Facilitating Strategies for Solving Work-Energy Problems in Graphical and Equational Representations

Dong-Hai Nguyen, Elizabeth Gire and N. Sanjay Rebello

Department of Physics, 116 Cardwell Hall, Kansas State University, Manhattan, KS 66506-2601

Abstract. Our previous research has suggested that the major difficulty students have when solving physics problems posed in graphical and equational representations is due to students' inability to appropriately activate the required mathematical knowledge in the context of a physics problem. Based on these results, we developed problem sets for each major topic in introductory mechanics. Each set consisted of one or two pairs of matched math and physics problems, debate problems, and problem posing tasks. We conducted focus group learning interviews with two groups of students working in pairs: a treatment group working on our research-based problem sets and a control group solving isomorphic textbook problems on the same topics. We present here a description of one of our problem sets on Work-Energy problems as well as a comparison of the performance of the two groups on transfer problems on Work-Energy involving graphical and equational representations.

Keywords: problem solving, representation, interview, transfer, physics education research
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INTRODUCTION

In a previous study [1] we conducted individual teaching/learning interviews with 20 students in a calculus-based introductory physics course, covering topics in mechanics, to investigate the kinds of difficulties students encountered when solving problems in graphical and equational representations and the kinds of scaffolding that might help students overcome those difficulties. Results from this previous study indicate that students encounter a variety of difficulties when solving problems in graphical and equational representations. These difficulties can be attributed primarily to students' inability to activate the required mathematical knowledge in the context of a physics problem. This result suggests that exercises that facilitate the activation of required mathematical skills needed to appropriately use information provided in graphical and equational representations may help students improve their problem solving skills.

In this study, we develop problem sets which consist of pairs of matched math and physics problems, debate problems and problem posing tasks [2]. We test the impact of these sequences of problems on students' ability to solve problems in graphical and equational representations. The problems in these sets were created such that they targeted the common difficulties we observed in our previous study. The overarching research question is: Can a research-based sequence of math, physics and non-traditional problems improve students' ability to solve physics problems in graphical and equational representations?

Given the limited scope of this paper, we focus specifically on the topic of work-energy. We present a problem set focusing on this topic and its effect on students' performance on problems in graphical and equational representations.

METHODOLOGY

We conducted five focus group learning interviews (FOGLIs) [3] with two groups of students randomly selected from a pool of 88 volunteers enrolled in a first-semester calculus-based physics course. The format of the FOGLI sessions is described in the next paragraph. Most participants were freshmen or sophomores majoring in engineering. The participants were randomly assigned into either a control group or a treatment group. The number of students in each group varied with each FOGLI session, ranging from eight to 10 students in the control group and from 12 to 14 students in the treatment group.

The topics for the FOGLI sessions were one-dimensional kinematics in session 1, Newton's second law in session 2, work and energy with friction in session 3 and rotational energy with friction in session

4. Session 5 was comprehensive, covering all topics from sessions 1 through 4. In each of these 90-minute FOGLI sessions, for the first 15-20 minutes students attempted a pre-test that consisted of two physics problems: one graphical and the other equational. In the next 40–50 minutes, students worked in pairs on the problem sets prepared by us. Students in the treatment group worked on a problem set which included two pairs of matched math and physics problems, a debate problem and one or two problem posing tasks. Students in the control group worked on isomorphic textbook problems covering the same concepts. Finally, in the last 15-20 minutes, students worked individually on the post-test which differed from the pre-test only in numerical values of physical quantities given in the problem statements.

Students in both groups were encouraged to discuss their problem solving strategies with their partners. Students in the control group were provided with a printed solution of each problem before proceeding to the next problem. Students in the treatment group were required to check-in with a facilitator before proceeding to the next problem. The facilitator engaged in Socratic dialog [4] with the students to elicit their ideas and facilitate them to solve the problems in the problem set. The problem sets for the control and treatment groups as well as the pre-test and post-test problems are shown in Figures 1, 2 and 3.

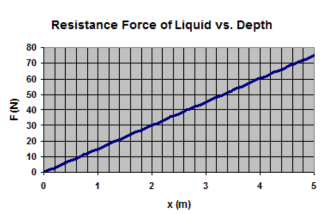
Rubrics were created to grade the pre-test and post-test problems (transfer tasks) in each FOGLI session. Each problem was graded separately on the *physics* aspect and the *representation* aspect. The maximum score on the physics aspect was 10 points and on the representation aspect was 8 points.

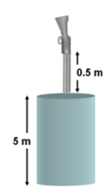
Problem A

A 0.05 kg bullet is loaded into a gun compressing a spring which has spring constant k = 5000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum.

What is the spring compression x?





Problem B

A 0.05 kg bullet is loaded into a gun compressing a spring which has spring constant k = 5000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force F (in Newtons) provided by the liquid changes with depth x (in meters) as per the following function:

$$F(x) = 8x + 0.5x^2$$

The bullet comes to rest at the bottom of the drum.

What is the spring compression x?

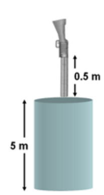
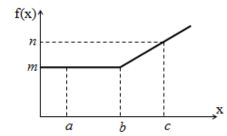


FIGURE 1. Transfer tasks in FOGLI session 3.

Problem 1

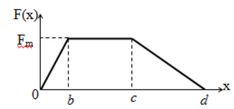
The graph of a function f(x) is given below.

Find the value of the integral $\int_a^c f(x)\,dx$ in terms of the constants a, b, c, m, n.



Problem 2

The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newton and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.



Problem 3

Find the area of the region surrounded by the graphs of the following functions: $f(x) = x^3 + 2x + 1,\ f(x) = 0,\ x = x_1,\ x = x_2$.

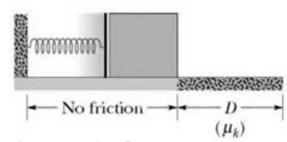
Problem 4

A block is pulled on a horizontal frictionless floor by a force F whose magnitude (in Newton) depends on the displacement x of the block (in meters) as per the function: $F(x) = ax^2 + bx + c$ (a, b, c are constants). Find the work done by force F when the block has been moved from x_1 to x_2.

Problem 5

Five students are discussing their strategies to solve the following problems.

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction μ_k = 0.25. The frictional force stops the block in distance D = 7.8 m.



What was the spring compression x?

Which student is correct? Comment on each student's ideas. Explain who you agree with most and why. For the students who make statements you disagree with, try to identify what went wrong in the student's reasoning.

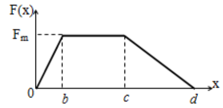
Student	Strategy	Comments
David	Energy is conserved so all the changes in energy add to zero. The block starts from rest and then comes to a stop, so there is no change in kinetic energy. The only energy that changes is the spring's potential energy and that's good because that involves the compression of the spring. You can calculate the change in potential energy and solve for the compression.	
Mary	Friction is involved so you need to use $\Delta K + \Delta U = W$, where W = -μkmgD is the work done by friction. ΔK is zero because initial and final speeds are zero. The initial U is that of the spring and final U is zero. Then put everything into the equation and solve for x.	
Eric	Isn't the work +μkmgD, because W in that equation is the amount of work done and therefore it must be positive?	
Susan	But the spring does work on the block too and you have to take that into account. Work is force times distance, and since the force of the spring is -kx and the spring pushes the block a distance x, the work done by the spring is -kx^2. That's the formula you should use to find the compression.	
Mike	All you have to do to calculate the work done by the spring is to plug in the total distance the spring pushes the block into the force -kx. So, if the initial compression is L, the work done by the spring is -kL.	

Problem 6

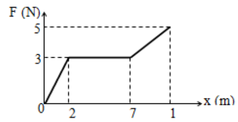
a. Start with the physics problem in problem 5, modify it by including in it the physics ideas in problem 2 to create a new solvable problem of your own. Write your instructions to solve that new problem.
b. Start with the physics problem in problem 5, modify it by including in it the physics ideas in problem 4 to create a new solvable problem of your own. Write your instructions to solve that new problem.

FIGURE 2. Problem set for the treatment group in FOGLI session 3.

Problem 1
The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newtons and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.



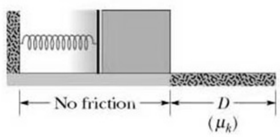
Problem 2
The graph below shows the magnitude of a force F (in Newtons) acting on an object with respect to the displacement x (in meters) of the object. Find the work done by force F on the object over the displacement from 0 m to 10 m.



Problem 3
A block is pulled on a horizontal frictionless floor by a force whose magnitude F (in Newtons) depends on the displacement x of the block (in meters) as per the function: $F(x) = ax^2 + bx + c$ (a, b, c are constants). Find the work done by force F when the block has been moved from x_1 to x_2.

Problem 4
A block is pulled on a horizontal frictionless floor by a force F whose magnitude depends on the displacement of the block as per the function: $F(x) = 2x^3 - 3x + 2$ (x is in meters, F is in Newtons). Find the work done by force F when the block has been moved from 0 m to 2 m.

Problem 5
A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m.



What was the spring compression x?

FIGURE 3. Problem set for the control group in FOGLI session 3.

The rubric for the ***physics*** aspect rated five dimensions: *approach* (i.e. was correct principle used?), *equations* (i.e. were correct equations used ?), *values* (i.e. were correct values of quantities used?), *manipulation* (i.e. were the equations correctly manipulated?), and *units* (i.e. were the units correct?).

The rubric for the ***representational*** aspect also rated five dimensions: *gathering* (i.e. was the correct information gathered from the representation?), *mapping* (i.e. was the information correctly mapped on to the physics problem), *setting up* (i.e. was the information correctly used in the physics problem?), *manipulation* and *units* which are same as above.

In this paper, we discuss the problems and results of FOGLI session 3. In this session, the physics aspect of the transfer tasks includes the application of conservation of energy, while the representation aspect involves the calculation of work done from the graph of force vs. depth or from the equation of force. Our previous study [1] shows us that students have difficulties finding work in these problems although they know that work equals force times distance and are able to calculate an integral. There seems to be a gap between students' understanding of Work = Force x Displacement and a recognition that when the force is a function of displacement, they need to integrate force over displacement. We developed a set of problems for the treatment group as presented in Fig. 2, which targeted two key ideas:

(i) $\int F(x)dx$ is the area under the graph of F(x) vs. x.

(ii) Work equals $\int F(x)dx$.

Problems 1 and 3 in the problem set are math problems targeting the first idea, while problems 2 and 4 are physics problems targeting the second idea. These problems are organized in pairs: 1 & 2 and 3 & 4, to emphasize the use of mathematical skills (integral and area) in calculating a physical quantity (work). Note that in the problem set for the control group, the math problems are replaced by physics problems.

Problem 5 is a debate problem in which fictitious students discuss the physics of the solution to the problem. The reasoning of these fictitious students contains common errors that students displayed in our previous study. The goal of this problem is to prepare students with the physics knowledge needed to solve the transfer tasks by recognizing the errors other students make. The debate aspect of this problem is supposed to foster reflection on various problem solving approaches.

Problem 6 is a problem posing task [2] which asks students to embed the idea they learn from previous problems into a physics context to pose more complex physics problems. The goal of this problem is to prepare students to integrate the math and physics ideas they had learned in previous problems in this sequence. The problem posing and solving aspect of this task is designed to foster metacognition.

RESULTS

We present results of FOGLI session 3 in Tables 1 and 2. There were eight students in the control group and 12 students in the treatment group. The inter-rater reliability for scoring the physics aspect was 92%, while the inter-rater reliability for scoring the representation aspect was 89%. The means and standard deviations of the scores of each group in the pre-test and post-test are presented below..

TABLE 1. ***Physics*** score out of 10: Mean (± S.D.)

Problem	Group	Pre-test	Post-test
Graph	Control	8.25 (± 2.25)	7.88 (± 2.80)
	Treatment	8.08 (± 2.78)	9.08 (± 1.31)
Equation	Control	8.13 (± 2.59)	8.50 (± 2.00)
	Treatment	8.33 (± 2.27)	9.17 (± 1.11)

TABLE 2. ***Representation*** score out of 8: Mean (± S.D.)

Problem	Group	Pre-test	Post-test
Graph	Control	4.88 (± 2.75)	6.13 (± 1.89)
	Treatment	5.33 (± 2.84)	7.58 (± 0.90)
Equation	Control	4.25 (± 2.82)	4.88 (± 2.80)
	Treatment	4.08 (± 2.64)	7.00 (± 1.60)

Given the small number of participants in each group, the non-parametric Mann-Whitney U Test [5] was employed to test the significance of the difference between the scores of two groups on the pre-test and post-test. The null hypothesis is that the scores of the two groups are not statistically significantly different. SPSS was used to perform the Mann-Whitney test. We present below the results of the Mann-Whitney U test for the physics aspect and the representation aspect of each of the problems in the pre-test and post-test.

(i) *Physics* scores: Table 3 indicates that the score on the physics aspect of the treatment group is not statistically significantly different from that of the control group, on both the pre-test and post-test.

TABLE 3. Mann-Whitney for ***physics*** scores.

Problem	Pre-test	Post-test
Graph	U = 47.5, z = - 0.04 p = 0.97, r = - 0.01	U = 37.0, z = - 0.90 P = 0.37, r = - 0.20
Equation	U = 44.5, z = - 0.29 p = 0.78, r = - 0.06	U = 39.0, z = - 0.74 p = 0.46, r = - 0.17

Although the effect sizes are slightly higher in the post-test (r = -0.19 in the Graph problem and r = -0.16 in the Equation problem) than in the pre-test (r = -0.01 and r = -0.06 respectively), the effects are still weak. This implies that the treatment does not appear to improve students' ability to solve work-energy problems compared to the control. This result might suggest that the treatment should be refined to increase students' practice with the underlying physics knowledge of the problems.

(ii) *Representation* scores: Table 4 indicates a promising result. The score on the representation aspect of the treatment group is not statically significantly higher than that of the control group on the pre-test, but it is statistically significantly higher in the post-test. The effect sizes, r = -0.46 in the graph problem and r = -0.44 in the equation problem in the post-test suggest that these are strong effects.

TABLE 4. Mann-Whitney for ***representation*** scores

Problem	Pre-test	Post-test
Graph	U = 44.0, z = -0.32 p = 0.75, r = -0.07	U = 21.5, z = -2.26 *p = 0.02*, r = -0.51
Equation	U = 46.5, z = -0.12 p = 0.91, r = -0.03	U = 22.5, z = -2.05 *p = 0.04*, r = -0.46

This result implies that the treatment problem set significantly improves students' ability to work with graphical and equational representations more than the control problem set does.

CONCLUSIONS

This study investigates the effect of a sequence of research-based exercises on students' performance on physics problems posed in graphical and equational representations. In this paper, we focus specifically on Work-Energy problems. This sequence includes pairs of matched physics and math problems, a debate problem and problem posing tasks. Initial results suggest that such a sequence of problems has a positive effect in improving students' performance on the representation aspect of problems, while it is not as effective in improving students' performance on the physics aspect of problems.

Pedagogically, the promising result on the representation aspect of problem solving appears to suggest a strategy to improve students' representational skills in physics. The proposed strategy leads students through a sequence of problems which is structured to emphasize the activation and application of mathematical knowledge and skills in physics contexts.

LIMITATIONS & FUTURE WORK

Due to the limited time of each FOGLI session, we were unable to include many different types of problems into the treatment problem sets. The relatively small number of participants in each group also limits the generalizability of this study.

The problem sets used in this study will be refined based on what we have learned from the study and will be tested again with a larger group of students. Problem sets designed to facilitate strategies for solving problems in electromagnetism posed in graphical and equational representations will also be developed, tested and refined.

ACKNOWLEDGEMENT

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