Vector Addition: Effect of the Context and Position of the Vectors

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Abstract. In this article we investigate the effect of: 1) the context, and 2) the position of the vectors, on 2D vector addition tasks. We administered a test to 512 students completing introductory physics courses at a private Mexican university. In the first part, we analyze students' responses in three isomorphic problems: displacements, forces, and no physical context. Students were asked to draw two vectors and the vector sum. We analyzed students' procedures detecting the difficulties when drawing the vector addition and proved that the context matters, not only compared to the context-free case but also between the contexts. In the second part, we analyze students' responses with three different arrangements of the sum of two vectors: tail-to-tail, head-to-tail and separated vectors. We compared the frequencies of the errors in the three different positions to deduce students' conceptions in the addition of vectors.

Keywords: Vector addition, graphical vector addition, isomorphic problems, vectors arrangements.
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INTRODUCTION

The understanding of vectors is important for science and engineering students, not only to understand introductory-level physics concepts but also to understand more advance topics in their curriculum. In recent years, researchers have investigated students' difficulties with the addition of vectors [1-6]; this work contributes further to the understanding of those difficulties.

This work has two objectives: to analyze the effect of 1) the context and 2) the position of vectors on two-dimensional vector addition problems. In the first part, we analyze the effect of the context on isomorphic problems. Some researchers [4, 5] have investigated this effect. However, our investigation has two particular features: 1) we use three different isomorphic problems, displacements, forces, and no physical context; and, 2) we do not present a sketch of the adding vectors, they are only described. This permits us to establish relations between students' representations and the context of the problem.

In the second part, we analyze the effect of the position of the vectors on vector addition problems with no physical context. Hawkins et al. [6] began analyzing this effect with a qualitative study. The authors showed that most students stick to one method when working with different problems with different visual representations. Instead, we take a quantitative approach, in which we analyze students' responses solving a problem with three different representations: 1) vectors apart from each other, 2) tail-to-tail, and 3) head-to-tail.

In the following section we present the details of the methodology of this study. Later, we divide the Results and Discussion section in subsections covering each one of the two objectives. At the end, we present a Conclusions section with a review of the main results of the study.

METHODOLOGY

This research was conducted in a large private Mexican university. Problems were administered to 512 students in their last calculus-based introductory physics course in this institution. Fig. 1 shows the problems used in this study. To address the first objective, we designed three different isomorphic problems: one with displacements, another with forces, and the other with no physical context (Problems 1-3). To address the second objective, we used problems with different representations (Problems 4-6). Problem 4 was designed by Nguyen and Meltzer [1], and Problems 5 and 6 are our modifications of this problem.

To make comparisons in this study, we decided to divide the sample in three different groups (each of approximately 170 students), following the methodology used by Barniol & Zavala [7]. The selection of these three populations was made randomly. Population A solved Problems 1 and 4, Population B solved Problems 2 and 5, and Population C solved Problems 3 and 6.

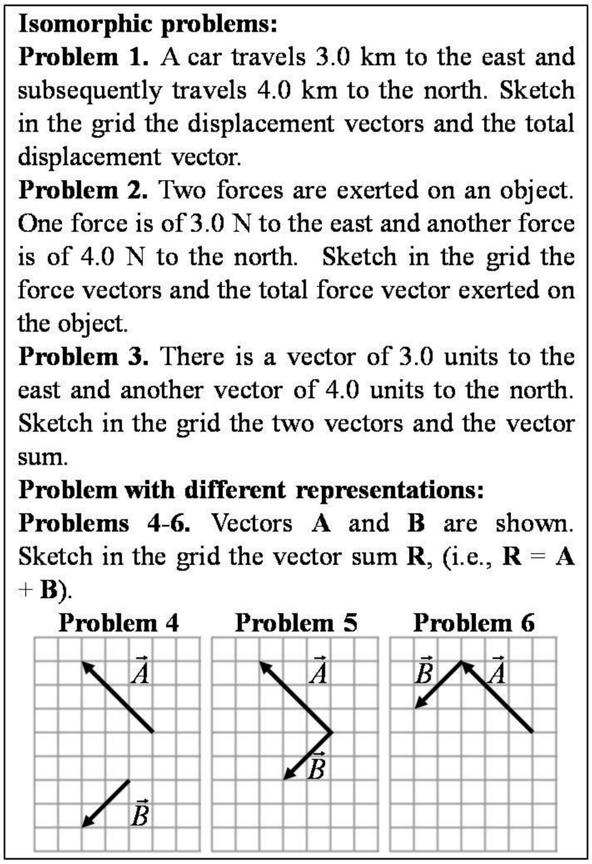

FIGURE 1. Complete set of questions administered to students in the study.

RESULTS AND DISCUSSION

This section is divided into two subsections addressing the two objectives of the study.

1. Effect of the context

In this subsection, we analyze the effect of the context on isomorphic vector addition problems (Problems 1-3). These problems showed the four cardinal directions and asked students to make a drawing to scale. Table 1 shows the frequencies of the different representations, in which the two vectors are sketched by students in each one of the contexts, and Fig. 2 shows these representations graphically.

Table 1 shows that in the displacement context, most of students draw the vectors in a head-to-tail representation, and that in the force-context most of them sketch the vectors in a tail-to-tail representation. This is the first difference of these two contexts. These results could be explained by the mental model students have to make to draw the vectors. That mental model in most cases corresponds to the vector context. In the no-context problem, students split in two significant percentages; however, the tendency is to draw the vectors in a tail-to-tail representation. It is interesting that in the no-context problem, 6% of students sketch the vectors in the “separate” representation. This is an indication that students have difficulty to make a mental representation of this problem.

TABLE 1. Differences in representations by students when sketching the vectors in Problems. 1-3.

Two vectors	Probl.1 Displ.	Probl. 2 Force	Probl.3 No context
Head-to-tail	92%	10%	29%
Tail-to-tail	7%	85%	60%
Separate	1%	1%	6%
Others	0%	4%	5%

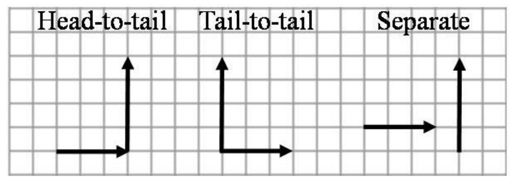

FIGURE 2. Representations by students when sketching the vectors in Problems 1-3.

Table 2 shows the frequencies of the different representations in which students draw the vector sum and Fig. 3 shows the errors made graphically. These errors exist in the literature [1-4]. The tip-to-tip error appears also with an opposite direction.

TABLE 2. Vectors sum sketched in Problems 1-3.

Vector sum	Probl.1 Displ.	Probl.2 Force	Probl.3 No context
Correct	80%	64%	70%
No direction	7%	0%	2%
Short-Bisector	2%	17%	10%
Long Bisector	2%	13%	7%
Closing the loop	3%	1%	4%
Tip-to-tip	1%	1%	5%
Others	5%	4%	2%

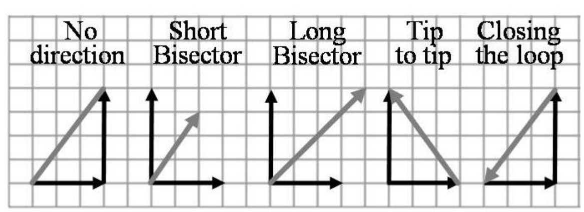

FIGURE 3. Errors in Problems 1-3.

Table 2 shows that the displacement-context problem is the one with the most correct answers. This could be due to the fact that this is the most familiar context to students. The most common error in this context (7%) is to sketch a line representing the vector with a correct magnitude but without specifying the direction by an arrow (see Fig. 3). This could be due to confusion between displacement and distance or a simple oversight of the student.

In the force-context and no-context problems, the most common error is to draw a bisector vector, also mentioned by Van Deventer [4]. In this error, students draw a vector sum that goes between the two vectors but lacks the precision to be considered correct. The bisector vectors detected in this study have different magnitudes and directions. It is possible to distinguish between short bisectors and long bisectors (similar to the ones shown in Fig. 3). It is feasible that this error is due to the fact that in the force-context and no-context problems the vector sum is less familiar and more abstract than that in the displacement-context problem.

Table 2 also shows that, in the no-context problem, the errors tip-to-tip and closing the loop are extended. This is another indication that the context influences the answer and that students have difficulties constructing mental models with no-context problems.

2. Effect of the position of the vectors

In this subsection, we analyze the effect of the position of the vectors on a vector addition problem presented in three different representations (Problems 4-6). Before comparing the error distribution in these problems, it is necessary to explain some of the errors found. Fig. 4 shows these errors graphically. The tip-to-tip error appears also with an opposite direction.

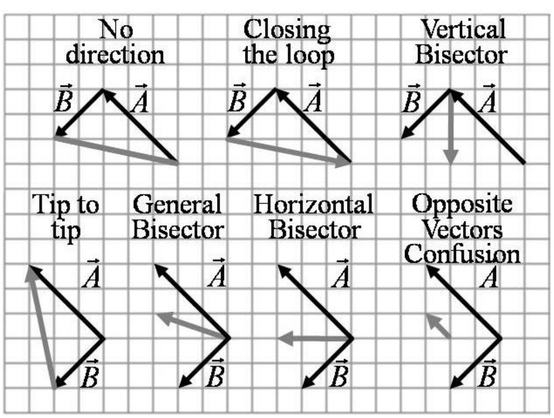

FIGURE 4. Errors in Problems 4-6.

Usually the students who make these errors do not explicitly show the procedure they follow to obtain their results. Many students directly sketch the vector sum. This makes the analysis more difficult. In this subsection, there are many types of bisector vectors (Fig. 4). We decided not to distinguish between short bisector and long bisector (like in the last subsection). Instead, we decided to distinguish among general bisector, horizontal bisector and vertical bisector. All these bisector vectors have in common that they are drawn between the two vectors.

The general bisector error is a vector that lacks precision and shows different magnitudes of the *x*- and *y*-components. Fig. 4 shows a common representation. A student wrote an explanation (as a part of his procedure) that exemplified this error: "***A*** *is greater than* ***B****, so* ***R*** *goes more tilted towards* ***A***."

In the horizontal bisector error students draw a vector (with different magnitudes) exactly in the negative *x*-axis. Fig. 4 shows a common representation. A student wrote a reasoning that exemplified this error: "***A*** *and* ***B*** *with their directions cancel each other to the center (to the left), and the magnitude is between 3 and 2, that is 2.5*". Note also that some students draw a horizontal vector with a magnitude of 5, which is the correct *x*-component of the vector sum. It seems that these students add the *x*-components, without adding the *y*-components. The general bisector and the horizontal bisector errors were also detected by Nguyen and Meltzer [1], but in our study we elaborate these errors using the bisector error definition stated by Van Deventer [4]. We confirm that many students add vectors by sketching a vector sum that goes between the two vectors but lacks precision, so we decided to identify these two errors as bisector vectors.

In our data, another error also appears: a vertical bisector vector (Fig. 4). This vector also goes between the two vectors, but the students seem to not realize that it is a head-to-tail representation. The vertical bisector vector error also shows different magnitudes and, in some cases, a slight inclination. This error had not been reported in the literature.

The opposite vectors confusion shown in Fig. 4 had not been reported in the literature either. Students usually sketched the vector sum directly, so it is difficult to make a complete analysis of this error. We observed two incorrect procedures that resulted in this particular incorrect answer. In the first procedure, students use a component addition algorithm subtracting (not adding) the *x*-components. In the second procedure, students either write this kind of explanations "***R****=3-2*", "***R****=3****A****-2****B***" or make sketches that seem to suggest that these students think that the vectors are "opposite", so to add them it is necessary to do a subtraction of them. This error will be analyzed in a future study using interviews.

After analyzing the errors, we are able to present the effect of the position of the vectors on vector addition tasks. Table 3 shows the differences in the frequencies of the errors in the three representations (Problems 4-6). In the two first representations ("separate" and tail-to-tail) 59% of students draw the correct sum vector and in the third representation (head-to-tail) this percentage increases to 65%. There are a small number of students in the three representations who draw a line representing the vector with a correct magnitude but without specifying the direction by an arrow. (See Fig. 4.)

TABLE 3. Vectors sum sketched in Problems 4-6.

Vector sum	Probl.4 Separate	Probl.5 Tail-to-tail	Probl.6 Head-to-tail
Correct	59%	59%	65%
No direction	2%	1%	3%
Closing the loop	2%	2%	10%
Tip-to-tip	9%	9%	3%
General bisector	6%	8%	4%
Horizontal bisector	9%	12%	5%
Vertical bisector	0%	0%	5%
Opposite vectors confusion	5%	4%	1%
Others	8%	5%	4%

There are clear tendencies in the tip-to-tip and closing the loop errors. In the two first representations the percentage of the tip-to-tip error is significant (9%), but in the third representation this error is only 3%. On the other hand, the closing the loop error is in the third representation (head-to-tail) significant (10%) and in the other two representations is only 2%. The students who made the closing the loop error in problem 5, which is a tail-to-tail representation, moved the vectors to a head-to-tail representation and then made the closing the loop error. On the other hand, the students who made the tip-to-tip error in problem 6, which is a head-to-tail representation, moved the vectors to a tail-to-tail representation and then made the tip-to-tip error.

The bisector error (general, horizontal and vertical) is important in the three representations. If we compare the frequencies of these three errors, we see interesting tendencies. The general bisector and horizontal bisector error are more common in the second representation, than in the first and, finally, in the third one. On the other hand, the vertical bisector error appears only in the third representation. The opposite vectors confusion appears with a significant percentage only in the first and second representations. It seems that these two representations trigger in some way this error. In general, we observe that the frequencies of error of the first two representations are very similar. This could be explained by the fact that the first representation is "closer" to the second representation than it is to the third.

CONCLUSIONS

We found important differences in responses of students in three isomorphic vector addition problems. The context has an influence on the representations used when sketching the two vectors needed to be added and on the vector sum. We found that the context matters, not only compared to the context-free case, but also between contexts. The results indicate that the context helps most of the students to make a mental model and then they solve the problem with their own sketch. One can argue the need to teach vectors using a context; however, students should be able to transfer knowledge among different contexts, and that probably is better achieved with a context-free approach. We are currently investigating this.

On the other hand, we found significant differences between responses to problems in the tail-to-tail and head-to-tail representations, and some similarities between those in the "separate" and the tail-to-tail representations. One can hypothesize that the results would have been different if the separate representation would have been closer to the head-to-tail representation instead. What is interesting is that each representation has its own difficulties and the instructor should be aware to plan the instruction. Finally, it is important to note that even though most students solve the problems correctly, some students, even after taking introductory physics courses, still show difficulties with basic vector operations.

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