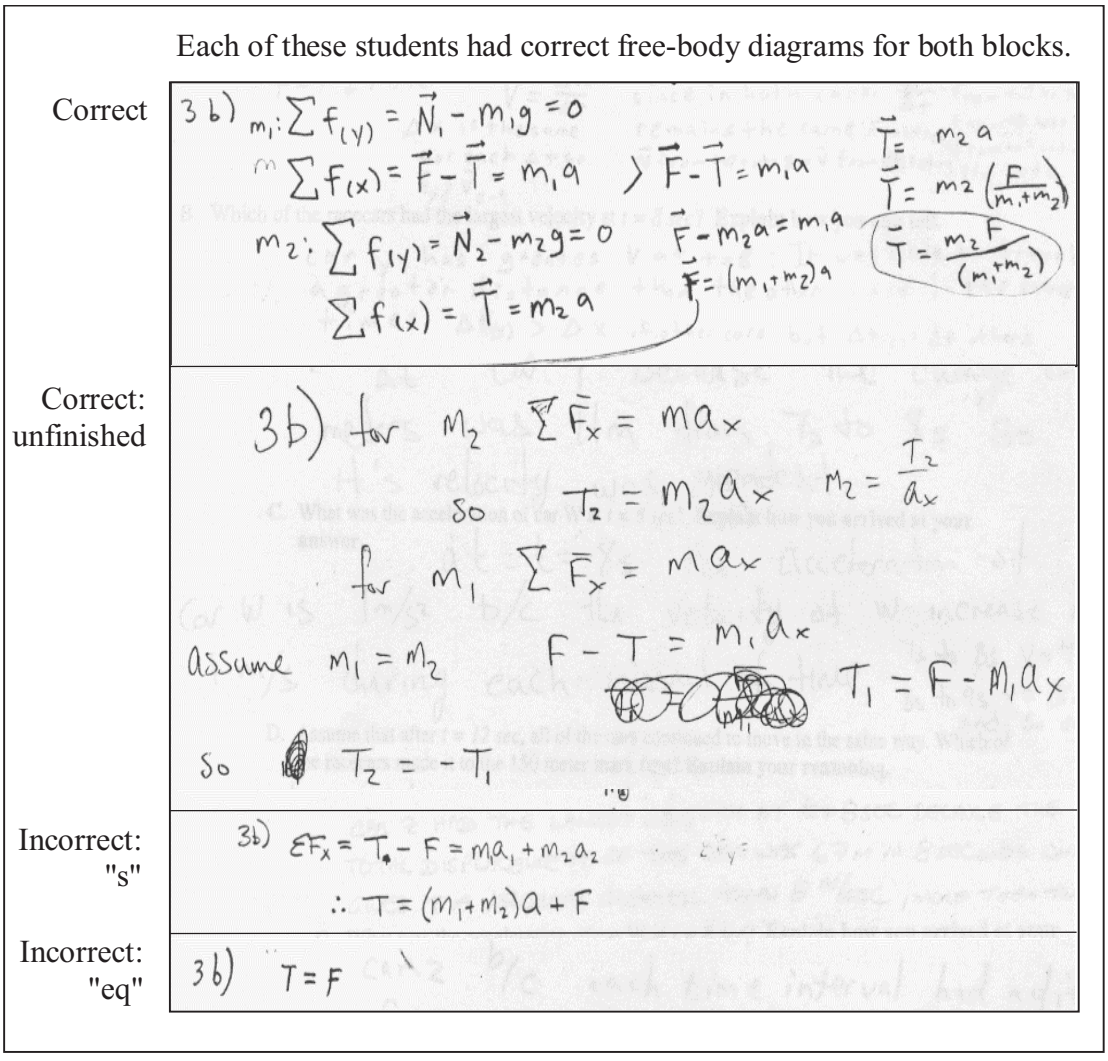


Figure 7 - 5

Examples of four different types of responses for the quantitative part of the question. Despite the different responses, each of these students had the correct free-body diagram for part (a). Note that the correct general category includes both the correct responses shown above.

semester. In particular the FCI provides a somewhat standardized measure of overall performance in the class. Students were given the FCI both before and after instruction. Students in all three classes performed about the same on the pre-test, indicating that the three populations were similar. The quantity often used to evaluate instruction is the gain on the FCI from pre-instruction to post-instruction. Classes A and B had a gain of 0.32 and class C had a gain of 0.25. So although class C performed worse on the exam question, since they performed worse on other measures also, we believe that seeing the question twice is not the main cause of the improvement we observe in class A's performance.

Calculate the tension in the string.	Correct: General	Incorrect: eq/s	Incorrect: Other	
Exam 1 (pre)	29% ± 5%	24% ± 4%	48% ± 5%	**N=92**
Exam 3 (post)	49% ± 6%	25% ± 5%	26% ± 5%	**N=76**

Table 7 - 2

Class A's performance on the quantitative part of the problem, where students are asked to calculate the tension in the string.

Case Examples

To better understand how student's progress from pre tutorial (pre-T) to post tutorial (post-T) it will be helpful to look at some sample student responses. Figure 7 - 6 shows the responses of two different students on exam 1 and exam 3. These two students show two different types of progression from the first exam to the third exam.

Student A's response on the free-body diagram is incorrect on exam 1; he neglects to include the tension force pulling back on the first block (block 1). His response on the quantitative part, on exam 1, is also incorrect; he states that the tension force is equal to the applied force. These responses also show that he is not relating the free-body diagram to the quantitative form of NII.

Student A performs much better on exam 3. He correctly sketches the free-body diagrams and seems to connect the quantitative representation of NII with the diagram.

On exam 1student B also answers the qualitative part of the question incorrectly. He makes the same error on the quantitative part that student A made. Again the qualitative representations is inconsistent with the quantitative representation.

On exam 3, student B does include correct free-body diagrams for each block. (Note that the blocks for student B are reversed compared to student A). He also seems to use NII to answer the quantitative part, by setting the force equal to the mass times the acceleration, unlike his response on exam 1 where his answer seemed to

have little to do with NII. Unfortunately this student seems to have difficulty isolating the relevant system, and he states that the tension force is equal to the applied force. This student makes the same error on the quantitative part of the question on exam 3 that he made on exam 1, despite his now correct free-body diagram.

The responses on exam 1 for the qualitative and the quantitative questions are similar for the two students. It is clear that the free-body diagrams are not being used correctly, if at all, when the students are trying to solve for the tensions.

Pre-post Progression

At this point we examine the pre-post progression of the entire class. Students' responses from exam 1 (pre-Tutorial) to exam 3 (post-T) were classified into categories representing the progression from pre-T to post-T. We did this for the students' responses on the qualitative and the quantitative parts of the question. The categories are:

- "c" meaning that the student responded correctly on both pre-T and post-T,
- "+" meaning better on post-T than pre-T,
- "0" meaning same (but incorrect) on pre-T and post-T, and
- "-" meaning worse on post-T than pre-T.

It should be noted that these categories are general. For instance, assigning a "0" to a student's quantitative response means the student answered incorrectly pre-T and post-T; it doesn't necessarily mean that the student made the same type of error both times. Classifying each type of response according to the particular answer would require too many categories and therefore make the interpretation of results very difficult.

We introduce a *pre-post progression table* to show how students perform on the qualitative part and the quantitative part of the questions both before and after the tutorial curriculum. The pre-post progression table therefore tells us about the change in the student responses. Student performance is characterized by four categories: correct, same, better, and worse. A matrix is then formed showing the dynamics of how students perform pre-tutorial (pre-T) and post-tutorial (post-T)[11] on the qualitative part and the quantitative part. This allows us to get a picture of the types of links students are developing after going through the tutorial curriculum.

The table shows how the students in the class progressed both qualitatively and quantitatively from the beginning of the course (Exam 1) to the end of the course (Exam 3). The qualitative responses are listed in the rows and the quantitative responses are listed in the columns. At this point it will be helpful to show where our case studies, shown in Figure 7 - 6, belong in the pre-post progression table.

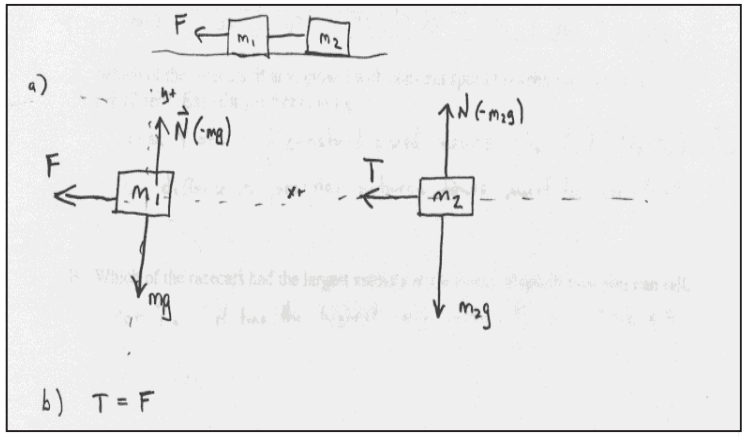
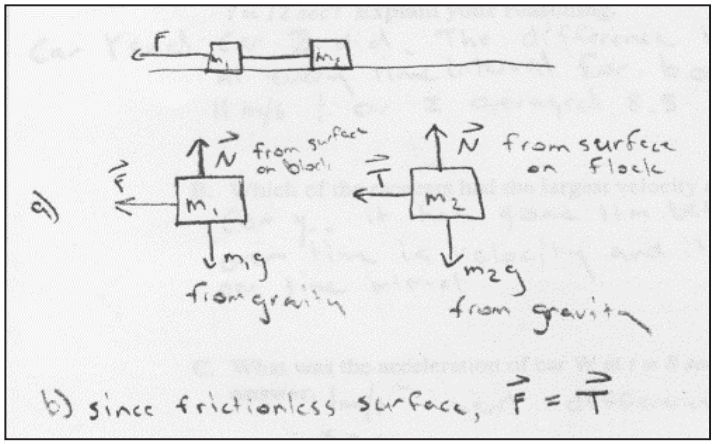
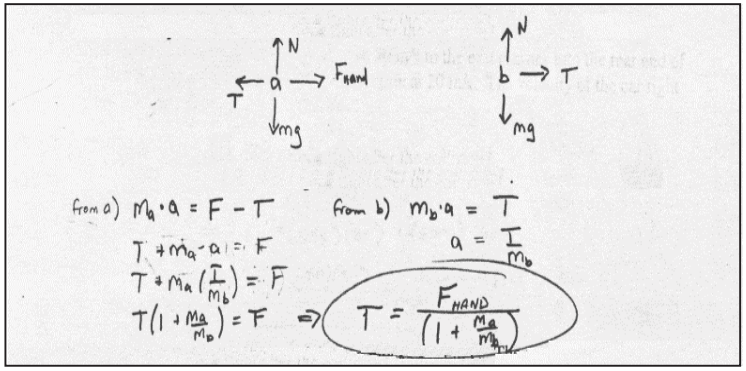
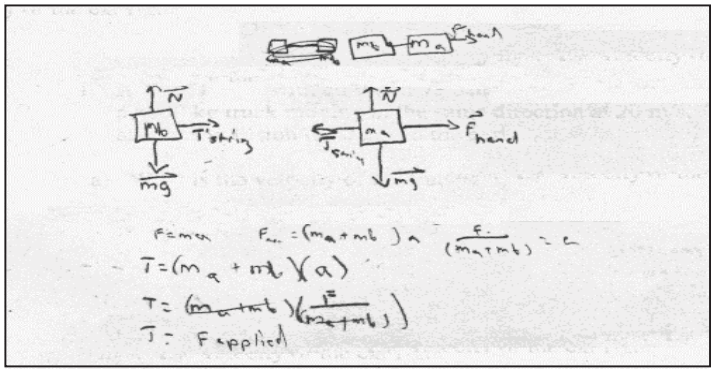
	Student A	Student B
Exam 1 Response Pre-Tutorial		
Exam 3 Response Post-Tutorial		

Figure 7 - 6

Examples of how two students responded on exam 1 and exam 3.

Student A's free-body diagrams improved from the pre-T version to the post-T version. In the pre-T version student A's free-body diagram was missing the tension force on the first block, while on the post-T version all the forces are included on the free-body diagrams. We would therefore record a "+" for the qualitative response. The quantitative response also improved. In particular we see student A correctly using the free-body diagram to obtain the algebraic form of NII on exam 3. This is coded as a "+" for the quantitative question. This student would therefore be in the "+" • "+" section of the table. The table shows that 9% of the students were in this category. Student B would instead be coded as a "+" • "0" indicating that his free-body diagram was better on post-T, but his quantitative response did not improve. On both the pre-T and post-T version the student stated that the tension is equal to the applied force. The "+ " • "0" section of the table includes 16% of the students in the class.

The table gives us an indication of how the student's qualitative schema and quantitative schema are linked. In particular the students who answered the qualitative question correctly both pre-T and post-T provide some useful insights. Despite the fact that so many students had correct free-body diagrams in both versions of the question, many students were unable to use the free-body diagrams correctly in order to solve for the tension in the string. These students had trouble making the connection between the free-body diagram representation and the quantitative representation of NII.

Pre-post progression	**Quantitative Question**				**N=76**
Qualitative Question	c	+	0	-	TOTAL
c	1%	24%	27%	5%	57%
+	1%	9%	16%	5%	31%
0	0%	0%	7%	0%	7%
-	0%	4%	1%	0%	5%
TOTAL	2%	37%	51%	10%	**100%**

Table 7 - 3

Pre-post progression table showing how students answered both the qualitative part and the quantitative part of the problem on both exam 1 and exam 3.

One would hope that conceptual exercises such as the tutorials help the students make the bridge between these two representations. Table 7 - 3 shows that some students are improving on the quantitative question. In particular, 24% of the students had the qualitative question correct both pre-T and post-T, and improved their quantitative responses on the post-T. There are a number of factors that could have led to these improvements and although this result is encouraging, we cannot say what factors were responsible for the improvements. It is possible, although unlikely, that students are simply remembering the correct answer on the first exam. Students in the "c " • "0" give us more information. These students answered the quantitative question wrong on both the pre-T and the post-T versions despite having the correct free-body diagrams on both versions. This result points out a limitation in the tutorial curriculum. The links between the qualitative representation and the quantitative representation are not being made by these students.

There were also a large number of students who improved on the qualitative part of the problem but did not improve on the quantitative part. These students are represented by the "+" • "0" and the "+" • "-" elements in Table 7 - 3. This type of progression accounted for 21% of the students in the class. This result is consistent with the results on the electric potential problem, discussed in chapter 5. In that example, many students had a good qualitative understanding of the electric field but had trouble linking it to the equation for the potential. These results show that even if qualitative understanding improves, it does not guarantee an improvement in quantitative problem-solving.

From their written responses it is hard to determine why the students performed so poorly on the quantitative question. Some of the students may be using NII haphazardly and not attaching meaning to the algebraic representation. In addition, some of the students may be attaching an incorrect qualitative understanding to either the free-body diagram or the algebraic representation. One-on-one interviews could be used to probe deeper into some of these issues.

Summary

We were able to determine the dynamics of coherence between qualitative knowledge and quantitative knowledge by looking at student responses to a question containing both qualitative and quantitative parts, both before and after a modified curriculum. We examined whether the tutorial curriculum, developed by the University of Washington PEG, helped students with the concepts of NII and tension and the application of these concepts to quantitative problem-solving.

Although the tutorial curriculum is effective in helping students obtain a qualitative understanding, there is only a small improvement in quantitative problem-solving on this problem; and this improvement may not be due entirely to the tutorial.[12]

Performance on the qualitative question was good both before and after tutorial instruction. We saw that 57% of the students answered correctly on the free-body diagram both pre-T and post-T and 88% of the students answered correctly post-T. On the quantitative question only 2% of the students answered correctly both pre-T and post-T and 39% of the students answered correctly on the post-T.

Our study shows that the tutorial curriculum is not sufficient for many of students to make the connections between their qualitative knowledge and their quantitative knowledge in this context. Many students still made errors on the quantitative part of the question, where they are asked to calculate the tension, despite having correct free-body diagrams. There are 27% of the students in the category "c" • "0" indicating that even though they had little difficulty with the free-body diagrams both pre-T and post-T they could not apply the free-body diagram correctly to solve for the tension in the string.

But some students are making the connections between their qualitative knowledge and their quantitative knowledge by the third exam. Students who have correct free-body diagrams both pre-T and post-T and show improvement on the quantitative part of the question are the strongest evidence for these links. Although most students taking exam 1 had a correct free-body diagram, they had difficulty using the free-body diagram to solve for the tension.

[1] E. F. Redish, J. M. Saul, and R. N. Steinberg, "Student expectations in introductory physics," Am. J. Phys. **66** (3), 212-224 (1998).
[2] L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).
[3] For more information on the Tutorial curriculum see chapter 3 of this dissertation and P. S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, (1993).
[4] For a research paper on student understanding of the concept of tension which includes an identical question see L. C. McDermott, P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. 62 (1) 46-55 (1994).
[5] R. A. Serway, *Physics for Scientists and Engineers*, 4th edition, (Saunders College, Philadelphia, 1996).
[6] For the actual topics see L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).
[7] See Ref. 4.
[8] See Ref. 4.
[9] Many researchers have documented these difficulties and others in introductory mechanics. The reader is referred to Ref. 4 for additional sources.
[10] For more detail see Ref. 4.
[11] Note that pre-T means that the question was asked before most of the tutorials were given. The students did have a single tutorial on forces before exam 1. See the timeline on page 160 and the context for more detail.
[12] Others have begun to study these issues also. Steve Kanim has designed supplemental homework assignments that have proven effective in helping students link the concepts from tutorial to quantitative problem-solving. See S. Kanim, "An investigation of student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999).

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Chapter 8: Student Performance on Quantitative Exam Problems in Two Instructional Modes

Introduction

In this chapter we compare performance on quantitative exam problems for students in a tutorial class and students in a recitation class. The previous chapter showed that even though almost all the students answered the qualitative question correctly on exam 3, only 49% of the students could answer the corresponding quantitative question correctly. Those results indicate that in certain contexts the tutorial curriculum does not seem to help students make the connection between their qualitative knowledge and their quantitative knowledge. In this chapter we look at performance in two new contexts: heat transfer and physical optics. While the last chapter evaluated the role of the tutorial section in problem-solving, this chapter will evaluate the role of the tutorial and the role of the traditional problem-solving recitation and compare the two.

Research results from The University of Maryland Physics Education Research Group (PERG), as well as results from the University of Washington Physics Education Group (PEG), have shown that tutorials[1] can improve student qualitative understanding of various topics.[2] But there is little published work demonstrating how students perform on more traditional type problems after going through tutorials.[3] The traditional recitation, where a teaching assistant shows students how to solve problems at the board, is usually employed to teach students how to solve problems. If these problem-solving sessions are replaced with conceptual activities such as tutorials, will the students' problem-solving ability degrade? Or will conceptually based exercises foster the development of expert-like problem-solving skills?

As we have demonstrated, students often view quantitative problems differently from qualitative problems. When given a qualitative problem in a class using tutorials, students usually apply the qualitative knowledge that they have developed. When the question is quantitative, even though the underlying physics may be the same, students do not automatically apply a qualitative analysis to the problem. We would therefore like to see whether the tutorial curriculum and the recitation curriculum help students develop coherence between qualitative knowledge and quantitative knowledge.

The two studies discussed in this chapter were conducted in the engineering physics course (Physics 262 and Physics 263) at UMd. The questions were asked as open-ended exam questions in a class with tutorials and in a class with traditional recitations. The first study presented in this chapter is on the topic of heat transfer in which the recitation students had instruction on the material and the tutorial students had none. In our second study a physical optics question was asked in the tutorial class and the recitation class. Students in the tutorial section had more instruction on the topic of physical optics than the students in recitation. We find that students in the two classes performed about the same on the heat transfer problem despite the recitation class spending more time on the material. On the physical optics question

students in the tutorial class performed significantly better than the students in the recitation class.

It should be noted that the tutorial curriculum is not designed to improve student problem-solving although it is sometimes assumed that improved problem-solving skills would be a side-effect of improved qualitative understanding. The results from the last chapter and the results from this chapter show that this is only true in certain cases. The last part of this chapter focuses on innovative curricula that use physics education research (PER) to specifically address problem-solving skills. The reform curricula that are presented in this dissertation all use PER as a guide to curriculum development.

Comparison of performance: Two examples

Heat Transfer Problem

To examine the effectiveness of the problem-solving recitation we asked a heat transfer question in the second semester engineering course (Physics 262) at the University of Maryland in two different instructional settings. One section of the Physics 262 class had tutorials, whereas the other class had a traditional recitation. Although the topic of heat transfer was lectured on in both classes and the students were assigned homework on the topic in both classes, the students in tutorial did not cover the topic in the tutorial class. In contrast, the teaching assistant[4] in the recitation class did go over many of the homework problems that were assigned in the recitation and in a review for the exam. We were able to put a heat transfer problem on the

Tipler Problem

Two metal cubes, one copper and one aluminum, with 3 cm edges are arranged as shown below. Find (a) the thermal resistence of each cube, (b) the thermal resistence of the two-cube system, (c) the thermal current I, and (d) the temperature at the interface of the two cubes.

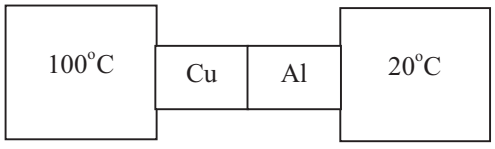


Figure 8 - 1

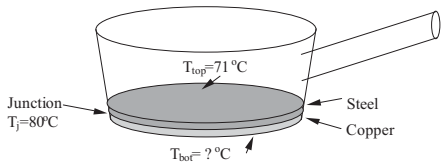
Tipler question on heat transfer asked as part of the homework assignment.

exam in both classes that was similar to a problem both sets of students were given on the homework assignment. The TA for the recitation section solved a similar homework problem in each of his recitation sections. The TA for the course estimated attendance in the recitation section to be between 70% and 80%.

The homework problem that was similar to our exam question came from Tipler's[5] introductory physics text and is shown on the previous page in Figure 8 - 1. If students were benefiting from the recitation we would expect that their performance on the exam question would be better due to the extra exposure to the subject. After the assignments were due in class, a solution to the homework problem was posted for both classes. The exam question was written by the author and is shown in Figure 8 - 2. A solution to the exam problem involves applying the same physics concepts and principles that were applied in the homework question. In addition, students can solve the exam problem by applying steps almost identical to the steps used on the homework problem. The exam problem can be solved in fewer steps if the student applies more conceptual arguments.

The correct solutions to the homework problem and the exam problem are shown in Figure 8 - 3. In the homework question the problem is broken down into four parts. Students are first asked to calculate the resistance of each block, then the equivalent resistance for the two blocks, then the thermal current that flows in the blocks, and finally the temperature at the junction. Students can apply the same four steps to calculate the temperature at the bottom of the pan in the exam problem. A

Suppose the base of a 40 cm^2 pan is made of two different materials each with thickness equal to 0.20 cm. The top side is made of steel [k = 0.46 W/cm*K)] and the bottom side is made of copper [k = 4.01 W/(cm*K)], as shown below.



a) Calculate the temperature of the bottom surface.
b) What is the thermal current through a 1 cm^2 cross section?

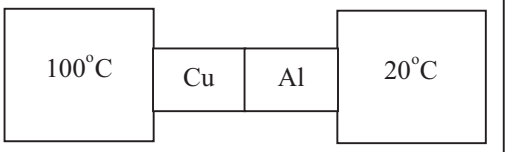
Figure 8 - 2

Heat transfer exam question.

more direct method of solution requires more of a conceptual understanding of the problem. Since the materials are in series, the currents in the two materials are the same, therefore one can obtain the temperature at the bottom of the pan in two steps. An important aspect of this research is examining the methods the students use in solving the problems.

We conducted a careful analysis of student responses after the students (N = 111 in the tutorial class and N = 73 in the recitation class) took the exams. Fewer than 50% of the students answered the exam question correctly in both the recitation and the tutorial class. Some student responses indicate that the students either used a pattern matching approach or solved the problem without a deep conceptual understanding. For example, 13% of the students in the tutorial class (and 11% of the students in the recitation class) incorrectly used the change in temperature between the top and the junction and the equivalent resistance to calculate the current through the base of the pan. In addition 5% (1%) of the students obtained a non-physical answer where the temperature at the bottom was less than the temperature of the junction. In part (b), where the students were asked to calculate the current through a cross section of the pan, 12% (14%) of the students made the mistake of calculating the thermal current through the entire base of the pan. Another common error made by 17% (11%) of the students was to find the thermal current in each material separately and then add the currents.

In part (a) of the exam problem the students are asked to calculate the temperature at the bottom of the pan, whereas in the homework problem students had to calculate the temperature at the junction. The results shown in Figure 8 - 4 show that students in the two sections performed equivalently within the uncertainty of the measurements.



(1) Calculate resistances:

$R_{Cu} = \Delta x / k_{Cu} A$ and $R_{Al} = \Delta x / k_{Al} A$

(2) Calculate R_{eq}:

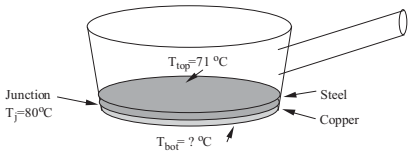
$R_{eq} = R_{Cu} + R_{Al}$

(3) Calculate I:

$I = \Delta T_{lr} / R_{eq}$

(4) Calculate T_j:

$\Delta T_{lj} = IR_{Cu}$



(a) $I_{steel} = I_{Cu}$ and $\Delta T = IR$

And since $R = \Delta x / kA$ we have

$\Delta T_{tj} / R_{steel} = I = 828 Watts$ and

$\Delta T_{jb} / R_{Cu} = I = 828 Watts$ so

$T_b = 81^o C$

(b) $828 Watts / 40 cm^2 = 20.7 Watts / cm^2$

Figure 8 - 3

Solutions to the heat transfer homework problem and the exam problem.

This result leads us to question the role of the traditional problem-solving recitation. The recitation seems to have had little effect on the students even after solving multiple problems from the chapter and solving an almost identical problem to the one asked on the exam. Although the students in the tutorial class did not have extra instruction on this material, one might hope that the emphasis on concepts in the tutorial section would improve their performance on this question, but this does not seem to be the case.

Part (b) of the exam is different from anything posed on the homework assignment. On this section of the problem, the recitation class performed slightly better, but not significantly better.

In order to get a better idea of student problem-solving ability we analyzed the responses in greater detail, looking for the methods used in solving the problem. As was mentioned earlier, a solution to the exam question could be obtained in two steps instead of the four steps that were done in the problem assigned for homework.

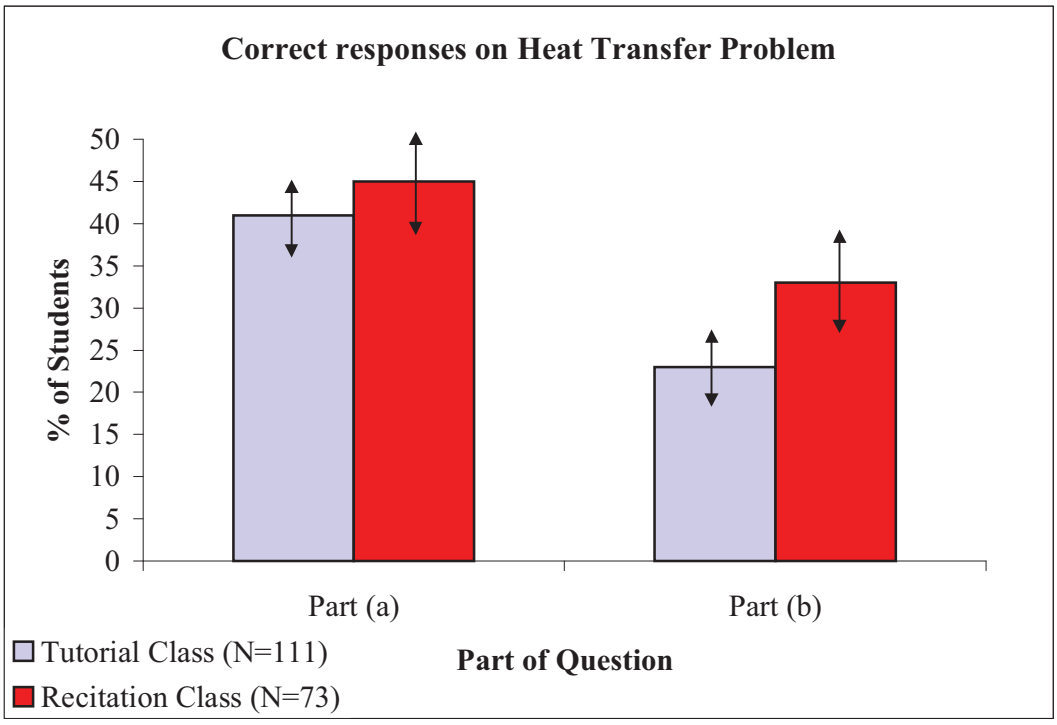


Figure 8 - 4

Performance on the heat transfer exam problem in the tutorial and recitation classes.

The two-step solution requires a qualitative understanding of the subject and also requires a conceptual understanding of current. Work that has been done concerning student understanding of electric current shows that students often lack a deep understanding of current, even after traditional instruction. Two examples of difficulties with current are that (1) students believe that current gets used up in a circuit after going through a light bulb or resistor, and that (2) a battery is a constant

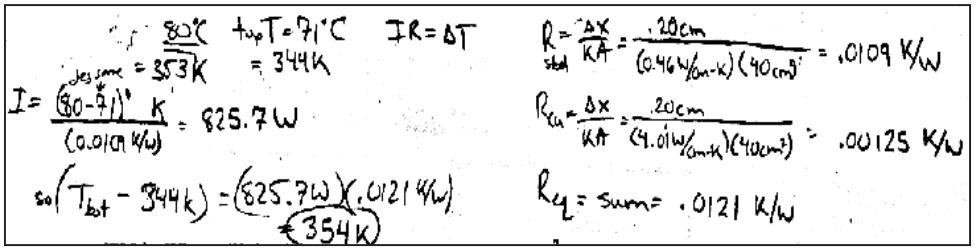
supply of current (independent of the circuit).[6] Without a good conceptual understanding of electric current it is reasonable that students will not posses a good conceptual understanding of thermal current though there is no evidence that the same or even similar misconceptions will appear.

The four-step solution indicates more of a pattern-matching approach to the exam problem. By applying each of the four steps correctly students will obtain the correct answer. Some instructors may feel that this is enough for students but one of our goals in physics education research is to give students an understanding of the materials that will allow them to think critically about a problem and allow them to transfer knowledge from one situation to another situation.

In order to compare how the students in the two classes solved the problem we classified each students' response into one of the two categories. Figure 8 - 5 shows two correct student responses.

The graph shown in Figure 8 - 6 shows the percentage of students in the tutorial section and recitation section that attempted to solve the problem by using the equivalent resistance. Note that the percentages shown are not necessarily students who answered the problem correctly. The graph indicates that although the solution using R_{eq} is more indirect, many students attempted to apply that method to solve the problem. One possible explanation for this is that students are applying a pattern matching approach to solve the problem. The graph also indicates that students in the tutorial section were less likely to apply this approach. These results may indicate that students in the recitation class are more likely to apply patterns of solution that they recall using on similar problems instead of applying the underlying concepts to the problem.

4-step solution (using R_{eq})



2-step solution (using $I_{Cu}=I_{Al}$)

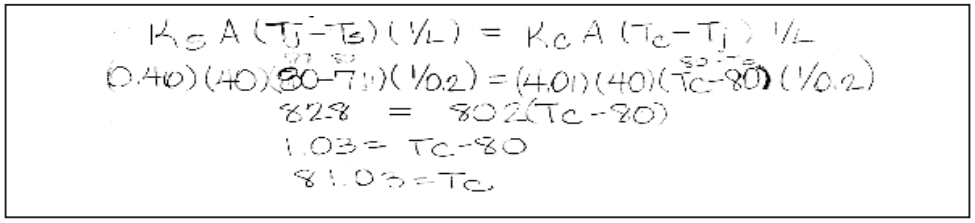


Figure 8 - 5

Two sample student responses on the exam problem. The first method of solution is more indirect and follows the same steps as the homework problem.

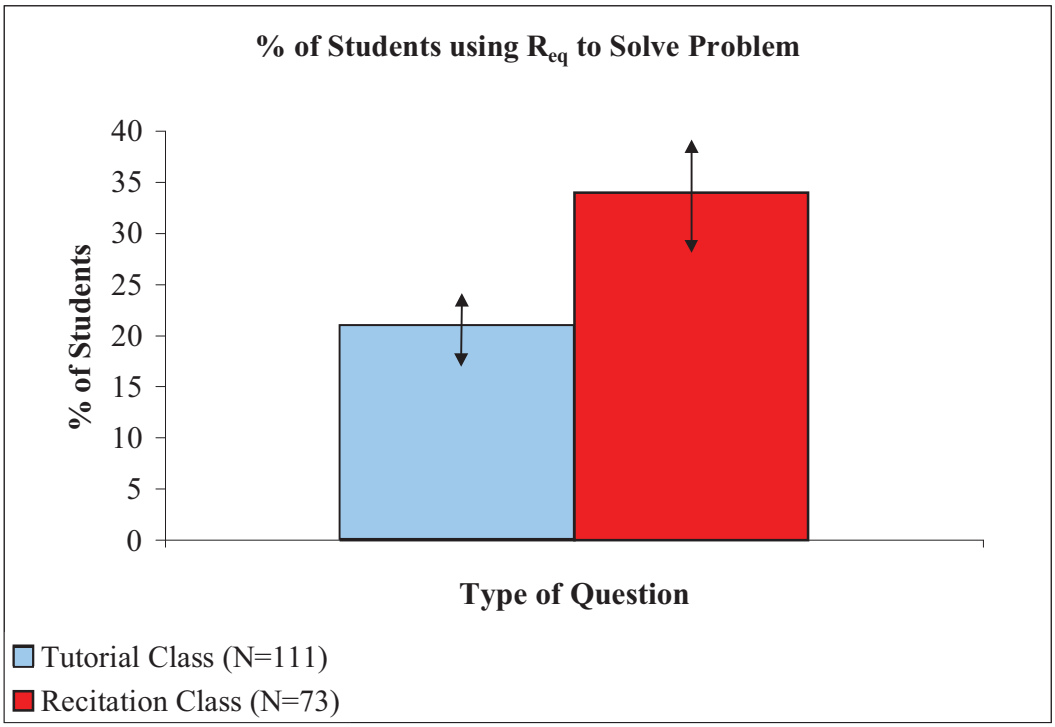


Figure 8 - 6

Graph showing how the students solved the problem in the two classes.

Physical Optics Problem

In our second study we presented the students with a problem in physical optics. In this study both the tutorial class and the recitation class had preparation on the material. The work was conducted by the PEG at the University of Washington and the PERG at the University of Maryland and has since been published.[7]

The UW-PEG has done an in-depth study of student understanding in optics, including the topics of geometric and physical optics. The physical optics paper cited above reports on a study conducted at the University of Maryland where a comparison between a tutorial class and a recitation class was done. A double-slit interference problem was posed on an exam in both classes. The first part of the problem was very similar to a traditional textbook type problem. We will focus on the student responses to that part of the problem.

The question was asked in the Physics 263 class at UMd. Physics 263 is the third semester of a three-semester sequence for engineering students. Students were given the problem shown in Figure 8 - 7 on an exam in both the class with traditional recitations and the class with tutorials.

Light with λ = 500 nm is incident on two narrow slits separated by d = 30μm. An interference pattern is observed on a screen a distance L away from the slits. The first dark fringe is found to be 1.5 cm from the central maximum. Calculate the distance, L, to the screen. Show all work.

Figure 8 - 7

Physical optics exam problem asked in a tutorial and a recitation class.

Students in the tutorial section performed significantly better than the students in the recitation class on this problem. Figure 8 - 8 shows the comparison between the two classes on this problem. The most common error made by the students in the recitation class seemed to be in applying remembered formulas in a haphazard manner. Many of the students in the recitation class used the condition for a maximum instead of a minimum, therefore obtaining an answer of 0.9 m. The students in the tutorial tended to attach more conceptual meaning to the formula for the path difference. Almost none of the students answering the question correctly in

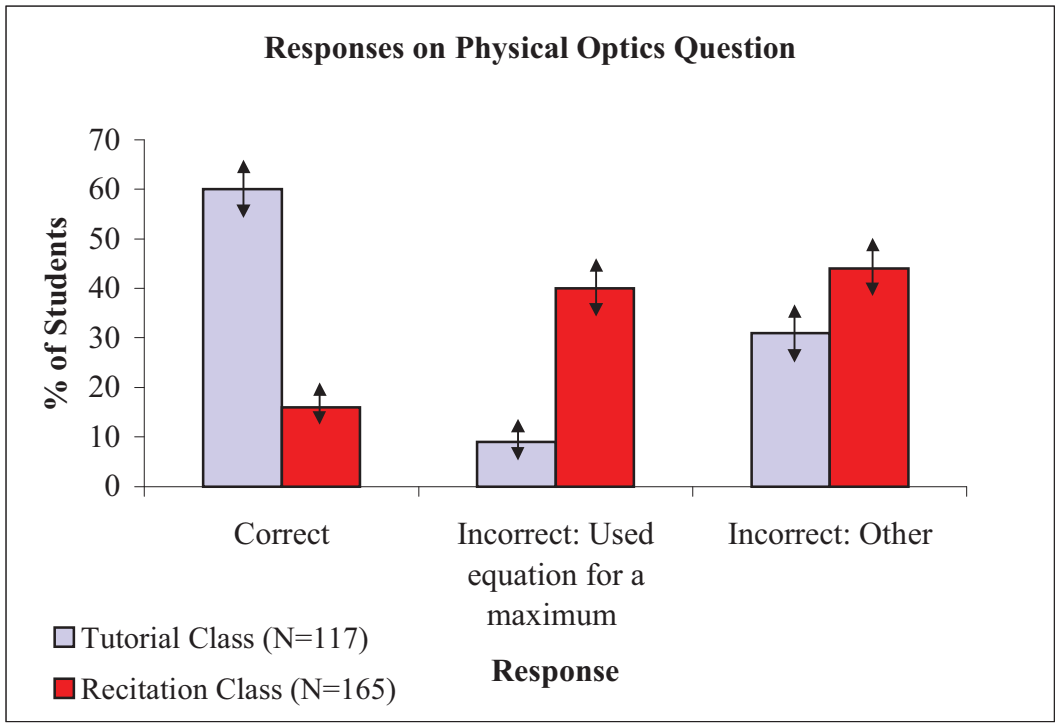


Figure 8 - 8

Performance of students on the physical optics question in the two instructional modes. Students in the tutorial class performed significantly better on this quantitative problem.

the tutorial section started with the derived formula, $L = {yd}/{n\lambda}$. Instead they began with the more fundamental equation $\left({1}/{2}\right)\lambda = d\sin\theta$ using the fact that $\Delta D = \frac{1}{2}\lambda$. This shows that there was most likely some level of qualitative reasoning in the application of the equation for the tutorial students who answered correctly. The difference in performance for the two groups indicates that increased conceptual understanding can improve quantitative problem-solving in certain contexts. For this to be the case, qualitative knowledge and quantitative knowledge must be linked.

Curricula that address problem-solving

We have shown in this chapter that tutorials can help student problem-solving. But recall that in the previous chapter there was little improvement in the quantitative performance despite a large number of tutorials on the subject of dynamics. But the emphasis of the tutorial curriculum is on conceptual understanding and not on problem-solving.

Others have focused specifically on improving student problem-solving ability. Issues related to problem-solving have been addressed by a number of physics education researchers, and are discussed in chapter 2. Innovative curricula have been implemented at a number of colleges and universities based on that research. In the rest of this chapter we will discuss some of the curricula that are currently being used to address problem-solving. The curricula we discuss are:

- Cooperative group problem-solving (UMinn)
- Qualitative strategies for problem-solving (UMass)
- Overview case study physics (OSU)
- Bridging problems and problem-solving tutorials (UMd)
- Lecture homework worksheets (UW)

A number of approaches designed to help students become better problem solvers include a set of problem-solving steps. Polya presents a general framework for solving problems in his 1945 book *How to Solve It*.[8] This framework consists of four steps, which include understanding the problem, devising a plan, carrying out a plan, and looking back. Since then, others have developed similar frameworks that range from general, like Polya's, to more specific. Reif, Larkin, and Brackett used a similar four-step process that included description, planning, implementation, and checking.[9] Reif describes a revised version of the four step process in his 1994 Milikan Lecture.[10] A schematic from the written version of the lecture has been reproduced and is shown in Figure 8 - 9. Heller, Keith and Anderson[11] and Heller and Hollabaugh[12] used a five-step approach to problem-solving in their cooperative group problem-solving.

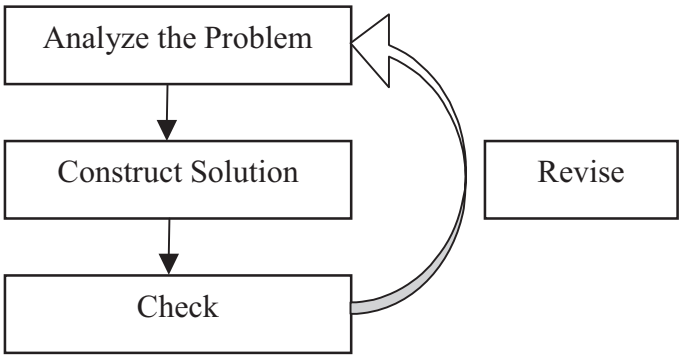


Figure 8 - 9

Problem-solving steps from Reif's Milikan lecture.

Cooperative Group Problem-solving (UMinn)

The Physics Education Research and Development Group at the University of Minnesota has implemented cooperative group problem-solving (CGPS) sessions in their introductory courses. The introductory course consists of lectures, TA-led recitations, and TA-led laboratories. The recitations and laboratories are designed to address student problem-solving. The two specific goals of the Minnesota curriculum are "(1) learn the fundamental principles and (2) learn general qualitative and quantitative problem-solving skills that they can apply to new situations."[13] Their curriculum is therefore designed not only to teach students how to solve problems but to teach students physics concepts through problem-solving. In order to achieve these goals the group at Minnesota relies on cooperative groups, problem-solving steps, guided questioning by TA's, and context rich problems.

The structure of the cooperative groups at Minnesota is similar to the structure of the groups we have discussed earlier when describing the tutorial curriculum. The group at Minnesota has investigated how to structure groups so that they function well. They found that groups of three or four students worked best. Groups of two tended to focus on only one approach to a problem solution and often had one member who would dominate the group.[14] Their study investigated the number of contributions each member of the group made, for groups of three and four students. They found that in the group of four there was usually one member of the group who did not participate as much as the others. In addition they found that the gender and the difference in the ability of the students strongly affect how a group will function. One method that is used to help a group function well is the assignment of roles. Each role has specific duties that are outlined for the students. For groups of three they used the roles of manager, skeptic, and checker/recorder. Students are asked to rotate the roles throughout the semester so each member would have an opportunity to take on each role. Students are also given 5 minutes at the end of the class to discuss how well their group functioned.[15]

The problem-solving instructional approach begins with a problem-solving strategy. This strategy is similar to the strategy described by Reif and Heller[16] and

earlier by Polya.[17] The five steps used by Minnesota are (1) visualize the problem, (2) construct a physics description, (3) plan a solution, (4) execute the plan, and (5) check and evaluate. Students are first asked to translate the problem statement into a visual and verbal understanding. This often includes a sketch, identification of the relevant information, and identification of the general approach they would apply to the situation. The physics description takes the visualization and represents it in physics terms. For instance a sketch of the situation may turn into a free body diagram. This step is similar to the step experts take in going from the naïve representation to the physical representation in the work of Larkin (described in chapter 2).[18] Once this is done students begin to plan a solution, where the physical representation is turned into a mathematical representation. At this stage students make sure they have enough information to solve the problem. Once all the information is available and the mathematical description is completed, students obtain an answer and check to see whether their answer is reasonable.

An important feature of this approach is the type of problems students will solve in the sessions. If a problem is too easy or simply requires manipulating formulas it will not foster group interaction and the general steps discussed earlier will not be important to solve the problem. For this reason the Minnesota Group has developed problems that are more challenging for the students. These *context-rich* problems have a number of characteristics that separate them from the traditional problems usually found in textbooks. (1) The problem statement will not always identify the unknown variable; (2) the problem may include information that is not important to the solution; (3) the problem may require the student to make estimations; and (4) the problem may require the student to make reasonable assumptions. These characteristics tend to make the context-rich problems too difficult for an individual student to solve, but appropriate for the three or four member groups.[19]

Graded questions provided the students with motivation for participating and working earnestly in the groups. Students are evaluated by group performance as well as individual performance. During each class test, each group submits one solution and each member of the group receives the same grade for that solution. A group is given credit for using the problem-solving steps as well as for the correctness of the solution. The individual component insures that students can not simply sit back and allow the rest of their group to do all the work. It also aids the instructor in identifying students that need additional help with the material. One way individuals were held accountable is that students are selected at random to present their group's solutions to various problems.

Heller et al. performed three types of studies to evaluate the CGPS.[20] The first study looked at whether the cooperative group's solutions were better than the individual solutions of the top scoring students. In the second study they looked at the dynamics of student problem-solving performance to see whether individual problem-solving performance improved over time. In the third study they compared the performance of students in the CGPS sections to students in a traditional section on standard problems.

They based their evaluation of problem solutions on six criteria: evidence of conceptual understanding, usefulness of description, match of equations with

description, reasonable plan, logical progression, and appropriate mathematics. This scoring criteria was checked for reliability and validity.

To compare the solutions of the group to the solutions of the top individuals Heller et al. used matched problems. Students first solved a context rich problem in a group. Individual students were then given a similar problem that was less difficult than the group problem. Group solutions were then compared to the solutions of the individuals from each group who scored the highest on the individual exams and the final exam problems. Scores on the group solutions and the individual solutions showed that the performance of the group surpassed the performance of the individuals, despite the fact that the individual problem was easier than the group problem and the fact that the group problem was completed first. This result led Heller et al. to conclude that the group work is not simply the work of the best problem solvers in the class.[21]

Heller et al. also looked at the quality of solutions on the least difficult individual problems throughout the course. The class was divided into the top third, middle third, and bottom third, based on total individual grades on the tests and final exams. They found that all students benefited from the CGPS on all six criteria except conceptual understanding, where there was no appreciable gain.[22] Saul has reported that performance on the FCI in the CGPS class was comparable to performance in a tutorial class but better than performance in a traditional recitation.[23]

The final investigation compared performance of students in the CGPS class with performance of students in a traditional class on standard problems. (Instructors in the standard class judged the context rich problems to be too difficult for their students.)[24] Students in the CGPS class scored significantly better on the criteria outlined by Heller et al. The biggest different was in the qualitative analysis of the problem. All CGPS students drew free-body diagrams compared to 57% in the traditional class. Heller et al. therefore conclude that students in the CGPS curriculum exhibit more expert-like characteristics in their problem solutions.[25]

Qualitative Strategies for Problem-solving (UMass)

The Physics Education Research Group at the University of Massachusetts, Amherst has developed curricula which encourage the application of qualitative analysis to quantitative problems. In 1991, Dufresne, Gerace, Hardiman, and Mestre presented questions to students using a computer-based environment called the Hierarchical Analysis Tool, or HAT. HAT begins by asking the student to select a principle that could be applied to a problem. HAT then proceeds to ask additional questions that are more specific, therefore employing a top down approach to problem-solving. After HAT completes an analysis, based on the user's input, it provides a set of equations that are consistent with the user's input. These equations may therefore be inappropriate for solving the particular problem. HAT's analysis is independent of the content of problem. It merely asks a series of general questions which aid the user in adopting a top-down approach to problem-solving.

Evaluation of HAT was done using forty-two undergraduate students. Each student took part in eight problem-solving interview sessions. The forty-two students were divided into three groups each having different instruction. The first group used HAT for a qualitative analysis before solving the problems, the second group was

given a textbook they could use to solve the problems and the third group used an Equation Sorting Tool (EST) which contained a database of 178 equations that could be searched. Students who used the HAT material tended to categorize problems by principles more often than the other groups. Dufresne et al. also concluded that HAT was more effective in improving student problem-solving than traditional methods.[26]

Based on earlier research, Leonard, Dufresne, and Mestre discussed a problem-solving curriculum that could be implemented on a large scale.[27] Their work describes how qualitative strategies could be used to highlight the role of conceptual knowledge in solving problems. They describe a strategy that can be employed in large introductory physics classes without making large changes to the course. In solving problems, they stress the use of identifying the concept or principle, justifying why the principle is appropriate, and describing the procedure by which the principle or concept could be used. Although the authors claim that a large restructuring of the course is not required, they do make some significant revisions in the course. In particular, one sample problem is worked out during each lecture. Students were also encouraged to apply strategies in every problem they attempt in the course.

Leonard et al. performed two types of studies to evaluate the effectiveness of the modified instruction. They looked at categorization tasks and students' recall of important ideas.

In the categorization tasks they administered five multiple-choice questions on a final exam. Students were given a problem and asked to select the major principles (from five choices) that should be used to solve the problem. Their results show that the students in the modified curriculum were less inclined to choose principles based on the surface features of the problem.[28] These results were true for all students in the modified curriculum.

The recall task occurred six months after students in the traditional class had completed the course and 11 months after students in the modified class had completed the course. The students were "asked to name the most important physics ideas and principles used to solve problems in mechanics."[29] Both populations of students were found to identify Newton's three laws with about the same frequency but the students in the modified curriculum cited the remaining four principles (conservation of energy, work-energy theorem, linear momentum conservation, and angular momentum conservation) at a consistently higher frequency than the other students. The authors caution that this data does not mean that the students who went through modified curriculum knew more than the students in the traditional class. But the data does suggest that focusing attention on the principles does help students retain the major ideas.

The researchers found that the modified curriculum they presented did help students improve their explanations. In addition, the grading of strategies helped the instructors become aware of how well students understood the ideas in the course.[30,31]

Overview, Case Study Physics

Alan Van Heuvelen at the Ohio State University is the project director of the Overview Case Study (OCS) curriculum. The OCS curriculum is designed to help students develop coherent knowledge by dividing the physics course into conceptual blocks. Students first qualitatively construct the basic ideas of a particular block.[32]

They then learn to build mathematical representations and then they apply their knowledge to complex case study problems. OCS tries to first lay the foundation and frame and then incorporate the details.[33] In addition, students are active participants in constructing this knowledge.

The frame and foundation are the overview, where the students are qualitatively constructing the concepts in a particular block. The details start getting filled in during the exposition, where the student translates the concepts into mathematical form. At this stage students are also investigating different representations for the problems, such as pictorial or physical representations. The next stage is the case study, where students integrate a number of concepts that were introduced earlier, in order to solve problems.[34] A case study can last between several days and several weeks. At the end of the semester a week-long review is conducted in which students are shown a hierarchical chart of the different blocks of knowledge. Throughout the semester students utilize a set of Active Learning Problem Sheets (ALPS). ALPS are designed to make the lecture parts of the course more interactive. The students complete an activity and then talk to their neighbors about their reasoning.

We will discuss two examples of the type of evaluation done on the OCS curriculum at New Mexico State University.[35] In the first study they compared the performance of students who went through the OCS curriculum to students who went through the conventionally taught curriculum. Students in the OCS curriculum used only one overview lasting about two weeks that covered material on NII and work-energy concepts. Students were evaluated on responses to quantitative problems. In the second study they compared the same types of populations (different students) on qualitative questions. Students in the OCS curriculum used the first edition of the OCS study guide and a standard calculus text. They compared student responses on questions involving NII (24 questions) given as a pretest and a posttest.

Analysis of the quantitative problems asked on the final exam showed that OCS students performed better than the students in the conventionally taught class. 51% of the OCS students answered correctly while only 13% of the conventionally taught students answered correctly on the dynamics problem. In addition 94% of the students in the OCS class included a free-body diagram in their solutions compared to only 9% including the diagram for the conventionally taught population. Van Heuvelen states that the conventionally taught students tried to solve the problems by applying special memorized equations rather than applying the fundamental concept to the problem.[36]

On the qualitative questions about NII both populations scored about the same on the pretest. The conventionally taught students took the posttest at the end of the semester, while the OCS students took the posttest after NII had been covered. Conventionally taught students had a posttest score of 53% while OCS students had a posttest score of 73%. On an additional posttest given at the end of the semester to the OCS students, which included slightly more difficult questions, the OCS students had a score of 86%.[37]

Research results show that the OCS curriculum does help students develop better qualitative skills and quantitative skills.[38] The integration of qualitative representations into quantitative problem-solving was shown to improve performance

on qualitative questions. As we saw in the work done by the group at Minnesota, curricula that have a strong problem-solving component can be used to improve qualitative understanding as well as quantitative problem-solving.

Bridging Problems and Problem-Solving Tutorials (UMd)

Our results show that concept-based curricula like the UW Tutorials can sometimes improve students' quantitative problem-solving. The results also indicate that this improvement depends on the context and the topic. Although this chapter provided results showing that tutorials did play a role in improving student problem-solving the last chapter showed that the tutorials had little effect on problem-solving ability. In addition we have observed that when asked qualitative questions prior to a quantitative question students sometimes performed worse. The results point out some limitations in the tutorial curriculum.

The nature of the tutorial curriculum makes it difficult for the students to see the connection between the concepts they are developing and the quantitative questions they are given on textbook homework assignments and exams. To aid the students in making the link between their qualitative knowledge and their quantitative knowledge, the PERG at the University of Maryland has been developing materials to try to address some of these transfer issues. Two methods we have developed are *bridging problems* that supplement the tutorial homework assignments and *problem-solving (PS) tutorials*.

Bridging problems are problems that contain both qualitative and quantitative parts. The bridging problems are discussed briefly in chapters 3 and 5 of this dissertation. Students are asked to answer a series of qualitative questions before a final quantitative question. Success on the quantitative part of the problem requires a deep conceptual understanding of the material. By asking qualitative questions along with the quantitative question, we are attempting to foster the acquisition of links between students' qualitative and quantitative schema. Bridging problems are given as supplements to the tutorial homework each week. They involve ideas and concepts that were covered in the tutorial for that week. Students are provided with solutions to the problems after they are due. The bridging problems I have been involved in constructing are included in Appendix A.

Problem-solving (PS) tutorials were designed by the PERG at the University of Maryland to meet goals similar to the goals of the bridging problems. Problem-solving tutorials are implemented in the same way the University of Washington-style tutorials are implemented.[39] Students work in groups of three or four on the PS-tutorials with two facilitators in the room asking the students guided questions. The PS-tutorials do not concentrate on a particular subject. Instead, they include problems based on the material in preceding tutorials. Three or four problems are constructed that require a deep qualitative understanding of the material. The problems are similar to the bridging problems in that the students are asked qualitative questions before they are asked to answer the quantitative part of the problem. Although most of the problems on the PS-tutorials were designed by the PERG at UMd, some were adopted from other sources.[40] The PS-solving tutorials I have been involved in constructing are included in Appendix B.

These curricula are small perturbations to the course, therefore their evaluation is difficult. Only one or two of the thirteen tutorials in a given semester are problem-solving tutorials. In addition, only one of the problems out of about nine problems the students solve on homework assignments each week are bridging problems. In order to have an effect on creating links between qualitative knowledge and quantitative problem-solving we expect that bridging problems and problem-solving tutorials must be implemented on a much larger scale. Unfortunately, due to time constraints and logistic constraints, a larger scale implementation was impossible at the University of Maryland.

Lecture Homework Worksheets (UW)

Steve Kanim and the University of Washington Physics Education Group (PEG) have been developing *lecture homework worksheets* to supplement the textbook homework. These worksheets are designed to address the difficulties students had in applying conceptual models to standard problems.[41] The worksheets attempt to accomplish goals that are similar to the goals of the bridging problems[42] designed by the PERG at Maryland. The homework worksheets are assigned weekly, consist of about four pages, and include both qualitative and quantitative questions. Because of this supplemental homework, instructors in the course decrease the amount of textbook homework assigned.

Kanim has seen gains in performance on quantitative circuits problems for students in the upper half of the class. On paired qualitative and quantitative questions, his research shows that the performance on the qualitative questions remains about the same yet the performance on the quantitative question improves for these students.

The performance on quantitative problems in electrostatics and Gauss's Law did not improve with the homework worksheets. Research into why students were not performing better on quantitative problems in electrostatics indicated that students had conceptual difficulties that had not previously been addressed.

Common Elements

There are a number of elements that these curricula have in common that separate them from the traditional physics curriculum. Students in these instructional settings are actively participating in constructing their own knowledge. In the traditional course students often sit passively watching instructors solve problems for them. Even when instructors attempt to make the class interactive by asking questions, only a small number of students participate in the discussion. In addition, the students that participate are usually the students that already know the material. These research-based curricula described above also rely on the students solving problems that require them to use concepts and principles and not simply facts and formulas.

As stated by Dufesne et al. traditional textbook problems often foster the use of

> memory taxing strategies . . . to solve problems. Such strategies use much of the novice's available working memory to attend to the details of executing a problem's solution . . . there is little working memory left to extract from a solution generalizable principles and procedures.[43]

Ward and Sweller, [44] as cited by Maloney,[45] suggest that standard end of the chapter textbook problems can actually be counterproductive. Our results, presented in this chapter, suggest that a traditional problem-solving recitation where students watch an instructor solve problems at the board may actually encourage students to apply a pattern matching approach. Students sometimes think less about the underlying physics and more about how similar a problem is to problems they have already solved. My own experience in graduate school suggests that many physics graduate students engage in similar activities. For instance, many students perform well on the physics qualifier by solving many older problems, and being able to recognize similarities between different questions. Although this may be a good method to perform well on a test it is not necessarily the best way to learn physics.

Summary

In this chapter we compared responses given by students in two instructional modes. The analyses of the responses to the two exam problems presented in this chapter suggest some tentative conclusions. We have seen that the traditional problem-solving recitation that is common at most large universities did not help students on the problems presented in this chapter. An identical heat transfer question was given, on an exam, to students in a physics 262 class with tutorials and a class with traditional recitations. Even though students had additional instruction on heat transfer in the recitation class, and solved a similar problem in the recitation section, they performed no better than the tutorial class, which received no additional instruction on heat transfer.

The recitation section seems to help the students apply pattern-matching approaches to problems. This encourages students to simply look at the surface features of a problem and not the underlying principles and concepts. The data from the heat transfer problem also shows that simply using tutorials to develop conceptual understanding does not mean students will develop more expert-like characteristics; i.e. think qualitatively about quantitative problems.

When students have explicit instruction on a particular topic in the tutorial class it is possible that they will perform better on quantitative problems than students who go through a traditional recitation class. This was true in the physical optics question. Students in the tutorial class performed much better than students in the recitation class. It seemed as though students in the tutorial class were applying the concepts more often than the students in the recitation class. Unfortunately, this improvement for tutorial students, in applying the concepts, does not happen in topics throughout the introductory sequence (as we saw in chapter 7.) It therefore depends on the type of question.

The tutorial curriculum is not designed to address student problem-solving. The PERG at the University of Maryland has supplemented the tutorial curriculum with bridging problems and problem-solving tutorials to help students develop

coherence in their qualitative and quantitative content knowledge. Because these supplements were implemented on such a small scale it was difficult to evaluate their effectiveness.

Other curricula have been developed, using PER as a guideline, which place a much greater emphasis on student problem-solving. Three well-established curricula which teach problem-solving and use problem-solving to teach concepts are discussed in some detail in this chapter. Each curriculum shares a number of common elements including a strong research base, students actively participating in constructing their knowledge, and the use of problems which require and encourage the use of conceptual understanding. Research on the effectiveness of these curricula show that they can be effective in helping students improve their problem-solving ability.

[1] The tutorial curriculum is discussed in chapter 3.
[2] See L.C. McDermott, and the Physics Education Group, A perspective on physics education research as a guide to the improvement of instruction, unpublished collection, (1998) and E.F. Redish, J.M. Saul, and R.N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **66** (3) 212-224 (1998).
[3] S. Kanim, "An investigation of Student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999); B. Thacker, E. Kim, K. Trefz, S. Lea, "Comparing problem-solving performance of physics students in inquiry-based and traditional introductory physics courses," American Journal of Physics 62 (7), 627-633 (1991); and B. A. Ambrose, P. S. Shaffer, R. N. Steinberg, L. C. McDermott, "An investigation of Student Understanding of Single Slit Diffraction and Double Slit Interference," Am. J. Phys., **67** (2) 146-155 (1999).
[4] There was a single TA for the recitation class.
[5] P. Tipler, Physics for Scientists and Engineers, 4th edition, (W. H. Freeman and Co., NY, 1999).
[6] For more detail see L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993).
[7] B. A. Ambrose, P. S. Shaffer, R. N. Steinberg, L. C. McDermott, "An investigation of Student Understanding of Single Slit Diffraction and Double Slit Interference," Am. J. Phys., **67** (2) 146-155 (1999).
[8] G. Polya, *How to solve it*, (Doubleday, NY, 1945).
[9] F. Reif, J.H. Larkin, and G.C. Brackett, "Teaching general learning and problem-solving skills," Am. J. Phys. **44** (3) 212-217 (1976).
[10] F. Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," Am. J. Phys. **63**, 17-32 (1995).
[11] P. Heller, R. Keith, and S. Anderson, "Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving," Am. J. Phys. **60** (7), 627-636 (1992).
[12] P. Heller and M. Hollabaugh, "Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups," Am. J. Phys. **60** (7), 637-644 (1992).
[13] from their web site; see http://www.physics.umn.edu/groups/physed/Research/MNModel/MMt.html
[14] See ref. 11. (Heller, Keith, and Anderson)
[15] See ref. 12. (Heller and Hollabaugh)
[16] See F. Reif and J. I. Heller, "Knowledge structures and problem solving in physics," Educational Psychologist, **17** (2), 102-127 (1982)
[17] G. Polya, *How to solve it*, (Doubleday, NY, 1945).
[18] J.H. Larkin, "The role of problem representation in physics." In D. Gentner and A. L. Stevens (Eds.), *Mental models*, (Lawrence Erlbaum, NJ, 1983), pp. 75-98

[19] See Ref. 12.
[20] See Ref. 11.
[21] See Ref. 11.
[22] See Ref. 11.
[23] J. M. Saul, "Beyond Problem Solving: Evaluating introductory physics courses through the hidden curriculum," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).
[24] See Ref. 11.
[25] See Ref. 11.
[26] R. J. Dufresne, W. J. Gerace, P. T., Hardiman, and J. P. Mestre, "Constraining novices to perform expert-like problem analyses: Effects on schema acquisition," Journal of the leaning Sciences, **2** (3) 307-331 (1992).
[27] W. J. Leonard, R. J. Dufresne, and J. P. Mestre, "Using Qualitative Problem-Solving Strategies to Highlight the role of conceptual knowledge in solving problems," Am. J. Phys., **64** (12), 1495-1503 (1996).
[28] See Ref. 27.
[29] See Ref. 27.
[30] See Ref. 27.
[31] For more information see the group's web site at http://www-perg.phast.umass.edu/. They have developed a course at the high school level which ties together many of these ideas called *Minds on Physics*. Kendall/Hunt publishes the curriculum.
[32] A. Van Heuvelen, "Overview, Case Study Physics," Am. J. Phys. **59** (10), 898-907 (1991)
[33] See ref. 32 in which Van Heuvelen makes a reference to a statement made by Steve Kanim. The statement compare physics instruction to building a house room by room versus foundation and frame first - then the details.
[34] See Ref. 32.
[35] See Ref. 32.
[36] See Ref. 32.
[37] See Ref. 32.
[38] See Ref. 32.
[39] Implementation of tutorials is discussed in detail in chapter 3 and in P. S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, (1993).
[40] A. B. Arons, *Teaching introductory physics* (Wiley, New York, 1997).
[41] S. Kanim, "An investigation of student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999).
[42] Bridging Problems are discussed in chapter 3
[43] See Ref. 26.
[44] D. S. Ward, and J. Sweller, "Structuring effective worked examples," Cognition and Instruction. **7,** 1-39 (1990).
[45] D. P. Maloney, "Rule-governed approaches to physics: Newton's Third Law," Phys. Educ. **19**, 37-42 (1984).

Chapter 9: Summary and Speculations for the Future

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Chapter 9: Summary and Speculations for the Future

Through a series of studies on student problem-solving we provide evidence that students solve problems using locally coherent sets of knowledge we call schemas. Activating these schemas bring a set of declarative knowledge and procedural rules that students use to accomplish the goal of a problem. Unfortunately the schemas our students activate contain small sets of knowledge, and are isolated from other schemas. If the schema they activate is not sufficient for solving the problem many of our students have difficulty accessing the relevant knowledge.

Specific Examples

In our first study we compare student responses on open-ended exam problems and responses on corresponding multiple choice questions. Results show that even though the corresponding questions test identical physics concepts, students often answered differently on the exam problem and the multiple-choice question. For instance on a question asking students to compare the magnitudes of the two forces acting on object moving at constant speed, 90% of the students answer correctly on the open-ended problem, yet only 54% answer correctly on the corresponding multiple choice question. (This is shown in Figure 9 - 1.) Each question triggers a different set of physics knowledge, which in this case, are inconsistent with each other.

Summary Example

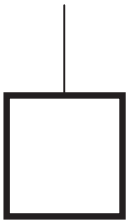
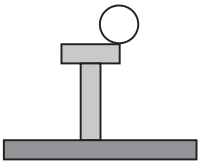
Elevator moving down at constant speed. (Real-world context)	Steel ball moving up at constant speed. (Physics context)
Multiple-choice question	Open ended problem
	
54% correctly state that the force of gravity is equal to the force from the rope.	**90%** correctly state that the force of gravity is equal to the force from the platform.

Figure 9 - 1

Example from chapter 4.

Our next study involved responses to a dynamics and work-energy problem. In this problem students had to use knowledge from multiple topics and both qualitative and quantitative understanding. Interviews with advanced students and undergraduate students in engineering physics showed that students had difficulty activating knowledge from work-energy. Most students only used dynamics knowledge to solve the problem. When they were stuck, they were rarely able to activate the relevant schema to solve the problem. Qualitative questions designed to aid the students in solving the problem actually caused many of the students to perform worse because they were less likely to bring up the ideas of work and energy. On the version of the problem with the qualitative questions 15% of the students answered correctly, while 30% of the students correctly answered the version without the qualitative questions. (This is shown in Figure 9 - 2.)

Summary Example

A hand applies a constant force to a block along a surface with friction and a surface without friction. Calculate the coefficient of kinetic friction. (paraphrased)

Version with no qualitative parts	Version with qualitative parts
30% answer correctly • **19%** used work-energy • **11%** used other methods	**15%** answer correctly • **4%** used work-energy • **11%** used other methods

Figure 9 - 2

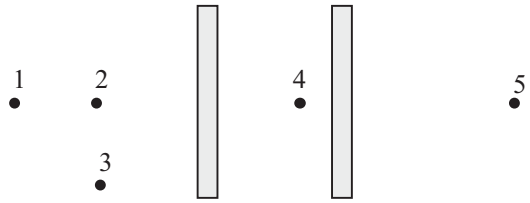
Example from Chapter 5.

Student responses to open-ended exam problems indicated that students often respond to qualitative questions differently from the way they respond to quantitative questions. The students rarely integrated their qualitative and quantitative knowledge. We have provided results from the three semesters of the engineering physics sequence showing that even in a single problem, some students would give contradictory answers to qualitative questions and quantitative questions. Sometimes students are successful on the quantitative questions and other times they are successful on the qualitative questions. For instance on a problem asking students to calculate the potential between two parallel plates 27% of the students treated the

electric field as constant in the equation $\int_1^5 E \cdot dl = -V$ even though over three-quarters of those students drew electric field vectors that were not constant in the three regions (This is shown in Figure 9 - 3.)

Summary Example

Two large conducting plates, each with a charge +Q, are places as shown.
Draw electric field vectors.
Calculate the potential from point 1 to point 5.



27% of the students treated the E field to be constant in $\int_1^5 E \cdot dl = -V$. Most of these students (21% out of the 27%) correctly stated that the field at point 4 was zero.

Figure 9 - 3

Example from Chapter 6.

The tutorial curriculum designed by the University of Washington Physics Education Group (PEG) has been shown to be effective in promoting qualitative understanding of various topics in physics. But there is little work documenting how students perform on traditional quantitative problems after going through the Tutorials. Our results show that after going through the tutorial curriculum performance on traditional problems improves in certain cases, but not others. In the first case we see only modest improvements in a quantitative problem about NII and tension. These results point out some of the limitations in solely relying on the tutorial curriculum; students are still not connecting their qualitative understanding to quantitative problem-solving. In another example involving a quantitative exam problem on physical optics, students who went through the tutorial curriculum performed much better than students enrolled in a class with problem-solving recitations. We observed that 60% of the students answered correctly after tutorial instruction and 16% answered correctly after a traditional problem-solving recitation. (This is shown in Figure 9 - 4.)

Summary Example

Light with λ = 500 nm is incident on 2 narrow slits. An interference pattern is observed on a distant screen. The first dark fringe is located 1.5 cm from the central maximum. Find L. (paraphrased)

	Tutorial	Recitation
Correct response	**60%**	**16%**
Used formula for maximum	**9%**	**40%**
Other	**31%**	**44%**

Figure 9 - 4

Example from chapter 7.

Summary

We have observed that our students tend to activate particular schemas in particular contexts. Often these schemas are centered around isolated topics such as in the example with dynamics and work-energy. Students in the introductory calculus-based physics course only sometimes activate schemas containing integrated topic knowledge. In addition students often view questions and problems as either qualitative or quantitative. They therefore activate qualitative schemas for qualitative questions and quantitative schemas for quantitative questions. In a single problem we have observed students directly contradicting their responses on the qualitative parts with their responses on the quantitative parts.

Evaluation of students' understanding in the introductory class often focuses on evaluating knowledge on a particular topic (for example, with end of chapter problems). Exam and homework questions are usually either quantitative or qualitative and rarely ask students to integrate their qualitative and their quantitative knowledge. Evaluation therefore focuses on the quality of particular schemas but not necessarily on the quality of physics knowledge and the coherence of physics knowledge.

Our data shows that although students may possess local knowledge that is correct and coherent they often do not possess the global coherence that characterizes expert problem solvers. In this dissertation we have provided a number of methods that can be used to evaluate coherence in student understanding of physics.

How can instructors and researchers evaluate coherence?

Evaluation of performance in physics classes is often one-sided. Instructors in the physics course usually use traditional problems, which test how well students can manipulate formulas rather than how well students can apply their qualitative knowledge. In a revised course consisting of a conceptual component, such as the tutorial curriculum, students may be given questions and problems that test their qualitative understanding. But, it is rare that students are given problems requiring both qualitative skills and quantitative skills.

An example of this is the engineering physics course at the University of Maryland with tutorials. On each exam in the tutorial class the students are given a qualitative question (the tutorial question) and usually three quantitative questions (similar to the questions students find in the textbook.) The qualitative question rarely requires the use of quantitative skills and the quantitative problems rarely require integrating qualitative skills. These questions therefore give the instructor a measure of how students answer the qualitative question and how well students answer the quantitative question but they usually do not tell us about how well qualitative knowledge is integrated with quantitative knowledge. By having questions segregated like this, instructors may actually be encouraging their students to treat the two types of questions in different ways.

In this dissertation we have outlined a number of ways that an instructor or a researcher can evaluate coherence. The following list provides a brief summary of the different methods we have used.

- The use of *problem-solving interviews* provides us with a detailed account of how our students solve problems. After transcribing these interviews we can create *interview maps* showing the types of knowledge students use to solve complex problems. Color coding the maps by the topic can help us understand whether students are able to go back and forth between different physics topics.[1]
- Asking open-ended *bridging problems* that require both qualitative and quantitative responses testing the same material can help us see how these two types of knowledge are integrated. Our problems often contain three to four qualitative parts about the situation and then a quantitative part. By examining how the students respond to the different questions we can look for consistency in their responses. In an instructional setting these problems can help the student recognize inconsistencies in their work. Many bridging problems are too difficult and require too much time for an exam context. They are more appropriate as in-class activities or homework assignments.[2]
- Asking different versions of problems (with and without qualitative parts) and comparing solutions on the two versions provide us with information on how students integrate their qualitative and quantitative knowledge. We have observed that the qualitative questions often degrade performance for our introductory students. This indicates that students can get cued by the qualitative questions into certain schemas where they get trapped. Because their schemas are not linked they do not employ methods that they may

apply if there were no qualitative questions or cues. Asking two versions of problems therefore allows us to see if students are linking different schema.[3]

- Using open-ended and multiple choice questions testing identical physics topics provide information about student coherence within a specific topic. We have observed that different types of questions can cue our students into different schemas, causing them to answer differently even though the questions test the same physics ideas and concepts.[4]
- Asking an identical question on two different exams and constructing a *pre-post progression* table helps us understand the dynamics of coherence in physics knowledge. We asked students questions after the students had traditional instruction but before they went through a modified curriculum. The question was then repeated after the students had the modified curriculum. By doing this we can examine how a particular class progresses and how individual students progress in developing coherence.[5]

In this dissertation we have demonstrated that the context of problem-solving can tell us about the links our students make between different physics concepts and principles and between their qualitative and quantitative knowledge. Constructing problems that that tie together different physics topics and require both qualitative and quantitative skills can help us evaluate coherence and also help our students develop global coherence.

Speculations for Future Work

Schema theory provides us with a useful framework to understand why student qualitative knowledge and quantitative knowledge is only weakly connected. In this dissertation we have presented evidence for the weak connection by examining many physics contexts. Qualitative and quantitative schemas were viewed in terms of the specific questions and problems we asked. The next step in this research would be to identify characteristics of qualitative schemas and quantitative schemas that cut across many contexts. By doing this we may be in a better position to address some of the issues concerning student coherence in physics.

The previous research in problem-solving has characterized general methods and procedures that experts and novices use to solve problems. In this work we have focused on the schemas students use to solve qualitative questions and quantitative questions in specific physics topics. The next step is tying these two areas of research together. Some relevant questions are:

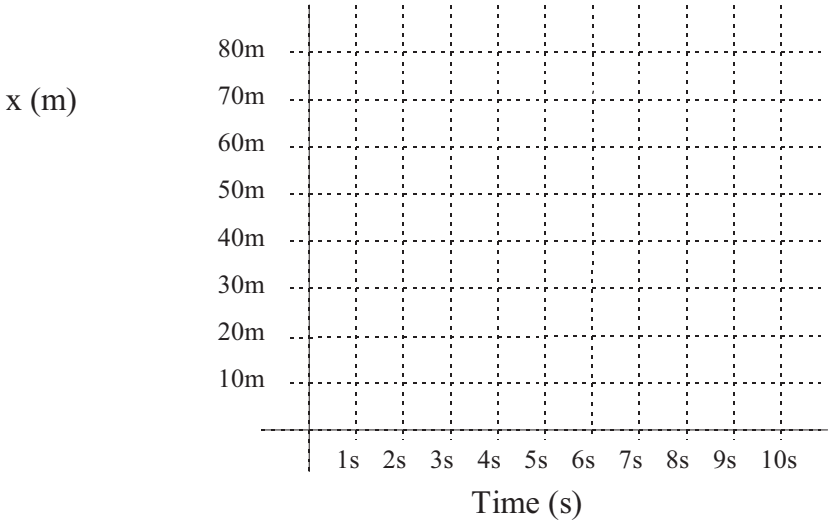
- Can we identify general schemas students use for qualitative questions and quantitative questions? Or are the schemas students used dependent on the topic?
- Do the general schemas experts use for qualitative (and quantitative) questions differ from the general schemas novice's use for qualitative (and quantitative) questions?
- What methods can we use to identify general schemas?

[1] See chapter 5 to see how problem-solving interviews and interview maps are used in a research context.
[2] See chapter 3 for a description of bridging problems; see chapter 5,6 to see how these methods are used in a research context.
[3] See chapters 5 and 6 to see how this method is used in a research context.
[4] See chapter 4 to see how this method is used in a research context; see also R. S. Steinberg and M. S. Sabella, "Performance on multiple-choice diagnostics and complimentary exam problems," Phys. Teach. **35** (3), 150-155 (1997).
[5] See chapter 7 to see how this method is used in a research context.

Appendix A: Selected Bridging Problems

Two cars starting from rest at the same position are facing east on a straight road. The 1st car starts moving at t = 0 seconds and accelerates at 5 m/s^2 for 2 seconds then travels at a constant speed. The 2nd car starts moving 3 seconds after the 1st car started and accelerates at 20 m/s^2 for 1 second and then travels at constant speed.

a) In the space below sketch a position vs. time graph for each car, labeling each car's curve. Explain how you determined the shape of the graph.

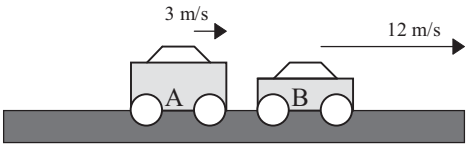


b) State whether the second car catches up with the first car in the first 10 seconds. If the second car does catch up with the first car, state when and where the 2nd car catches up, and explain how you know. If the second car does not catch up explain how you know.

c) State whether or not the two cars are ever at the same speed at the same time after t = 0 seconds. Explain how you could tell from the graph.

Two carts move toward each other on a tabletop. Initially (at t=0), cart A has a velocity of 2 m/s and an acceleration of 2 m/s^2 and cart B has a constant velocity of –6 m/s as shown below. The two cars hit at t=2.5 s. (Assume that the carts are in contact for only an instant.) Right after the two carts hit, cart A and cart B are seen to be going at constant velocities of 3 m/s and 12 m/s, respectively.

t=0 s (before collision)



(after collision)

a) Draw **and** calculate the magnitudes of the velocity and acceleration vectors for **cart A** at the times indicated (be sure to label the velocity and acceleration vectors).

i. t = 1 s

	vector	magnitude
velocity		
acceleration		

ii. t = 2 s

	vector	magnitude
velocity		
acceleration		

iii. t = 3 s

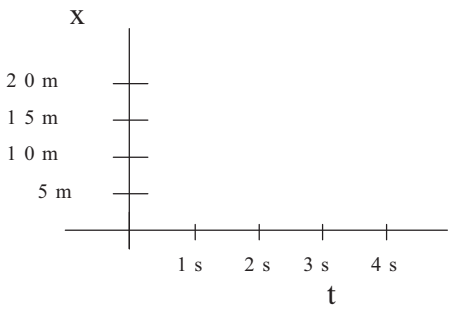
	vector	magnitude
velocity		
acceleration		

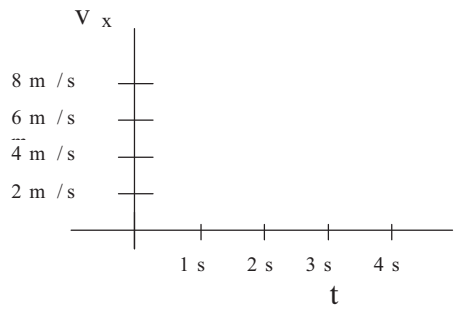
iv. t = 4 s

	vector	magnitude
velocity		
acceleration		

b) Find the average acceleration for **cart B** between t=2 s and t=3 s. Show all work.

c) Sketch the following graphs for **cart A** from t=0 s to t=4 s



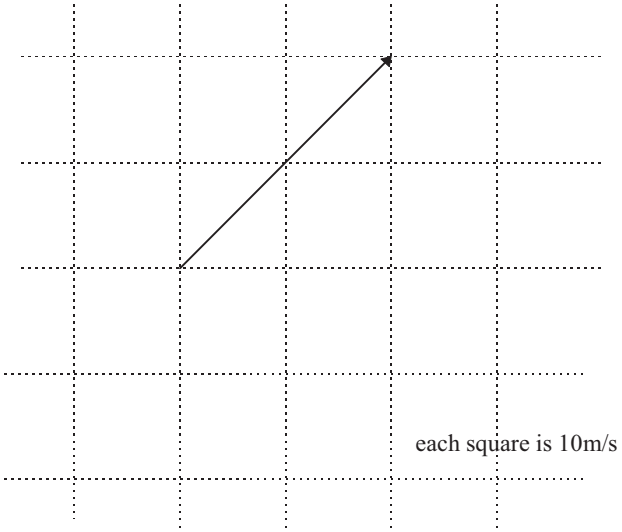


A cannon ball is fired from a cannon at an angle of 45° with respect to the horizontal. The ball has an initial velocity of 28 m/s. (Ignore air resistance.)

a) What is $\Delta \vec{v}$ within the following time intervals. Be sure to indicate magnitude and direction and explain your answer.
   i. t=0 s and t=1 s

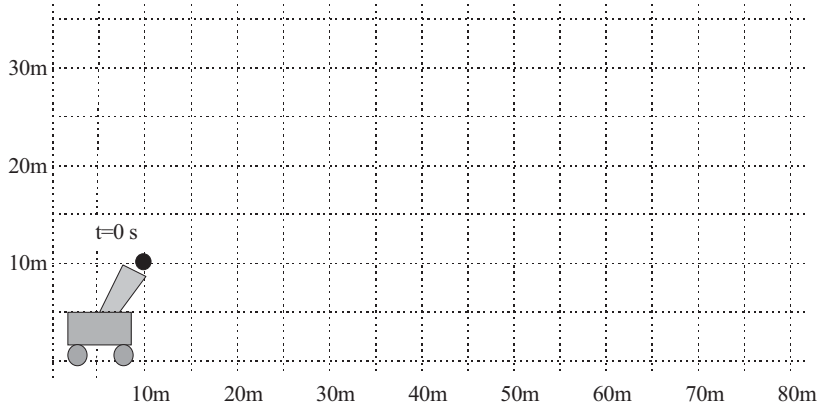
   ii. t=1 s and t=2 s

b) Draw the velocities vectors for t=1 s, t=2 s, t=3 s, and t=4 s on the grid. The velocity vector at t=0 s is shown. Explain how you determined the vectors.

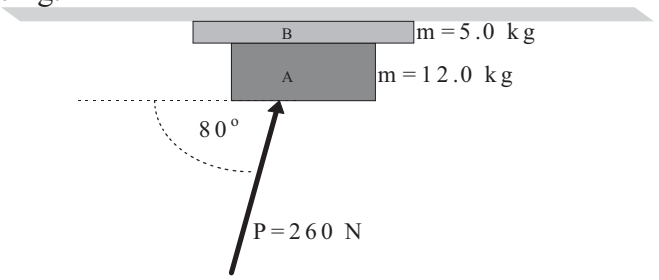


c) Draw and label the *average* velocity vectors on the grid above with dotted lines, for the following time intervals. Explain how you determined your answer.
   i. t=0s to t=1 s
   ii. t=0s to t=2 s

d) Sketch a strobe photograph of the ball showing its location at 1 second time intervals. Note that the position at t=0 s has already been done. Show your work.



A pole is used to hold two blocks at rest on a ceiling as shown below. The pole exerts a force of 260 N at an angle of 80° on block A. The mass of block A is 12.0 kg and the mass of block B is 5.0 kg.



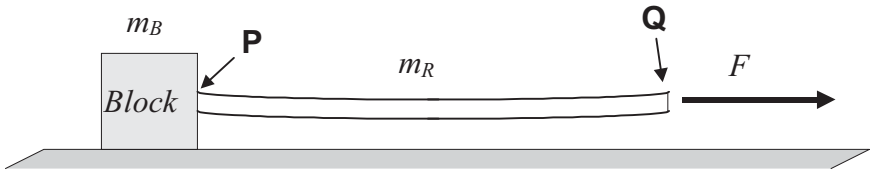
a) Sketch the free-body diagrams of block A and block B. For each force on your diagrams, state the type of force (e.g., gravitational, frictional, etc.)

b) Are there any forces in your free-body diagrams that are equal in magnitude to each other? If so, which ones are they and how do you know they are equal?

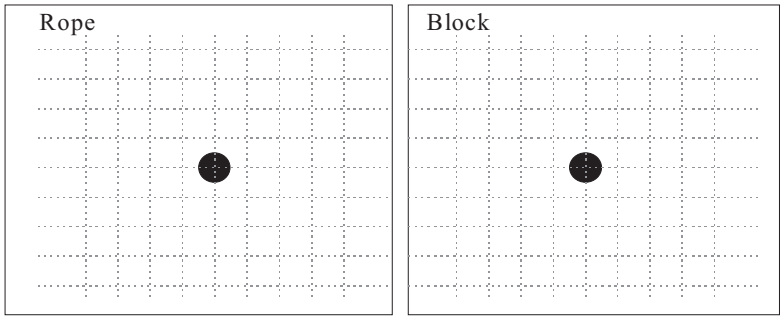
c) Calculate the normal force exerted by block B on block A. Show your work.

d) Rank all the forces on your free body diagram for block A from largest to smallest. Explain your reasoning.

Suppose a hand applies a force F to the end of a rope attached to a block so that the entire system has an acceleration of a toward the right. The rope has a mass m_R, and the block has a mass m_B. Assume that the friction between the block and the surface is negligible.



a) In the space at right sketch a free-body diagram for the rope and the block. Be sure the relative magnitudes of the forces are consistent with the physical situation. For each force in your free body diagrams indicate the object exerting the force and the object upon which the force is exerted.

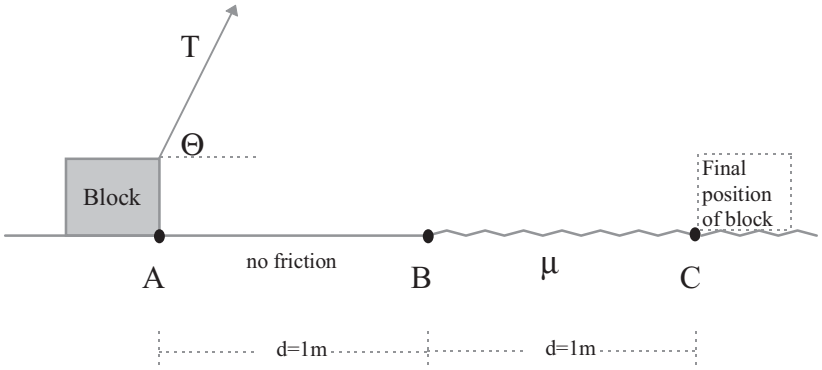


b) Compare the x-component of the tension at point **P** to the x-component of the tension at point **Q**. Explain your reasoning.

c) Calculate the tension in the rope at points **P** and **Q** if m_B=5kg, m_R=0.2kg, and the system accelerates at 2m/s^2. Assume that the rope is nearly parallel with the surface.

d) If the rope is replaced with a massless string, compare the tension at points **P** and **Q** to each other. Explain your reasoning.

A block is pulled from point A to point C by a tension force, T, as shown at right. The block starts off at rest at point A and speeds up to point B. The block then slows down finally stopping at point C (as shown). The magnitude and direction of the tension are constant throughout the motion. Note that the block is first pulled across a surface with no friction and is then pulled an equal distance across a surface with friction.



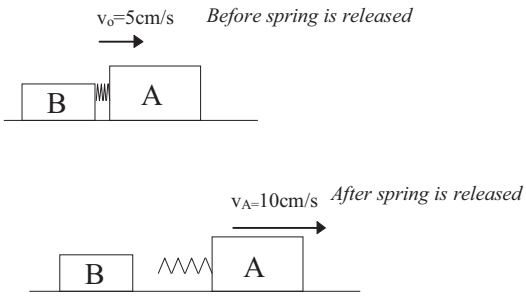
a) Draw a free-body diagram at right for the block when it's between B and C.

b) Is the magnitude of the net force acting on the block from A to B greater than, less than, or equal to the magnitude of the net force acting on the block from B to C? Explain your reasoning.

c) Calculate the coefficient of kinetic friction between the surface and the block between B and C if $\Theta=60^{o}$, $m_{block}=1.5kg$, and T=5N.

d) Calculate the work done by friction to move the block from A to C. Show all work.

Two carts are moving together, with a velocity of 5 cm/s $\hat{i}$, on a frictionless surface as shown at right. Cart A has a spring attached to it and the spring is in a locked position. At a time t_o the locked spring attached to cart A is released and the spring expands until it is no longer in contact with cart B. Consider cart A and the spring as system A, cart B as system B, and all three as system C. System A has a mass of 5 g, system B has a mass of 3 g, and the final velocity of system A is 10 cm/s $\hat{i}$



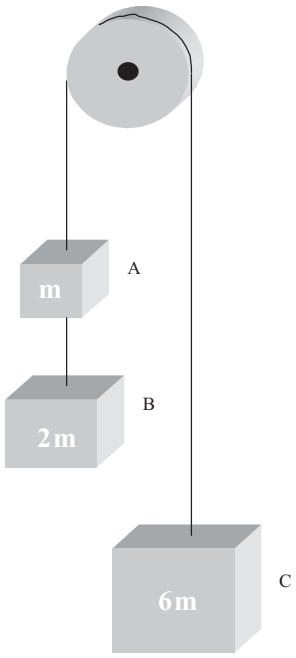
a) Compare the magnitude and direction of the force that each system exerts on the other as the spring expands. Explain how you arrived at your answer.

b) Compare the magnitude and direction of the impulse exerted on system A by system B to the impulse exerted on system B by system A. Explain how you arrived at your answer.

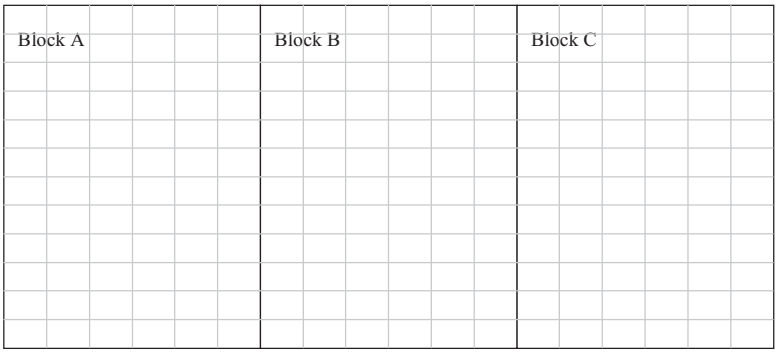
c) Find the final velocity of system B. Show all work.

d) Find the ΔKE of system C. Show all work.

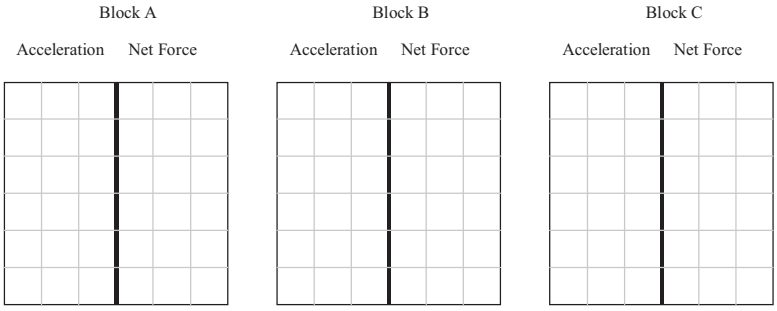
Three blocks are connected by a massless string, which goes over a massless pulley as shown at right. The mass of each block is given in the diagram.



a. Draw a free-body diagram for each block in the space provided. Label each force as you were instructed in tutorial.



b. For each block draw a vector representing the **net force** and the acceleration of each block. If any of the quantities are zero state that explicitly. Make your vectors for the three blocks consistent with each other.

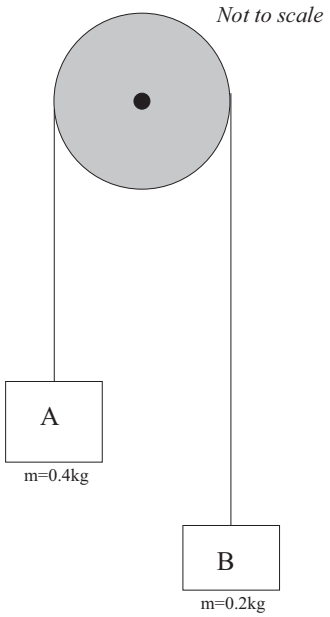


c. Rank the magnitudes of all the forces in your free-body diagrams. Explain your reasoning.

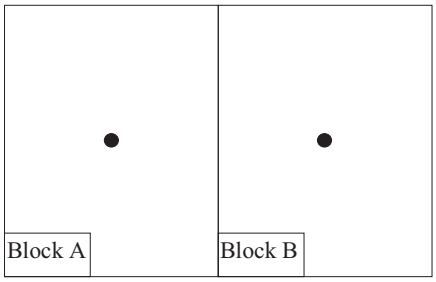
d. Calculate the acceleration of the large block. Show all work and explain your reasoning.

e. Rank the magnitudes of the net work done on the each block after they move a distance *d*. Assume they have not yet hit the pulley.

In the figure at the right is shown an Atwood's machine with two unequal masses attached by a massless string which does not slip. The pulley has a mass of 0.4kg, a radius of 0.2m, and a moment of inertia of 8×10^{-3} kg·m^2 about its center of mass. The masses are released from rest at t=0s.

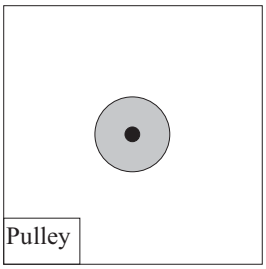


a) Draw a free body diagram for block A and block B after t = 0s. Make sure to label all the forces.



b) Rank the magnitudes of all the forces in your free body diagrams for the blocks. Explain your reasoning.

c) Draw a free body diagram for the pulley in the space provided.



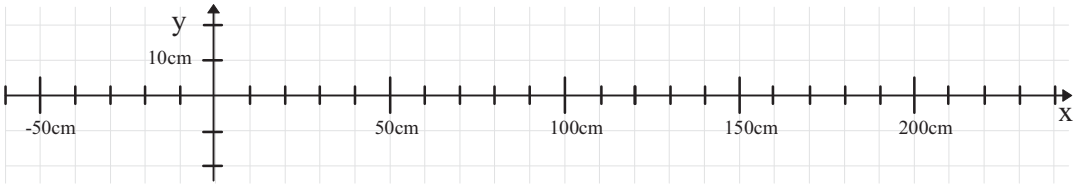
d) What force(s) acting on the pulley cause it to rotate about its center of mass? Explain.

e) Calculate the acceleration of block B. Show all work.

f) What is the angular speed of the pulley after 5s. Show all work.

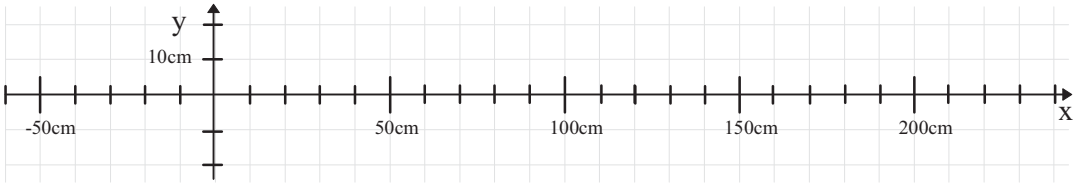
Consider a pulse traveling *to the right* with a speed of 600 cm/sec. The equation describing the displacement of the spring from equilibrium at time t = 0 sec is $y_1(x) = A_1 e^{-\left(\frac{x}{b_1}\right)^2}$, where A_1 = 20 cm and b_1 = 20 cm.

a) Sketch the shape of the spring at time t = 0 sec in the graph below. Use the indicated scale.



b) Write an equation that describes the displacement of any piece of the spring at any time for this pulse. Explain how you arrived at this answer.

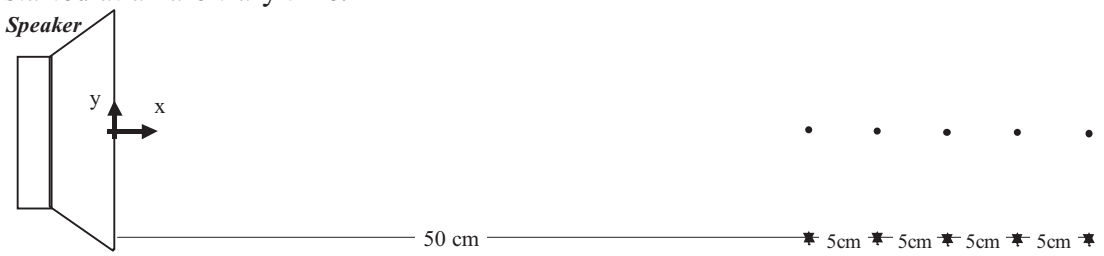
c) Now suppose a second pulse moving *to the left* is also present on the spring. At t = 0.1 sec the equation describing this 2nd pulse is $y_2(x) = A_2 e^{-\left(\frac{x-200cm}{b_2}\right)^2}$ where A_2 = 10 cm and b_2 = 10 cm. Sketch the shape of the spring at time t = 0.1 sec in the graph below, labeling the pulse going to the right "1" and the pulse going to the left "2".



d) At what time and position do the maxima of the two pulses meet? Show your work.

e) Write an equation that describes the shape of the spring when the maxima meet. Show your work.

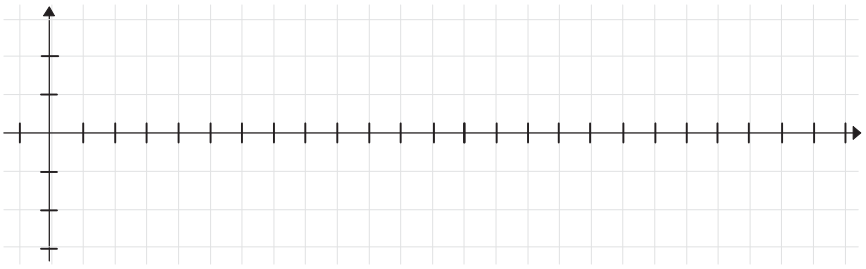
Five dust particles are placed in a row 5 cm apart beginning 50 cm from a loudspeaker (see figure). The speaker plays a note with a frequency of 1700 Hz. The speed of sound is 340 m/s. The maximum displacement of the first dust particle is s_{max} = 3 mm. Assume that the intensity of the sound wave is the same for all dust particles. In the indicated coordinate system, the origin is at the center of the loudspeaker. A clock is started at an arbitrary time.



a) At time t = 0 sec, the first dust particle is at equilibrium and moving away from the loudspeaker. Find t_0, the amount of time that elapses until the second dust particle is at its equilibrium position. Explain.

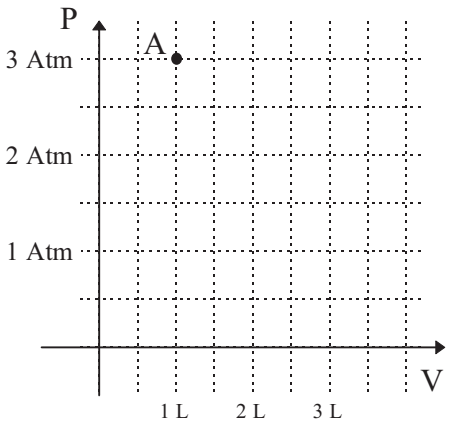
b) What is the displacement from equilibrium of the first dust particle at time t_0? Explain how you arrived at your answer.

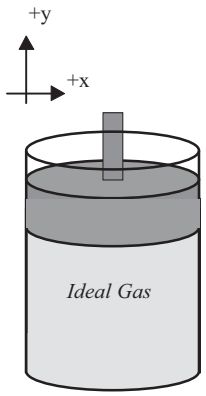
c) In the graph below, sketch a graph of s vs. x at time t_0. Define each axis clearly.



d) Find s(x,t) for x = 65 cm and $t = 2.941176 \times 10^{-4}$ sec. Show all work.

An ideal gas in a container with a piston starts at a pressure of 3 Atm and a volume of 1 L. The gas first goes through an isothermal process ending up with a pressure of 1 Atm and a volume of 3 L. The gas then undergoes a process at constant volume ending up with a final pressure of 3 Atm. (R=0.0821 L•Atm/mole•K)





a) Sketch the isothermal process on the PV diagram and label the resulting state "B."

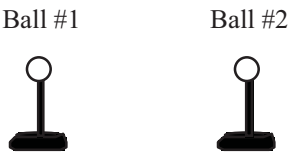
b) Sketch the constant volume process on the PV diagram and label the final state "C."

c) What is the direction of the force exerted on the piston by the gas in the container in the process A⇒B? Is the work done by the gas positive, negative, or zero? Explain.

d) What is the direction of the force exerted on the piston by the gas in the container in the process B⇒C? Is the work done by the gas positive, negative, or zero? Explain.

e) Calculate the work done by the gas in the process A⇒B⇒C. Show all work.

The metal balls placed 10 cm apart each have mass of 1 g and a charge of 2.0 x 10^{-5} C (see diagram). Show all work for all questions.



a) Assume that the balls can be treated as point charges.

What is the electrical force exerted by the first ball on the second?

What is the electrical force exerted by the second ball on the first?

b) Compare the gravitational force exerted by each sphere on the other to the electrical force exerted by each sphere on the other. ($G = 6.7 \times 10^{-11}$ m^3/s^2kg)

c) How do the following changes affect the force exerted by each ball on the other?

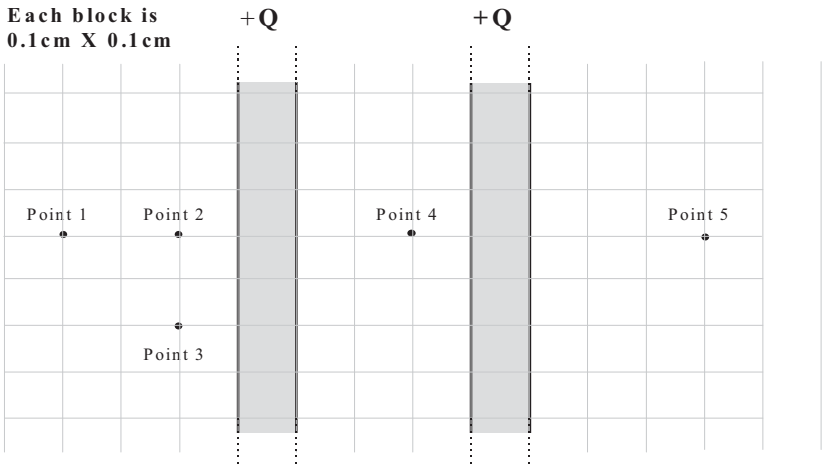
The distance between the balls is decreased by a factor of 2, to 5.0 cm.

The charge on the second ball is decreased by a factor of 2, to 1.0 x 10^{-5} C.

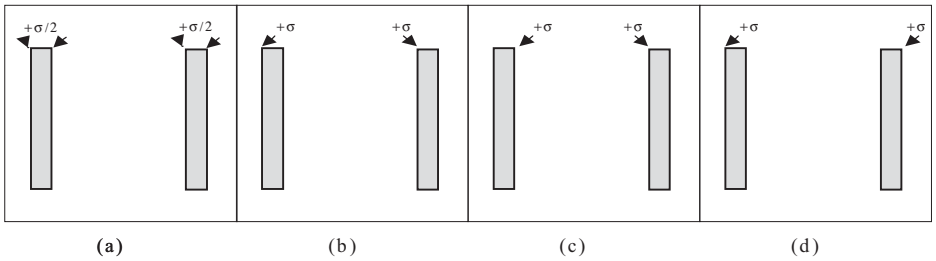
d) Suppose the balls cannot be treated as point charges. In this case, would you say that the force you calculated in part A is correct? If so, explain why it is correct. If not, explain why not and tell whether the force between the two balls would be greater than or smaller than your calculated value. (As part of your explanation, include a sketch that shows the distribution of charge on the two balls.)

Under what circumstances will the calculation you performed in part a) for the force between the two balls be *nearly* correct? Explain.

Consider two very large conducting plates a distance *D* apart. Both plates have a charge of +Q on them. See figure at right.



a) Which diagram below shows the correct charge distribution on the plates? Explain your reasoning.

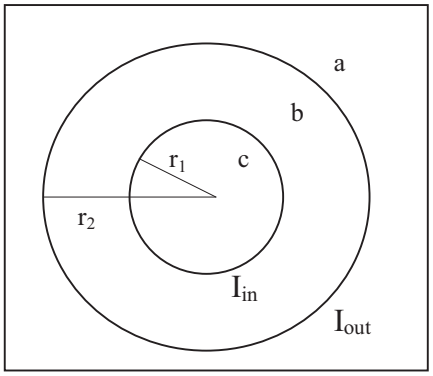


b) Draw vectors on the diagram above representing the electric field at points 1, 2, 3, 4, and 5. If the field is zero at any point indicate that explicitly.

c) A small charge q, initially at rest at point 1, is moved by a hand so that it comes to rest at point 3. Write an equation for the work done on the charge.

d) If the charge density, σ, is 2 X 10^{-9} C/m^2, determine the potential difference between points 1 and 5. Show all work.

Consider a coaxial cable, as shown in a head-on view in the figure to the right. A coaxial cable consists of two very thin-walled concentric cylindrical wires with currents traveling in opposite directions. In this problem, the inner cylindrical wire with radius r_1 has a current, I, traveling *into* of the page, and the outer cylindrical wire with radius r_2 has a current, I, traveling *out* of the page. Labels *a*, *b*, and *c* indicate three regions in space.



a) Draw a path in region *b* such that the value of $\frac{1}{\mu_0}\oint \overline{B} \bullet d\overline{l}$ equals I. Which direction is your path? Explain how you arrived at your answer.

Is this the only such path you could have drawn? Explain.

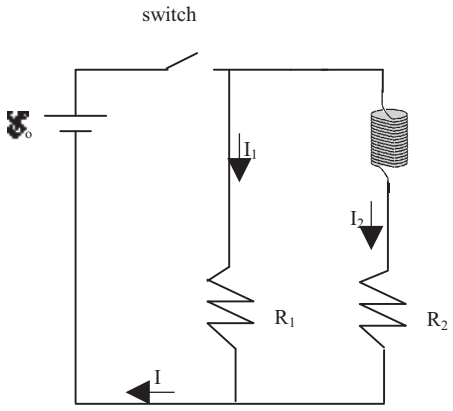
b) Find the magnetic field as a function of r and I in each of the three regions of space. Show all work.

c) Consider that the current on the inside wire equals zero, while the magnitude of the current on the outside is still I. How, if at all, does the magnetic field in each of the three regions differ from the values you found in question b? Explain how you arrived at your answer.

d) Consider that the current on the inside wire has a magnitude, I, while the current on the outside wire equals zero. How, if at all, does the magnetic field in each of the three regions differ from the values you found in question b? Explain how you arrived at your answer.

e) Consider that a current I flows through both coaxial wires (as indicated on the figure above). The value of r_1 is 1 cm and the value of r_2 is 4 cm. The value of I is 4 amps. Find the value of $\left|\overline{B}\right|$ a distance 3 cm from the axis of the wires.

Consider the circuit shown at right. Assume that the resistance across the solenoid and the wires is zero and that the battery is ideal.



A. Just after the switch has been closed, calculate the following. In each case explain how you determined your answer.

   i. the current I_1 through the resistance R_1.

   ii. the current I_2 through the resistance R_2.

   iii. the total current I through the battery.

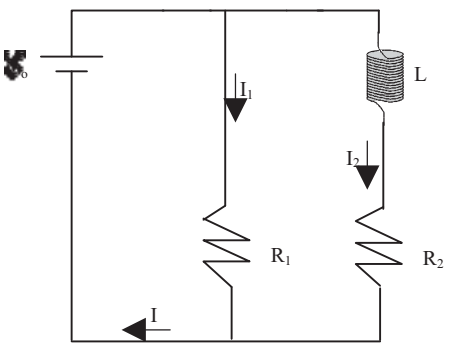
   iv. the potential difference across R_2.

   v. the potential difference across L.

   vi. dI_2/dt

(continued on next page)

B. After the switch has been closed for a long time, calculate the following. In each case explain how you determined your answer. The figure from page 1 has been reproduced at right.



i. the current I_1 through the resistance R_1.

ii. the current I_2 through the resistance R_2.

iii. the total current I through the battery.

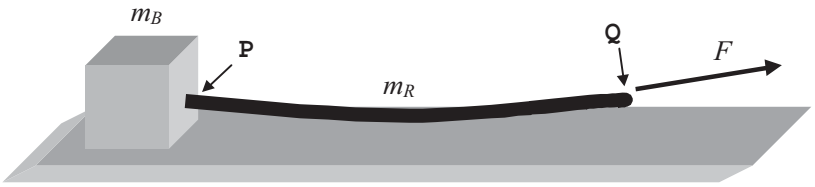
iv. the potential difference across R_2.

v. the potential difference across L.

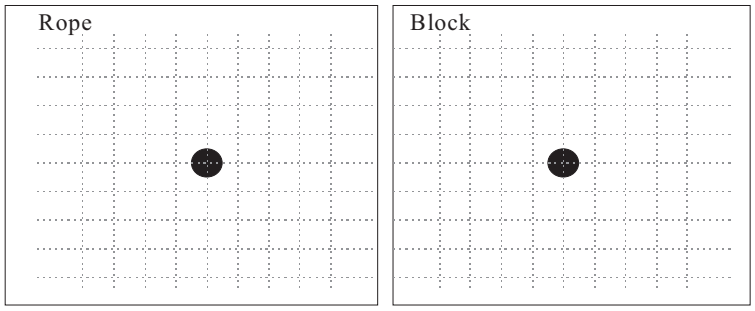
vi. dI_2/dt

Mechanics Problem Solving I

1. A hand applies a force ***F*** to the end of a rope attached to a block. The rope has a mass m_R, and the block has a mass m_B. Assume that the friction between the block and the surface is negligible.



a) In the space at right sketch a free-body diagram for the rope and the block. Be sure the relative magnitudes of the forces are consistent with the physical situation. For each force in your free body diagrams indicate the object exerting the force and the object upon which the force is exerted.



b) Compare the horizontal components of tension at point **P** and point **Q**. Explain your reasoning.

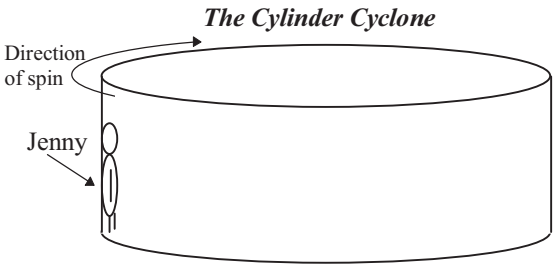
For the remainder of the problem assume the rope to be nearly parallel with the surface.

c) Calculate the tension in the rope at points **P** and **Q** if m_B=5kg, m_R=0.2kg, and the system accelerates at 2m/s^2. Assume that the rope is nearly parallel with the surface. Explain your reasoning and show all work.

d) If the rope is replaced with a massless string, calculate the tension at points **P** and **Q** and compare the two tensions. Explain your reasoning.

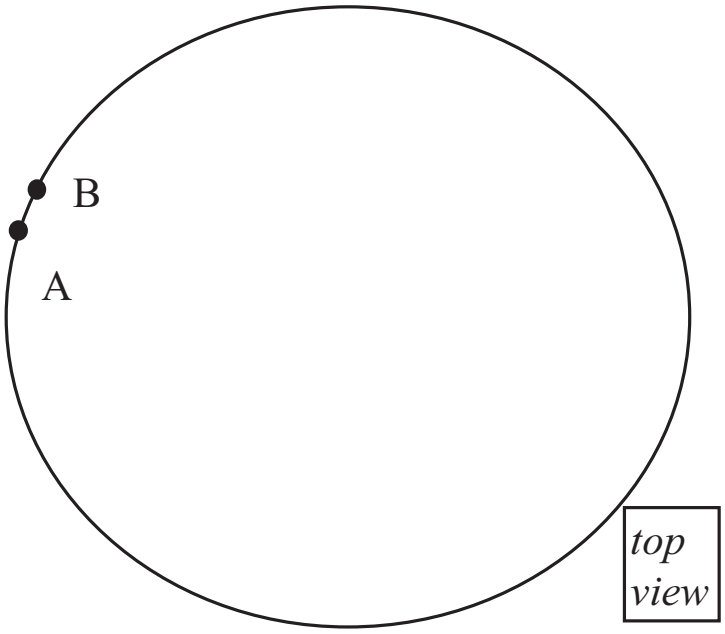
e) Compare the acceleration of the block using the rope with the acceleration using the massless string. Explain your reasoning.

2. At an amusement park, Jenny decides to take the Cylinder Cyclone ride, shown in the diagram at the right. In this ride, a cylinder spins at a constant speed and then suddenly the bottom drops out from underneath everyone's feet.



a) On the top view diagram provided at the right, draw Jenny's velocity at the indicated points.

Use your velocity vectors to find the direction of Jenny's average acceleration between points A and B. Explain.



Use your velocity vectors to find how the acceleration would change if Jenny's speed were doubled. Use your velocity vectors to find how the acceleration would change if the radius of the cylinder were doubled but the speed were held constant.

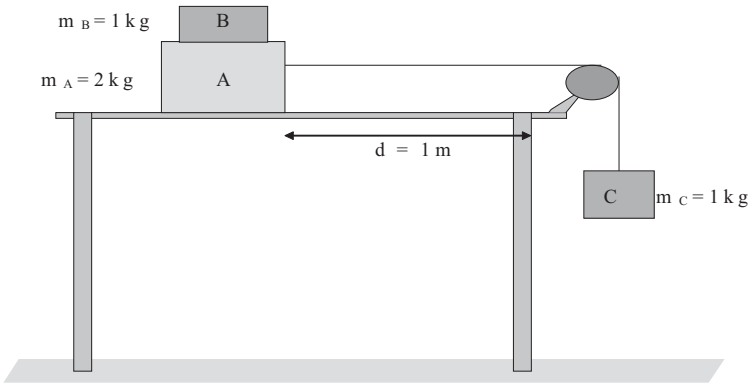
Are your answers consistent with the equation $a = v^2/r$? Resolve any discrepancies.

b) Draw a free body diagram for Jenny when she is at the position indicated in the top diagram. Label all forces clearly. What is the work done by *each force* acting on her? Relate your answer to the change in Jenny's kinetic energy. Explain.

c) The knowledgeable ride attendant says that the coefficient of static friction between cotton and the edge of the cylinder is 0.9. If Jenny has a mass of 50 kg, is moving with a speed of 10 m/s, and knows the cylinder is 10 m in diameter, should she worry about slipping when the bottom drops out? Should her boyfriend, who has a mass of 80 kg? Explain.

Mechanics Problem Solving II

1. Consider the situation below where two blocks, A and B are attached by a string attached to another block, C. At t=0s block C is released and blocks A and B move together. The blocks travel a distance d in time t_o. The values of the masses and the distance d are shown in the diagram. (Assume the string and pulley are massless and that there is no friction between A and the table.)



a) In the space below draw a free body diagram for block A, block B, and block C.

b) For each force acting on block B identify its NIII force pair.

Block A	Block B	Block C

c) Rank the magnitudes of the impulses exerted on carts A, B, and C between t=0s and t_o. Explain how you know.

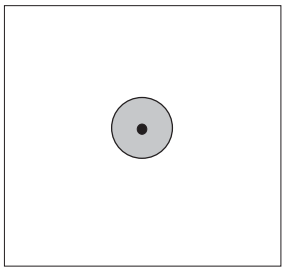
d) Rank the magnitudes of the change in momentum for carts A, B, C. Explain your reasoning.

e) Obtain the velocity of mass A at time t_o. Show all work.

f) Suppose the massless pulley is replaced with a pulley of mass m_p and radius r_p, where m_p=½kg and r_p=0.2m. (Assume the string does not slip on the pulley and that the pulley is a solid cylinder.)

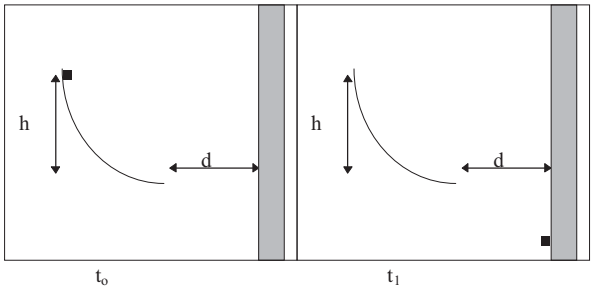
i. Will the velocity of A at t_o be greater than, less than, or equal to the velocity you found in part (f)? Explain how you know.

ii. Draw a free body diagram of the pulley in the space provided and identify the forces that exert a torque on the pulley. Explain your reasoning.

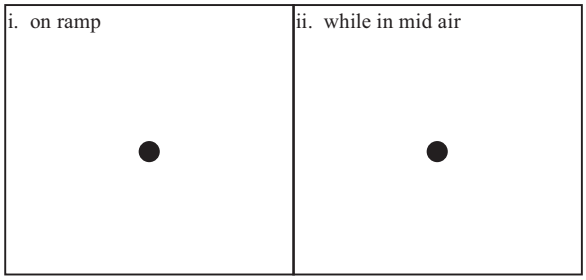


iii. Obtain the velocity of mass A at time t_o. Show all work.

2. A block slides down a frictionless track, flies through the air, and collides with a wall, sticking to it. The diagrams at right show the system at successive times.



a) Draw a free body diagram for the block:
   i. at some instant while it is on the ramp.
   ii. at some instant while in between the ramp and the wall.



b) What forces do work on the block at the following times?
   i. while on the ramp

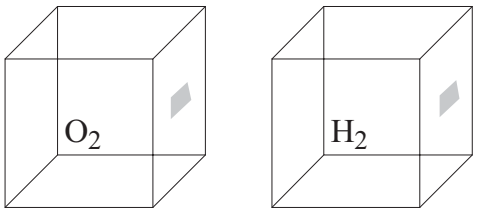
   ii. while in mid-air

c) Compute the vertical distance the block falls between t_o and t_1 in terms of h and d. Show all work.

d) What is the approximate direction of the impulse from the wall on the block during the collision. Explain how you found the direction.

Heat and Temperature Problem Solving

1. Two cubic containers of equal volume are kept at the same temperature. Both containers hold the same number of moles of gas. Container A holds O_2 and container B holds H_2. A shaded area of the same size is shown on each container.



a) On average, compare the amount of time it takes molecules of O_2 to cross the container to the amount of time it takes molecules of H_2 to cross the container. Explain your reasoning and discuss any assumptions you have made.

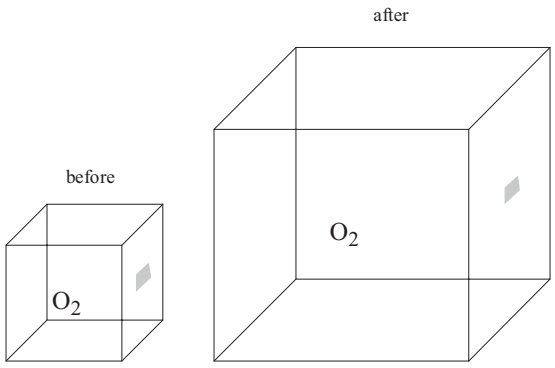
b) On average, compare the change in momentum of a single molecule of O_2 to the change in momentum of a single molecule of H_2 in a collision with the wall. Explain your reasoning and discuss any assumptions you have made.

c) Use the Kinetic Model of gases to compare the pressure on the shaded area in container A to the pressure on the shaded area in container B. Explain your reasoning.

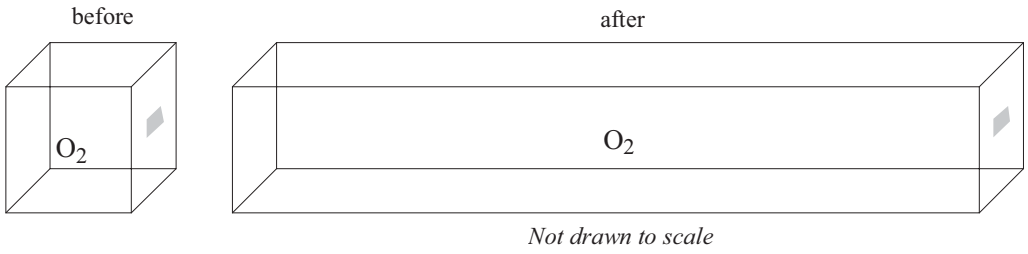
d) Is the pressure comparison using the Kinetic Model consistent with the Ideal Gas Law? If they are not consistent resolve any discrepancies.

e) If the original pressure on container A is 5 N/m^2 what is the new pressure on the shaded area when the temperature is doubled. Explain your answer in terms of the Kinetic Model **and** the Ideal Gas Law.

f) If the original pressure on container A is 5 N/m^2 what is the new pressure on the shaded area when the length of each side is doubled as shown in the figure. Explain your answer in terms of the Kinetic Model.



g) If the original pressure on container A is 5 N/m^2 what is the new pressure on the shaded area when the length of one side is increased eight times as shown in the figure. Explain your answer in terms of the Kinetic Model.



Not drawn to scale

2. (from Arons, 1997) Several small pieces of copper, having a total (combined) mass of 350 g are placed in liquid nitrogen and, when removed, are at a temperature of -180^{o}C. The pieces of copper are quickly transferred to a 200 g brass calorimeter containing 420 g of water. Both the calorimeter and the water are at 9.0^{o}C and the temperature of the room in which the experiment is conducted is 20^{o}C.

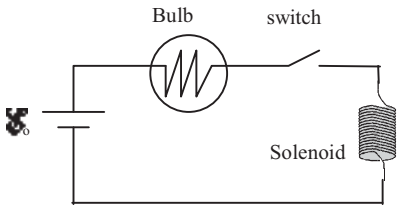
   a) Describe qualitatively what happens in the way of heat transfers and temperature changes after the copper has been placed in the calorimeter. Describe the conservation relation that governs the phenomena taking place, and indicate the idealization we make in applying this relation to the interaction between the copper and the calorimeter. What role does the concept of "closed system" play in the idealizations you invoke?

b) Now proceed to predict, numerically, the final equilibrium state that is attained within the calorimeter. Explain your reasoning as you set up expressions and interpret, in words, the physical meaning of each separate term that is present in the equation you end up with. (Hint: It is wise to make a preliminary, rough calculation to estimate the final physical condition the system attains and to identify the relevant unknown, or unknowns. Otherwise you may find yourself putting numbers into expressions that turn out to be irrelevant.)

c) Noting that it is, in fact, never possible to attain a perfectly closed system in these circumstances, analyze how interaction with the surroundings will, in this instance, affect the final result: Will the calculated value (of whatever you calculated) be greater than. equal to, or less than the "correct" value that would have been obtained in the ideal situation? Explain your reasoning.

Inductive Circuits

1. Consider the circuit shown at right. Assume that the resistance of the wires and across the solenoid are zero and that the battery is ideal. It is observed that when the switch is closed, the bulb is initially not lit, and then gradually increases in brightness. Once the brightness of the bulb is the same as its brightness when connected directly to the battery, it remains unchanged.



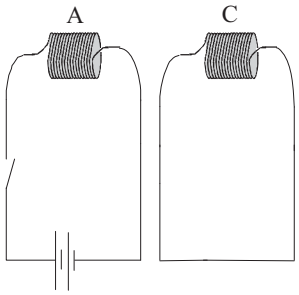
   a) Just after the switch is closed, what is the voltage across the bulb? Explain how you know.

   b) <u>Just after</u> the switch is closed, what is the voltage across the solenoid? How can you account for this in terms of Lenz's law?

   c) After a long time, what is the voltage across the bulb? Explain how you know.

   d) After a long time, what is the voltage across the solenoid? How can you account for this in terms of Lenz's law?

2. Two solenoids A and C are sufficiently close together that the magnetic field formed in A, in the presence of the electric current, also penetrates into C. The figure is shown at right.

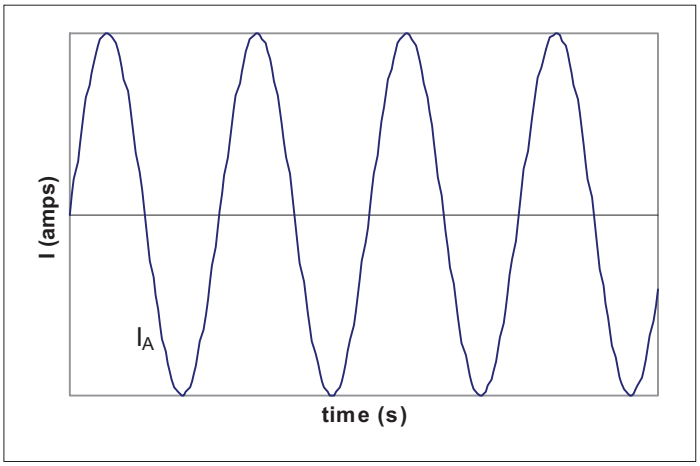


a) We start with switch S closed so that a steady current is present in solenoid A. Establish the direction of the B-field penetrating into solenoid C and show its direction. Explain how you arrived at your result. If the B-field is zero, say so explicitly and explain your reasoning.

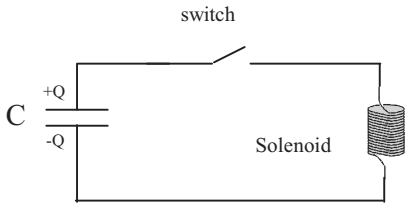
b) While the switch is closed and the steady current is present in A, what is the direction of the induced current in C? Explain how you arrived at your result. If the current is zero, say so explicitly and explain your reasoning.

c) The switch is now opened, and the current in A drops to zero. Describe what, if anything, happens in C, showing the direction of any possibly induced current. Explain how you arrived at your conclusion. What is the situation in C after the current in A has dropped to zero? Explain your reasoning.

d) Suppose the battery in the diagram above is replaced with an AC voltage source such that the current, I_A, through solenoid A is shown at right. A positive current on the graph indicates a clockwise current around the circuit. Sketch the current in solenoid C as a function of time on this graph and explain your reasoning.



3. Consider the circuit shown at right. Assume that the resistance across the solenoid and the wires is zero and there is an initial charge on the capacitor of Q.



a) At the instant after the switch is closed:

i. What is the voltage across the capacitor and the voltage across the solenoid? Explain how you know.

ii. Describe the current in the circuit. (In which direction, if any, is it? Is it increasing or decreasing?) Explain using Lenz's Law.

b) At some later time, t_1, the charge on the capacitor has been reduced to Q/2.

i. What is the voltage across the capacitor and the voltage across the solenoid? Explain how you know.

ii. Describe the current in the circuit. Explain using Lenz's Law. (Hint: Does the EMF across the solenoid correspond to an increasing current or a decreasing current?)

c) At some later time, t_2, the charge on the capacitor is 0.

i. What is the voltage across the capacitor and the voltage across the solenoid? Explain how you know.

ii. Describe the current in the circuit using Lenz's Law. Explain.

d) Describe what happens in the circuit after t_2. Explain your reasoning.

Appendix C: Full Interview Transcripts of the Interviews Used in the Dissertation: Advanced Students

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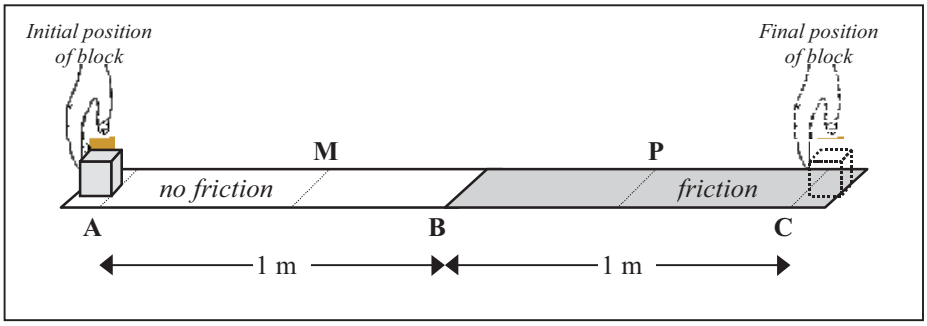
Picture from the problem shown in Figure 5 – 1.



"Peter" (11/12/98)

(reads question)

a) (Draw a free body diagram)

P: So this point represents a block — this is the normal force — weight and [pause — looks back at question] [draws in a force to the right and a force to the left labeled friction] ... *this is 2 N* [writes 2N on the force to the right] — *do I have to draw it to scale? I guess this is — it probably travels at a constant velocity — right? F equals F. F big equals F small amplitude.*

I: How did you know it travels at constant velocity?

P: Umm — actually I don't — there is no indication that it travels — it might be having some acceleration — there is not enough information so — it is not necessarily —

I: You are told that the block starts at rest at point A and the force of the hand remains constant — and it comes to rest at point C.

P: Okay.

I: And the hand applies that force throughout the whole time.

P: ... what is the question?

I: Calculate the coefficient of kinetic friction.

P: So — ... So we have to write out in components the NII law which would be — x component is — F minus F of kinetic friction equals the acceleration which is going to be negative because my x axis is to the right — so [draws an axis labeled +x to the right] *— since the object comes to rest moving from B to C the acceleration is in the negative x direction although the sign of the acceleration will — eventually determined at the end —*

I: ... Before you were saying the acceleration was zero —

P: No — you're right — because since ... you pointed out that the force was acting throughout the motion. So since there is no motion in y component the normal force ... is balanced by weight which is N equals mg — so we can write F minus μ_k N equals minus ma — so we have to find a now and — we are going to do it in the following way — I guess — So we are going to find the velocity at point B — this will be v_o + v squared will be equal to v_o squared plus 2a′ times the distance of AB times 1 meter where a′ is force over mass — since the force — since the only force acting on the segment of the distance from A to B is F equals to 2 Newtons — so we have 2 Newtons over 1 kilogram — so we have 2 Newtons per kilogram and we can find v_o — it's zero — it starts from rest. ... So therefore v_B squared is 4 ... So — and then find a ... we have — we can use the same formula — it will be — if v is zero minus v_B squared over 2 times 1 meter — so it will be minus 4 over 2 — which is the same acceleration — but opposite sign ... and now we can plug it back into this equation [points to F - μ_kN = —ma] *for the force and the coefficient of friction and so we have 2 Newtons — we are going to put numbers immediately — 2 minus μ_k will be equal to F plus ma over N — μ_k is 2 Newtons plus ... I'm using here the absolute values of the acceleration over the 10 ... so we are going to have 0.4.*

I: How does the net force at M compare to the net force at P?

P: Umm — the net force at P is smaller than at M — by the amount of the kinetic friction. The net force is smaller by this amount because — the y components of the two forces are canceled out. So the only difference — they will be cancelled out at point A too — and the only difference comes at point P because of the introduction of the force of friction, which is directed opposite to the applied force.

I: So can you draw me a vector for the net force at point P — how would that look?

P: It would look — almost caught me there — yeah the force of friction is bigger than the force — this is net force — since the acceleration is negative — so negative y direction — net force — according to the famous Newton's

second Law should ... I'll put it here to support my statement — the net force should be in the same direction as acceleration —

I: Same direction as acceleration — so this is the net force at P — how about at M?

P: It would be to the right because this is the only force — we have no friction here so this is P and this is M — without scale — this is 2 Newtons — this is the difference between 2 Newtons and force of friction — .4 times N which is 4 — actually it is 2 Newtons also —

I: So how does the magnitude of this guy [points to the net force at P] compare to this [points to the net force at M]*?*

P: The same — ... μ_k *is .4 — coefficient of friction is .4 and N is 10 and that is why we saw that the acceleration between A and B is positive 2 meters per second and the acceleration from B to C is negative 2. So far we did not find any discrepancies from Newton's second Law.*

I: Before you were saying that the net force at P was less because friction opposed —

P: It is not necessarily that — I had the impulse to say that the force was pointing to the right because I accidentally assumed 2 Newtons minus the frictional force would be still positive but I reversed the statement.

"Granola" (11/12/98)

(reads question)

a) Draw a free body diagram for the block when it is at "P."

G: Okay when it is a P — free-body diagram represented as a point mass — there will be a Normal force — no— a force of gravity acting down on the block — there will be a normal force acting up — there will be the force of the hand pushing it to the right — and there will be a force due to friction pushing the opposite direction — because it's in P which is the part with friction — I'm assuming — so four forces.

b) Calculate the coefficient of kinetic friction μ.

G: ... Alright — 1 kilogram — what's it moving — okay 2 Newtons — okay — you start at B so it's moving at point B — there will be 2 Newtons, normal, gravity and then the final position — does it come to rest? Ok it comes to rest at point C. So at point C it comes to rest which means that at point C the force of the hand.

I: So this is a free body diagram [I point to the new free body diagram that has no friction force].

G: — at B — then that is what is between B and C and then over here — free body diagram here would be this and the hand and friction would be the same — because it is at rest — and these will be the same. So what is it?

I: If I compare the hand and the friction here— how do they compare?

G: At P? The hand should be greater because it is still moving because it goes from B to C and it comes to rest at C. Trying to think what friction is — friction is — the coefficient of friction is force equal to force gravity — I forget which one it is —

I: That is actually the normal.

G: — normal right — they are almost always the same — right? The normal and gravity?

I: It depends.

G: ... so the force of the friction is equal to some coefficient of the normal force which will be 9.8 Newtons ... —

I: What is 9.8 Newtons?

G: 1 kilogram block — The force Normal should be 9.8 down so that is going to be 9.8 up and force of friction would have to be 2 Newtons — that's 9.8 so 2 over 9.8 equals μ. I'm all confused — maybe I'm sicker than I thought. I feel retarded now — that happens often. ... Something is not right —

I: ... why don't you like it?

G: I don't know — there should be the force of the hand in there somewhere —

I: What is that 2 Newtons [points to 2 N in the equation Granola has just written]*?*

G: That is the force of the hand — oh when that equals — we want to find when this is equal — oh no — that's not right at all — this is the problem here — that is what it looks like over here [points to point C on the diagram] —

I: Right at point C?

G: Yeah. So wait maybe that should be right. [pause] *That's not right at all — ... it doesn't seem right to me — just give me a second — I just started to solve it assuming it would be easy — then realized maybe it wasn't as easy as I thought. Could also do the work — the work from here* [points to point A] *to there* [points to point B] *— no that's got nothing to do with it — no — friction — 2 Newtons — the force of the hand remains constant — because according to this — this wouldn't be stopped there — it would stop there if this were the case — ... if this were the case if it was 9.8 Newtons and 2 Newtons because that is the same circumstance that we have at B — so it would stop at B the way I have it set up — why would it stop at C? There is something about the 1 meter that I'm not getting — I'm not thinking very well. There has to be something to do with ... the velocity — with the hand force I'm thinking — so I think — there is something to do with friction apparently — I've never done friction —* [pause] *I don't know — do you want to come back to this one.*

I: ... [I give him the long version] *— This is the same thing but it has parts ... — You did part A already so how about part B.*

b) Is the magnitude of the net force acting on the block at "M" greater than, less than, or equal to the magnitude of the net force acting at "P"? Explain your reasoning.

G: … well these cancel at M so it would just be like that so the net force would be like that at M [draws vector to the right] *— at P there should be the same normal and gravitational force should be the same because it is not moving up or down. But this should be less* [draws smaller net force vector to the right] *because it will be decreased by the … frictional force.*

c. i. Draw a vector representing the acceleration of the block at "P." If the acceleration is zero state that explicitly.

G: There is a force on it — there is a net force on it so it should be accelerating — I'm guessing …

I: How did you know it was to the right?

G: The greatest friction could be — friction can't push something backwards so friction can keep something at rest but it can't push something backwards — less than or equal to — it is a less than or equal to force — yeah — I don't know — … I've never known something to move backwards because of friction.

c. ii. Does the magnitude of the acceleration increase, decrease, or remain the same as the block moves from "B" to "C"? Explain.

G: It decreases — the acceleration — because the force decreases —

I: Which force decreases?

G: The net force … — it should — at some point the net force has to be equal to zero — the net force has to be equal to zero.

I: Why is that?

G: Because the hand and the frictional force will cancel out because it says it is at rest at point C … So decreases — cause …

d. Calculate the coefficient of kinetic friction μ.

G: I think it is the same problem — I'm drawing a complete blank here.

I: What is Newton's Second Law … ?

G: The second law?

I: The sum of the forces —

G: The sum of the forces is equal to zero for a body at rest. — I don't know the second law.

I: What is happening to the velocity as the block goes from B to C?

G: It is decreasing — wait — oh no — the velocity here [points to point A] *is going to be — whatever — 2 N over 1 meter —*

I: ... Why is that the velocity?

G: No — you can get the velocity — what is it v squared over ... d —

I: Equals 2 a.

G: Yeah so v squared will be 2 a d so 2 Newtons over the mass which is 1 kilogram — ... something like that — ... velocity at B — the velocity at C — at C the velocity is zero because it is not moving. What is the question?

I: Calculate μ_k.

G: Okay — it goes from root 2 meters per second to 0 meters per second — ... so if there was a constant acceleration from B to C it would look like . . . so it would have the same acceleration here [points in region between B and A] *as it did before — that's not the case —*

I: How do you know that is not the case?

G: Because it has the frictional force acting backwards so can't use that.

I: Newton's second law is sum of the forces equals mass times acceleration.

G: Ok —

I: ... Does that come into play here?

G: ... Sum of the forces equals mass times acceleration — so the acceleration — oh — ... this acceleration would be [pause] *this acceleration has to be the same as this one — if it goes from 0 to 2 — 0 to whatever here* [points to region from A to B] *and then goes from here to there* [points in region from B to C] *in the same distance — same force —*

I: ... that's different then what you said before — right?

G: Before I said it couldn't be the same.

I: Now you are saying the same force?

G: No — I guess — I guess it has to have the same net acceleration — if it goes from 0 to root 2 — or 0 to v here and then goes from v to zero that way — it should have the same acceleration going this direction — except the acceleration would be in the opposite direction — so —

I: So you're saying the acceleration would be this way now [gestures to the left] *in this region* [points at BC region]*?*

G: Yes — in this region — ... It is different then what I said over here — I said this coefficient of friction would be less than since it was still moving to the right — I'm really confused — I'm confusing myself — that was over here actually.

I: — and here for part C when you were drawing the acceleration.

G: Yeah — it's not accelerating at all in that direction —

I: How do you know it is not accelerating in that direction?

G: It would still be moving at this point [points to point C] *if it were accelerating in that direction — it would be moving faster in here then it was before — that was dumb — not dumb but ... so now we got it going backwards because it goes from v to 0 — the change in velocity is acceleration — all that — so sum of the forces — mass times acceleration — I guess this acceleration* [points to region BC] *has to be equal to the acceleration here* [points to region AB] *which is 2 Newtons —*

I: How did you know that?

G: It goes from zero to velocity and then from the same velocity to zero — this force — this acceleration has to be the same as it is up there — it's the same mass — same mass — same acceleration — it should be the same force as it is up here — which is the same as —

I: ... Is it always true that if I go from 0 to a certain velocity — I'll have a certain acceleration — and maybe I'm driving my car the next day and I go again from 0 to that same velocity — is my acceleration always the same?

G: No — not necessarily — it could be — I guess it would be — something like the same average acceleration — not average — net acceleration — if you didn't know anything about in between rest point and going fifty miles an hour or whatever and you didn't know anything about those two and you did it in the same amount of time — ... you could — if you didn't know anything else about in—between you could guess that it was constant acceleration — but you could speed up and slow down — and speed up and slow down — I don't know

— I'm trying to think if that has to do with anything here. This is actually one of the first times I've actually had to solve a problem with friction — if that helps at all. There is a story behind it.

I: ... I'd like to hear it — maybe after —

G: ... So now go back over here — this — the force friction is equal — to μ *times the force normal and this force of friction we got was equal to — this acceleration is the same as the acceleration from A to B which was 2 Newtons over —... this force was equal to 2N and ... meters per second squared is the acceleration and we got that multiplied by the kilograms so that is 1 Newton — I guess — I think — same kilogram — oh that's 2 — 2 meters per second — so the acceleration is 2 Newtons backwards — is the force normal — no — is the force friction —*

I: The force of friction is 2N backwards?

G: 2 Newtons ... — I said the acceleration has to be the same if it was constant — the acceleration had to be the same from here [points to point A] *to here* [points to point C] *— because the velocities — so we have 2 meters per second squared is the acceleration — it's the same mass so we have 2 Newtons is the force of friction — and the force normal is equal to 1 kilogram times gravitation, 9.8 meters per second squared so I got the same thing as I had before. Umm — ... same as before which is 2 N divided by 9.8 N — but still it doesn't seem —*

I: You're not happy with it?

G: I don't know. I guess that's right.

I: So here you have a free body diagram at P — so you are saying the force of friction is — can you put numbers on all of these [points to the forces on free body diagram.]

G: Yeah — before I had it increasing because I was initially thinking of friction being velocity dependent because — it always shows up in differential equations as a velocity ... dependent thing — ... here it's not really velocity dependent — it's just sort of a constant frictional force — so yeah — there should be — that's not right either — because — no — yeah — ... there is an initial velocity at B so — but then again there is no force so it should just keep moving — so this has to be — 4 Newtons ...

I: How did you know it had to be 4 Newtons?

G: Because it had to be 2 Newtons backwards.

I: What had to be 2 N backwards?

G: The force of friction in order to get it to stop ... the net force — the net force was 2 Newtons in order to get a net acceleration acting backwards to get the velocity to go from 0 to v and we want it to go from v to 0 so we had some acceleration here — make a backwards arrow and we had to have the same acceleration there in order to get that same acceleration — we had to have a greater net force in the opposite direction. So now I've got μ equals 4 N over 9.8 — ... that sounds better.

"Erica" (11/12/98)

(reads problem)

a) Draw a free body diagram for the block when it is at "P."

E: ... At P we have the friction and there's the hand so — this is the block and the hand is pushing it this way — and we have — this is the hand and this is mg and the friction is this way — ...

b) Calculate the coefficient of kinetic friction μ.

E: Okay — so friction equals μN — friction equals μ kinetic N and this is the force of the hand — and we would like to sum the forces in the x direction — so the force of the hand is 2 Newtons — so 2 Newtons equals Normal μ kinetic ...

I: How did you know that?

E: Because — You need to sum all the forces in the x direction — and if you sum them you just write 2 N minus F friction equals zero — sum of the forces in the x direction equals force of hand minus force of friction , equals — wait — is it moving? ... no — but it's not accelerating ... it's constant force so it's equal zero. Okay — so force normal is just — it's force normal that the table is providing so it's equal to mg — so that's mg — so μ_k equals 2 divided by mg — and mass is 1 kilogram — that's 2 divided by 9.8 which is about 0.2 — Newtons divided by ...

I: How does this force [friction] *compare to that force* [hand]*?*

E: Well if it's still moving forward then this [points to force of hand] *is bigger then this* [points to friction force] *— it's not enough to stop it.*

I: ... What can you tell me about the velocity as it goes from point B to C?

E: The velocity?

I: Velocity of the block —

E: Well the force is constant so — acceleration is constant — yeah — velocity is increasing.

I: So the velocity is increasing from B to C?

E: I'm not sure. Let me think —I think it's the same —

I: You think the velocity is the same?

E: But then you won't have acceleration. Umm — acceleration is constant so velocity is increasing …

I: So what happens at point C?

E: It stops — zero — velocity equals zero — … Oh — I see what your saying — but that is because the force is not acting anymore — the hand is not pushing anymore —

I: The hand acts all the way to point C.

E: Oh — so it just stops at point C — for no reason? — …

I: The force of the hand is remaining constant from point A to point C.

E: Then if there is no impediment there then friction is greater than the hand pushing it.

I: How did you know that? How did you know friction was greater?

E: How? Because the force of the hand is the same — so friction must have dominated that.

I: How did you know it dominated?

E: Because it came to a stop —

I: Before you said this force [points to force of hand] *was greater than the force of friction.*

E: Cause it was moving that way … — The coefficient must have been — No — Not the coefficient — I don't know — if the force of the hand is the same then the friction must have been different. No? (IA) maybe it's not zero — maybe it's not zero — I'm not sure. What if it's not — Hold on. (IA)

I: So why were you saying this was zero before?

E: It's not right. It's in equilibrium — this is — why was I saying zero? I don't know. — let me see if I can solve it. … — (IA) this velocity here [points to point C] *is zero. The velocity here* [points to A] *is zero — so v initial and v final are*

both zero. [Pause] *(IA)* ... [Pause] ... *The velocity of the block from A to B is different from the velocity of the block from B to C? ...*

I: What do you mean the velocity?

E: The speed?

I: So if I look from A to B — what's happening to the velocity?

E: It's the same.

I: What do you mean it's the same? It's the same throughout?

E: It's starting from zero and increasing —

I: And where does it start decreasing?

E: At B — so from v_o to B it's a certain velocity and from B to C again it's zero — at C it's zero — right? So we are going from zero to some number then from that number to zero— right?

I: Is that the way it would work out?

E: ... Starts from rest and then moves on frictionless from A to B and then on friction from B to C — it has to be slower from B to C — then from A to B. Right? But — ... if the force is constant — is this a simple problem or am I making it difficult? ...

I: What if I asked you to compare the net force at point M to the net force at point P?

E: You mean the summation of forces?

I: Yeah.

E: It's equal to ma.

I: Okay.

E: And if they are equal to ma — is a the same? Is it the same or different?

I: Good question.

E: Umm — I have no idea — no but — here [points to point M] *you have just the hand — here* [points to point P] *you have the hand minus friction — so*

here the hand force equals ma and here force hand minus force of friction equals ma. Okay — we can't equal them because that would make no sense.

I: How come ...?

E: Because then friction would be zero. Right? If I equal this [F_h = ma] *and this* [F_h - F_f = ma] *then I would say F of friction equals zero. — It's not zero here* [points to point P]. *Therefore we can't say a is the same over here and here.*

I: What if I asked you about the change in kinetic energy from A to B compared to the change in kinetic energy from B to C?

E: This would be — it's the same — it would go from zero to something and this would go to zero. Give me a hint.

I: Does that help in any way?

E: ... okay — ... I already said that it is the same velocity here [points to B] — *I already said that.*

I: What if I asked you about the net work done on the block in going from A to B compared to the net work for going from B to C?

E: The work?

I: Yeah — the net work.

E: Work is equal to force times velocity

I: Distance.

E: Oh — right — ... force times distance — it's the same.

I: How did you know?

E: Because the force is constant and the distance is the same.

I: So which force are you considering when you are thinking about the work?

E: Ooh — the summation of forces ... — you have to do that — so here it is just 2 Newtons times the distance, 1 meter, and here it is just this minus force times 1 meter but these two are not equal. And the work must be different.

I: Do you know how the work done on something relates to the change in kinetic energy?

E: I forgot — Wait — Oh god — Change in kinetic energy and the work — work ... [silence] ...

[Erica compares the dimensions of work and energy at this point.]

E: (IA) ... There're the same — oh — they are the same dimensions — so — change in work must equal change in kinetic energy —

I: Yeah — work equals change in kinetic energy.

E: So — ... Kinetic energy is — so for this — so it's like equaling these two together —

I: But you tried that before and you didn't like that.

E: It's saying that this [points to $F_h = ma$] *is equal to this* [points to $F_h - F_f = ma$] *— but — if I equal this to this then friction equals zero. What does that mean? Is it zero here* [points to M]*? It is zero here* [points to point M] *but not here* [points to point P]. *So — Okay what am I doing wrong?*

I: Can you draw me an acceleration vector at point M?

E: Acceleration?

I: Draw me an acceleration vector at point M and if it is equal to zero state that explicitly.

E: How can it be zero — if we start with v_o to v final? ... Well v is increasing so acceleration is constant — if v is increasing then acceleration is constant. Right? Or — it doesn't have to be. Like that [draws vector to the right labeled a] —

I: Okay — how about at point P?

E: Oh it has two components — it has this one and this one [draws two vectors pointing in opposite directions on x axis] *— and this one* [pointing right] *is bigger.*

I: How did you know the right one as bigger?

E: Because it's moving. — Still moving forward ... — but it's decelerating. ... It's slowing down — velocity is zero here [points to point C]. *But velocity is decreasing — velocity is decreasing — if velocity is increasing can a be constant?*

I: ... What is the definition of acceleration?

E: delta v over delta t — so t is increasing and v is increasing — can a be constant?

I: What do you think?

E: Yeah? — Can it? It can or it can't?

I: If I go at a constant acceleration what happens?

E: ... You're increasing velocity ... — If you are constantly accelerating you are constantly increasing velocity. No? Right?

I: What do these deltas mean?

E: Change in —

I: So when you say dv over dt what does that mean?

E: v final minus v initial over t final minus t initial —

[At this point we stray from the problem and talk about the concept of acceleration for the remainder of the interview. The remainder of the interview is not included in this thesis.]

"Eagle" (11/18/98)

(reads question)

a) Draw a free body diagram for the block when it is at "P."

E: So — I see there is no given friction force — so ... I'm drawing the block or something that looks like it. So these are the 2 Newtons coming from the hand — Now — of course — I suppose it is useless but this is the weight and the normal force and the resulting friction force — since we are on the part of the surface with friction —

b) Calculate the coefficient of kinetic friction μ.

E: Let's see— the block travels an equal distance (IA) with the force remaining constant — Let's see — (IA) — let me think — does it say anything about the speed — it doesn't — oh okay I see — I suppose the forces being applied until the end of the trajectory and the block stops due to the friction and not that the hand stops.

I: The hand keeps applying from A to C.

E: Okay — So I'm going to calculate the kinetic energy that the block has until point B —

I: How come you're doing that?

E: To find out what the total — what the energy it loses on the friction surface is — which should tell me —yes of course — what the force acting against it was. So that is going to be 2 Newtons times 1 meter, which is 1 Joule and that is equal to ½ m v squared — ... v being the velocity of the block — and that is exactly what it is going to lose which means the force — the friction force should be equal to 4 Newtons in the other direction — meaning ... in the direction C to A, or to the left — So that the net force being applied on the block is 2 Newtons in the other direction so that the loss of energy is equal to the gain of energy in the first half of the trajectory.

I: How did you know the loss in energy was the same as the gain in energy?

E: Because it started at rest and it ends at rest — I suppose. I assume that is what it means when it travels an equal distance — ... [Rereads part of the question.] So the force is 4 Newtons, which is equal to the magnitude of the normal force times μ — the kinetic friction coefficient— ... so the normal force

is equal to the weight of the block which is 1 kilogram times 9.8 so — μ is equal to 4 over 9.8 which is about .4.

I: ... Can you compare the magnitude of the net force at M to the net for at P — how would they compare?

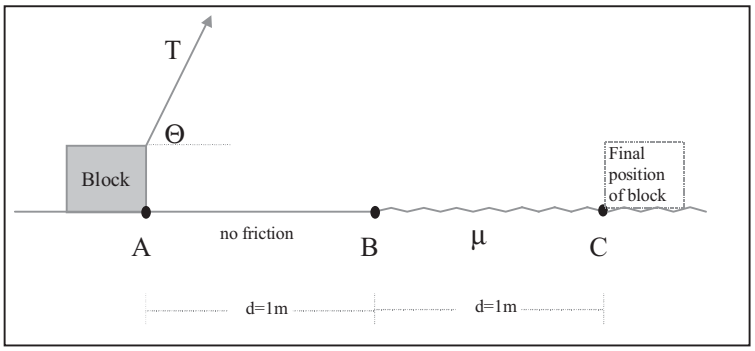
E: The magnitude of the net forces? — well they should be equal and opposite —

I: And how did you know that?

E: By the same argument — because I assume that the force due to friction — which is constant along the whole surface since the weight of the block doesn't change — I assumed that it was equal and opposite to the force — I'm sorry — I mean the sum of the force being applied by the hand and the friction should be totally equal and opposite to just the force applied by the hand on the block so that the loss of energy is equal along the same distance traveled — so they will be equal and opposite.

Appendix D: Full Interview Transcripts of the Interviews Used in the Dissertation: Undergraduate

Picture shown in problem from Figure 5 - 9.



"PINK" (3/24/97)

(reads question)

a) Draw a free-body diagram for the block when it's between B and C.

P: ... So that is the block and there is the force of gravity — friction is going this way — tension is going like that and that's it — this is theta — this is going to be T cosine theta and this is T sine theta — and there is a normal force.

b) Is the magnitude of the net force acting on the block from A to B greater than, less than, or equal to the magnitude of the net force acting on the block from B to C? Explain your reasoning.

P: Well from A to B there is no friction so the magnitude of the force acting on the block is going to be greater because the free body diagram is going to be like that ... [points to free-body diagram for block] *between A to B — where there is going to be no friction — this is friction. ... The net force between A and B is going to be greater than B and C ...Just because there is no friction so you are not going to have to subtract that from (IA) —*

c) Calculate the coefficient of kinetic friction between the surface and the block between B and C if $\Theta=60^{o}$, $m_{block}=1.5kg$, and T=5N.

P: So we know that friction is μ_k times the normal force — we know that the normal force is going to be the same as the weight, which is mg — because —

by NIII — ... each force has an equal and opposite reaction — oh actually it is by the second law — F equals ma and there is no acceleration down so the sum of the forces is zero. ... I am talking about the y direction.

I: So why isn't this a NIII force pair? —

P: That's each force has an equal and opposite reaction force — that's internal. Okay — (IA) so the weight is 1.5 times 9.8 [gets calculator] *— which is 14.7 —*

I: ... So you said the normal is equal to the weight —

P: Oh actually I guess we have T sine theta — so that plus the normal equals weight. So we have T sine theta plus normal equals mg so (IA) [Pink performs the calculation] *— so the normal force is 10.4 Newtons and then the kinetic friction — ... T cosine theta plus the friction is going to equal mass times acceleration. (IA) ... I guess it is a constant —* [Rereads problem] *(IA) So acceleration along the x direction is going to be zero because it is constant — ... wait the force is constant so the acceleration is constant. So it is zero.*

I: Why do you say zero?

P: I don't know — that doesn't make sense ... I was saying that the force was constant so the acceleration was constant but that doesn't make that zero. [Pause] —

I: It looks like you wanted to say zero — why did you ant to say zero?

P: Cause I don't know this (IA) but even if it is zero that would make that negative ...

I: You said this force was in the opposite direction?

P: Yeah.

I: So if you add this force to this force [points to T cosine theta and force of friction in equation] *— is this ever going to be zero?*

P: When it stops — When it gets to C the —

I: So when it gets to C —

P: The acceleration will be zero — ...[Fixes signs in the NII equation by putting a negative sign in front of the force or friction.]

I: How did you know that was going to be minus —

P: Because they are in opposite directions.

I: How did you know that the friction would be minus and not this force [T cosine theta]*?*

P: Well I could. (IA) ... I don't know that the acceleration is — [Rereads] ...

I: Anyway to figure out the acceleration?

P: I can't remember how —

I: So in the problem what information do we know?

P: we know the distance the angle — the tension — ...

I: And what is the block doing? It starts off at rest —

P: Starts off at rest — speeds up — and then slows down — I know acceleration is velocity over time ...Work equals force times distance

I: How does the work to get the block from point A to point B compare to the work to get the block from point C to point D?

P: They are equal — ... because the force pulling it from A to B is the same and the distance is the same.

I: Which force? —

P: The tension is the same pulling it from A to B as B to C — ... but I guess the work is more — it takes more work to go from B to C ... because friction is applying a force in the opposite direction.

I: Which force is doing work on the block to get it from A to C?

P: Well tension and friction — ... It's just tension from A to B.

I: Remember the Work-Energy theorem — [pause] *work equals change KE.*

P: Force times the distance is ½ m v squared. [pause] *... force equals mass times acceleration times distance which is the work and that equals 1/2 times the mass times the final velocity squared. — so between A and B the final velocity is —* [pause]

I: How does the change in kinetic energy from A to B compare to the change in kinetic energy from B to C?

P: … The change in kinetic energy from A to B is going to be 1/2 times the mass times the velocity and from B to C — its going to be — well you're staring out with a velocity so — They will be the same because you are starting from rest and you get to a final v and that is the initial velocity from here to here and you're stopping. So the velocity is the same and the mass is the same so the change in kinetic energy is the same.

I: does that help — [pause] *so you said that the change in kinetic energy is the same —*

P: So the acceleration is zero.

I: So how do you get that?

P: Because it is not acceleration … because it is not accelerating because the velocity is not changing.

I: What do you mean the velocity is not —

P: Well, it is starting and ending at the same velocity so the change in the velocity is zero — and the acceleration is change in velocity over change in time so its zero.

I: So you said acceleration is zero because it starts off at rest and it ends at rest … so the change in velocity from A to …

P: A to C is zero —

[Pink then proceeds to solve the problem, setting the sum of forces equal to zero. She therefore has $T\cos\theta - \mu_k n = 0$ and gets $\mu_k = .24$. She then tries to move on to the next part.]

I: This equation here — this F cosine 60 plus the force of friction equals ma — when does this equation apply?

P: Between B and C.

I: Between B and C— so between B and C —

P: And between A and B it is just T cosine theta.

I: T cos equals —

P: Mass times acceleration.

I: ... Can you write sum of the forces in the x direction between A and B?

P: ... It's just T cosine theta cause that is the only force in the x direction.

I: And that's equal to ma?

P: Yeah —

I: And the acceleration?

P: Well, it's going to be ... cause it starts from rest and its got a final velocity at B so its just change in velocity over change in time.

I: So it is accelerating —

P: Between A and B — and its decelerating between B and C.

I: ... so when you wrote this equation here [points to $T \cos\theta - F_r = ma$] — *this is for — you said this is for the part of the motion between B and C?*

P: Yeah — So if you have ... I guess if you add T cosine theta then it would be for the whole thing

I: Say that again.

P: If you add T cosine theta again that would add all the forces in the x direction from here to that [points to A and C]. *So this would be* [writes a 2 in front of the tension force — she has therefore changed $T \cos \theta - \mu_k n = ma$ into $2 T \cos \theta - \mu_k n = ma$]

I: so you would get 2 T ... so what is this equation again?

P: If we add T cosine theta — that's the forces between A and B plus the forces between B and C it would be the whole thing.

I: So what do you mean the whole thing?

P: I mean the sum of the forces ... from A to C.

I: The sum of the forces from A through C is this guy [points to $2T\cos\theta - \mu_k = ma$.]

P: Right.

I: Now why do you say that?

P: Its just the sum of the force from A to B plus the forces from B to C.

I: So when you say this [points to $2T\cos\theta - \mu_k = ma$] *what's the acceleration?*

P: Well, there it is zero.

I: There its zero because . . .

P: Because the change in velocity over the change in time is zero —

I: ... So, is it zero here? [Points to old sum of forces from B to C which is $T\cos\theta - F_r = ma$.]

P: Umm — That's between B and C — well it's decelerating so it can't be zero.

d) Reads question.

P: Work is force times distance and from A to C we said that — the work is done in the x direction because it moves in the x direction — the work between A and B is going to be T cosine theta times 1 meter cause the distance is one meter and between B and C it is going to be T cosine theta minus friction times 1 m — and it is just going to be those two added together —

I: And what work is this?

P: This is between A and C —

I: By what force? —

P: Tension ... Tension plus friction from B to C —

I: So this is the work done by —

P: the work done by the sum of the forces ... [Reads question] *From friction from A to B it is zero and from B to C it is just μ times the normal force times the distance so its (IA) ...* [writes answer].

"Michelle" (5/9/97)

(reads question)

a) Draw a free-body diagram for the block when it's between B and C.

M: Between B and C you have weight of the block times gravity — you have constant tension force at angle alpha and your going to have friction between B and C — and a normal force from the table.

b) Is the magnitude of the net force acting on the block from A to B greater than, less than, or equal to the magnitude of the net force acting on the block from B to C? Explain your reasoning.

M: Net force from A to B is greater than net force from b to c because there is no friction from A to B.

I: So why does having no friction mean —

M: You're looking at horizontal forces so tension is off to the right and without friction this would be gone … the friction cancels out part of the tension force.

c) Calculate the coefficient of kinetic friction between the surface and the block between B and C if $\Theta=60^{o}$, $m_{block}=1.5kg$, and T=5N.

M: Friction — I usually start with any definition or formula I can think of — so that is μN *and in this case it is not moving off the table or surface so you are going to have the normal plus the Tension times sine theta will equal m of the block times gravity. And we can solve for N so we can put in this equation* [points to $\mu N = F_r$] *— They give us mass of block — is 1.5 times 9.8 meters per second squared minus the tension which is 5 N times sine sixty degrees. … I'd get a number for this and plug that in here* [points to $\mu N = F_r$] *— Friction — because it is going to come to a stop at point C you are going to take friction at rest will be equal to tension times cosine of theta. … Because at the end it is going to be at rest so it is not going to be moving in the horizontal direction — I took that at point C.*

I: So you are saying that at point C the force of friction has to be equal to that [points to Tcos60] *because it is at rest.*

M: So they are equal — 5 N times cosine sixty — that will give me another number which can go into there [points to $\mu N = F_r$] *and I can solve for* μ.

I: ... So at point C you said it was at rest so these two forces are equal?

M: Yes — I said it's got a velocity coming this way and the friction is eventually going to slow it to a stop and the μ isn't changing at all so —

I: So the acceleration at point C is what?

M: The acceleration at point C is zero because it is not moving.

d) Calculate the work done by friction to move the block from A to C. Show all work.

M: Start with a definition again — Work equals change in kinetic energy or Force dot distance — [pause] *I'm thinking — I'm using this one — I'm not sure that will work but I'll try it. Force and direction and distance are in the same direction so this is just force times distance.*

I: Which force?

M: The net force — which I haven't found yet. I am going to split it up between A and B and B and C because it is a different net force — So Work from A to B is —

I: How did you know it was a different net force?

M: A to B — the only force is going to be tension in the horizontal direction — so it's tension cosine theta times distance which is 1 meter which will be 5 N times cosine sixty — .5 times — 1 — will be 5/2 — work is in Joules — I usually check units if I have enough time — I do that with the homework. That is the work done from A to B. The work done from B to C is going to be the same tension force minus the friction force times distance — so that is going to be 5/2 minus this thing [points to F_r in the work form B to C equation] *— wait a second — I would get zero — I don't think that makes sense* [pause] *— I don't know —... That would be the final work done — it seems to me that there is more work done from B to C.*

I: Why do you say there is more work done from B to C?

M: — just because there is friction ...

I: What work are you finding here?

M: That is just the work done by tension to pull it from A to B.

I: When you did the work from B to C — what work were you finding?

M: I took the net force — the work of the net force over that distance — I guess that could make sense because tension is pulling towards the right and friction is pulling towards the left so they cancel out — but I don't know if that cancels out work. ...

I: So what is the net force in this [points in BC region] *region?*

M: There is still going to be some tension force — friction only cancels out at point C — it's not between B and C.

I: So the friction force and the T cosine sixty are equal at point C.

M: ... I get μ from part C — no it will still get me the same thing — never mind — ... I get zero — is that right — that's not right is it? —

I: ... you said the friction force and the and the T cosine sixty are equal at point C — what about here [points to region in between B and C]*?*

M: Well friction stays the same because μ is not changing between B and C and neither is the normal — ... friction is just taking away the momentum of the block — the velocity —

I: So are they equal here [points to region between B and C]*?*

M: Could be — I guess they are — because the tension's constant too and friction would be constant — so I'll leave it at zero then. ...

I: How does the change in kinetic energy from A to B compare to the change in kinetic energy from B to C?

M: Well looking at this if work equals the change in kinetic energy I would get change in kinetic energy here is 5/2 and here it is zero. —

I: What do you think about that?

M: I would probably want to look at the formula for KE — ½ m v squared — KE initial would be zero because it started from rest so this is final and — this is from A to B [writes $\Delta KE_{AB} = \frac{1}{2} m v_f^2$] *— change in KE from B to C — the final would be zero since it comes to rest minus KE initial which is at point B — mass is the same and the final velocity at B is the initial velocity so it is the same velocity — so the change in KE —* [writes $\Delta KE_{BC} = -1/2\ m v_f^2$] *so change in KE from A to C — if you add those two up you get zero. That conflicts with my earlier answer. I think there is something wrong with this* [points to equation for work — $W_{BC} = 0$ J] *— I think I should get — 5/2 J —*

I: How do you know that?

M: It has to be zero — this way tells you it has to be zero —

I: Knowing that — would you keep your answer to (b) the same?

M: I would think that is good — [pause] *The distance is the same and if the net work is the same that means the net force would have to be the same but opposite directions — which tells me that the friction is going to be greater than tension right here* [points to region between B and C] —

I: Does that make sense?

M: Yeah.

I: Why does that make sense?

M: From A to B you have no friction so you have the tension force off to the right and that's the net force from A to B and from B to C it has to be the same net force but is opposite directions so friction is going to have to cancel that out and double it to get the net force the same. ...

I: The way you find μ *— is that ok?*

M: Probably not — the normal is alright. Friction is going to equal two times the tension cosine theta — ...The net force from A to B is going to be tension cosine theta and because the works turn out to be zero — you are going to have the net force from B to C — it is going to have to be the same except in the opposite direction —

I: ... before you looked at point C and said that the acceleration is zero there —

M: I guess I shouldn't have.

I: How does the acceleration here [points to C] *compare to the acceleration here —* [points to location between B and C]*?*

M: It is decelerating —

I: How about the magnitude?

M: The magnitude is greater here — ... The easy answer is that it is moving forward and it eventually comes to a stop — if you look at NII the mass stays the same and the forces — friction is going to be slowing it down so the magnitude is going to be smaller hen it gets to C —

I: And then at point C — is it zero?

M: I'd say yes because it is not moving (IA).

Appendix E: The Force Concept Inventory

The FCI, contained on pages 208-212 of the original dissertation, are redacted.

Bibliography

B. A. Ambrose, P. S. Shaffer, R. N. Steinberg, L. C. McDermott, "An investigation of Student Understanding of Single Slit Diffraction and Double Slit Interference," Am. J. Phys., **67** (2) 146-155 (1999).

Y. Anzai and T. Yokoyama, "Internal Models in Physics Problem Solving," Cognition and Instruction, **1** (4) (1984).

A. B. Arons, *Teaching introductory physics* (Wiley, New York, 1997).

E. Bagno and B. Eylon, "From Problem Solving to a knowledge structure: An example from the domain of electromagnetism," Am. J. Phys. **65** (8) 726-736 (1997).

R. J Beichner, "Testing student interpretation of kinematic graphs," Am. J. Phys. **62**, 750-762 (1994).

R. Bhaskar and H. A. Simon, "Problem Solving in Semantically Rich Domains: An example from engineering thermodynamics," Cog. Sci., **1** 193-215 (1977).

M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).

M. T. H. Chi, N. de Leeuw, M.H. Chiu, and C. LaVancher, "Eliciting Self-Explanations Improves Understanding," Cognitive Science, **18**, 439-477 (1994).

C. A. Chinn and W. F. Brewer, "The role of anomalous data in knowledge acquisition: A theoretical framework and implications for science instruction," Review of Educational Research, 63 (1), 1-49 (1993).

J. Clement, "Students' preconceptions in introductory mechanics," Am. J. Phys. **50** (1) 66-71 (1982).

A. A. diSessa, "Knowledge in Pieces," In *Constructivism in the Computer Age,* G. Forman and P. Pufall (Eds.) (Lawrence Erlbaum, NJ, 1988), pp. 1-24.

R. Elio and P.B. Scharf, "Modeling Novice-to-Expert Shifts in Problem Solving Strategy and Knowledge Organization," Cognitive Science, **14**, 579-639 (1990).

H. C. Ellis and R. R. Hunt, *Fundamentals of Cognitive Psychology*, (Brown & Benchmark, WI, 1993).

B. Eylon and R. Reif, "Effects of knowledge organization on task performance," Cognition and Instruction," Cognition and Instruction, **1** (1), 5-44 (1984).

R. J. Dufresne, W. J. Gerace, P. T., Hardiman, and J. P. Mestre, "Constraining novices to perform expert-like problem analyses: Effects on schema acquisition," Journal of the leaning Sciences, **2** (3) 3 07-331 (1992).

M. G. M. Ferguson-Hessler and T. de Jong, "The quality of knowledge in the field of electricity and magnetism," Am. J. Phys., **55** (6) 492-497 (1987).

D. Gentner and A. L. Stevens, *Mental Models*, (Lawrence Erlbaum Associates, NJ, 1983).

R. Hake, "Interactive engagement versus traditional methods: a six-thousand-student survey of mechanics test data for introductory physics courses,: Am. J. Phys., 65 (5) 418-428 (1997).

D. Halliday and R. Resnick, *Fundamentals of physics*, (Wiley, NY, 1974).

D. Hammer, "Two approaches to learning physics," Phys. Teach. **27** (9) 664-670 (1989).

D. Hammer, "Epistemological beliefs in introductory physics," Cognition and Instruction, **12** (2) 151-183 (1994).

D. Hammer, "Students' beliefs about conceptual knowledge in introductory physics," Int. J. Sci. Ed., **16** (4) 385-403 (1994).

D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).

I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. **53** (11) 1043-1055 (1985).

I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. **53** (11) 1056-1064 (1985).

M. Hegarty, "Knowledge and processes in mechanical problem solving," In *Complex problem solving: Principles and mechanisms,* R. J. Sternberg and P. A. Frensch (Eds.) (Lawrence Erlbaum, NJ, 1991), pp. 283-285.

J. L. Heller and F. Reif, "Prescribing effective human problem-solving process: Problem description in physics," Cognitive and Instruction, **1** (2), 177-216 (1984).

P. Heller, R. Keith, and S. Anderson, "Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving," Am. J. Phys. **60** (7), 627-636 (1992).

P. Heller and M. Hollabaugh, "Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups," Am. J. Phys. **60** (7), 637-644 (1992).

P. Heller and D. Huffman, "Interpreting the force concept inventory, a reply to Hestenes and Halloun," Phys. Teach. **33**, 503 (1995).

D. Hestenes, "Wherefore a science teaching?" Phys. Teach. **17**, 232-242, (1979).

D. Hestenes, M. Wells and, G. Swackhammer, "Force Concept Inventory," Phys. Teach. **30** (3) 141-153 (1992).

D. Hestenes and I. Halloun, "Interpreting the force concept inventory, a response to the March 1995 critique by Huffman and Heller," Phys. Teach. **33**, 502 (1995).

D. Huffman and P. Heller, "What does the Force Concept Inventory actually measure?" Phys. Teach. **33**, 138-143 (1995).

D. Huffman, "Effect of Explicit Problem Solving Instruction on High School Students' Problem Solving Performance and Conceptual Understanding of Physics," Journal of Research in Science Teaching, **34** (6) 551-570 (1997).

S. Kanim, "An investigation of Student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999).

J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979).

J. H. Larkin, "Skilled problem solving in physics: A hierarchical planning model," Journal of Structural Learning, **1**, 271-297 (1980).

J. H. Larkin, J. McDermott, D. P. Simon, and H.A. Simon, "Expert and Novice Performance in Solving Physics Problems," Science, **208** (20) 1335-1342 (1980).

J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.

W. J. Leonard, W. J. Gerace, R. J. Dufresne, and J. P. Mestre, "Concept-based Problem Solving in Physics," Internal Report PERG, (1994).

W. J. Leonard, R. J. Dufresne, and J. P. Mestre, "Using Qualitative Problem-Solving Strategies to Highlight the role of conceptual knowledge in solving problems," Am. J. Phys., **64** (12), 1495-1503 (1996).

D. P. Maloney, "Rule-governed approaches to physics: Newton's Third Law," Phys. Educ. **19**, 37-42 (1984).

D. P. Maloney, "Research on Problem Solving: Physics. In *Handbook of Research on Science Teaching and Learning*, D. L. Gabel (Ed.), (Macmillan Publishing Co, NY, 1994), pp. 327-354.

S. P. Marshall, "Assessing Problem Solving: A short term remedy and a long term solution," In *The teaching and assessing of mathematical problem solving*, R. I Charles and E. A. Silver (Eds.) (L. Erlbaum Associates and the National Council of Teachers of Mathematics, VA, 1988), pp. 159-177.

S. P. Marshall, *Schemas in Problem-solving*, (Cambridge University Press, NY, 1995).

E. Mazur, *Peer Instruction*, (Prentice Hall, NJ, 1997).

J. Minstrell, "Facets of students' knowledge and relevant instruction," In Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop, R. Duit, F. Goldberg, and H. Neidderer, (Eds.) (IPN, Kiel Germany, 1992), pp. 110-128.

L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned – closing the gap," Am. J. Phys. **59** (4) 301-315 (1991).

L. C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993).

L. C. McDermott, P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. 62 (1) 46-55 (1994).

L. C. McDermott, P. S. Shaffer, M. L. Rosenquist and the PEG, *Physics by Inquiry*, (Wiley, NY, 1996).

L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).

L. C. McDermott, and the Physics Education Group, A perspective on physics education research as a guide to the improvement of instruction, unpublished collection, (1998).

L. C. McDermott and E. F. Redish, "Resource Letter on Physics Education Research," to be published in Am. J. Phys. (1999)

C. McMillian, and M. Swadener, "Novice Use of Qualitative versus Quantitative Problem Solving in Electrostatics," Journal of Research in Science Teaching, **28** (8), 661-670 (1991).

A. Newell and H. A. Simon, *Human problem solving*. (Prentice Hall, NJ, 1972).

T. S. O'Brien Pride, S. Vokos, and L. C. McDermott, "The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems," Am. J. Phys. **66** (2) 147-157 (1998).

G. Polya, *How to solve it*, (Doubleday, NY, 1945).

E. F. Redish, "Implications of cognitive studies for teaching physics," Am. J. Phys. **62** (9), 796-803 (1994).

E. F. Redish and J.S. Rigden (Eds.), AIP Conference Proceedings No. 399 The changing role of physics departments in modern universities*: Proceedings of the international conference on undergraduate physics education*, (AIP Press, NY, 1997).

E. F. Redish, J. M. Saul, and R. N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **66** (3), 212-224 (1998).

E. F. Redish, J. M. Saul, and R. N. Steinberg, "Student expectations in introductory physics," Am. J. Phys. **66** (3), 212-224 (1998).

E. F. Redish, "Millikan Award Lecture (1998): Building a Science of Teaching Physics," Am. J. Phys. **67** (7), 562-573 (1999).

F. Reif, J. H. Larkin, and G.C. Brackett, "Teaching general learning and problem-solving skills," Am. J. Phys. **44** (3) 212-217 (1976).

F. Reif, "Teaching problem solving – A scientific approach," The Physics Teacher, 310-316 (1981).

F. Reif and J. I. Heller, "Knowledge structures and problem solving in physics," Educational Psychologist, **17** (2), 102-127 (1982).

F. Reif and S. Allen, "Cognition for Interpreting Scientific Concepts: A study of acceleration," Cognition and Instruction, **9** (1), 1-44 (1992).

F. Reif, "Millikan Lecture 1994: Understanding and teaching important scientific thought processes," Am. J. Phys. **63**, 17-32 (1995).

D. E. Rumelhart, "Schemata: The building blocks of cognition," In *Theoretical issues in reading comprehension*, R. Spiro, B. Bruce, and W. Brewer, (Eds.) (Lawrence Erlbaum, NY, 1980), pp. 33-58.

J. M. Saul, "Beyond Problem Solving: Evaluating introductory physics courses through the hidden curriculum," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).

R. A. Serway, *Physics for Scientists and Engineers*, 4th edition, (Saunders College, Philadelphia, 1996).

R. C. Shank, *Tell me a story*, (Charles Scribner's Sons, NY, 1990).

P. S. Shaffer and L. C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity, Part II: Design of instructional strategies." Am. J. Phys. 60 (11), 1003-1013 (1992).

P. S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, (1993).

D. P. Simon and H. A. Simon, "Individual differences in solving physics problems," In *Children's Thinking: What develops?* R. S. Siegler (Ed.) (Lawrence Erlbaum, NJ, 1978), pp. 325-348.

R. S. Steinberg and M. S. Sabella, "Performance on multiple-choice diagnostics and complimentary exam problems," Phys. Teach. **35** (3), 150-155 (1997).

B. Sherin, "The Symbolic Basis of Physical Intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, School of Education, University of California, Berkley, (1996).

B. Thacker, E. Kim, K. Trefz, S. Lea, "Comparing problem-solving performance of physics students in inquiry-based and traditional introductory physics courses," American Journal of Physics 62 (7), 627-633 (1991).

R. K. Thornton and D. R. Sokoloff, "Assessing student learning of Newton's laws: The Force and motion concept evaluation and the evaluation of active learning laboratory and lecture," A. J. Phys. **66** (4) 338-351 (1998).

J. Torney-Purta, "Schema Theory and Cognitive Psychology: Implications for Social Studies," Theory and Research in Social Education, **19** (2), 189-210 (1991).

D. E. Trowbridge and L. C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," Am. J. Phys. **49** (3), 242-253 (1981).

A. Van Heuvelen, "Overview, Case Study Physics," Am. J. Phys. **59** (10), 898-907 (1991).

S. Vosniadou, and W. F. Brewer, "Mental Models of the earth: A study of conceptual change in childhood," Cog. Psych., 24, 535-585 (1992).

S. Vosniadou, "Capturing and modeling the process of conceptual change," Learning and Instruction, (4) 45-69 (1994).

D. S. Ward, and J. Sweller, "Structuring effective worked examples," Cognition and Instruction. **7,** 1-39 (1990).

M. C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).