

IMPROVING STUDENTS' UNDERSTANDING OF ELECTRICITY AND MAGNETISM

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Electricity and magnetism are important topics in physics. Research shows that students have many common difficulties in understanding concepts related to electricity and magnetism. However, research to improve students' understanding of electricity and magnetism is limited compared to introductory mechanics. This thesis explores issues related to students' common difficulties in learning some topics in electricity and magnetism and how these difficulties can be reduced by research-based learning tutorials. We investigated students' difficulties in solving problems involving light bulbs and equations involving circuit elements. We administered multiple choice questions and essay questions to many classes and conducted individual interviews with a subset of students. Based on these investigations, we provide suggestions to improve learning. We also developed and evaluated five tutorials on Coulomb's law, Gauss's law and the superposition principle to help students build a robust knowledge structure and firm understanding of these concepts. Students' performance on the corresponding pre- and post-tests indicates that these tutorials effectively improved their understanding. We also designed a Magnetism Conceptual Survey (MCS) that can help instructors probe students' understanding of magnetism concepts. The validity and reliability of this MCS is discussed. The performance of students from different groups (e.g. female students vs. male students, calculus-based students vs. algebra-based students) was compared. We also compare the MCS and the Conceptual Survey of Electricity and Magnetism (CSEM) on common topics.

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PREFACE

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1.0 INTRODUCTION

Electricity and Magnetism (E&M) are two important topics in physics. They play an important role in understanding the world around us. They are also fundamental to most current and emergent technologies. Electronics, power generation, and sensors all involve electricity and magnetism. E&M is also the simplest example of unification in science. However, many students find learning E&M to be very difficult including those who have done well in learning mechanics (McDermott & Shaffer 1992, Pepper et al. 2010).

1.1 MOTIVATION

1.1.1 Traditionally taught E&M is challenging

From cognitive point of view, it is not difficult to understand why E&M is challenging for students. In mechanics, many important concepts, for example force, velocity and acceleration, are related to everyday experience and many situations involve objects like cars, balls, boats that can be seen in everyday life. However, in E&M, it is the first time student experience abstract and sophisticated mathematical problems. Many concepts like electrons, electric field and electric force are microscopic and invisible. Students' intuition in mechanics (which can often be incorrect but can be improved with research-based instruction) does not work in E&M. Students

often believe that in mechanics everything is mainly common sense whereas it is not common sense to believe that the electric field anywhere inside a hollow conducting sphere is zero. In mechanics, students can directly observe almost everything they study; in E&M, they should make measurements to determine, judge and calculate physical quantities they cannot directly observe.

In most traditional introductory E&M courses (both calculus-based and algebra-based), the sequence of teaching the course involves going through new concepts at high speed and spending most of the course on rote problem solving. The ideas of charge, electric force, field, flux, and Gauss's law are often presented within the first couple of weeks of the course. These ideas are quickly followed by the concepts of potential, potential difference, and electric current, which appear to be only slightly related to the previous set of concepts (Chabay & Sherwood 2005). Students have little "conditioning" about electric and magnetic fields which are new for them as well as less intuitive than mechanics topics. They usually have not yet had enough exposure and experience with these ideas to have become skilled and comfortable with them. The conceptual and mathematical complexity of the field is exacerbated by the extraordinarily rapid introduction of a long sequence of new and increasingly abstract concepts.

In calculus-based courses, calculus becomes an important mathematical tool in E&M. For algebra-based courses, students' algebra skills need to be on solid ground throughout. For calculus-based courses, the ability to apply calculus in unfamiliar ways like calculating the line integral or surface integral of a quantity is challenging for students while processing the relevant physics at the same time. Simply teaching or reviewing relevant calculus to students who have previously taken it or are concurrently taking it is not sufficient to help students learn physics. An introductory E&M course is also the first time for students to think in three dimensions and

consider symmetry in their reasoning. Students have little experience in three dimensional visualization and symmetry arguments which are both required in correctly understanding and applying concepts in E&M.

The introduction of Gauss's law is a good example that manifests the student difficulties in learning E&M (Chabay & Sherwood 2005, Pepper et al. 2010, Singh 2006). Gauss's law is often introduced during the first few weeks of the course. While physics teachers lament that students do not understand Gauss's law despite their efforts, they simply lecture about these concepts at a rapid pace in a traditional course. When students are still struggling with distinguishing the concepts of charge and field, they are introduced to Gaussian surface and the complicated Gauss's law equation which embodies a complex relationship between charge and field in three-dimensional space. Before the students have not gotten familiar with the concept of field, they are lectured on how to apply the laws of E&M in diverse situations without success. In traditional courses, students are not provided modeling, coaching and fading approach (Collins, Brown & Newman 1989) and sufficient time and practice to understand and discriminate these concepts and apply them correctly. Many students are unable to connect physics and mathematics and make sense of the physics involved when the calculus involved is challenging. They often memorize a collection of derived algebraic expressions (which they believe are disconnected formulas) and apply them in the tests without understanding whether they are applicable and why they are applicable in a given situation and not in another situation. Moreover, students' lack of ability in three dimensional visualization and symmetry arguments which play a major role in the applications of Gauss's law contribute to the difficulty in understanding this concept.

Thus, in traditional classes, the lack of modeling, coaching and weaning in addition to the unfamiliarity with the concepts, the students' lack of mathematical background, and their unfamiliarity with visualizing in three dimension and symmetry arguments make the learning of E&M frustrating. To help improve students' learning of E&M and provide effective learning and assessment tools which can be used in conjunction with traditional instruction, we focus our research on developing and evaluating conceptual research-based learning tools (tutorials) and assessment tools (conceptual tests) in introductory E&M. The conceptual test on magnetism and the tutorials for Coulomb's law, superposition principle and Gauss's law have been developed and administered to a large number of students. Details will be discussed in the next few chapters.

1.1.2 Developing reasoning skills is important for student learning

Reasoning is often referred to as the ability to analyze information and solve problems on a complex, thought-based level. When people reason, they must go "beyond the information given (Bruner 1957)". They can do reasoning by the following two ways: 1. "They attempt to infer (either automatically or deliberately) concepts, patterns, or rules that best characterize the relationships or patterns they perceive among all the elements (e.g., words, symbols, figures, sounds, movements) in a stimulus set"; 2. "They attempt to deduce the consequences or implications of a rule, set of premises, or statements using warrants that are rendered plausible by logic or by information that is either given in the problem or assumed to be true within the community of discourse (Lakin 2009)." Reasoning procedures require the skills of forming theories, understanding subjects, applying knowledge and interpreting relationships.

“(Reasoning) skills help students think clearly and logically, as answers to issues and problems usually entail making careful distinctions in arguments and as solutions to these issues also require logical and critical thinking (Moore & Bruder, 1990).” Reasoning skills are good reflections on students’ learning. By tracking their reasoning skills and learning outcomes, we can understand how to provide effective instructions and instructional adaptations to help students’ learning. Reasoning skills are also dependent on knowledge and expertise. Expertise is rooted in knowledge, and experts reason differently about problems than do novices (Feltovich et al. 2006). Experts are more likely to build a robust and hierarchical knowledge structure than novices. Since helping students think like an expert is one of the goals of physics education research, it is crucial to investigate students’ reasoning skills during problem solving to learn about their learning and understanding.

1.2 INFLUENCE FROM COGNITIVE SCIENCE

Cognitive psychology is a field of psychology focused on studying mental processes such as problem solving, memory, reasoning, learning, attention, perception and language comprehension. Some of the interesting findings of cognitive psychology carry important implications for physics learning and problem solving even though they are not directly applicable to improving classroom instruction (Pollock and Chasteen 2009). In investigating students’ understanding of concepts and assessing students’ performance, cognitive theories and findings are carefully integrated, e.g., Piaget’s “optimal mismatch”, Vygotsky’s “zone of proximal development”, the Preparation for Future Learning model of Bransford and Schwartz and the knowledge related to memory (Smith 1985, Piaget 1964, Raymond 2000, Bransford &

Schwartz 1999, Schwartz et al. 2005). The following is a short review of the relevant cognitive theories and concepts that helped my research.

1.2.1 Memory and cognitive load

Human memory is also known as human information processing system, which refers to the brain's ability to store, retain and retrieve information. It consists of two major components: short-term memory (or working memory) and long-term memory (Simon, 1974). Short-term memory is where the information is processed and long-term memory is where the prior learned knowledge is stored. While problem solving, short-term memory uses input from the sensory buffers (e.g., eyes, ears, hands) and the knowledge retrieved from long-term memory to rearrange and synthesize ideas to reach the goal.

George Miller showed that the storage ability of short-term memory is limited to 7 ± 2 bits, which indicates that if people attempt to process many disparate bits of information at the same time, they experience cognitive overload and are unable to complete the task (Miller, 1956). However, short-term memory can be "extended" by chunking disparate bits of information with specific association into the same group. For example, it is much easier for people to remember a phone number in the US by dividing the string into three "3digits – 3 digits – 4 digits" chunks than by memorizing ten digits together.

Research shows that experts have better chunking skills. Experts can retrieve their compiled knowledge from long-term memory and use one bit of working memory to process the information without noticing that they have processed many related concepts. On the other hand, novices whose knowledge chunks are smaller than experts must use many "slots" in their

working memory to process the same information, have a higher cognitive load and are more likely to experience cognitive overload.

One major goal of most introductory physics classes is to help students develop thinking skills of an expert physicist. In the schema or knowledge structure of a physics expert, the most fundamental principles are at the top of the hierarchy and the secondary and tertiary concepts are lower. On the other hand, novices don't have their knowledge organized hierarchically. When solving a problem, unlike experts who discern the deep feature of the problem which provides an overall plan for solving the problem, novices often notice the superficial features and are likely to apply a concept even without thinking if it is applicable in a given situation.

To decrease the possibility of a cognitive overload and improve students' thinking skills, a variety of research-based methods, e.g., carefully designed curriculum, scaffolding using guided inquiry and working in groups, can be applied. Research-based curricula can improve students' thinking skills, and help students organize their knowledge hierarchically. This can improve their chunking ability and cognitive overload can be avoided and managed appropriately.

1.2.2 Alternative conceptions or Misconceptions

The goal of instruction is to guide students from their current knowledge state to the desired knowledge state. Students' knowledge state after instruction depends not only on the instruction but also on their initial knowledge state. The same instruction can produce very different final knowledge states for different students. Students are not blank slates (Schauble 1995). It is important to assess and be familiar with students' initial knowledge state. Everyone is constantly trying to make sense of the world around them based upon his/her existing knowledge. When

people encounter new circumstances, they attempt to build “micro” knowledge structures which they are satisfied with. However, the “micro” knowledge structures usually are only locally consistent and often lack global consistency if people are not experts in that domain. People also have a tendency to over-generalize the knowledge acquired in one context and believe that the knowledge is valid in other contexts in which they are not applicable without noticing the similarities and differences between those contexts. For example, some students believe that a battery is a constant current source because they over-generalize the fact that a battery is used to produce current. This tendency of over-generalization often leads to alternative conceptions or misconceptions.

Alternative conceptions or misconceptions are often very robust and difficult to change without proper intervention. Even if it is removed after instruction, it can re-emerge after sometime. They interfere with learning during and after the learning process. Students often interpret and mould physics concepts to suit their alternative conceptions. For example, when children who believe that earth is flat are told that it is round, they infer that it is round like a pancake (Vosniadou & Brewer, 1992). When they are told that it really is round like a ball, they infer that it is hemispherical and we are standing on the flat side. The confusion between physics terms and their everyday interpretation is also a common hindrance in learning physics. In everyday life, how fast we walk refers to the term speed in physics. On the other hand, the term velocity not only refers to the speed of the walk but also its direction. Novices have difficulty in discriminating these concepts which can lead to alternative conceptions or misconceptions.

In most physics classes, instructors’ awareness of alternative conceptions is not enough to help students learn the correct concepts. In fact, even during instruction, alternative conceptions can emerge. Indeed, these issues must be discussed within a coherent curriculum. Without

providing an opportunity to focus on the knowledge structure, students can misinterpret or modify what they are told or what they observe based upon prior knowledge if they are not guided appropriately. Thus, it is very important for the students to get an opportunity to connect new and prior knowledge and learn to build knowledge coherently and hierarchically. Piaget's "optimal mismatch" idea and Vygotsky's "zone of proximal development" theory which are discussed in the next section can help develop curriculum that bridges the gap between the new and prior knowledge.

1.2.3 Optimal mismatch and the zone of proximal development

Learning is incremental. The new knowledge builds on the prior knowledge of an individual. Piaget suggested that "optimal mismatch" strategies can create a state of disequilibrium in students' minds and help students learn new concepts effectively. In particular, students can realize a contradiction between their initial prediction and something they observe by working on the tasks instructors pose in which common difficulties and misconceptions are elicited (Smith, 1985). By noticing the discrepancies between their observation and prior predictions, they realize there is inconsistency in their reasoning and they are in a state of disequilibrium in which they are eager to resolve the discrepancies (Piaget 1964, p. 29). At this point, it is suggested that students should be provided with systematic tasks commensurate with their prior knowledge to help them resolve the discrepancies and accommodate and assimilate new knowledge. The accommodating and assimilating of knowledge requires the instructional approach to not only help students understand why the new ideas are applicable, but also why the old ideas do not apply.

Similar to the optimal mismatch idea of Piaget, another cognitive model that emphasizes the importance of building new knowledge on the prior knowledge is the zone of proximal development (ZPD) attributed to Vygotsky in the early twentieth century. ZPD is commonly defined as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers” (Vygotsky, p. 86). It refers to what students can do on their own vs. with the help of an instructor who is familiar with their prior knowledge and skills. The heart of ZPD is scaffolding which can be used to stretch students’ learning process and help them develop independence. Base on the student’s knowledge state at a given time, the instructor can manipulate the current instructional strategies and keep the knowledge difficulty level within the zone of proximal development to facilitate student’s understanding of the new material. This can help students to connect with their current knowledge with prior knowledge and help them develop an organized and solid knowledge structure.

1.2.4 Preparation for future learning

The cognitive model of Bransford and Schwartz on preparation for future learning (PFL) suggests that the transfer of knowledge from the acquired situation to new situations is optimal if both the elements of innovation and efficiency are included in instruction (Bransford & Schwartz 1999). In their model, efficiency and innovation are two orthogonal coordinates.

People with high efficiency can “rapidly retrieve and accurately apply appropriate knowledge and skills to solve a problem or understand an explanation” (Schwartz et al. 2005).

Generally speaking, the best way to be efficient is to practice appropriate and useful skills so that they become “routine” (Anderson 1999).

However, prior research shows the disadvantages of over-emphasis of efficiency. Hatano and Inagaki’s study (1986) discusses “routine experts” who are good at solving similar problems and have difficulty in acquiring new knowledge to solve non-routine problems (Hatano & Oura 2003). With over-emphasis of efficiency, people can behave “functional fixedly” while solving routine problems instead of trying to re-conceptualize learning and transfer knowledge to new situations appropriately (Schwartz et al. 2005).

The fact that a focus on efficiency alone in instruction (which is typical for traditional instruction) cannot help students become physics experts is also observed in our study discussed later. For example, in research related to circuit problems discussed later, many students believe that the current is always the same through the battery regardless of how resistors are connected to it because that is the case in some situations.

Therefore, to effectively transfer knowledge and prepare for future learning, the element of “innovation” should be included in instructional design. Being different from efficiency of repeating a behavior to tune speed, innovation involves reaching beyond the immediately known (Schwartz et al. 2005). The creativity of problems forces students’ cognitive engagement and learning. Working through numerous rote exercises or reading a physics textbook like a novel may not help students become adaptive experts. Appropriate combination of efficiency and innovation in instruction can help students break non-routine, difficult-to-solve problems into routine problems that can be solved easily (Schwartz et al. 2005).

When students’ prior experiences do not work and foster a state of disequilibrium or curiosity, it is best to design instruction that lets the procedure of innovation to work out.

Connecting the PFL model to ZPD, we note that within the zone of proximal development, innovation helps students develop a better grasp of knowledge and conceptual interpretation. However, innovation can be too challenging if it is out of the zone of proximal development. Students may experience too much struggle that can inhibit problem engagement and they may lose confidence of learning. Thus, for meaningful learning and appropriate transfer of knowledge, instruction should focus on a combination of efficiency and innovation along a diagonal trajectory in the two dimensional space of innovation and efficiency (Schwartz et al. 2005). With a good control and balance of efficiency and innovation, students can not only quickly and accurately solve routine problems but also apply knowledge to solve novel problems.

1.2.5 Guided inquiry

In most traditional physics classes, instructors design the curriculum based on their perspective of understanding as physics experts instead of from students' perspective (McDermott 1991). Without guidance from physics education research, many instructors fail to realize that knowing students' prior knowledge is very important to help design the instruction appropriately.. Moreover, many instructors do not model a systematic approach to problem solving and use their instruction as an opportunity for helping students repair and extend their knowledge structure. Their instructional approach does not necessarily focus on the importance of reflection and metacognition.

To overcome the disadvantages of traditional instruction, several inquiry-oriented science instruction have been developed. The National Committee on Science Education Standards and Assessment (1992) has noted that one goal of science education is “to prepare students who

understand the modes of reasoning of scientific inquiry and can use them.” Inquiry is defined as follows (National Research Council 1996, p.23):

“Scientific inquiry refers to the diverse ways in which scientists study the natural world and propose explanations based on evidence derived from their work. Inquiry also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as an understanding of how scientists study the natural world.”

Effective learning must involve students actively engaged in the process. From a science perspective, inquiry-oriented instruction engages students in the investigative nature of science (Haury 1993). It helps students reflect on how science is developed and how people understand the world. It focuses on the active search for knowledge or understanding to satisfy a curiosity and involves activity and skills. Thus, it is a more natural and effective way to foster student motivation, develop research competency and construct their knowledge structure. Guided inquiry is a commonly used learning process in science education. In the guided inquiry approach, students are provided with course materials and “guiding” questions and they try to investigate the questions or generate an explanation (Colburn 2000).

When we develop our tutorials on Coulomb’s law and Gauss’s law, we incorporated the guided inquiry approach to help improve student learning. When students work on the tutorials, they start working on questions using their prior knowledge of the concepts so that they develop their own explanations based upon their current understanding. Then they discuss their reasoning and explanations with their classmates to see if their interpretations are consistent with others. The students must also answer questions in various situations and asked to evaluate if their reasoning is consistent with what actually happens and with the guidance and perspective

provided by the instructor. If the students find that inconsistency exists between their and their classmates' interpretation or between their reasoning and the provided perspective, their raised curiosity forces them to examine possible misconceptions and gaps in their knowledge and to reconcile the difference between their prior reasoning and the correct perspective. After that, another question on another aspect of the concepts can be posed to the students for investigation. By repeating this guided inquiry learning cycle, students can be helped to build a robust knowledge structure and deep understanding of the relevant concepts.

1.3 A STUDY OF STUDENTS' UNDERSTANDING IN THE CONTEXT OF E&M

My studies that involve improving students' understanding of E&M are described in the following chapters. We look deep into students' understanding of relevant concepts and provide methods to improve and assess their understanding. The following is a short description of my studies on investigations involving circuit elements, Coulomb's law, Gauss's law and magnetism.

1.3.1 Investigation of students' difficulties with circuits involving light bulbs

Conceptual reasoning is severely underemphasized in many traditional courses. Most traditional courses do not explicitly teach students problem solving strategies and only emphasize plug and chug approach. Students often solve physics problems by applying concepts without thinking if it is applicable or not. Thus, investigation of students reasoning is very important to help students learn better. Being familiar with students' thinking process and knowledge state can help

instructors use appropriate research-based strategies to improve student learning.

We investigate introductory physics students' conceptual difficulties with the brightness of two non-identical light bulbs connected in series or parallel to each other in a circuit. Students were asked which light bulb will be brighter when connected in series or parallel with given wattage or resistance. We compare students' performance of the wattage version and resistance version and find that the students are more capable of answering the resistance version and have more difficulty with the wattage version. We also compare the performance of the introductory students on written free-response questions with those of the physics graduate students on multiple-choice questions. Possible reasons for students' difficulties and misconceptions are discussed.

1.3.2 Student difficulties with equations involving circuit elements

In the second investigation, we explore students' conceptual difficulties in understanding equations involving circuit elements. The way students view physics equations as plug-and-chug tools is not only limited to circuit elements questions, but it can be a general difficulty in introductory physics learning. We expected students to internalize that each equation is a constraint that relates variables and constants represented by symbols. However, students had great difficulty with these.

We investigate the difficulties by analyzing calculus-based introductory physics students' performance on questions about circuit elements (cylindrical resistor, parallel plate capacitor and solenoid inductor) both in the free-response and multiple-choice formats and by comparing their performance to that of physics graduate students. We also conducted formal paid interview with six introductory physics students individually to understand their thought processes better. We

discuss the difficulties we observed in our investigation and provide instructional strategies to help improve learning.

1.3.3 Coulomb's law, superposition principle and Gauss's law

Followed by the procedure of investigating students' conceptual difficulties, developing effective methods for improving learning is also important in physics education research (PER). Being informed by the knowledge of zone proximal development and alternative conceptions, we designed and assessed five tutorials which address difficulties found via research and provide helpful tools for students developing a coherent understanding of Coulomb's law, superposition principle, symmetry and Gauss's law.

We administered pre-/post-tests in four different calculus-based introductory classes. Three of the classes were given tutorials and the other class was used as a control group without tutorials. In pre-/post-tests, we investigated students' difficulties on Coulomb's law, superposition principle and Gauss's law in these classes. Out of the five tutorials, the first two focused on Coulomb's law, superposition and symmetry. The first tutorial started with electric field due to a single charge and then extended to two or more charges. The second tutorial continued the conceptual discussion that started in the first tutorial to continuous charge distributions. The third tutorial was designed to help students learn to determine the electric flux. The fourth tutorial was designed to help students exploit Gauss's law to calculate the electric field at a point. The fifth tutorial revisited superposition principle after Gauss's law.

Comparing the results of pre-/post-tests, it is very encouraging to observe that students in tutorial classes have improved understanding these concepts. Students from the non-tutorial class did not perform as well as the tutorial class students on the post-tests. Students were also

separated into three levels based on their pre-test scores to observe how they performed on the post-tests. Results suggest that the tutorials help students from all three levels.

1.3.4 Magnetism conceptual survey

From the model of learning we know that students' final knowledge states depend on both the instructional design and their initial knowledge states. We need assessment tools to become familiar with students' initial knowledge state and to learn how much they learned after instruction. Our research-based Magnetism Conceptual Survey (MCS) covers topics in magnetism discussed in a traditional calculus- or algebra-based introductory physics curriculum up to Faraday's law. Multiple-choice is chosen as the format of the test. MCS was administered both as a pre-test and a post-test to a large number of algebra- or calculus-based students at Pitt. Our analysis of the reliability index KR-20, the item difficulty and discrimination indices, and point biserial coefficient of the items suggest that our test is reliable and valid (Ding et al. 2006).

Although it is not easy to capture student thought process by looking at their choices on a multiple-choice test, we developed the distracter choices for the multiple-choice questions to conform to common misconceptions found via research. Also, we observed that the introductory physics students have difficulty with 3D visualization and the right hand rule. Other common difficulties are also discussed in a later chapter of this thesis.

We also perform analysis of variance (ANOVA) to investigate the gender differences in the pre-test and the post-test MCS data. The results for algebra-based classes and calculus-based classes are not the same. However, the results for different calculus-based classes were different. We also compare students' performance in the calculus-based classes and algebra-based classes on the post-test and pre-test. The result suggests that significant difference appears at the

beginning of learning and persists on the post-test.

We have also collected data for the CSEM test (Maloney et al. 2001) from Pitt. We first compare the Pitt data on CSEM with the data reported in Maloney et al. 2001. We then compare the students' performance on CSEM on certain questions with their performance on comparable questions on MCS.

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2.0 STUDENTS' CONCEPTUAL DIFFICULTIES WITH LIGHT BULBS CONNECTED IN SERIES AND PARALLEL

2.1 ABSTRACT

Conceptual learning and sense making is at the heart of developing a robust knowledge structure and becoming an expert in physics. Unfortunately, conceptual reasoning is severely underemphasized in traditional physics classes all the way from the introductory level to the graduate level. Prior research has shown that if students are reasonably comfortable with the mathematical manipulations required to solve a quantitative problem, they may perform better on quantitative problems using an algorithmic approach than on the corresponding conceptual questions requiring sense making. Here, we discuss an investigation of students' conceptual difficulties with the brightness of light bulbs connected in series and parallel. The questions about the light bulbs could be solved quantitatively but a majority of students chose not to write down any equations while answering the questions. We discuss the conceptual difficulties in the context of introductory physics students' performance on these questions in the free-response format in which students were asked to explain their reasoning and compare their performance to that of a set of graduate students. We also discuss the findings of individual interviews that provided further insights into student reasoning.

2.2 INTRODUCTION

In physics, there are very few fundamental laws. They are expressed in compact mathematical forms and can provide students tools for organizing their knowledge hierarchically. Such organization is crucial for easy retention and retrieval of knowledge and can help students in reasoning and deciding which concept is applicable in a particular context. The goal of an introductory physics course is to enable students to develop complex reasoning and problem solving skills and use these skills in a unified manner to explain and predict diverse phenomena in everyday experience. However, numerous studies show that students do not acquire these skills from a *traditional* course (McDermott & Shaffer 1992). The problem can partly be attributed to the fact that the kind of reasoning that is usually learned and employed in everyday life is not systematic or rigorous. Although such hap hazardous reasoning may have little measurable negative consequences in the everyday domain, it is insufficient to deal with the complex chain of reasoning that is required in the relatively precise scientific domain.

Instruction can help students develop their scientific reasoning skills in two broad ways: first, students can be taught to reason conceptually without equations; second, they can learn to reason by drawing conceptual inferences from symbolic equations. Due to a high level of math anxiety and lack of relevant experience, physics courses geared towards non-science majors resort to the first route. Use of quantitative tools in such courses can increase students' cognitive load to the extent that very little cognitive resources may be available for drawing conceptual inferences. But most introductory physics courses are tailored to science, engineering, and pre-professional students. These students are supposed to be reasonably comfortable with mathematics and are expected to learn to reason by drawing conceptual inferences from quantitative problem solving. However, in order to learn physics and build a robust knowledge

structure with quantitative tools, students must interpret symbolic equations correctly and be able to draw conceptual inferences from them. This implies that students must not treat quantitative problem solving merely as a mathematical exercise but as an opportunity for sense making, learning physics and developing expertise. This requires that students engage in effective problem solving strategies.

Unfortunately, students often solve physics problems using superficial clues and cues, applying concepts without doing sense making and thinking whether they are applicable or not. Also, most traditional courses do not *explicitly* teach students effective problem solving strategies. Rather, they reward inferior problem solving strategies that many students engage in. Instructors implicitly assume that students know that analysis, planning, evaluation, and reflection phases of problem solving are as important as the implementation phase. Consequently, they do not explicitly discuss these strategies while solving problems during the lecture. Recitation is usually taught by the teaching assistants who present homework solutions on the blackboard while students copy them in their notebooks. There is no mechanism in place in a traditional physics course to ensure that students make a conscious effort to interpret the concepts, make conceptual inferences from the quantitative problem solving tasks, relate the new concepts with their prior knowledge and build a robust knowledge structure.

Moreover, conceptual problem solving can often be more challenging than quantitative problem solving because quantitative problems can be solved algorithmically by constraint satisfaction. For example, if a student knows which equations are involved in solving the problem, he or she can combine them in *any order* to obtain a quantitative answer. On the contrary, while reasoning conceptually, the student *must* understand the physics underlying the given situation and generally proceed in a *particular order* to arrive at the correct conclusion.

Therefore, the probability of deviating from the correct reasoning chain increases rapidly as the chain of reasoning becomes long.

In a study on student understanding of diffraction and interference concepts, the group that was given a quantitative problem performed significantly better than the group given a similar conceptual question (McDermott 1999). In another study, Kim et al. examined the relation between traditional physics textbook-style quantitative problem solving and conceptual reasoning (Kim and Pak 2001). They found that, although students in a mechanics course on average had solved more than 1000 quantitative problems and were facile at mathematical manipulations, they still had many common difficulties when answering conceptual questions on related topics. When Mazur gave a group of Harvard students quantitative problems related to power dissipation in a circuit, students performed significantly better than when an equivalent group was given conceptual questions about the relative brightness of light bulbs in similar circuits (Mazur 1997). In solving the quantitative problems given by Mazur, students applied Kirchhoff's rules to write down a set of equations and then solved the equations algebraically for the relevant variables from which they calculated the power dissipated. When the conceptual circuit question was given to students in similar classes, many students appeared to guess the answer rather than reasoning about it systematically. For example, if students are given quantitative problems about the power dissipated in each (identical) headlight of a car with resistance R when the two bulbs are connected in parallel to a battery with an internal resistance r and then asked to repeat the calculation for the case when one of the headlights is burned out, the procedural knowledge of Kirchhoff's rules can help students solve for the power dissipated in each headlight even if they cannot conceptually reason about the current and voltage in different parts of the circuit. To reason without resorting explicitly to mathematical tools (Kirchhoff's

rules) that the single headlight in the car will be brighter when the other headlight is burned out, students have to reason in the following manner. The equivalent resistance of the circuit is lower when both headlights are working so that the current coming out of the battery is larger. Hence, more of the battery voltage drops across the internal resistance r and less of the battery voltage drops across each headlight and therefore each headlight will be less bright. If a student deviates from this long chain of reasoning required in conceptual understanding, the student may not make a correct inference.

Here, we discuss introductory physics students' conceptual difficulties with the brightness of two light bulbs which are not identical and connected in series or parallel to each other and to a battery with no internal resistance in a circuit. Students were either given the wattage (for the standard power supply) or the resistance of each light bulb connected in series or parallel in the circuit and asked which bulb will be brighter. Students were told that they can assume that the light bulbs are ohmic and that the brightness of the light bulbs is proportional to the power dissipated. We also compare the performance of the introductory students on written free-response questions with those of the physics graduate students who were given the questions in the multiple-choice format. We also conducted individual interviews with a subset of introductory students to get a better understanding of the origins of their difficulties. Although these questions could be answered using quantitative tools (equations), a majority of introductory students wrote down no equation to answer them. Moreover, while one may predict that students may have less difficulty in reasoning about the brightness of the light bulb in the circuits we used in our research compared to those used in the earlier studies (e.g., in which the battery to which the identical light bulbs were connected had an internal resistance), students had great conceptual

difficulties with these questions discussed here. The fact that the light bulbs were not identical made the questions we discuss here quite challenging for most students.

2.3 METHOD OF INVESTIGATION

In the preliminary administration of the questions in the free-response format, introductory students in the calculus-based courses were asked to explain their reasoning after answering the questions. We then developed the multiple-choice questions and refined the free response questions based upon student responses to the preliminary version of the free-response questions. Here we only discuss student responses to the final version of the questions administered in the free-response format to introductory physics students and multiple-choice format to the physics graduate students. The multiple-choice questions are useful because the analysis of data is not time consuming. The free-response questions are useful for understanding the students' thought processes since students were asked to explain their reasoning. There were two versions of both the free-response and multiple-choice questions. In one version of the questions (which we will call the wattage version), students were given the wattage of the light bulbs when connected to a standard power supply. In the other version of the questions (which we will call the resistance version), they were given the resistance of each light bulb. Before attempting to answer the questions in both the free-response and multiple-choice formats, students were told to assume that the brightness of the light bulbs is proportional to the power dissipated. Here are the multiple-choice questions in the wattage version:

1) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which one of the

following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 100W light bulb will be brighter.
- b) 25W light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
- e) None of the above

2) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 100W light bulb will be brighter.
- b) 25W light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
- e) None of the above

The following are the multiple-choice questions in the resistance version:

1) Two light bulbs have 200 ohm and 500 ohm resistances. They are connected in parallel to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 200 ohm light bulb will be brighter.
- b) 500 ohm light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 200 ohm bulb will be brighter.
- e) None of the above

2) Two light bulbs have 200 ohm and 500 ohm resistances. They are connected in series to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 200 ohm light bulb will be brighter.
- b) 500 ohm light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
- e) None of the above

The following are the free-response questions in the wattage version:

1) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be ohmic? You must explain your reasoning.

2) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be ohmic? You must explain your reasoning.

Similar free response questions were also administered to some introductory physics students in the resistance version. We note that the questions in the free-response format were administered in different calculus-based introductory physics courses. All students involved in this study received traditional lecture-based instruction on circuits in their relevant courses. The questions were administered in pairs on written quizzes in the recitations after relevant instruction. We administered the wattage version of the questions in the multiple-choice format to 300 calculus-based introductory physics students but we will not discuss these data in this paper in detail because the results are very similar to those obtained for the free-response version. The resistance version of the free-response questions were administered to 241 students from two calculus-based classes. The results for the two classes are very similar (differences are not statistically significant). The wattage version of the free-response questions was administered to 103 students from a different calculus-based class. One of the classes which were given the resistance version of the free-response questions had the same instructor as the class which was given the wattage version of the free-response questions. We note that the resistance version is easier than the wattage version because students have an additional step to work out in the

wattage version and have to figure out the resistance of the bulbs from the wattage provided for the standard power supply. We hypothesized that even the students who can answer the resistance version correctly may not be able to answer the wattage version correctly due to the difficulty in this conversion. By comparing the student performance on these two versions of the questions, we expected to obtain a crude measure of the percentage of students who had difficulty converting from the wattage of the bulb in the wattage version to the corresponding resistance.

We also conducted individual interviews to get a better insight into student reasoning. The interviews were conducted with six students from a calculus-based course in which the written test was not administered. They were all volunteers whose first midterm exam grades were around the class mean. During the interviews, students were asked to work on both versions of the questions and also articulate if there is any difference in the difficulty level of the two versions from their perspective. Interviewed students were first asked about the wattage version and then about the resistance version. The analysis of student responses to the interview questions yielded further insight into student difficulties with the brightness of light bulbs.

2.4 RESULTS

Before discussing the results, we reiterate that all students were specifically given that the brightness of the light bulbs is proportional to the power dissipated and that the bulbs have ohmic resistances. For the wattage version, students have to exploit the fact that the standard wattage rating is for light bulbs connected in parallel to the standard power supply (and to the other appliances). They had to make use of the relation between the power dissipated, voltage and

resistance to calculate the resistance of each light bulb. When the bulbs are connected in parallel as in the standard household connection, the higher wattage bulb (the lower resistance bulb) will have a higher power dissipated. Then, to determine which light bulb is brighter when connected in series, students must realize that the bulbs have the same current through them in series. Therefore, the bulb with a higher resistance (lower wattage rating for the standard power supply) will have a higher power dissipated.

2.4.1 Light bulbs connected in parallel or series with resistance provided

Table 2.1 shows that for light bulbs connected in parallel given as a free response question, 71% of the students correctly answered that the 200 ohm light bulb is brighter. About 18% of them thought that the one with a higher resistance will be brighter and 5% of them claimed that the two bulbs have the same brightness. For the case for which the light bulbs are connected in series, 53% of them answered the question correctly and 17% of them believed that the brightness of both bulbs should be the same.

Table 2.1. Introductory students' responses to each free-response question with known resistances

	Light bulbs connected in parallel (N=241)	Light bulbs connected in series (N=241)
200ohm is brighter	<u>71%</u>	19%
500ohm is brighter	18%	<u>53%</u>
same brightness	5%	17%
Whichever one is first	0%	5%
Other Responses	6%	6%

To better understand the patterns in answering these questions correctly, we analyzed the distribution of students' responses for both (series and parallel) situations together. The results are shown in Table 2.2. Table 2.2 shows that 46% of the students answered both questions

correctly and noted that for light bulbs connected in parallel, the one with lower resistance will have a higher current and thus be brighter. Moreover, 15% of them answered the question about the light bulbs connected in parallel correctly but claimed that the light bulbs connected in series should have the same brightness. According to some of them, *only* the current through the bulb determines the brightness. Some of them noted that for the light bulbs connected in series, the current is the same and therefore the brightness should be the same. 10% of the students believed that the 500ohm light bulb will be brighter when connected in parallel and the 200ohm light bulb will be brighter in series.

Table 2.2. Introductory students' responses to both free-response questions with known resistances

Light bulbs connected in parallel or series	Problems with known resistances (N=241)
200ohm, 500ohm	<u>46%</u>
200ohm, same	15%
500ohm, 200ohm	10%
200ohm, 200ohm	7%
500ohm, 500ohm	5%
Other responses	17%

2.4.2 Light bulbs connected in parallel or series with standard power supply provided

The correct answer for these questions requires the ability to determine which light bulb has a larger resistance. Once the students find the resistance, the questions are the same as the ones discussed above.

Table 2.3. Introductory students' responses to each free-response question with known wattage rating for a household power supply.

	Light bulbs connected in parallel (N=103)	Light bulbs connected in series (N=103)
100W is brighter	<u>49%</u>	54%
25W is brighter	29%	<u>8%</u>
same brightness	12%	23%
Whichever one first	0%	7%
Other responses	10%	8%

Table 2.3 suggests that the question about the light bulb in series is much more difficult than when they are connected in parallel. Table 2.3 shows that 49% of the students answered the question about the light bulbs in parallel correctly but only 8% of them answered the question about the bulbs in series correctly. More than half of the students believed that the 100W bulb is brighter when the bulbs are connected in series. Written explanations suggest that many students expected the brightness to be the same as in parallel when the bulbs are connected in series. These results for the free-response questions are qualitatively similar those for the multiple choice questions.

Table 2.4. Introductory students' responses to both free-response questions with known wattage rating for each bulb for a household power supply.

	Problems with known power (N=103)
100W, 25W	<u>6%</u>
100W, 100W	24%
25W, 100W	22%
100W, same	19%
100W/25W, whichever one first	7%
Other responses	22%

Table 2.4 shows that if we consider introductory physics students who answered both the series and parallel questions correctly in the wattage version, only 6% of them answered both questions correctly. Table 2.4 also shows that 24% of them thought that the 100W light bulb is

always brighter no matter how they are connected (parallel or series). Moreover, 22% of the students chose answers for both questions to be incorrect. Interviews and written explanations suggest that some of these students believed that the 100W light bulb has a higher resistance than the 25W bulb. Table 2.4 also shows that 19% of the students claimed that the 100W light bulb is brighter when it is connected in parallel and it has the same brightness when it is connected in series. Interviews and written explanations suggest that some of them regarded the current as the only factor that determines the brightness and argued that when the bulbs are in parallel, the 100W bulb has more current flowing through it and when the bulbs are in series, they have the same current.

Table 2.1-2.4 together show that students who worked on the resistance version performed much better than those who worked on the wattage version. Since the two versions were given to similar calculus-based introductory physics classes (with the instructor being the same for two sections of the course, which were administered different versions), it is likely that students performed poorly on the wattage version because they had difficulty in finding the resistance of each light bulb. Our interviews with individual students support this conclusion.

2.4.3 Comparison of the performance of introductory students with physics graduate students

As noted earlier, we administered both versions of the questions in the multiple-choice format to physics graduate students in their first semester in a mandatory semester long teaching assistant training course. These questions were administered in the multiple-choice format to the graduate students because they were given as a part of a large number of multiple-choice questions. The performance of the graduate students is helpful in benchmarking the performance of the

introductory students. The wattage version of the questions was administered to 42 graduate students in the TA training courses in two consecutive years (26 students in one class and 16 in another class). The 26 graduate students in one of the classes were also given the resistance version of the questions on a separate day. As can be seen from Table 2.5, graduate students' performance in most situations is better than that of the introductory students. However, even the graduate students had difficulties in answering some of these questions. Moreover, if we only consider the performance of the 26 graduate students who answered both versions, 85% answered the question about bulbs in parallel and 46% answered the question about bulbs in series correctly in the resistance version. In the power version, 65% of the graduate students answered the question for bulbs in series correctly and 35% answered the question for bulbs in parallel correctly. Overall, the performance of graduate students suggests that these questions are quite challenging.

Table 2.5. Graduate students' responses to multiple-choice questions.

	Light bulbs connected in parallel (with known wattage rating, N=42)	Light bulbs connected in series (with known wattage rating, N=42)		Light bulbs connected in parallel (with known resistances, N=26)	Light bulbs connected in series (with known resistances, N=26)
100W is brighter	<u>71%</u>	40%	200ohm is brighter	<u>85%</u>	31%
25W is brighter	12%	<u>43%</u>	500ohm is brighter	8%	<u>46%</u>
same brightness	17%	12%	same brightness	8%	12%
Other responses	0%	5%	Other responses	0%	12%

2.5 DISCUSSION

2.5.1 Students' difficulties

Written explanations and individual interviews suggest that students had many common difficulties. Here, we discuss student difficulties in answering both versions of the questions.

Current alone determines the brightness of a light bulb: This is one of the most common misconceptions. Due to this misconception, 15% -18% of the students answered the question with light bulbs connected in parallel correctly but the one with light bulbs connected in series incorrectly. These students claimed that the light bulb with more current will always be brighter and the light bulbs with the same current through them will be equally bright. In the

interviews, even those students who noted that they have to find the power dissipated in each light bulb to compare their brightness, sometimes only focused on current. For example, to answer the question about the two light bulbs connected in series correctly, students must know that the current is the same for both light bulbs and also the resistance of each bulb (or the voltage across each bulb). Sometimes the students ignored the information about the resistances of the bulbs (especially in the wattage version) and only focused on the current to determine the brightness. The following statements are from a student who answered both versions of the questions in the interview: For the wattage version in series, the student ignored the resistance and incorrectly noted that “the bulb brightness will be equal, because current is equal in series. With the current equal, the brightness has to be the same.” For the resistance version, the same student correctly noted that “the current is the same through both bulbs. Since the current is the same, the voltage drop across the 500ohm resistor will be greater. The 500ohm light bulb is brighter.” Further probing suggests that this student did not know how to compute the resistance for the wattage version of the questions so he simply chose to ignore it and assumed that the current determines the brightness in the wattage version.

Consideration of resistance of the light bulbs alone determines their brightness: The difficulties of the students who incorrectly believed that the resistance alone determines the brightness of the bulbs fall in several categories. One group of students believed that the current is the same through both light bulbs without considering how the bulbs are connected in a circuit. Another group of students believed that the bulb with less resistance always has more current flowing through it even if it is connected in series to the other bulb. Some students were confused about whether they should take into account the equivalent resistance of the entire circuit or the resistance of each light bulb especially when connected in series to determine the

brightness of each bulb. These students asserted that when the light bulbs are connected in series, the resistance applies to the entire circuit and not to each bulb separately. Therefore, the brightness of both bulbs in series should be the same. For example, one student claimed the following: “the brightness of the light bulbs is the same. In series, the resistance is evenly distributed.”

The light bulb with a larger wattage rating for a standard power supply will always be brighter: 24% of the students believed that the 100W light bulb is always brighter than the 25W bulb no matter how they are connected to each other (in parallel or in series). In the interviews, some students explicitly mentioned that the 100W light bulb is always brighter than the 25W bulb because that is how they are manufactured and that fact cannot be changed by changing the circuit. When interviewed students were asked explicitly about how the bulbs are connected to the standard power supply in everyday life, none of them could articulate that they are connected in parallel to the power supply. Moreover, few students who provided written explanation for the free-response wattage questions mentioned that the light bulbs are connected in parallel to the standard power supply as part of their explanations.

Failure to recognize the relation between resistance of a bulb and power dissipated: As mentioned earlier, the wattage version is much more difficult because students have to first figure out which of the two light bulbs has a higher resistance. During the interviews, students were encouraged to articulate the connection between the two versions of problems. However, not a single interviewed student noted that there is an extra step involved in going from the wattage version to the resistance version. They could only state the literal difference between the two versions, namely, one provides the wattage of the bulbs for the standard power supply and the other provides the resistances. Even asking students explicitly to relate the wattage of the

bulbs to their resistance and thus explore the connection between the two versions rarely helped. For the resistance version with light bulbs in parallel, some students correctly claimed that since $P=IV$ but the voltage V is the same for both bulbs in parallel, the light bulb with a higher wattage rating for the standard power supply will have more current through it. However, they often incorrectly believed that this “fact” also applies to the bulbs connected in series.

Wattage rating of a bulb is proportional to the resistance: Some students incorrectly believed that the bulb with a higher wattage for a standard power supply will have a higher resistance. The reasons used by these students were sometimes simple and sometimes quite convoluted. Some of the students who believed that the wattage rating of a bulb is proportional to the resistance claimed that a higher resistance of a tungsten filament in the bulb will make the bulbs glow brighter. They noted that higher resistance implies a higher power dissipated and hence a brighter bulb. Therefore, the higher wattage bulbs which glow brighter must have a larger resistance. The following student responses to the wattage version manifest the difficulty due to different reasons:

“In series, the 100W bulb will be brighter. It has a higher resistance, which will use more voltage. The 25 watt bulb has less resistance, but the same current. This will produce less heat/light than the 100W bulb.”

“The 25W light bulb will be brighter. In series, more current will flow through the path of least resistance. Power is measured in watts. $\text{Power} = I^2 * R$. Assuming each has equal currents, the 100W bulb will have a higher resistance. Therefore, the 25 watt bulb will have more current when connected in parallel.”

Failure to understand the basic features of circuits in parallel and series: From the interviews, we find that when many introductory students see an electrical circuit, the first

feature that comes to their mind is the relation between the equivalent resistance and individual resistances even though it may not be relevant for the given question. Even when students were asked explicitly about these issues, many interviewed students did not understand that the voltage is the same across resistances in parallel and current is the same for the resistances in series. Students who were explicitly asked sometimes admitted that they did not know these things while others incorrectly noted that the current is the same for resistances in parallel and the voltage is the same across resistances in series. Some students noted that when light bulbs with 200ohm and 500ohm are connected in parallel, the 200ohm is brighter because it has more current flowing through it and hence has a higher voltage across it. They believed that a larger current means a higher voltage without realizing that the resistors connected in parallel have the same voltage across them.

In series the light bulb that is close to the power supply has a higher voltage across it: Some students incorrectly believed that the voltage across the first light bulb is the entire potential difference of the power supply and the voltage across the second light bulb is only part of it because the current flowing through the first light bulb will cause a “voltage drop”. For example, one student wrote that “It would depend on which one came first in the series because the voltage would drop after going across the resistances of the bulbs. So the first bulb would be the brightest because it got the most voltage.”

Failure to distinguish the rated power and the dissipated power: Students written explanations on the wattage version questions suggest that many students assume that the rated power of a light bulb equals the dissipated power of that light bulb no matter what the supply voltage is or how the bulbs are arranged. This leads to confusion when they try to solve for the resistance. For example, when light bulbs are connected in parallel, many students use the 20V

voltage and the provided wattage to solve for the resistance instead of using the standard power supply 120V.

Failure to realize that it is the power dissipated that determines the brightness:

Although it was specifically written that the brightness of the light bulbs is proportional to the power dissipated, many students did not consider the power explicitly when working on either version of the questions.

Confusion between resistance, current, voltage and power: Some students used resistance and power interchangeably. Others used current and power interchangeably. Yet, others used voltage and power interchangeably.

2.5.2 Are these questions quantitative or conceptual?

We note that the questions with the light bulbs discussed here are such that students are given the actual wattage for a standard power supply or the resistance of the light bulbs and asked which bulb will be brighter. These questions can be answered quantitatively by actually calculating the power dissipated in each light bulb for each case (parallel or series). In the wattage version, students will have to first figure out the resistances of each of the light bulbs using the relation between power, resistance and voltage. One interesting finding is that for the wattage version questions, only 26% of the students answering the question in parallel and 14% of the students answering the question in series by writing any equations (e.g., $R=V^2/P$, $P=V^2 \cdot R$, $P=i^2 \cdot R$). For the resistance version, approximately 40% of the students wrote any equation to answer the question for both series and parallel cases. Students who used equations were more likely to answer the question correctly.

One relevant question is why more students did not invoke equations to solve the problem. There are many advantages of learning to reason with equations if math anxiety is not dominant. Equations can provide the necessary constraints and act as roadmaps to navigate and help students draw meaningful inferences. When reasoning without quantitative tools, it is significantly more difficult to recreate the correct reasoning chain if the student is rusty about a concept. Equations can provide a pivot point for reconstructing it. For example, if a student does not remember whether the resistance of a bulb with a higher wattage is lower, the student can use the relation between the power, voltage and resistance to infer it. As noted in this study, for the wattage version of questions discussed here, only 26% of the students who answered the questions in parallel wrote down any equation and only 14% of those who answered the questions in series used any equation. Also, as students develop expertise, their dependence on equations for drawing conceptual inferences may gradually decrease. For example, while a novice student who has learned to reason with equations will invoke the equation relating power, voltage and resistance *explicitly* to conclude that for a standard parallel power supply, a higher wattage implies a bulb with lower resistance, an expert can use the same relation *implicitly* to conclude the same thing.

Discussions with several instructors who have taught introductory physics recently suggests that many students may not have used equations to answer these questions because they were asked which light bulb will be brighter and not “what is the power dissipated in each bulb”. Several professors claimed that many introductory students only think of using an equation when they are asked to calculate a physical quantity explicitly. For example, in the questions discussed here, if the students were asked to calculate the power dissipated (instead of being asked which bulb is brighter), they would have resorted to a quantitative analysis and thought of relevant

equations. Some of these professors claimed that if a question asks which bulb is brighter, even if the relevant numerical values are provided, students treat it as a conceptual question and often use their gut feelings rather than explicitly invoking a relevant physics concept or principle. These discussions suggest that if we want students to do sense making and answer these questions using physics concepts, one strategy would be to combine quantitative and conceptual questions so that students learn to make sense of a physical situation gradually. As the students develop expertise, their explicit dependence on physics equations may decrease.

2.6 SUMMARY AND CONCLUSION

We find that many introductory students have difficulty in determining the brightness of light bulbs under different conditions despite being told explicitly that the brightness of a light bulb is proportional to the power dissipated and that the bulbs have ohmic resistances. To investigate student difficulties, we developed and administered both free-response and multiple-choice questions and interviewed a subset of introductory students in-depth. One version of the questions provided the resistance of each light bulb while the other version provided the wattage rating of each light bulb when connected to a standard power supply. By comparing student performance on the two versions, we found that the wattage version of the questions was much more difficult than the resistance version because it involved an additional step. In particular, to answer the wattage version correctly, students must first figure out the resistance of the light bulbs and then the power dissipated under different conditions.

We also find that these questions are quite challenging for first year graduate students enrolled in a training course for teaching assistants.

Many students did not understand that the standard wattage rating of a light bulb corresponds to the case when they are connected in parallel to the standard power supply and they assumed that the light bulbs will have the same brightness regardless of how they are connected in a circuit. Students need explicit guidance in comprehending that the brightness of a light bulb can change depending on how it is connected in a circuit and the only thing that would not change is the resistance of the bulb. Also, some students believed that a higher current always implies a higher voltage across the bulb (even if the bulbs are in parallel) or a higher resistance always implies a lower current through a bulb (even if both bulbs are in series). Moreover, some students had a tendency to associate the brightness of a light bulb only with the current or the voltage which often lead to incorrect inferences. Curricula and pedagogies developed to improve students' understanding should take into account these difficulties.

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3.0 STUDENT DIFFICULTIES WITH EQUATIONS INVOLVING CIRCUIT ELEMENTS

3.1 ABSTRACT

We discuss an investigation exploring students' difficulties with equations involving circuit elements. We find that introductory physics students have great difficulty understanding the physical meaning of equations. For instance, they know that the resistance of an ohmic resistor can be written in terms of the potential difference across it and the current through it, but they fail to see that the resistance does not change when the potential difference across the resistor is varied. Similar confusions arose in problems relating to capacitors and inductors. We discuss these difficulties with equations in the context of introductory physics students' performance on questions about circuit elements both in the free-response and multiple-choice formats and compare their performance to that of graduate students. The student difficulties with equations discussed here in the context of circuit elements are likely to be prevalent even in other physics contexts.

3.2 INTRODUCTION

In order to become an expert in physics, students must learn to regard an equation as a relation between physical quantities, and not merely as a plug-and-chug tool or a formula that only requires numerical substitution to obtain a solution. They must internalize that each equation is a constraint that may relate variables and constants written in symbolic form, and that there may be many constraints relating one physical quantity to different physical quantities. They must also learn that some symbols represent universal constants, some are constant under certain conditions (e.g., the resistance of an ohmic material of a given length and a given cross sectional area at a fixed temperature), and some have a truly functional relationship (e.g., current and voltage across a resistor). Here, we discuss an investigation exploring students' difficulties with equations involving circuit elements (Engelhard & Beichner 2004, McDermott & Shaffer 1992). We hypothesize that the difficulties discussed here in the context of circuit elements may be more generally applicable across different topics since many introductory students view physics equations solely as plug-and-chug tools.

3.3 METHODOLOGY

These difficulties were investigated by analyzing calculus-based introductory physics students' performance on questions about circuit elements both in the free-response and multiple-choice formats and by comparing their performance to that of physics graduate students. We also discussed the responses individually with a subset of introductory students to understand their thought processes better. We note that students who participated in the research had all received

traditional lecture-based instruction on relevant content. The problems discussed here were administered in the recitations either as part of quizzes in courses taught by different instructors. In addition to the discussions with a subset of students, we conducted formal paid interviews with six volunteers from a calculus-based course whose first midterm exam scores were close to the class average. In addition to the written explanations, the analysis of the responses from the interviews yielded further information about student reasoning pertaining to the factors on which the various circuit elements depend.

One question administered in the multiple choice format to 237 calculus-based introductory students and 42 physics graduate students was the following:

The resistance of a cylindrical ohmic resistor at a fixed temperature depends on:

- (I) the current;
- (II) the potential difference across it;
- (III) the cross-sectional area;
- (IV) the length of the resistor.

- Answers:
- A. (I) and (II) only;
 - B. (III) and (IV) only;
 - C. (I), (II) and (III) only;
 - D. (I), (II) and (IV) only;
 - E. All of the above

In the corresponding problem in the free-response format, various factors were listed and students had to choose all of the factors on which the resistance of an ohmic resistor at a fixed temperature depends and explain their reasoning. The free-response questions were given to 430

students from four calculus-based introductory physics courses which were different from those in which the multiple-choice questions were administered. We wanted students to reason that although the resistance of a cylindrical ohmic resistor can be defined by the equation $R = V/I$, it is an intrinsic property of the resistor and is given by $R = \rho l/S$, where V is the voltage, I is the current, ρ is the resistivity, l is the length and S is the cross-sectional area. We wanted them to argue that the resistance does not depend on the potential difference or current. If the potential difference across the resistor is changed, the current will change correspondingly because the resistance remains fixed.

Students were asked analogous questions in both multiple-choice and open-ended formats about the capacitance of a parallel-plate capacitor and the inductance of a solenoid. Similar to the resistance question, we wanted students to reason, for example, that although the capacitance is the charge on each plate per unit voltage, the ratio will remain unchanged when the voltage across the plates is changed because the charge on the plates will change correspondingly. We wanted them to argue that the capacitance is an intrinsic property of a parallel plate capacitor and will depend only on the dielectric constant of the dielectric between the plates, the distance between the plates and the area of cross section of the plates.

Analogous to the resistance question, students had to choose from the following factors on which the capacitance of a parallel plate capacitor may depend:

- (I) the charge on the plates;
- (II) the potential difference across the plates;
- (III) the area of the plates;
- (IV) the distance between the plates.

The choices provided to them for the factors that determine the inductance of an inductor are:

- (I) the current;
- (II) the magnetic flux through the coil and number of ideal turns of coil;
- (III) the cross sectional area of the coil;
- (IV) the number of turns per unit length.

Here, the Roman numerals assigned to the factors (on which the capacitance and inductance depend) refer to their order in the multiple-choice questions and they will be referred to in the results section. We note that the resistor and capacitor questions were always administered in the same recitation but the inductor question was sometimes given in a separate recitation class after the instructor had covered the material on inductors in the course.

3.4 RESULTS

Table 3.1 shows the introductory students' responses to the multiple choice questions about resistance, capacitance and inductance. The bold numbers represent the percentage for the correct answer. It shows that 54% of the students answered the question about resistance correctly; 25% incorrectly believed that the resistance depends on only the current and voltage; and 12% thought that the resistance depends on all of the factors given. Only 35% of the students correctly answered the question about capacitance; 29% incorrectly believed that capacitance depends on the charge on the plates, the voltage and the distance between the plates; and 27% believed that all of the given factors determine the capacitance. For the question about inductance, the answers were almost equally distributed across various choices suggesting

students may be guessing the answer. Individual discussions with students also suggest that their knowledge about inductors was often shaky and many students admitted not understanding this topic.

Table 3.1. Distribution of introductory students' responses to the multiple-choice questions.

	I&II	III&IV	I,II&III	I,II&IV	ALL
RESISTANCE	25	<u>54</u>	7	3	12
CAPACITANCE	6	<u>35</u>	4	29	27
INDUCTANCE	25	<u>17</u>	15	25	17

Table 3.2. Distribution of introductory students' responses to the free-response questions.

	I&II	III&IV	ALL	I,III&IV	OTHER
RESISTANCE	18	<u>53</u>	12	3	14
CAPACITANCE	11	<u>53</u>	16	5	15
INDUCTANCE	7	<u>36</u>	11	6	40

Table 3.2 shows that introductory students' response to the free-response and multiple choice questions are similar for resistance but the performance is better on the capacitance and inductance questions. Moreover, in the free-response questions, some students selected other combinations of the possible factors provided. For the inductor free-response question, 8% chose II, III & IV, 7% chose I, II & III, and 5% chose I, II & IV. The wide variety of responses for the inductor question again suggests guess work.

Table 3.3. Distribution of physics graduate students' responses to the multiple-choice questions.

	I&II	III&IV	I,II&III	I,II&IV	ALL
RESISTANCE	2	<u>93</u>	0	0	5
CAPACITANCE	0	<u>86</u>	0	5	10
INDUCTANCE	8	<u>76</u>	3	3	10

In order to compare and benchmark introductory students' responses with physics graduate students, we administered the same three multiple-choice questions to 42 graduate students enrolled in a second semester mandatory TA training course two years in a row. Table 3.3 shows that while the graduate students perform significantly better than the introductory students, they have similar difficulties. Also, the inductor question is relatively difficult even for them.

3.5 DISCUSSION

Although students performed somewhat worse on the multiple-choice questions than on the free-response ones, the results from both versions have similar trends. Written responses and individual discussions both suggest that, at least within the context of resistance, students were often more familiar with $R = V/I$ than its relation with the resistivity, length and the cross sectional area of the resistor. During individual discussions, many students were surprised that there are “two” equations for the resistance ($R = V/I$ and $R = \rho l/S$) because they felt that one should be able to plug numbers in *only* one special formula that epitomizes resistance.

Some of the students who questioned how there can be two equations for the same physical quantity (e.g., resistance) were reminded by the interviewer that the acceleration of an object can be described in terms of the net force per unit mass or the rate of change of velocity with time. In response to this comment, students often noted that they had not thought about the fact that more than one equation can be used to calculate the acceleration. Very often, however, they still continued to express their concern about the fact that a physical quantity can be calculated using two totally different equations. Discussions suggest that students often believed that if one is given a formula for calculating a physical quantity, all the physics must be buried in that formula and one should be able to calculate everything about that physical quantity using that unique formula. We note that some of these students noted that the capacitance of a capacitor can depend on charge, voltage, as well as the area of cross section and the distance between the plates. Moreover, some of them were even able to recite the formula $C = Q/V$ and note that there was some formula that relates the capacitance to the distance between the plates and cross sectional area. These students were inconsistent in their assertion that there should be only one formula for a physical quantity. In particular, they felt that there should be only one formula for resistance but they mentioned more than one formula or relation for capacitance often without realizing that there was an inconsistency in their reasoning.

Another lesson learned both from the written tasks and discussions is that students often did not think of the resistance of an ohmic resistor at a fixed temperature, the capacitance of a capacitor and the inductance of an inductor as properties of the resistor, capacitor and inductor respectively. They incorrectly believed that the resistance of an ohmic resistor at a given temperature should change when the voltage or current is changed because of the definition $R=V/I$. Similarly, they believed that the capacitance of a capacitor must depend on the potential

difference across it and on the charge on the capacitor plates. As one student summarized it “if I rearrange $V=IR$ I get $R=V/I$ which means that the resistance depends on V and I ...how can it not be true?” Some of the students who stated that the resistance depends on voltage incorrectly interpreted that “a higher voltage will resist the flow of charge and cause a higher resistance”.

During individual discussions, when students were explicitly told that the resistance (of an ohmic resistor at a fixed temperature), the capacitance and the inductance are intrinsic properties of a resistor, capacitor and inductor, respectively, and were provided the relevant relationships to illustrate these points and asked to explain how they would explain relations such as $R=V/I$ or $C=Q/V$, students were confused. They were in general unable to explain, e.g., that when V increases I must increase proportionately in order to keep R constant. Moreover, during individual discussions, in a familiar Newtonian mechanics context, the same students who correctly claimed that Newton's second law implies that increasing the net force on an object will increase the acceleration but *not* change the mass of the object, had difficulty understanding how the resistance of an ohmic resistor will not depend on the voltage and current when $V=IR$ is an abstract context.

As noted earlier, students had similar difficulties with the capacitor question and were confused about why $C = Q/V$ does not necessarily imply that C depends on Q and V . Even when the interviewer discussed both equations $C = Q/V$ and $C = \epsilon A/d$ and asked students to interpret what C should depend on using both equations, they often believed that it should depend on all of the variables occurring on the right hand side of both equations. For example, one student noted "variables that occur in an equation affect each other". Another student noted that C should depend on Q and V because “charge and potential difference determine whether C is fully charged resulting in its ability to hold additional charge.” Discussions also suggest that some

students believed that the capacitance depends on the energy stored in the capacitor because the voltage and the charge affect the stored energy. Even when students were told during the discussions that the capacitance is an intrinsic property of a capacitor and asked to interpret what should happen when the voltage across a capacitor is changed, it was difficult for them to exploit $C = Q/V$ to infer that Q must change proportionately when V changes to keep C fixed. We note that these types of scientific reasoning skills are very important but the traditional physics courses are unable to help students develop them.

Similarly, many students incorrectly claimed that the inductance of an inductor depends on the current through the coil or the magnetic flux through the coils because of the equation relating the inductance to the current and the flux (although students in general admitted during individual discussions that they did not know about inductance as well as they knew about resistance and capacitance). Some students noted that the inductance depends on the current and the magnetic field because the current running through the inductor creates an induced magnetic field and the magnetic field produces the inductance. They believed that the inductance is non-zero only when an inductor is connected in a circuit and there is a current.

During individual discussions, students were most likely to answer the inductance question using a formula perhaps because this topic was most unfamiliar to them. However, they often mixed up the definitions of magnetic flux, magnetic field, induced emf and inductance. For example, some students who used the equation for Faraday's law of electromagnetic induction, confused the induced emf with the inductance of the inductor. A typical response from a student is the following: " $\mathcal{E} = -Nd\phi/dt$ ". The inductance depends on the number of turns and the change in magnetic flux. The flux depends on the magnetic field which depends on the current. Therefore, the inductance depends on the flux, the current and the number of turns."

3.6 SUMMARY AND CONCLUSION

We find that many students believe that there should be only one equation for a physical quantity that should provide the one and only “solution” for that physical quantity. For example, they were confused when presented with two separate equations for the resistance: one relating it to the potential difference and current; and the other relating it to the resistivity, length and area of cross-section of the resistor. Similar confusions arose in problems relating to capacitors and inductors. Students struggled with the fact that each equation is a constraint that may relate variables and constants written in symbolic form and there may be many constraints relating one physical quantity to other ones. The introductory students had great difficulty understanding concepts such as how the resistance of an ohmic resistor can be written in terms of the potential difference across it and the current through it, but it does not change when the potential difference across the resistor is varied. It was difficult for them to understand that some symbols represent universal constants, while others are constant under certain conditions.

Instructional strategies to improve students’ understanding of these issues related to interpreting equations should take into account these difficulties found in the context of circuit elements. Such difficulties are likely to be prevalent across different topics. Instruction should help students learn to reason appropriately about equations rather than viewing them as plug-and-chug tools. Physics topics (e.g., related to electrical circuits discussed) should not simply be taught as algorithmic exercises but rather should be used to help students develop reasoning skill. Conceptual and quantitative questions can be combined to help students think about the issues related to equations discussed here in more depth.

Prior research shows that knowing students' current knowledge and designing instruction to build on it is important (McDermott & Shaffer 1992). While it may be easy for an instructor to

understand that when the voltage increases, the current increases proportionately so that the resistance of an ohmic resistor does not change at a fixed temperature, it is very challenging for students. Research suggests that it is not sufficient to simply tell students that $R = V/I$ does not imply that the resistance depends on the voltage or current. Because students can quickly revert back to interpreting these equations incorrectly (McDermott & Shaffer 1992).

One strategy to help students with these concepts is to provide them with guided exploration activities within a coherent curriculum that challenge these incorrect notions so that students have the opportunity to organize and extend their knowledge (McDermott et al. 2002, Singh 2002b). It may be beneficial to ask students to first predict what should happen in a given situation before performing an exploration so that if the prediction and observation do not match, students would notice that there is a discrepancy between their prediction and observation. This would be an opportune time to provide them with guidance and support to help them accommodate and assimilate new concepts. For example, within a coherent curriculum, students can be given a resistor and asked to connect it to batteries with different voltages and measure the current through it. They can be asked to calculate the ratio of V/I for different cases and interpret why this ratio, which is the resistance, does not change when the voltage changes. They can perform similar explorations with a capacitor where they can increase the charge on the plates and observe how it affects the voltage across the plates and whether the ratio of the charge to the voltage is the same for different cases. They can also be asked to comment on the correctness of several statements provided to them only one of which is correct, and discuss their reasoning with their peers and instructor.

3.7 REFERENCES

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4.0 IMPROVING STUDENTS' UNDERSTANDING OF ELECTROSTATICS I. COULOMB'S LAW AND SUPERPOSITION PRINCIPLE

4.1 ABSTRACT

We discuss the development and evaluation of two research-based tutorials on Coulomb's law and superposition principle to help students in the calculus-based introductory physics courses learn these concepts. The tutorials were developed based upon research on students' difficulties on relevant topics. During the development of the tutorials, we obtained feedback both from physics faculty who regularly teach introductory courses and from introductory students for whom the tutorials are intended. We discuss the performance of students in individual interviews and on the pre-/post-tests given before and after the tutorials, respectively, in three calculus-based introductory physics courses. We also compare the performance of students who used the tutorials with those who did not use them. We find that students performed significantly better in classes in which tutorials were used than in the classes where students learned the material via traditional lecture only.

4.2 INTRODUCTION

Electrostatics is an important topic in most calculus-based introductory physics courses.

Although Coulomb's law, the superposition principle, and Gauss's law are taught in most of these courses, investigations have shown that these concepts are challenging for students (Maloney et al. 2001, Ding et al. 2006, Engelhard et al. 2004, Rainson et al 1994, Sadaghiani 2011, Savelsbergh et al. 2011, Guisasola et al. 2010, Wallace & Chasteen 2010, Pollock 2009, Sayre & Heckler 2009). Despite the fact that students may have learned the superposition principle in the context of forces in introductory mechanics, this learning does not automatically transfer to the abstract context of electrostatics and students get distracted by the different surface features of the electrostatics problems. Effective application of Coulomb's law or Gauss's law to find the net electric field due to a charge distribution requires understanding the principle of superposition for electric fields. Moreover, beyond the mathematical facility (Nguyen & Rebello, 2011), discerning the symmetry associated with a given charge distribution is critical for applying Gauss's law to find the magnitude of the electric field due to highly symmetric charge distributions. Helping students learn these concepts can improve their reasoning and higher-order learning skills and can help them build a more coherent knowledge structure.

In this chapter, we discuss the development and evaluation of two research-based tutorials (I and II) and the corresponding pre-/post-tests to help students develop a functional understanding of Coulomb's law and the superposition principle. We find that classes in which students worked on the tutorials did significantly better than those which did not use them. In the next chapter, we focus on similar issues while developing and evaluating three tutorials (III-V) on Gauss's law and symmetry considerations, which build on tutorials I and II.

4.3 OTHER INVESTIGATIONS RELATED TO ELECTRICITY AND MAGNETISM

Investigation of student difficulties related to a particular physics concept is important for designing instructional strategies to reduce them. The origin of student difficulties in learning physics concepts can broadly be classified in two categories: gaps in students' knowledge, and misconceptions. Cognitive theory suggests that learning is incremental and new knowledge builds on prior knowledge (Simon 1989, Anderson 1999). Knowledge gaps can arise from many sources, e.g., a mismatch between the levels at which the material is presented in a course and student's prior knowledge. Deep-rooted misconceptions can also seriously impede the learning process at all levels of instruction. Prior investigations related to electricity and magnetism have included difficulties with general introductory concepts, electrical circuits, and superposition of the electric field. Maloney et al. (2001) developed and administered a 32 item multiple-choice test (the Conceptual Survey of Electricity and Magnetism) that surveys many important concepts covered in the introductory physics courses and is suitable for both calculus- and algebra-based courses. Ding et al. (2006) developed another multiple-choice test that surveys topics covered in introductory electricity and magnetism courses. These investigations found that students have common difficulties with fundamental concepts related to electricity and magnetism. Engelhard et al. (2004) have developed conceptual assessments related to electrical circuits. Rainson et al. (1994) investigated difficulties with the superposition of electric fields by administering written questions. McDermott et al. (1992) performed an in-depth investigation of the difficulties students have with electrical circuits and developed tutorials and the inquiry-based curriculum that significantly reduce these difficulties among introductory physics students and pre- and in-service teachers (McDermott et al. 2002). Some tutorials on Coulomb's law and Gauss's law

topics have been developed by the University of Washington group (McDermott et al 2002). Those tutorials are complementary to the ones we have developed, which focus on achieving competency with symmetry ideas in Coulomb's Law, the superposition principle, and Gauss's Law. Belcher et al. and Belloni and Christian (2004) have developed visualization tools to improve student understanding of physics concepts including those related to electricity and magnetism.

4.4 TUTORIAL DEVELOPMENT AND ADMINISTRATION

The development and assessment of research-based tutorials to minimize student difficulties was carried out with the following core issues in mind: (1) the tutorials must build on students' prior knowledge so it is important to investigate the difficulties students have related to a particular topic before the development of the tutorials, (2) the tutorials must create an active learning environment where students get an opportunity to build a robust knowledge structure in which there is less room for misconceptions, (3) the tutorials must provide scaffolding support, guidance and feedback to students and opportunity to organize, reconstruct, and extend their knowledge.

Before the development of the tutorials, we conducted an investigation of student difficulties with these concepts (Singh 2006) by administering free-response and multiple-choice questions and by interviewing individual students using a think aloud protocol (Chi 1994). In this think aloud protocol, we initially ask students to verbalize their thought processes while they answer questions but do not disturb them except asking them to keep talking. Only at the end, we ask for clarifications of points that the students had not made clear earlier. We found that many

students have difficulty distinguishing between the electric charge, electric field and force. Students also have difficulty with the principle of superposition and in recognizing whether sufficient symmetry exists to predict whether the magnitude of the electric field should be the same at various points for a particular charge distribution.

We then developed the preliminary version of the tutorials and the corresponding pre-/post-tests based upon the universal students' difficulties with these concepts found in research and a theoretical task analysis of the underlying concepts. Theoretical task analysis involves making a fine-grained flow chart of the concepts involved in solving a specific class of problems from the perspective of an expert. Such analysis can help identify some stumbling blocks where students may have difficulty. Investigation of students' difficulties using written tests and interviews is critical for developing tutorials because theoretical analysis from the perspective of an expert often does not capture all of the difficulties students have with relevant concepts.

During the investigation of students' difficulties with relevant concepts, we gave written free response and multiple-choice questions to students in many courses and also interviewed a subset of students. Then, the two tutorials were developed to help students learn about Coulomb's law, the superposition principle and symmetry in the context of discrete and continuous charge distributions (conceptually). The first tutorial started with the electric field due to a single point charge and then extended this discussion to two or more point charges. The second tutorial further continued the conceptual discussion that started in the first tutorial (which was mainly about discrete charges) to continuous charge distributions. The tutorials guided students to understand the vector nature of the electric field, learn the superposition principle and recognize the symmetry of the charge distribution. Students worked on examples in which the symmetry of the charge distribution (and hence the electric field) was the same but the charges

were embedded on objects of different shapes (e.g., four equidistant charges on a plastic ring vs. a plastic square). Sometimes common misconceptions were explicitly brought out by having two characters in a tutorial discuss an issue in a particular context. Students were asked to identify the person with whom they agreed and justify their reasoning. Then the students were given guidance and support to develop a knowledge structure related to these topics so that there is less room for misconceptions.

We administered each pre-test, tutorial and post-test to several students individually, who were asked to talk aloud while working on them. After each administration, we modified the tutorials based upon the feedback obtained from student interviews. The tutorials also went through several iterations with four physics faculty members who had taught introductory electricity and magnetism courses. These individual evaluations helped to fine-tune the tutorials and improve their organization and flow. Then, the tutorials were administered to four different calculus-based introductory physics classes with four lecture hours and one recitation hour per week. Students worked on each tutorial in groups of two or three either during the lecture section of the class or in the recitation depending upon what was most convenient for an instructor. The pre-tests and post-tests were taken individually by the students.

Although the pre-test and post-test accompanying a particular tutorial assess student understanding of concepts learned in the tutorials, the same test was not used as both the pre-test and post-test for a given tutorial. Research has shown that, in some situations, students performed better on the post-test because they had seen the pre-test, and we wanted to minimize this effect (Kulik et al 1984, Benedict & Zgaljardic 1998). We also note that the pre-tests were neither returned to the students nor were they discussed with the students.

All pre-tests and tutorials were administered after traditional instruction in relevant

concepts. Instructors often preferred to alternate between lectures and tutorials during the class and give an additional tutorial during the recitation. This way all of the five tutorials from Coulomb's law to Gauss's law (Gauss's law tutorials III-V are discussed in the next chapter) were administered within two weeks. For the tutorials administered in lecture section of the class, pre-tests were given to students right before they worked on the tutorials in groups. Since not all students completed a tutorial during the class, they were asked to complete them as part of their homework assignment. At the beginning of the next class, students were given an opportunity to ask for clarification on any issue related to the part of the tutorial they completed at home and then they were administered the corresponding post-test before the lecture began. Each pre-/post-test counted as a quiz. The pre-tests were not returned but the post-tests were returned after grading. When a tutorial was administered in the recitation (tutorial II in this chapter and tutorial V in the next chapter), the teaching assistant (TA) was given specific instruction on how to conduct the group work effectively during the tutorial. Moreover, since the TA had to give the post-test corresponding to the tutorial during the same recitation class in which the students worked on the tutorials (unlike the lecture administration in which the post-tests were in the following class), the pre-tests were skipped for some of these tutorials due to a lack of time. Sometimes, the instructors gave the pre-tests in the lecture section of the class for a tutorial that was administered in the following recitation.

In all of the classes in which the tutorials were used, 2-2.5 weeks were sufficient to cover all topics from Coulomb's law to Gauss's law. This time line is not significantly different from what the instructors in other courses allocated to this material. The main difference between the tutorial and the non-tutorial courses is that fewer solved examples were presented in the tutorial classes (students worked on many problems themselves in the tutorials). We note that since

many of the tutorials were administered during the lecture section of the class, sometimes two instructors (e.g., the instructor and the TA) were present during these “large” tutorial sessions to ensure smooth facilitation. In such cases, students working in groups of three were asked to raise their hands for questions and clarifications. Once the instructor knew that a group of students was making good progress, that group was invited to help other groups in the vicinity which had similar questions. Thus, students not only worked in small groups discussing issues with each other, some of them also got an opportunity to help those in the other groups.

4.5 DISCUSSION OF STUDENTS’ DIFFICULTIES

The process of the development and evaluation of the tutorials started with the investigation of common difficulties that students have about these concepts. These difficulties were explicitly addressed in the tutorials. While students were less likely to have these difficulties in the post-tests after the tutorials than in the pre-tests, some students still had difficulties with these concepts. The difficulties on the pre-test and post-test were similar in nature for both the tutorial and comparison groups, but students in the tutorial group were less likely to have these difficulties. Below, we discuss examples of the common difficulties found about topics related to the tutorials without separating performance of the tutorial groups and comparison group on pre-/post-tests. For reference the pre-tests and post-tests for both tutorials are included in the Appendix.

Students often have difficulty with the principle of superposition and in calculating the vector sum of the electric fields due to the individual point charges to obtain the net electric field at a point if more than one point charge is present in the region. All of the questions on the pre-

/post-tests of both tutorials require the use of the superposition principle to compare the electric field at various points for a given charge distribution. Therefore, the performance of many students on the pre-/post-tests was closely tied with their understanding of the principle of superposition. Below, we discuss common difficulties found.

4.5.1 If distances from different charges add up to the same value at two points, the electric field will be the same at those points

When asked to compare the electric field at different points due to more than one charge in situations where there wasn't sufficient symmetry to claim that the magnitudes of the electric field are the same, some students incorrectly claimed that the magnitudes of the net electric field will be the same at those points because the electric field produced by different charges will somehow compensate for each other. These kinds of arguments made no mention of the vector nature of the electric field and were solely based upon arguments about the distances of various charges from those points adding up to the same value.

The pre-test questions for tutorial I involve a situation with three identical point charges on a straight line. Our prior study about this situation shows that 13% of the students in the calculus-based introductory physics sequence believe that the magnitude of the electric field is the same at points A, B and C (see the Appendix C) but the directions are different (Singh 2006). Interviews suggest that these students believed that the magnitude of the electric field at these points should be the same because they were the same perpendicular distance from the straight line joining the three charges. Some students provided more detailed reasoning. Instead of viewing it as a problem involving the addition of three electric field vectors, these students often made guesses by looking at the distances of points A, B, and C from the three charges and

hoping that the electric field will somehow work out to be the same at the three points. They often claimed that point A is closer to one charge and farther away from the other two charges than point B, which is equidistant from the two charges and not as far away from the third charge as point A. Therefore, the electric field at points A and B will have the same magnitude if we take into account all the three charges. In one on one discussion, some students with such difficulties were explicitly asked about the case with only two point charges. When they were asked to compare the electric field at two different points such that the distances from these two point charges added up to the same value at each point but the symmetries of those points were different with respect to the two charges, those students continued to claim that the magnitude of the electric field produced by the two charges must work out to be the same at both points.

In the same spirit, the most common difficulty with question (2) on the post-test of tutorial I was that they agreed with the statement. Some students claimed that since one charge is closer and the other two charges are (for most points) farther from a point on the dashed triangle, the magnitude of the net electric field will work out to be the same at all points on the dashed triangle. The following sample student responses illustrate this type of claim:

- I agree because on the dashed triangle, the closer you get to one charge, you must move that much farther from the other charges. This keeps everything in equilibrium.
- I agree because everywhere on the dashed line the sum of the distances is the same from all 3 charges
- Agree. Since it is a symmetrical situation, net electric field is equal everywhere
- Agree because when the magnitude from one charge weakens because of increase in

distance, the next charge evens it out. So the electric field magnitude is equal.

Interviews suggest that some students had great difficulty in adding the electric field vectors due to the three charges at various points of the dashed triangle and realizing that the magnitude of the electric field cannot work out to be the same at all points of the dashed triangle. After the individual interviews, some students stayed back and discussed the correct answers for each question. The interviewer tried to explain to a student that the magnitude of the electric field cannot work out to be the same at all points by choosing two points on the dashed triangle, one point at the vertex of the dashed triangle and another equidistant from two vertices on the dashed triangle, drawing individual contributions to the electric field due to the three charges and arguing that the vector sum need not have the same magnitude at the two points. However, the student argued that the interviewer had not convinced him of anything. He said: “I do not see how you can convince anybody that those two points on the dashed triangle will not have the same magnitude electric field without using numerical values for charges and distances.” The interviewer further tried to explain to the student by taking a limiting case such that the ring with three charges was very small compared to the dashed triangle so that the three point charges can approximately be lumped into a single point charge. The interviewer tried to explain that in this case the electric field magnitudes cannot be the same at different points on the dashed triangle because the distances of different points on the dashed triangle from the single point charge “lump” are different without even accounting for the fact that a vector addition is required to find the net electric field. The student was still not convinced and claimed that he did not see how the three point charges can be lumped into a single point charge if they are actually supposed to be arranged in a triangular shape no matter how large the dashed triangle is compared to the ring on

which the charges are located. The interviewer had further discussion with the student about what a point charge is and the student initially said that it was probably a single proton or electron. When the interviewer pointed out that the point charges can have magnitudes much larger than the magnitude of charge on a single electron or proton, which is 1.6×10^{-19} C, the student was a little confused and said that perhaps a point charge can be more than one proton or electron and he needs to think about it. The arguments from the student show the difficulty in conceptual reasoning and in making approximations. If students are given a quantitative problem with numerical values of charges and distances of the charges from the point where they have to find the electric field, students may succeed by using an algorithmic approach to solve for the electric field without even thinking about the conceptual implications of those results. The conceptual questions can sometimes be more challenging because students cannot use a recipe and must use their conceptual understanding (McDermott 2001, Mazur 1997, Kim and Pak 2002).

Some students claimed that the electric field will have the same magnitude at every point on the triangle except at the three vertices. Here is a sample response from a student: “disagree. The electric field is radial so for this effect to equal at every point on the dashed triangle, the corners would need to be rounded slightly.” The use of the word “slightly” also shows that the student may not have understood that the laws of physics are precise and words like “slightly” do not have a quantitative definition.

Some students who correctly disagreed with the statement in question (2) on the post-test for tutorial I did not explicitly mention anything about the vector nature of the electric field and claimed that the magnitude of the electric field cannot be the same at all points on the dashed triangle because different points of the triangle are closer from some charges and farther from the other charges. In the written responses, students got full credit if they noted that the magnitude of

the electric field will not be equal at all points on the triangle and explained this by mentioning that the vector sum of the electric field due to the three charges at different points cannot be the same or invoked reasoning involving the distances of different points on the dashed triangle not being the same from the three charges. For example, the following response was given full credit although the explanation is far from perfect: “Disagree because some parts of the dashed lines are closer to the charges than other parts. Thus some parts will be less affected by some charges and the magnitude of the electric field at different points on the dashed triangle cannot be the same.”

In response to question (2) on the post-test of tutorial I, one of the interviewed students noted that the electric field magnitude is not the same at every point on the dashed triangle because the electric field is proportional to $1/r^2$. He incorrectly claimed that the sum of the distances from the three point charges of any point on the dashed triangle is the same and the electric field magnitude would have been the same at every point on the dashed triangle if it were proportional to $1/r$ instead of $1/r^2$. Further prodding showed that the student was not able to distinguish between a quantity proportional to r vs. $1/r$ and thought that the functional dependence $1/r$ is a linear dependence on the distance r . Moreover, the student ignored the vector nature of the electric field. They assumed that if the electric field were proportional to $1/r$, the net electric field due to the three charges will work out to be the same at all points.

As expected, the principle of superposition is also challenging for students in the context of a continuous charge distribution. In question (4) on the post-test for tutorial II, charge is uniformly distributed on a finite square sheet and students were asked to compare the electric field at points B and C. In fact, a similar question was asked to 541 introductory students in the multiple-choice format and some students incorrectly claimed that the electric field at points B

and C have the same magnitude and same direction (20%), same magnitude but different directions (15%), and different magnitudes but same direction (7%) (Singh 2006). Those who claimed that the magnitude of the electric field is the same at both points but not the direction often justified it by citing that the vertical distance of points B and C from the finite uniform sheet of charge is the same so the magnitude is the same but the direction of the electric field at point C above the center is perpendicular to the sheet but not at point B near the sheet edge. Those who stated that the direction of the electric field at points A and B are the same often incorrectly claimed that the direction of the electric field is perpendicular and outward from the sheet at both points. Students who claimed that both the magnitudes and directions of the electric field must be the same at both points claimed that this was true because the sheet has a uniform charge and points B and C are at the same heights above the sheet. Figure 4.1 shows the reasoning of a student who believed that both the magnitudes and directions of the electric field must be the same at both points.

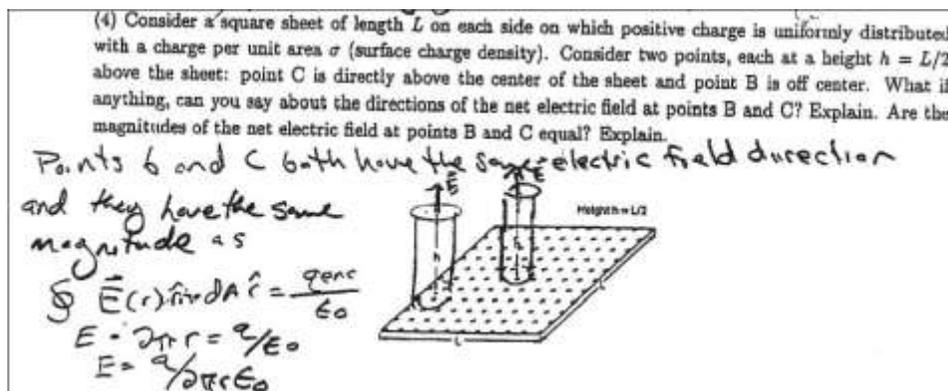


Figure 4.1: A sample response for the post-test question (4) on tutorial II

4.5.2 Only the nearest charge contributes to the electric field at a point

Some students claimed that only the nearest charge will contribute to the electric field at a point (McDermott & Shaffer 1992). When asked to find the net electric field at a point, they only took into account the electric field at that point due to the nearest charge. However, most of these students invoked this idea selectively and claimed that only the nearest charge will contribute to the electric field if they “felt” that the other charges are sufficiently farther away and could not contribute to the electric field at the point. For example, in question (2) on the post-test for tutorial I, the following are sample responses provided by the students: “The triangle will have less electric field at its corner because only one charge will act on it.” “The center of dashed side of the triangle will have more electric field because more than one charge will act on it.” Similarly, on question (3) on the post-test of tutorial I, some students claimed that only the nearest charge will produce the electric field at points A, C or E.

4.5.3 Charges in a straight line that are blocked by other charges do not contribute to the electric field

Some students claimed that if several charges are in a straight line, the effect of the charges that were “blocked” by other charges do not contribute to the electric field at a point. For example, in response to question (3) on the pre-test of tutorial I, some students claimed that the electric field at point D is zero because the field due to the two charges on the two sides cancel out. Interviews confirm that some of these students were ignoring the effect of the third charge assuming it was blocked by the other charge and could not influence the electric field at point D.

4.5.4 Confusion between the electric field due to an individual charge and the net electric field due to all charges

Some students had difficulty differentiating between the electric field due to individual charges at a point and the net electric field. These students often drew several arrows to show the contributions of various point charges to the electric field at a point but did not realize they had to add them vectorially to find the net electric field. The responses to the pre-/post-test questions for tutorial I show that some students could not distinguish between the electric field at a point due to the individual charges and the net electric field. These students drew the electric field at a point from the individual point charges present and did not address the net electric field even though they were asked about the net electric field.

4.5.5 Confusion between electric field, electric force and electric charge

Some students had difficulty differentiating between the electric field, electric force and electric charge. They used the words “electric force”, “electric field” and “electric charge” interchangeably. For example, in response to various questions about the electric field at a point, some students claimed that the “charge” or “force” at that point is in a particular direction. Before the development of tutorials, we gave several open-ended and multiple-choice questions to students. In a multiple-choice question given to 541 introductory students, 10% of the students identified electric charge as a vector (Singh 2006). To justify why the electric charge is a vector in one-on-one interview situations, these students claimed that the positive charges point outward and the negative charges point inward. It was clear from the responses that students were often referring to the electric field but calling it “charge.”

4.5.6 Electric field can only be found at points where there is a charge present

Some students believed that there must be a charge present at the point where they are asked to find the electric field. This type of confusion is coupled with the difficulty in differentiating between the electric force and electric field. While discussing the pretest question in tutorial I about the electric field due to three identical point charges in a straight line, one interviewed student said that the charge at point A will be repelled from other charges. He was explicitly asked by the interviewer why there is a charge at point A. The student first appeared a little surprised but then argued that there could not be any attraction or repulsion if there was no charge present at point A and it would not make sense to talk about the electric field at that point. Further discussion shows that the student was confusing “electric field” at a point produced by charges in that region with the “electric force” on a charge placed at that point. Since $F = qE$, students are often taught that in order to find the direction of the electric field at a point, they should place an imaginary positive test charge at that point. Then, the direction of the net force on that charge and the electric field at that point will point in the same directions. Interviews with individual students suggest that over-generalization of this explanation may be partially responsible for students believing that there must be a point charge present at the point where the electric field is to be calculated. While the electric field is defined as the force per unit charge, the electric field and electric force do not even have the same units. In response to the pre-test question (1) on tutorial II, one student drew the diagram shown in Figure 4.2 and claimed that “the electric field due to point charges at points A, B and C will point in all directions so the electric field cannot be directed perpendicular to the finite line of charge everywhere”.

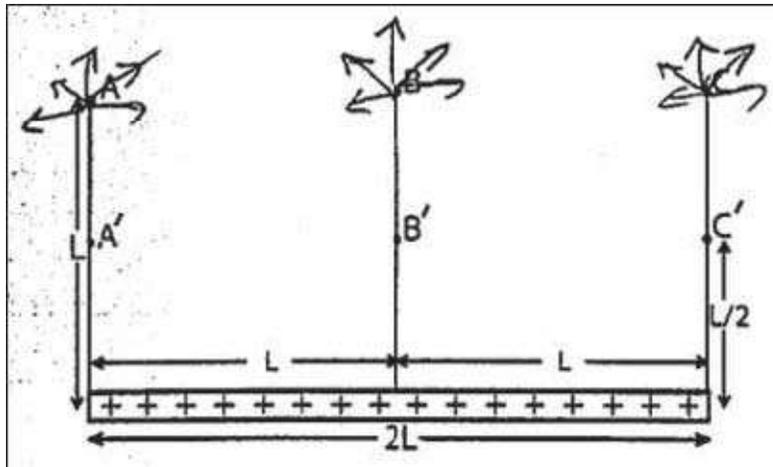


Figure 4.2: A sample drawing for the pre-test question (1) on tutorial II

4.5.7 Assuming that a positive charge attracts all points around it so that the electric field due to the charge points towards it

Some students believe that the electric field due to a positive point charge points towards the charge. For example, when asked to draw the direction of the electric field at points A, B and C in the pre-test question (1) for tutorial I, one difficulty was drawing the electric field arrows towards the three point charges as though the three charges shown in the figure are negative. One interviewed student explicitly explained his drawing by incorrectly arguing that the positive charges will attract points A, B and C because positive charges attract all points around them. When asked to elaborate, the student could not explain the reasoning but said that this is what he remembers. Further discussions with him suggest that this confusion may be due to the fact that the student had done some problems in which negative charges were present and they were attracted towards a positive charge and the student interpreted that the arrows for the electric field were always towards a positive charge. The student was also not making a distinction between the electric field and electric force.

4.5.8 Confusion about electric field line representation and interpretation of electric field using it

The electric field line representation is used as a tool to obtain the direction of the electric field and to get a qualitative feel for the magnitude of the electric field at various points for a given charge distribution. In this representation, the electric field is tangent to the electric field line at any given point. If the field lines are closer together in a region, the electric field is stronger in that region. Unfortunately, this representation can be very misleading for introductory students. Some students claimed that the direction of the electric field at a point is given by the curved electric field lines rather than the tangent to those lines at each point. A common difficulty was discerning the connection between the electric field line representation, the electric field at a point due to individual charges and the net electric field at that point. For example, in response to question (1) on the pre-test of tutorial I, two sample responses are shown in Figure 4.3.

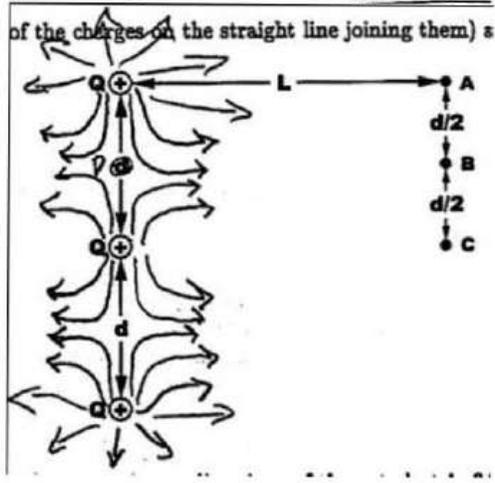
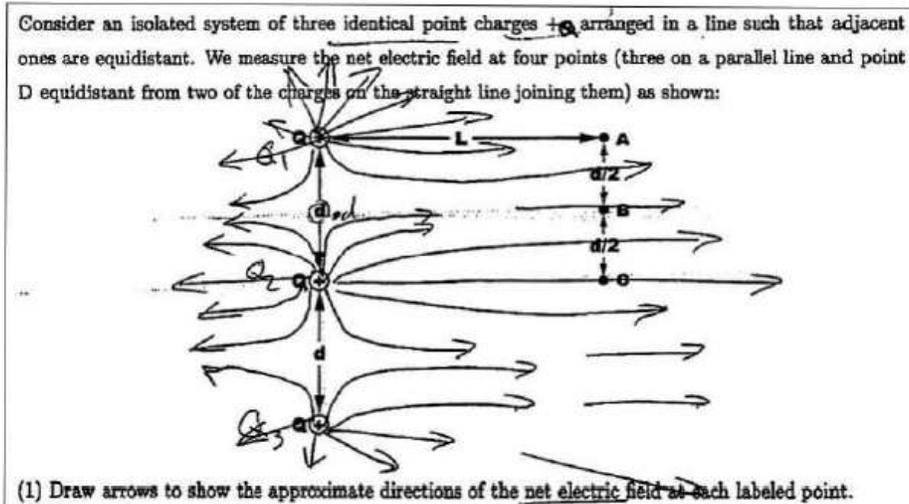


Figure 4.3: Sample responses for the pre-test question (1) on tutorial I

Similarly, in response to question (1) on the post-test for tutorial I, some students drew electric field lines of a dipole. If students had interpreted this electric field line representation correctly, they would have predicted the direction of the electric field correctly at both points A and B. Some students who correctly drew the electric field lines for the electric dipole did not know how to interpret the direction of the electric field at points A and B using the electric field line representation. For example, in response to question (1) on the post-test of tutorial I, a sample

response shown in Figure 4.4 points to this difficulty.

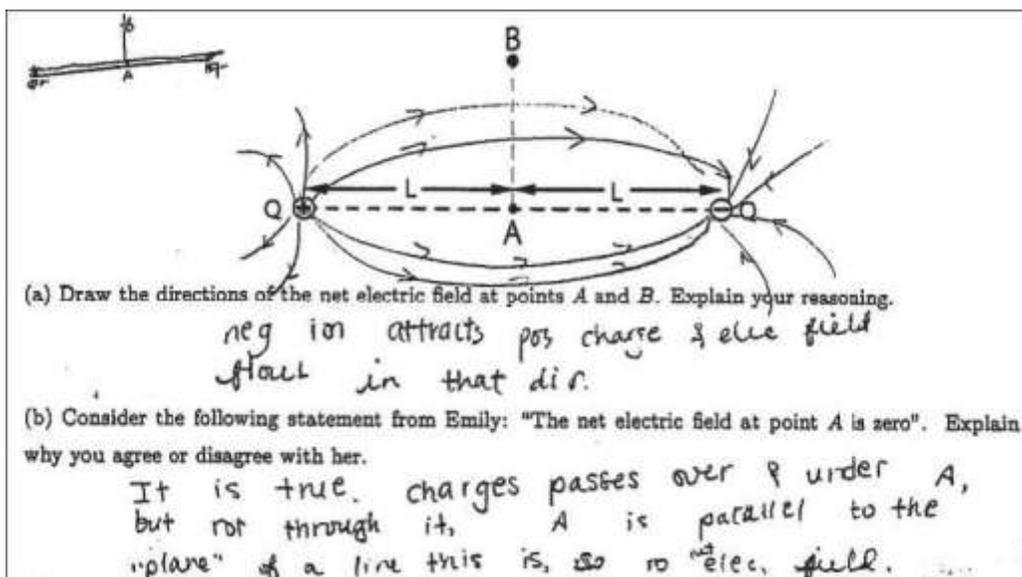


Figure 4.4 : A sample response for the post-test question (1) on tutorial I

The student whose response is shown in Figure 4.4 and who drew the electric field lines for an electric dipole and claimed that the “neg(ative) ion attracts pos(itive) charge & elec(tric) field flows in that dir(ection)” was interviewed. When the student was specifically asked to draw the direction of the electric field at point B, the student drew a field line passing through point B. He kept pointing to the electric field line he had drawn passing through point B and said that the whole curve gives the direction of the electric field at point B. When the interviewer insisted that the student should say something specifically about the direction of the electric field at point B, the student incorrectly added that point B will get pulled towards the negative charge. As can be seen from the student response to question (1b) on the post-test for tutorial I, the student believes that the electric field at point A is zero. Many other students who drew electric field lines used this representation inappropriately in a similar manner and focused on the whole curve connecting, e.g., point B to the two charges as representing the direction of the electric field at

point B rather than drawing the tangent to the electric field line at point B where they were asked to find the electric field. Even those students who correctly noted that the field at point A points towards the negative charge were often confused about the direction of the electric field at point B. They claimed incorrectly (similar to the interviewed student) that the electric field at point B is pointing towards the negative charge because the field line curves towards the negative charge or because point B is attracted to the negative charge.

Incidentally, some the students who drew the electric field lines correctly and had a field line pointing to the right at point A incorrectly claimed that the net electric field at point A is zero in response to Question 1(b) on the post-test of tutorial I. The most common reason cited was that the effect of the positive and negative charges will cancel at the midpoint. It was clear from student responses that there was a disconnect between the electric field line they had drawn connecting the positive and negative charges passing through point A and what it implied for the electric field at point A. One interviewed student drew arrows showing the electric field lines emanating in all directions from a single point charge and claimed that the electric field due to a point charge cancels out since it points in all directions.

4.5.9 Electric field cannot be zero at any point in a region if only positive charges are present

Some students believe that both the positive and negative charges must be present in a region for the electric field to be zero at a point in that region. In response to question (3) on the pre-test of tutorial I, many students correctly noted that the electric field cannot be zero at any of the points shown because all the three charges present are positive. While the electric field is not zero at any point shown in the figure, a large number of students provided reasoning that was incorrect

for why the electric field cannot be zero. They incorrectly argued that, in order for the electric field to be zero at a point, there must be both positive and negative charges present in the region and the electric field cannot be zero at any location in situations where only positive charges were present as in the pre-test question (3) of tutorial I. This statement is incorrect, e.g., the electric field will be zero on a straight line joining two identical positive charges equidistant from each charge. An interviewed student who said that the field cannot be zero at any point in pre-test question (3) on tutorial I explained: “can’t cancel the effect because the effect of positive needs to be canceled by negative”. The student further argued that this must be true because opposite charges repel and cancel each other out. As noted below in the discussion related to the electric field due to an electric dipole, students often invoked the idea that the positive and negative charges negate each other so both must be present in a region to make the net electric field zero at a point.

4.5.10 Difficulty with the electric field due to an electric dipole at points on the perpendicular bisector

Question 1(a) in the post-test for tutorial I is about the direction of the electric field at point A midway between the straight line joining the two charges of an electric dipole and the direction of the electric field at point B on the perpendicular bisector (but not on the straight line joining the two charges). Students who incorrectly claimed that the electric field is zero at both points A and B ignored the vector nature of the electric field. The most common difficulty with the electric field at point A was assuming that the electric field is zero at that point. The three most common difficulties with the direction of the electric field at point B were the following in order of their prevalence:

(I) It points towards the negative charge because point B is positively charged and is pulled towards the negative charge or the electric field line pulls point B towards the negative charge.

(II) It is zero because the effects of the negative and positive charges cancel out at the perpendicular bisector of the straight line joining the two charges (which are equal in magnitude and opposite in sign).

(III) It is vertically downward because both charges will pull point B towards them with an equal magnitude force so that the net electric field is downward. The following are sample incorrect responses for post-test Question 1(a) for tutorial I:

- B will be drawn into the middle of the field by opposing forces and point down. A will remain stationary due to the canceling forces.
- They cancel at both points because charges are opposite and they attract. Being equal they are equally attracted to one another.
- Net field for both points is zero because they are equidistant from opposite forces of the same magnitude.
- A has no net electric field and B is downward.
- The charges emitted from +Q will be drawn towards the -Q charge so at point B they will be at an angle towards -Q.

Question 1(b) on the post-test of tutorial I asked students to consider the following

statement about point A between the two charges of an electric dipole: “The net electric field at point A is zero”. The students were asked to explain why they agree or disagree with the statement. The following are sample responses from students who agreed with the statement:

- Agree because the forces cancel each other out.
- Agree, the two charges cancel each other out.
- Agree, charges of equal magnitude but opposite sign will pull against each other so point A is pulled in each direction equally.
- Agree assuming point B is not charged. The charges have equal magnitude so they cannot dump a net charge on A.
- Agree since opposites attract. They are putting the same force on A that cancels out.
- Agree as long as A remains equal distance from the two charges.

4.5.11 Invoking the dynamics of charges in an electrostatics problem

In electrostatics problems, students sometimes invoked the dynamics of charges and discussed how charges would accelerate. For example, when students were asked to calculate the electric field for the charge distributions shown in the pre-test and post-test of tutorial I, some students described how charges will move around due to the attraction and repulsion between them. None of these students who described the dynamics of charges in this manner and how it will affect the electric field mentioned that the positive and negative charges will essentially collide and the same type of charges will move infinitely far away from each other if the electrostatic force was the only force acting on them. In the post-test of tutorial I, a student who noted that the opposite

charges will move towards each other in problem (1) incorrectly agreed with the statement in problem (2) about three point charges. He agreed with the statement claiming that the magnitude of the net electric field is the same everywhere on the dashed imaginary triangle and provided the following reasoning: “I agree because like charges repel and so these charges will maintain the same distance between them”.

4.5.12 Confusion about symmetry of charge distribution vs. symmetry of the object in which charges are distributed

Students often have difficulty in evaluating the symmetry of the charge distribution in a given situation and confuse the symmetry of the charge distribution with the symmetry of the object in which charges are embedded. The most common difficulty with question (3b) on the post-test of tutorial I was the assumption that all the points shown have the same magnitude of the net electric field as point A. Students often justified this by incorrectly citing that the problem had circular symmetry confusing the symmetry of the object on which charges are embedded with the symmetry of the charge distribution. Question (3c) on the post-test was very difficult for students and many students agreed with the statement. They claimed that the electric field is radially outward everywhere on the dashed circle. The following are examples of student responses:

- Agree. At any point on the circle the three point charges will cause there to be a radially outward electric field.
- Agree. This has to do with the fact that the three charges are in a circle making the final outcome radial.

- Agree, the tangent to any point on the imaginary circle would point to the center.
- I agree because there are only positive charges which push out.
- Agree. This region is infinitely concentric so the electric field vector cannot point anywhere else but radially outward.
- Agree because charges will always balance out and produce the same electric field going radially outward.

We note that students were explicitly told on the test that the meaning of “radially” outward is straight out from the center. Interviews suggest that those who believed that all points on the circle have an electric field that is radial also believed that the magnitude of the electric field is the same at all points on the dashed circle.

4.5.13 Difficulties in generalizing from discrete to continuous charge distribution

Many students have difficulty realizing that they should invoke the principle of linear superposition and vectorially add the electric field at a point due to the individual point charges to find the electric field due to a charge distribution. The principle of linear superposition can be applied to the continuous distribution of charge and students can break up the continuous charge distribution to infinitesimal elements of charge (length, area or volume) and find the net electric field at a point by vectorially adding (or integrating over) the electric field due to the whole charge distribution. We find that the difficulty in using the principle of linear superposition correctly was exacerbated for cases where the charge distributions are continuous. Incidentally,

students were never asked to perform any difficult integrals. They only had to draw the arrows showing the electric field qualitatively for the continuous charge distributions or predict whether the magnitude of the electric field will be the same at two different points due to a given charge distribution based upon symmetry considerations.

Interviews suggested that sometimes a student who knew how to calculate the net electric field at a point due to an electric dipole by drawing the electric field due to individual charges and then finding their vector sum did not realize that the same procedure could apply for a charge distribution which is continuous. One interviewed student who found the direction of the electric field correctly for the dipole said that he never understood how to handle the continuous charge distributions and found those problems difficult. He did not apply the principle of linear superposition to the continuous charge distribution and used his memorized knowledge to guess the answers. For example, for a uniformly charged finite rod, he claimed that the electric field is radially outward even if the rod is finite in length. A similar response on pre-test question (1) for tutorial II is shown in Figure 4.5. Discerning the symmetry of the charge distributions was difficult for him.

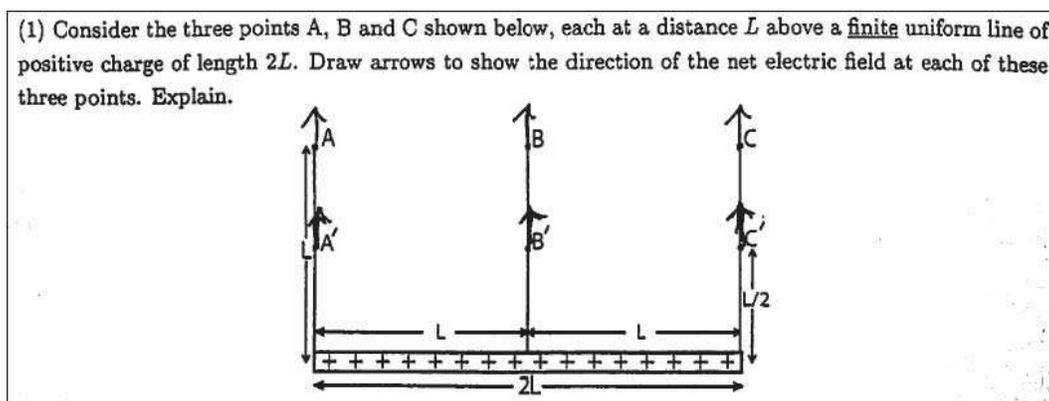


Figure 4.5 : A sample response for the pre-test question (1) on tutorial II

It appeared from the interviews that students either found the continuous charge distribution problems too cognitively demanding and did not know how to deal with them or they did not understand that the same superposition principle that can be exploited for finding the electric field due to a dipole can also be used for a continuous charge distribution. Interviews suggested that some students knew the procedure for calculating the electric field for the electric dipole by rote, but they had difficulty in extending this procedure even to situations with more discrete charges. These students had a lack of conceptual understanding about the procedural knowledge they were applying correctly for calculating the electric field due to an electric dipole. Interviews also suggested that some of the students had never carefully thought about what it means to calculate the electric field due to a charge distribution. These students treated the problems requiring the direction of the electric field for discrete charge distribution differently from the direction of the electric field for a continuous charge distribution. The lack of transfer of knowledge from one situation to another is a common difficulty in physics where the same concept is applicable in diverse situations with different surface features (Gick and Holyoak 1987, Dufresne et al. 2005, Lobato 2003, Bassok and Holyoak 1989, Bransford 1999).

4.6 PERFORMANCE OF TUTORIAL AND CONTROL GROUPS

The pre-tests and post-tests (shown in the Appendix C) were graded by two individuals based upon an agreed rubric, and the inter-rater reliability was better than 85%. Table 4.1 shows the pre-/post-test data on each question from three of the classes in which the tutorials were administered. In the fourth class, the post-tests were returned without photocopying them and we only have data on student performance on the cumulative test (administered after all tutorials),

which will be discussed in the next chapter. As shown in Table 4.1, for tutorial I, an additional question was included in the pre-test for classes 1 and 2 after analysis of data for class 3 (which was the first class in which the tutorials were administered). As noted earlier, the pre-/post-tests were not identical but focused on the same topics covered in a tutorial. Table 4.1 shows the student performance (on each question and also overall) on the pre-test and post-test in each of the two tutorials (I-II) for each class. We find that the average performance was significantly better on the post-tests compared to the pre-tests for both tutorials. We note that the classes utilizing each tutorial may differ either because additional pre-/post-test questions were added or the pre-test for tutorial II was not administered to all classes. The differences in the performance of different classes may also be due to the differences in student samples, instructor/TA differences or the manner in which the tutorials were administered.

Table 4.1 : Average percentage scores obtained on individual questions on the pre-/post-tests in tutorial classes.

Tutorial	Class	n	Pre-test					Post-test					
			1	2	3	4	Pre-total	1	2	3	4	5	Post-total
I	1	83	65	58	47	45	53	92	85	93	-	-	90
	2	61	53	41	50	-	49	87	79	88	-	-	85
	3	59	51	58	38	46	49	98	99	96	-	-	97
II	1	87	-	-	-	-	-	77	83	77	79	93	79
	2	64	65	6	41	-	38	90	96	88	88	98	92
	3	64	-	-	-	-	-	68	84	70	72	91	77

Table 4.2 shows the pre-/post-test data from a comparison group which consists of a class in which tutorials I and II were not used. The total class time devoted to these topics was equivalent to the time spent by the instructors in the tutorial groups. The pre-tests were given to the students in the comparison group immediately after relevant instruction. The post-tests were

given in the following week as part of the weekly recitation quizzes after the students had the opportunity to complete all the homework problems from those topics.

Table 4.2 : Average percentage scores obtained on individual questions on the pre-/post-tests in non-tutorial class.

Tutorial	n	Pre-test					Post-test					
		1	2	3	4	Pre-total	1	2	3	4	5	Post-total
I	66	44	57	34	52	47	42	45	60	-	-	49
II	57	63	9	41	-	38	45	61	63	45	59	55

Table 4.3 shows the results of t-test comparing the performance of the tutorial classes and non-tutorial class on the pre- and post-tests. The results show that regardless of which class the students belong to (i.e., whether they belonged to the tutorial or comparison groups), their performance on the pre-test was poor after traditional instruction. Moreover, students in the comparison group did significantly worse on the post-test than the classes in which students worked on the tutorials. This finding may not be surprising considering conceptual understanding is generally under-emphasized in the traditional classes.

Table 4.3 : P values for the t-tests comparing performance of tutorial classes and non-tutorial classes on the pre-/post-tests.

P value	Tutorial I	Tutorial II
pre-test	0.23	0.95
post-test	0.00	0.00

Table 4.4 shows the performance of students on the pre-/post-tests for each of the two tutorials partitioned into three separate groups based upon the performance on the pre-tests (see the Range column). As can be seen from Table 4.4, tutorials generally helped all students

including those who performed poorest on the pre-tests. Table 4.5 shows the performance of students in the comparison group on the pre-/post-tests for each tutorial partitioned into three separate groups based upon the pre-test performance. As can be seen from comparing Tables 4.4 and 4.5, students in the comparison group did not perform on-par with the tutorial groups on the post-tests for any of the three pre-test ranges.

Table 4.4 : Percentage average pre-/post-test scores (matched pairs) in tutorial classes for each of the two tutorials (I-II), divided into three groups according to the pre-test performance.

N	Tutorial	Range (%)	n1 (class 1)	pre	post	n2 (class 2)	pre	post	n3 (class 3)	pre	post
204	I	All	83	54	90	61	49	85	59	49	97
		0-34	24	19	78	28	21	75	21	16	95
		34-67	29	53	93	21	58	92	18	51	98
		67-100	30	82	97	12	98	94	20	83	99
64	II	All				64	38	92			
		0-34				28	19	88			
		34-67				34	51	95			
		67-100				2	85	95			

Table 4.5 : Percentage average pre-/post-test scores (matched pairs) in non-tutorial class related to each of the two tutorials (I-II), divided into three groups according to the pre-test performance.

Tutorial	Range (%)	n	pre	post
I	All	66	47	49
	0-34	20	17	40
	34-67	29	49	50
	67-100	17	78	59
II	All	57	38	55
	0-34	27	22	39
	34-67	29	51	68
	67-100	1	70	90

4.7 SUMMARY

Students have many common difficulties about Coulomb's law and the superposition principle. We developed and evaluated research-based tutorials to help calculus-based introductory students learn Coulomb's law and the superposition principle. Pre-/post-tests for each tutorial show that the tutorials can be effective in improving student understanding of these concepts. Moreover, the tutorials appear to be helpful for students who obtained low scores (0 – 33%) on the pre-test after traditional instruction. The tutorials did not increase the class time devoted to these topics significantly. While all the tutorials were administered in-class in all of the classes in this study, instructors may choose to give pre-/post-tests in class and ask students to work on the tutorials in small groups outside of the class as part of the homework assignment. If post-tests count for grade, students will have an incentive to work on the tutorials even outside of the class.

4.8 ACKNOWLEDGMENTS

We are very grateful to Z. Isvan for help in grading and analyzing data related to the pre/post-tests. We thank F. Reif, R.P. Devaty and J. Levy for helpful discussions. We thank all the faculty who helped in this study by administering the tutorials and/or pre-/post-tests to their classes.

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5.0 IMPROVING STUDENTS' UNDERSTANDING OF ELECTROSTATICS II. SYMMETRY AND GAUSS'S LAW

5.1 ABSTRACT

In this chapter, we discuss students' difficulties with symmetry and Gauss's Law and the development and evaluation of three research-based tutorials related to these topics to help students in the calculus-based introductory physics courses learn these concepts. The tutorials were developed based upon research on students' difficulties in learning these concepts. During the development of the tutorials, we interviewed students individually at various stages of development and gave written tests in the free-response and multiple-choice formats to learn about the difficulties with these concepts. We also obtained feedback from physics faculty members who teach introductory physics regularly. We discuss the performance of students on individual interviews and on the written pre-/post-tests given before and after the tutorials. We find that classes in which students used the tutorials outperformed those in which tutorials were not used.

5.2 INTRODUCTION

In this chapter, we discuss the development and evaluation of tutorials on symmetry and Gauss's

law to help students develop a functional understanding of these concepts. Gauss's Law allows us to relate the net electric flux through a closed surface to the net charge enclosed by the surface:

$$\Phi = Q_{\text{enclosed}} / \epsilon_0 \quad (1)$$

This means:

- If we know the net electric flux through a closed surface, we can readily find the net charge inside it.
- If we know the net charge inside a closed surface, we can readily find the net electric flux through it.

It does not mean that, in general, we can use Gauss's Law to readily find the magnitude of the electric field \vec{E} at a point. Only in situations where the charge distribution has very high symmetry can we find, $|\vec{E}|$ from the net electric flux. Although there are only three types of symmetry (spherical, cylindrical, and planar) for which Gauss's law can readily be exploited to determine the electric field at various points from the information about the electric flux, students need help in learning to identify when these symmetries are present.

The net electric flux through a closed surface is given by:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint |\vec{E}| \cos\theta |d\vec{A}| \quad (2)$$

where θ is the angle between the electric field \vec{E} and infinitesimal area vector $d\vec{A}$. The net electric flux Φ over a closed (Gaussian) surface can be exploited to determine the electric field

magnitude $|\vec{E}|$ at an arbitrary point P, on the surface readily only if the following conditions are met:

- We can determine the direction of \vec{E} relative to the area vector everywhere on the closed surface by symmetry (only $\theta = 0^\circ, 180^\circ$ or $\pm 90^\circ$ are associated with sufficiently high symmetry).

In some cases, we can divide the closed surface into sub-sections (for each sub-section the electric flux can be readily calculated e.g., side and two caps of a cylinder) such that one of the following is true:

1. $|\vec{E}|$ is the same everywhere on the sub-section by the symmetry of the charge distribution.
2. $|\vec{E}|$ and the area vector (outward normal to the surface) are perpendicular ($\theta = 90^\circ$) so that there is no electric flux through that sub-section.

Thus, to determine if the information about the net electric flux through a closed surface can be exploited to determine $|\vec{E}|$ at a point P, we may cleverly choose a Gaussian (closed imaginary) surface such that:

- it contains the point P where we want to determine $|\vec{E}|$
- $|\vec{E}| \cos \theta$ is known (by symmetry) to have a constant value on each sub-section of the surface so that it can be pulled out of the flux integral in Eq.(2). Then, $\int |\vec{E}| \cos \theta d\vec{A} = |\vec{E}| \cos \theta \int d\vec{A} = |\vec{E}| \cos \theta \times \text{total area}$

of the sub-section of the surface.

Learning to reason whether Gauss's law can be exploited in a particular situation to determine the electric field, without having to evaluate complicated integrals, can provide an excellent context for helping students develop a good grasp of symmetry considerations and develop their reasoning and meta-cognitive skills (National Research Council 1999, Simon & Kaplan 1989, Newell 1990, Schoenfeld 1987, Brown 1978). Unfortunately, students often memorize a collection of formulas for the magnitude of the electric field for various geometries, without paying attention to symmetry considerations. Many students have difficulty identifying situations where Gauss's law is useful and overgeneralize results obtained for highly symmetric charge distributions to situations where they are not applicable. Most textbooks do not sufficiently emphasize symmetry considerations or the chain of reasoning required to determine if Gauss's law is useful for calculating the electric field. Distinguishing between electric field and flux is often difficult for students. Choosing appropriate Gaussian surfaces to calculate the electric field using Gauss's law when sufficient symmetry exists is also challenging. Many students were confused about the symmetry of the charge distribution vs. the symmetry of the object on which the charges were embedded.

5.3 TUTORIAL TOPIC

The two tutorials on Coulomb's law and superposition principle (I and II) discussed in chapter 4 were administered before the three tutorials (III-V) related to Gauss's law so we will call the three tutorials discussed here tutorials III-V. Tutorial III focused on distinguishing between

electric flux and field, and the fourth and fifth tutorials dealt with symmetry and Gauss's law and on revisiting the superposition principle after Gauss's law. The development and administration process of all the tutorials is described in the previous chapter. Here we focus on the findings and discussions about tutorials III-V related to Gauss's law. The pre-tests and post-tests for these tutorials are in the Appendix D.

Tutorial III, which was designed to help students learn to distinguish between the electric field and flux, strived to help students learn that the electric field is a vector while the electric flux is a scalar. Also, the electric field is defined at all points in space surrounding a charge distribution while the electric flux is always through an area. Through the tutorial, students learn about Gauss's law and how to relate the electric flux through a closed surface to the net charge enclosed. Rather than emphasizing the symmetry considerations, this tutorial focused on helping students use Gauss's law to find the net electric flux through a closed surface given the net charge enclosed and vice versa. Common misconceptions were explicitly elicited, e.g., by having two hypothetical people discuss an issue in a particular context. The students were asked to identify the statement they agreed with and provide an explanation (Posner et al 1982).

Tutorial IV was designed to help students learn to exploit Gauss's law to calculate the electric field at a point due to a given charge distribution if a high symmetry exists. Students were helped to draw upon the superposition and symmetry ideas they learned in the first two tutorials (discussed in the previous chapter) to evaluate whether sufficient symmetry exists to exploit Gauss's law to calculate the electric field. Then, students learn to choose the appropriate Gaussian surfaces that would aid in using Gauss's law to find the electric field. Finally, they use Gauss's law to calculate the electric field in these cases.

Tutorial V revisits the superposition principle after students have learned to exploit

Gauss's law to calculate the electric field. For example, students learn to find the electric field at a point due to two non-concentric uniform spheres of charge or due to a point charge and an infinitely long uniform cylinder of charge.

5.4 DISCUSSION OF STUDENT DIFFICULTIES

Before the development of the tutorials, we conducted investigation of students' difficulties with these concepts (Singh 2006) by administering free-response and multiple-choice questions and by interviewing individual students using a think aloud protocol (Chi et al 1994). Most of these difficulties were explicitly addressed in the tutorials. While students were less likely to have these difficulties in the post-tests after the tutorials than in the pre-tests, some students still had difficulties with these concepts. Moreover, the difficulties on the pre-test and post-test were similar in nature for both the tutorial and comparison groups, but the students in the tutorial group were less likely to have these difficulties. Below, we discuss examples of the common difficulties found about topics related to the tutorials without separating the performance of the tutorial groups and the comparison group on pre-/post-tests. Later, we will discuss the performance of the tutorial groups and the comparison group in Tables 5.1-5.5. For reference, the pre-tests and post-tests, for all three tutorials are included in the Appendix D.

5.4.1 Difficulty with the principle of superposition

In chapter 4, we discussed that the performance of many students was closely tied with their

understanding of the principle of superposition. The difficulty with the principle of superposition was also observable when students worked on the pre-tests and post-tests for tutorials III-V. The pre-tests and post-tests for tutorials III-V are included in the Appendix D.

Tutorial V deals with revisiting the superposition principle after learning to use Gauss's law to find the electric field due to a symmetric charge distribution. Some students were confused about the difference between the net electric field and its components. For example, when asked to find the net electric field in the pre-test question (3) in tutorial V, some drew electric field vectors due to the individual non-conducting uniform spheres of charge, but did not draw the direction of the net electric field. In the post-test question (3) for tutorial V, students had difficulty finding the electric field at points C and A. Some students incorrectly claimed that the electric field at point C is directed radially away from the closer sphere since only the closer sphere will produce an electric field at that point. In the post-test questions (4) and (5) for tutorial V, the most common mistake was assuming that the electric field is zero at point B because the point is not in-between the sheets. One interviewed student who was making a generalization for the case of a parallel plate capacitor to claim that the electric field at point B is zero (see post-test for Tutorial V) was explicitly asked to show why the electric field is zero at point B. The student was uncomfortable using the principle of superposition and said that he remembers that the electric field should be zero outside the parallel plates.

5.4.2 Measuring distances from the surface of a uniformly charged sphere or cylinder

At the beginning of the pre-test and post-test of tutorial V, students were explicitly given that the magnitude of the electric field due to a sphere of charge, with total charge $+Q$, outside the

sphere at a distance r from the center is KQ/r^2 . Some students appear not to have taken note of this instruction carefully. One common difficulty was assuming that the electric field outside a uniform sphere of charge is KQ/r^2 , but r is the distance measured from the surface. For example, in the pre-test question (1) of tutorial V, some students did not realize that the electric field at point B is zero because the electric field produced by the infinite non-conducting hollow cylinder with uniform charge exactly cancels the electric field due to an infinite line with uniform charge. One common reason for this difficulty was measuring the distance of point B from the cylinder to be from the surface (L) as opposed to from the center (3L). The fact that for point B outside the cylinder, the charge on the cylinder can be thought of as a line of charge on the axis of the cylinder was non-intuitive to many students who focused on the distance of the closest end of the cylinder from point B.

Similar difficulty was observed in the post-test question (1) of tutorial V. Many students incorrectly claimed that the electric field due to the sphere and the point charge will cancel each other out at point B. Their incorrect claim that the electric field is zero at point B was due to the fact that they measured the distance of point B from the surface of the sphere (rather than the center of the sphere) to account for the electric field due to a uniform sphere of charge at a point B outside.

5.4.3 Magnitude of the net electric field is the sum of the magnitudes of the components

Students had great difficulty with the pre-test questions (1) and (2) on tutorial V, which are related. In these questions, students had to draw arrows to show the direction of the net electric field and write its magnitude at two points A and B shown. The point A was inside an infinite

non-conducting hollow cylinder with uniform surface charge and point B was outside. There was also an infinite line with uniform charge present in the region. Few students provided the correct responses to both parts and many students (especially those without the tutorials) did neither part correctly. One common difficulty in question (2) about the magnitude of the electric field was that many students did not take into account the fact that the electric field is a vector so the directions of both vectors must be taken into account to calculate the magnitude of the net electric field. These students simply added the magnitudes of the electric field due to the infinite cylinder with uniform surface charge and the infinite line with uniform charge. One interviewed student who used this incorrect method was asked explicitly if the electric field is a vector. This student asserted that the electric field is indeed a vector but, since he is asked for the magnitude of the net electric field, he is adding the magnitudes of the electric field contributions coming from the two charged objects. Of course, the student has difficulty realizing that the magnitude of the net electric field is not the sum of the magnitudes of the contribution to the electric field from the two charged objects.

5.4.4 Confusion that a non-conductor completely shields the inside from the electric field due to outside charges

Students had difficulty with the electric field inside hollow non-conducting objects of different shapes due to the charges on their surface or charges outside. Many students claimed that such material will “isolate” or “screen” the inside completely from the electric field due to outside charges. Thus, the students were confused about the role of conductor vs. insulator in shielding the inside from the electric field produced by outside charges. For example, many students claimed that the electric field inside a non-conducting hollow cube with charge uniformly

distributed on its surface will be zero everywhere. Interviews suggest that these students often believed that a hollow region inside an object is always shielded from the charges on the surface or charges outside regardless of whether the object is a conductor or non-conductor. This notion of shielding was maintained by some of the interviewed students even when they were reminded by the interviewer that the object on which the charges are distributed is non-conducting. Some students incorrectly claimed that the net electric field due to any charge outside must work out to be zero everywhere inside a hollow region of any shape regardless of whether the hollow region was bounded by a conductor or an insulator. For example, some students drew spherical or cubic Gaussian surfaces inside a hollow cube made of non-conducting material to argue that because there is no charge enclosed, the electric field will be zero everywhere according to Gauss's law. One student went on to incorrectly claim that he has always been amazed at how Gauss's law can be used to prove that the electric field in the hollow region inside a closed object is always zero everywhere, a result that appears to be counterintuitive to him.

Question (5) on the post-test of tutorial III was extremely difficult and a majority of students incorrectly agreed with the statement. A very common notion was that since the sphere is made of an idealized non polarizable non-conducting material, it would shield the inside from any charges on the outside. When asked for an explanation, interviewed students often invoked Gauss's law but they were not able to explain how it implies that the inside of an insulator or non-conductor is shielded from the outside. The following are sample incorrect responses:

- Agree. Even with a positive surface charge and negative point charge, the inside of the ball remains nonpolar and therefore not charged.
- Agree. The ball does not contain a charge and so the net electric field everywhere inside

must be zero.

- Agree because all charge is located outside of the Gaussian sphere. By Gauss's law there is no electric field.
- Agree because the point charge has to be enclosed within the sphere for it to cause an electric field.
- Agree. as long as point P is not charged, no charges will penetrate the non-polarizable non-conducting ball.
- Agree. Nothing will penetrate a non-conducting ball.
- The net flux inside the sphere would be zero and since there is some area to the Gaussian surface, so $\Phi_E = \oint \vec{E} \cdot d\vec{A}$ means $E = 0$.
- Agree. Materials that are non-conducting are not able to hold a charge.

For question (1) on the post-test of tutorial V, many students had difficulties with the principle of superposition and the electric field inside a hollow non-conducting sphere. When they were explicitly asked by the interviewer why the point charge near the sphere does not produce an electric field at point A, students who claimed that the net electric field at point A is zero often referred to the shielding of the inside of the sphere from the charges on the sphere and the charges outside of the sphere. Similar to the previous questions, even when the interviewer reminded students that the sphere was non-conducting, most students maintained that the point charge cannot have any influence inside the sphere. Some students said that they could not explain exactly why the non-conducting sphere will produce shielding, but that they remember that the electric field must somehow cancel out in the hollow region for all shapes and charge

distributions. Further prodding showed that due to a lack of thorough understanding, these students were often overgeneralizing or confusing two different facts: the symmetry argument that shows (using Gauss's law) that the electric field for a sphere with a uniform surface charge is zero everywhere inside regardless of whether the sphere is conducting or non-conducting, and/or the fact that the electric field inside a conductor is zero in equilibrium regardless of the shape of the conductor. In the pre-test given in conjunction with tutorial V, similar difficulties were found. For example, in pre-test question (1) of tutorial V, many students believed that the infinite line with uniform charge cannot produce an electric field at point A inside the infinite non-conducting hollow cylinder with uniform surface charge because point A is inside the cylinder and is shielded from charges outside. Most of them also claimed that in the pre-test question (3) of tutorial V, the second sphere cannot produce an electric field at point A inside the first sphere because point A is shielded from the charges on the second sphere.

We also gave to the introductory physics students and physics graduate students the following question that points to similar difficulty (Bilak & Singh, 2007). A small aluminum ball hanging from a non-conducting thread is placed at the center of a tall cylinder made with a good insulator (Styrofoam) and a positively charged plastic rod is brought near the ball in such a way that the Styrofoam wall is in between the rod and the ball (see Figure 5.1). Which one of the following statements is true about this situation?

- (a) The aluminum ball will not feel any force due to the charged rod.
- (b) The aluminum ball will be attracted to the charged rod and the force of attraction is the same as that without the Styrofoam cylinder.
- (c) The aluminum ball will be attracted to the charged rod but the force of attraction is more than that without the Styrofoam cylinder.

(d) The aluminum ball will be attracted to the charged rod and the force of attraction is less than that without the Styrofoam cylinder.

(e) None of the above.

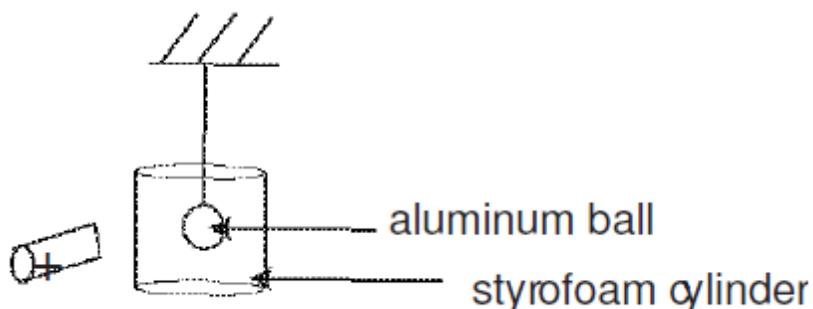


Figure 5.1 : The setup for the question given to both introductory students and graduate students

In response to this question, 32% of the graduate students and 27% of the introductory physics students chose option (a). Interviews suggest that students believed that the inside of an insulator is “insulated” or “isolated” from outside electric field. An introductory physics student who was interviewed drew an analogy with heat conduction and said that just like heat transfer can be made negligible with a good insulator, electric field penetration into the cylinder can be made negligible by choosing a good insulator. Discussions with the student suggest that the student incorrectly believed that an idealized non-polarizable insulator will ensure that the inside is completely screened from the outside. In contrast, the higher the dielectric constant, the more effective the screening will be.

5.4.5 Difficulty realizing that electric flux can be calculated without knowing electric field at each point on a closed surface

Sometimes students get too hung up on a particular equation and believe that it provides the only way to solve for an unknown. The sole dependence on one equation leads many students to claim that the electric flux cannot be calculated without knowing the electric field at each point on a closed surface. For example, for question (1a) on the pre-test of tutorial III, the most common difficulty was not realizing that one can calculate the flux through a closed surface from the knowledge of the charge enclosed with no need to know the electric field at each point on the surface. These students often incorrectly claimed that it is impossible to calculate electric flux because the electric field is not given. They often focused on the equation $\Phi = \int \vec{E} \cdot dA$ or $\Phi = EA$ or $\Phi = EA \cos \theta$ and thought that one must know the electric field and the area of the closed surface to be able to calculate the electric flux. We note that the surface area was provided in the problem description. In response to question (1a) on the pre-test of tutorial III, one interviewed student incorrectly claimed: “impossible because the electric field is not given”. After the interview, the student wanted to go over the correct answer to each question and said that he does not understand how there can be two ways of calculating the same quantity. He said he would really like to understand why there is more than one method. In this case, net flux through a closed surface can be calculated using $\Phi = Q / \epsilon_0$ or $\Phi = \int \vec{E} \cdot dA$ and if the charge enclosed is given, the first equation can be used to calculate the net flux through the closed surface without knowing the electric field at each point or the surface area. Then, the student asked how common it is that two equations can be used to calculate the same quantity. The interviewer responded that it is quite common in physics to have two equations to calculate the same quantity and he

has encountered several such examples before. The interviewer reminded the student that he can calculate the acceleration of an object, e.g., using a kinematics equation or using Newton's second law. Similarly, he can calculate the resistance of an Ohmic material, e.g., by using the definition of resistance (in terms of current and voltage) or by invoking the relation between the resistance, resistivity, area of cross section and the length of the conductor. Similarly, he can calculate the capacitance of a parallel plate capacitor, e.g., by using the fact that the capacitance is the charge on each plate per unit potential difference across the plates or by invoking the relation between the capacitance, area of cross section of the plates, the distance between the plates, and dielectric constant of the material between the plates. The student thought for some time and said that he had never thought about the fact that these equations give different ways of calculating the same quantity. In response to the post-test question (1a) on tutorial III, some students incorrectly used the equation $EA = Q/\epsilon_0$ to claim that while the area is given as 0.13 m^2 , we cannot calculate the charge enclosed because the electric field cannot be determined from the given information. Instead, they could have used $\Phi = Q/\epsilon_0$ to calculate the enclosed charge Q since the net electric flux Φ through the surface of the sphere was provided.

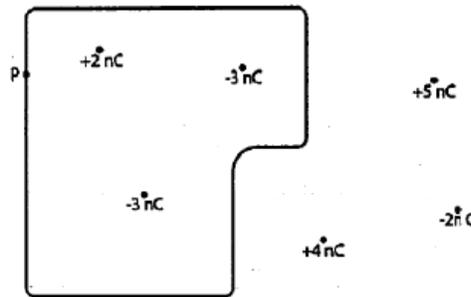
5.4.6 Ignoring the symmetry of the problem and assuming that the electric flux is always

$$\Phi = EA$$

Many students have difficulty realizing that the electric flux $\Phi = EA$ cannot always be written as $\Phi = EA$ but only in situations where there is sufficient symmetry. Many students did not account for symmetry consideration in determining whether $\Phi = EA$ is valid and used this equation even in cases where it is not applicable. For example, in post-test question (1b) in tutorial III, the most

common difficulty was using $E = \Phi / A$ to calculate the electric field. Some students explicitly mentioned that $\cos \theta = 1$ (where θ is the angle between \vec{E} and \vec{A} and used the electric flux Φ and the area of cross section A provided. Similarly, in the pre-test question (1) for tutorial III, Figure 5.2 shows that the student assumed $\Phi = EA$ although it is not true in the given situation to claim that the flux cannot be calculated from the information given. Instead, the student could have calculated the flux from the information about the net charge inside the closed surface. Some students who had correctly calculated the electric flux Φ in the pre-test question (1a) for tutorial III (using the charge enclosed) incorrectly claimed that $E = \Phi / A$ and calculated the magnitude of electric field at point P incorrectly in question (1b) using the information about the flux Φ from question (1a) and the area A .

(1) Consider the following sketch showing a cross-section of a closed surface with three charges inside and three outside the surface at various locations. The surface is known to have an area of $0.4m^2$.



(a) If possible, find the net electric flux through the surface. If it is not possible to find the net flux from the information given, explain why it is impossible.

$Q_{NET} = -4 \times 10^{-9} C$
 $E = \frac{-1}{4\pi\epsilon_0} \frac{Q_{ENC}}{R^2}$
 $\Phi_E = EA$
 $\Phi_E = \frac{Q_{ENC}A}{4\pi\epsilon_0 R^2} =$

$\phi = EA = R \text{ is NOT KNOWN}$
 No you can't because it is not a sphere + the R is unknown

Figure 5.2 : A sample response claiming $\Phi = EA$ on the pre-test question (1) on tutorial III.

5.4.7 Confusion about the underlying symmetry of a charge distribution

Similar to the difficulty comprehending the symmetry of the charge distribution discussed in the previous chapter about tutorials I and II, many students displayed difficulty realizing that it is the symmetry of the charge distribution (and not the symmetry of the object on which the charges are embedded) that is important in determining whether Gauss's law can be applied to calculate the electric field at a point. For example, in question (2) of the pre-test and question (1) of the post-test of tutorial IV, students had to identify the shape of the appropriate Gaussian surfaces that would make it easy to use Gauss's law to calculate the electric field due to an infinite uniform line of charge and an infinite uniform sheet of charge, respectively. For the uniform sheet of charge, many students believed that spherical surfaces will work because they are symmetric. However, the calculation with a Gaussian sphere is not easy because the area vector and the electric field make different angles for different infinitesimal areas on the sphere.

The pre-test and post-test questions for tutorial IV probe the extent to which students can discern the underlying symmetry of the charge distribution. In question (2) on the post-test for tutorial IV, Some students believed that we can use Gauss's law to find the electric field at a point outside due to a cube or finite cylinder with uniform surface charge. In the interviews students sometime recalled using Gauss's law for these surfaces. More prodding showed that they were either confusing the fact that those surfaces can be used as Gaussian surfaces for appropriate charge distributions or the fact that for an infinite uniformly charged cylinder (but not a finite cylinder) it is possible to exploit Gauss's law to find the electric field easily. It appears that many students have not thought carefully about the principle of superposition and its implication for the electric field due to a charge distribution and were applying memorized

knowledge whose correct applicability was forgotten. In question (2) of the pre-test of tutorial IV students had to choose the Gaussian surfaces that would help them determine the electric field at point P readily due to the infinite line of charge. All of the alternative choices were selected with an almost uniform frequency. Students were often unsure about the symmetry concepts relevant for making appropriate decisions and those who chose option (c) were often quite confident that the magnitude of the electric field due to the infinite line must be the same at every point on the cube as well.

5.4.8 Difficulty in determining how symmetric is symmetric enough to find the electric field using Gauss's law

Gauss's law is useful for finding the magnitude of the electric field only when there is sufficient symmetry to simplify the surface integral for the electric flux in terms of the electric field and area vector. Although students are always taught that spherical, cylindrical and planar symmetries are the only types of symmetries for which Gauss's law can be exploited to find the electric field, students have difficulty in figuring out when these symmetries exist. Both the pre-test and post-test questions (6) for tutorial IV were very difficult for students since students had difficulty discerning if there is sufficient symmetry to exploit Gauss's law to find the electric field in each of these cases. In the pre-test question (6) for tutorial IV, some students felt that the finite cylinder with a uniform charge on the surface had sufficient cylindrical symmetry so that Gauss's law can be exploited to easily find the electric field due to the uniform charge on the surface of the finite cylinder. Similarly, in the post-test question (6) for tutorial IV, many students felt that the cube with charge uniformly distributed on its surface was symmetric enough that one can use Gauss's law to find the magnitude of the electric field at a point P outside the

cube. Some of these students incorrectly claimed that the Gaussian surface should be a cube, or surfaces that are not closed such as a square or a circle around the actual cube with uniform surface charge.

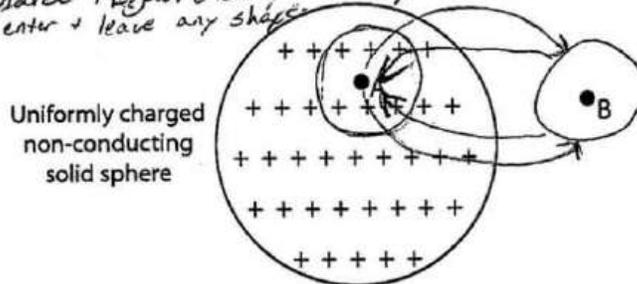
5.4.9 Difficulty in drawing a Gaussian surface to find the electric field at a point due to a symmetric charge distribution

In exploiting symmetry and using Gauss's law to find the magnitude of the electric field at a point due to a highly symmetric charge distribution, students should draw an appropriate hypothetical Gaussian surface consistent with the symmetry of the problem through the point where the electric field is desired.

The difficulty in drawing the Gaussian surface was evident, for example, in the pre-test and post-test questions (3) and (4) for tutorial IV. Written responses and interviews suggest that some students incorrectly believed that the shape of the Gaussian surface does not matter for determining the electric field at a point using Gauss's law (see sample Figure 5.3 for the pre-test question (3) on tutorial IV). Gauss's law is indeed valid for a closed surface of any shape and the knowledge of the charge enclosed by the surface is sufficient to yield information about the electric flux. However, information about the electric flux through a Gaussian surface does not yield information about the electric field at a point unless the charge distribution is symmetric and the Gaussian surface is chosen carefully.

(3) The following diagram shows the cross sectional view of a **solid** uniformly charged non-conducting sphere. To find the magnitudes of the electric field at points A (inside the sphere) and B (outside the sphere) using Gauss's law, what kind of Gaussian surface would you choose for each? Explain your reasoning. Can the same Gaussian surface be used for both points A and B? Explain.

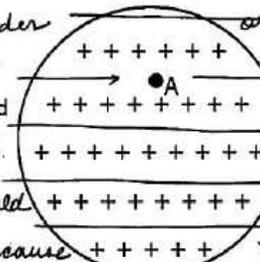
The shape does not matter, the electric field can be calculated regardless of shape because the flux lines will enter & leave any shape.



(4) Draw cross sections of your Gaussian surfaces for each of the points A and B on the cross-sectional view above.

(3) The following diagram shows the cross sectional view of a **solid** uniformly charged non-conducting sphere. To find the magnitudes of the electric field at points A (inside the sphere) and B (outside the sphere) using Gauss's law, what kind of Gaussian surface would you choose for each? Explain your reasoning. Can the same Gaussian surface be used for both points A and B? Explain.

You could contain the entire system in a single Gaussian surface such as a cylinder or cube. What is important is that you perform calculations for Electric charge flux based only on the charge inside your Gaussian surface. Personally, I would choose a sphere to encompass each point because this is already in that configuration & it would simplify your intuitive approach.

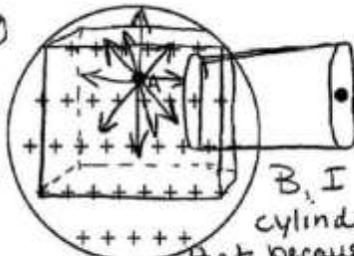


(4) Draw cross sections of your Gaussian surfaces for each of the points A and B on the cross-sectional view above. *I would place A in the sphere w/ same cross-section as the one enclosing it now. I would also enclose B in a sphere, not overlapping w/ A. (It is difficult to show & not interfere w/ my*

(5) Show the directions of the electric field at points A and B in the figure above due to the **solid** non-conducting sphere. Explain your reasoning. *Electric field shown by Arrows.*

A you could choose pretty much anything that is a 3D solid object so I chose a cube.

Uniformly charged non-conducting solid sphere



B, I chose a cylinder to show that because its on the face of the cylinder, outside the sphere

Figure 5.3 : Sample responses claiming the shape of the Gaussian surface is not important.

Many students did not even realize that the Gaussian surfaces should be through the point where they were supposed to find the magnitude of the electric field. Some students incorrectly situated the points where the electric field was to be determined at the center of a Gaussian sphere or on the axis of a Gaussian cylinder they drew. For example, on the pre-test question (3) for tutorial IV, some students claimed that points A and B should be at the center of a Gaussian sphere, circle (which is not even a Gaussian surface), or on the axis of a Gaussian cylinder (see Figure 5.4 as a sample response to the pre-test question (3) on tutorial IV).

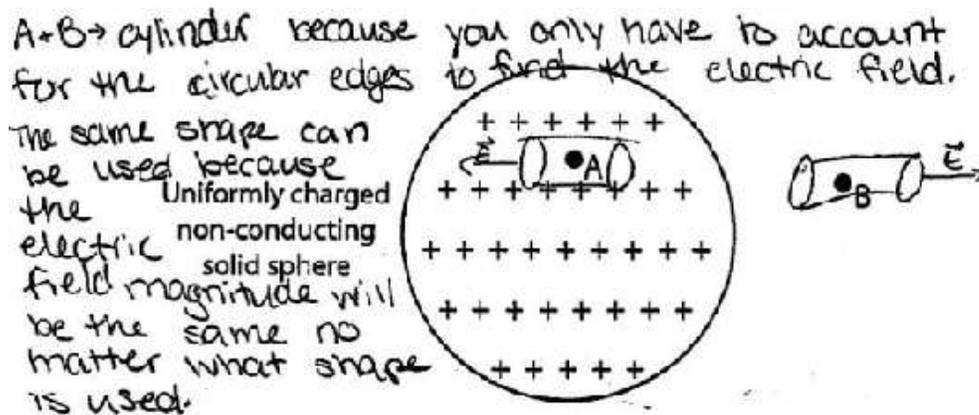


Figure 5.4: A sample response claiming point A and B should be on the axis of Gaussian cylinder

Another common misconception that students had is that if they have to find the electric field at two points A and B, e.g., inside and outside the sphere due to a uniform sphere of charge, then there should be one hypothetical Gaussian surface drawn through both points A and B. For example, the Gaussian surface that some students drew in pre-test question (3) for tutorial IV to find the electric field at points A and B due to a solid uniformly charged non-conducting sphere was a finite cylinder with both points A and B at the caps of the same cylinder (see Figure 5.5 as a sample response to the pre-test question (3) for tutorial IV). Other students used Gaussian

surfaces with incorrect symmetry to find the magnitude of the electric field at points A and B in the pre-test question (3) for tutorial IV as shown in sample Figure 5.6.

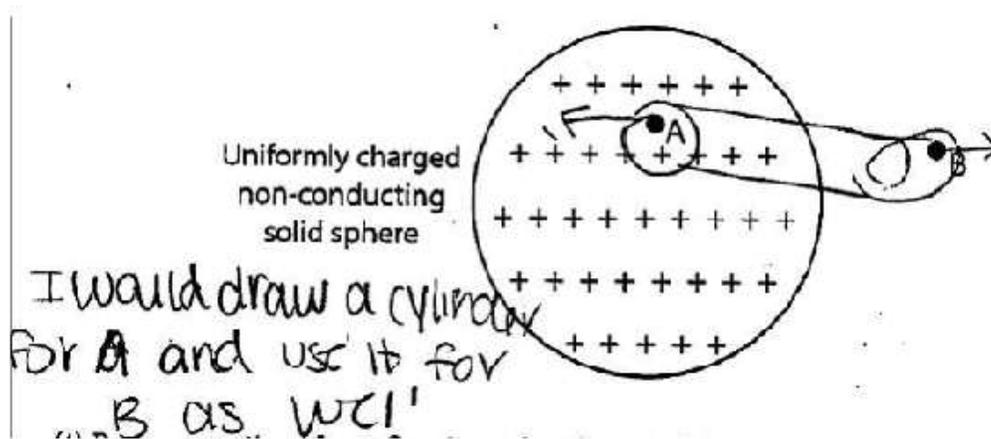
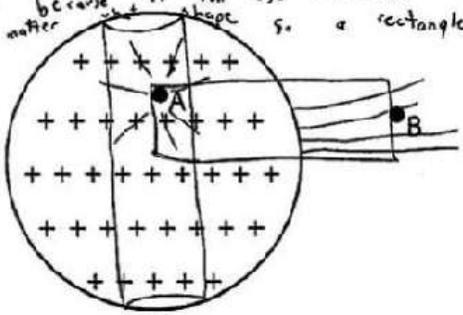


Figure 5.5 : A sample response claiming both points A and B should be on the same Gaussian surface.

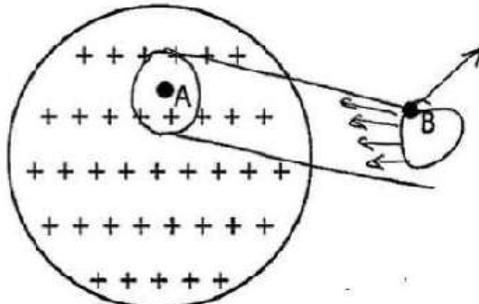
I would choose a cylinder to find A and a rectangle to find B.
 I would use the cylinder because it can use information about the radius
 and for B it does not matter so a rectangle is most simple.

Uniformly charged
 non-conducting
 solid sphere



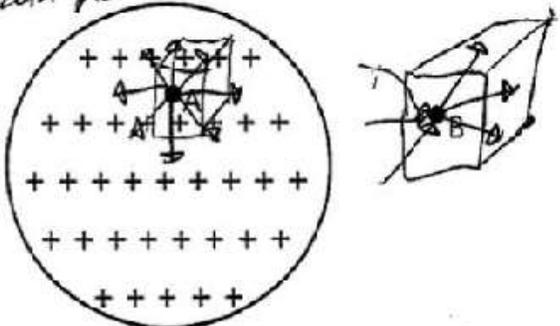
Gaussian Cylinder Solid sphere
 No the same surface cannot
 be used for both

Uniformly charged
 non-conducting
 solid sphere



- a cube, because the calculations would be easy
 - yes, can be used for both pts

Uniformly charged
 non-conducting
 solid sphere



a metal perpendicular gaussian surface is between the points, to see the flux or field

Uniformly charged
 non-conducting
 solid sphere

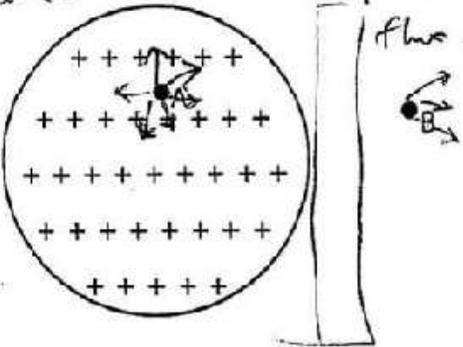


Figure 5.6 : Sample responses with Gaussian surfaces involving incorrect symmetry to find the magnitude of the electric field at points A and B

Similarly, in the post-test question (3), some students did not know that the hypothetical Gaussian surface to find the magnitude of the electric field due to a symmetric charge distribution should be such that the Gaussian surface passes through the point where they had to find the magnitude of the electric field. In the tutorial group, the number of students who had such difficulty was significantly less than in the pre-test.

5.4.10 Confusing electric flux for a vector

A common difficulty was mistakenly thinking that the electric flux is a vector. In interviews, students justified their response about why the electric flux is a vector by using the following facts: The expression for flux involves a scalar product of two vectors. Instead of identifying $\cos \theta$ with the angle between the electric field and the area vector, many students concluded that the flux is a vector because it involves a $\cos \theta$. Students pointed to the fact that the electric flux can have both positive and negative signs so it must be a vector. When asked if it would make sense to say that the electric flux points at 30° south of west, students often avoided a direct response. Their response implied that for a physical quantity to be a vector, it was not necessary to be able to specify the exact direction. Rather, because the electric field lines “going out” of a closed surface contribute positively and those “going in” contribute negatively to the total electric flux through a closed surface, electric flux must be a vector.

5.4.11 Confusion between electric flux and electric field

Students often have difficulty distinguishing between electric field and electric flux. While the electric field is a vector which is defined at each point, electric flux is a scalar quantity which is

defined through a surface. Of course, electric field and electric flux are related and students often have difficulty understanding this relation properly. It is difficult for students to understand that no net electric flux through a closed surface does not imply there cannot be a non-zero electric field at each point on the surface. For example, in response to the post-test question (3) on tutorial III, one student incorrectly noted: “Agree. No electricity with no net flux”

Tutorial III helps students learn to distinguish between electric field and electric flux. Some students were quite assertive during interviews and incorrectly claimed that if the magnitude of the electric flux through a closed surface is smaller than it is for another surface, then the magnitude of the electric field at points on the surface through which the electric flux is smaller must be smaller too. Interviews suggest that if two concentric Gaussian spheres were drawn with a single point charge at the center, some students claimed that the electric flux through the smaller sphere must be larger because it is closer to the positive charge at the center although the net charge enclosed is the same for both surfaces. These students were often confused about the distinction between electric flux and electric field. Interviews also show that some students incorrectly claimed that the electric flux through the larger sphere must be larger because it has a larger area although again the net charge enclosed is the same for both spheres.

A common difficulty with a question which was part of tutorial III, for which Figure 5.7 shows the setup, was assuming that the electric field is zero at point B on the side surface of the cube, although the question explicitly mentions that the cube is in a uniform electric field of 20N/C . During the development of the tutorial, in interviews and free-response questions, some students explicitly claimed that the area vector of the side surface is perpendicular to the direction of the electric field lines. Therefore, the electric field must be zero at point B. This kind of confusion between the electric field at a point and the contribution to the electric flux from a

certain area was quite common.

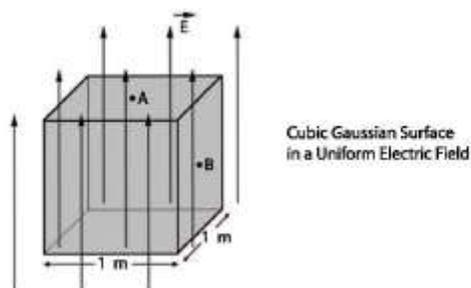


Figure 5.7 : Question on which students display confusion between electric field and electric flux.

Similar difficulty in differentiating and relating the electric field and electric flux was manifested in response to, e.g., questions (1) and (2) on the pre-test for tutorial IV. Responses to these questions suggest that many students are not comfortable with the statement of Gauss's law that relates the net flux through a closed surface to the net charge enclosed. They have difficulty differentiating between the electric flux through a closed surface and the electric field at a point on the surface. For example, in question (1) many students chose (I) (or (I) and (III)) and claimed incorrectly that only those surfaces can be used to determine the net electric flux through them because the other surfaces did not have the correct symmetry. A possible reason for this mistake is the confusion between electric flux and electric field.

A common incorrect response for question (2) on the post-test of tutorial III was that the net charge enclosed in a region is largest if the number of field lines penetrating the region is greatest. Students who made this mistake did not pay attention to the direction of the electric field lines which is crucial for determining the net flux through a closed surface and is related to the net charge enclosed via Gauss's law.

Question (5) on the post-test of tutorial III was extremely difficult and many students incorrectly agreed with the statement. The confusion was often between the electric field and flux and these students claimed that if there is no charge inside the Gaussian surface, the electric field must be zero everywhere inside the hollow region. Some students incorrectly argued that the effect of the positive charges on the surface of the sphere and the negative point charge outside will somehow mutually cancel each other's effect so that the net electric field inside the non-conducting sphere is zero everywhere.

5.4.12 Confusion between open and closed surfaces and Gauss's law

Students were sometimes unsure about the distinction between open and closed surfaces and that Gauss's law is only applicable to closed surfaces. For example, pre-test questions (1) and (2) in tutorial IV at least partly assess whether students understand this distinction. Some students incorrectly believed that Gauss's law applies to any symmetrical surface even if it is not closed. For example, in response to question (1), these students claimed that the electric flux due to an infinitely long line of charge (with uniform linear charge density λ) is $\lambda = L/\epsilon_0$ even for the two-dimensional square sheet.

5.4.13 There must be a charge present at the point where the electric field is desired

Similar to the difficulty found when students worked on tutorials I and II, some students claimed that if they are asked to find the electric field at a point A, there must be a charge present at that point even when working on symmetry and Gauss's law tutorials. For example, in the pre-test question (5) for tutorial IV in which students were asked to find the electric field due to a solid

non-conducting sphere with uniform surface charge at a point A, some students drew electric field lines coming out of point A as though there was a point charge located at point A and that was the only charge that was contributing to the electric field at point A. A sample response from a student who made such a claim is shown in Figure 5.8. Some students believed that there were two charges with equal magnitude but opposite signs present at points A and B and they drew the electric field lines from A to B as one would draw for a dipole because they were asked for the electric field at two points A (inside the uniform sphere of charge) and B (outside). On the other hand, some students claimed that the electric field at point B is zero because there is no charge at point B since it is outside the sphere of charge.

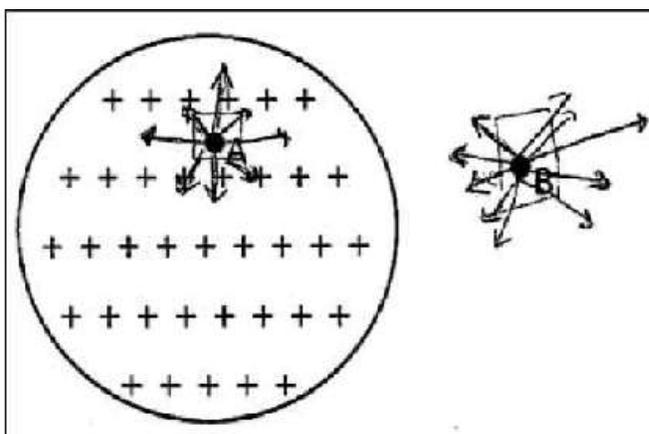


Figure 5.8 : A sample response for the pre-test question (5) on tutorial III

5.4.14 Assuming a point charge is present if the charge distribution in a region is not given explicitly

When students are asked questions without a concrete case (with a given charge distribution), they sometimes assume that the charge distribution must be the simplest possible, namely, a

point charge. Incidentally, students who made such an assumption never said that they were considering a very special case and in general the charge distribution could be more complex. For example, a common problem with post-test question (1b) in tutorial III was assuming that there was a point charge at the center of the sphere and using the given radius of the sphere to find the electric field using Coulomb's law.

In response to the post-test question (2d) for tutorial III, one common incorrect response was A and C because all lines are going in or out from the center and students assumed that there must be a point charge at the center of each of the regions A and C. For example, one student noted: "A and C because all the flux lines seem to be directed in and out of the center of the sphere." One interviewed student claimed that it is easy to tell from the figure that A and C have point charges at the center. However, there is no way to tell from the information given because even if the electric field lines are perpendicular to a spherical surface everywhere, the hidden charges inside could be uniformly distributed about the center of the sphere and not necessarily be localized as point charges at the center of the sphere.

5.4.15 Difficulty visualizing in three dimensions

Some students had great difficulty visualizing in three dimensions. Some students did not distinguish between two and three dimensional surfaces, e.g., some of them used the words sphere and circle interchangeably or used the words cube and square interchangeably. Some students incorrectly claimed that the Gaussian surface for a spherical charge distribution is a circle although a circle is not a closed surface and hence cannot be a Gaussian surface. In response to question (2) on the pre-test of tutorial IV, one interviewed student argued for a long time that the distances of all points on the surface of the Gaussian sphere must be the same from the axis of the infinitely long

line (with uniform charge) similar to the distance of all points on a coaxial Gaussian cylinder. It was clear from the discussion that the student had difficulty in visualizing how different points of the surface of the Gaussian sphere with its diameter coinciding with the infinite line of charge can have different distances from the axis of the cylinder. Visualizing this situation became more difficult because the student kept confusing what was relevant for the problem with the fact that all points on the sphere are at the same distance from the center of the sphere.

In some questions, we provided students with two views, e.g., cross-sectional view and side view for an infinite non-conducting hollow cylinder with uniform surface charge and infinite line with uniform charge in the pre-test question (1) of Tutorial V. Some students who used the cross-sectional view, referred to the cross-sectional view of the cylinder as a sphere or circle in their written explanations. There were also students who used either the cross-sectional view to find the electric field at a point A inside the cylinder and used the side view for the electric field at a point B outside or vice versa. Sometimes, these students incorrectly claimed that the appropriate Gaussian surface for point A inside the cylinder is a sphere and for the point B outside the cylinder is a cylinder (or vice versa).

5.4.16 Student difficulty with basic mathematical tools

Although the level of mathematics required to carry out Gauss's law is not very difficult, some students struggle with simple mathematical concepts such as the dot product of two vectors or the conceptual understanding of a surface integral. For example, in response to question (3) on the post-test of Tutorial III, one student wrote, "disagree, $\Phi = \int \vec{E} \cdot d\vec{A} = \int \vec{E} + \int d\vec{A} = 0 + A$." This addition, at least in this context, is surprising since the two terms that he is adding do not even have the same dimensions!

5.4.17 Student difficulty with scientific language

Physics is a discipline with precise meanings for various concepts. One common difficulty is the imprecise language students use to express physics concepts. Part of the reason Newton's third law is so difficult because students have not learned to differentiate between the concepts of "acceleration" and "force" (Halloun & Hestenes 1985, Hestenes et al 1992, Thornton & Sokoloff 1998, Hake 1998). For instance, students often believe that a large trailer truck will exert a larger magnitude force on a small car than the car will exert on the truck. While the forces on both objects will be equal in magnitude and opposite in direction, the acceleration is larger for the smaller object and hence the damage to the small car may in general be larger.

We find that students sometimes have difficulty in constructing sentences that are scientifically correct. In some cases, we could infer what students meant despite their imprecise language. But in other cases, it was difficult to make sense of their sentence structures. For example, in response to the post-test question (1) on tutorial III, one student incorrectly claimed: " $\Phi = EA \cos \theta$. All charges are pointing perpendicular to the surface so $\Phi = 0$." Prodding the student further suggests that the student believed that the electric field at each infinitesimal area of the surface will be parallel to the area vector which need not be true for the situation given. Similarly, in response to question 2 on the post-test of tutorial III, a student who provided the correct responses to parts (a) and (b), respectively, noted "[Figure] A because charge is leaving." and "[Figure] C because charge is coming towards the sphere". It is clear from the student's response that the student is referring to the electric field lines and calling them charges. In response to the post-test question 2(d) for tutorial III, the following examples also show similar difficulty with the language of physics.

- There could be multiple charges but the net charge is pointing in the direction indicated.

- Yes, A and C because they have a positive or negative electric field.

In the first statement, the student is correct that there could be multiple charges but the student talks about the net charge pointing in a certain direction which is semantically incorrect. The second student's response is incorrect but, additionally, the student is using scientifically inappropriate language while talking about the electric field being positive or negative without any coordinate axes and probably referring to electric field lines pointing outward or inward from the closed surface.

One student who correctly disagreed with the statement in the post-test question (3) of Tutorial III, drew a charge outside the cube as a counterexample and said, "electric field through the cube is zero because the charge passes through the cube". The idea the student was trying to convey was that, since the point charge is outside the cube, all of the electric field lines entering the cube also leave the cube so that there is no net flux through the cube. Such incorrect use of language can significantly impede student's ability to organize their knowledge hierarchically. In response to the same question (3) on the post-test of tutorial III, the following sample responses from the students also show difficulty with the language of physics:

- disagree. flux could be travelling opposite of field
- disagree because there must be some point on the surface of the cube that is not balanced.
- I disagree. The only flux that has any effect on the object is the enclosed flux or charge.
- Agree because the electric field should have the same charge as the flux running through it
- Agree. The only electric flux that has any effect on the object is the charge/flux enclosed.

None of these sentence structures are consistent with the expert view of relevant physics concepts.

In response to post-test question (4) for tutorial III, the following sample incorrect responses illustrate student difficulties with language.

- disagree. Flux could be negative thus exerting an inward force.
- disagree because there is a uniform electric field on the surface does not always mean there is zero flux. The sphere may contain charge and there would be flux even if field cancels out. (Note that in this example, the student initially refers to “zero” electric field as “uniform” electric field, which may technically be correct, but later makes an argument that the electric flux through a sphere can be non-zero even if the electric field everywhere on the surface is zero.)
- agree because flux would be distributed evenly throughout the sphere along with the electric field
- agree because points on the sphere are not tilted in some direction.

We found it difficult to change students’ language significantly in a short period of time and students did not easily give up using related words, e.g., “electric field lines” and “electric charge” interchangeably as in some of the examples above. Similarly, words such as electric force, electric field, electric flux, and electric charge were often used interchangeably by some students. Our preliminary analysis shows that students who do not speak the scientific language precisely and use related terms indiscriminately usually have a much more difficult time learning relevant concepts than others. However, further study is required to understand students’ facility with scientific language and their ability to learn concepts well within a fixed period.

5.4.18 If A implies B then B must imply A or other convoluted reasoning

Students sometimes use incorrect circular reasoning, e.g., if zero electric field at each point on the surface implies that the electric flux through the surface must be zero then zero electric flux through a closed surface implies that the electric field must be zero at each point. In response to post-test question (3) for tutorial III, one interviewed student incorrectly claimed: “flux is equal to the total charge divided by ϵ_0 . If the flux is zero, then charge is zero, and so the field is zero.” While his first statement is correct if the student is referring to the charge enclosed by the closed surface, the second statement is incorrect because the relationship between the electric flux through a closed surface and the electric field at various points on the surface is complicated. The following sample responses for the same question also illustrate the tendency to over generalize:

- Agree. Flux is what creates the field so if there is no flux there is no electric field.
- Agree. Electric flux and field are determined only by q enclosed. Flux outside does not matter.

In response to the post-test question (4) on tutorial III, while students correctly agreed with the statement, their reasoning is incorrect. If all the excess charge is outside the closed surface, it only implies that the electric flux through a closed surface will be zero according to Gauss’s law and not the electric field at various points on the surface. The following sample responses illustrate such responses:

- agree. The excess charge must be in the exterior.
- agree because if the net charge inside is zero then both the flux and field at the surface will be zero.

In response to the post-test Question (5) for tutorial III, here is an example of a circular reasoning: “agree. The ball must have a net charge inside of it for there to be a net flux which means there can’t be a net electric field if no charge inside.”

Such convoluted reasoning is common in physics especially amongst introductory physics students because physics is abstract and proper reasoning about abstract concepts is difficult. Students need guidance and support with concrete examples in different contexts to internalize various abstract physics concepts. The following Wason tasks (Wason 1968) are examples from cognitive psychology of abstract and concrete problems which are conceptually similar, but the abstract problem is cognitively more demanding.

- Abstract Task: You will lose your job unless you enforce the following rule: “If a person is rated K, then his/her document must be marked with a 3”.

Each card on the table for a person has a letter on one side and a number on the other side.

Indicate only the card(s) shown in Figure 5.9 that you definitely need to turn over to see if the document of any of these people violates this rule.

- Concrete Task: You are serving behind the bar of a city center pub and will lose your job unless you enforce the following rule: “If a person is drinking beer, then he/she must be over 18 years old”.

Each person has a card on the table which has his/her age on one side and the name of his/her drink on the other side. Indicate only the card(s) shown in Figure 5.10 that you definitely need to turn over to see if any of these people are breaking this rule.



Figure 5.9 : Figure for Wason Task in abstract context (Wason 1968).



Figure 5.10 : Figure for Wason Task in concrete context (Wason 1968).

The correct answer for the abstract case is that you must turn the cards with K and 7 (to make sure that there is no K on the other side). Note that the logic presented in the task is one sided in that it is okay for a document with a 3 to have anything on the other side. The correct answer for the concrete case is “beer” and “16 years old”, and it is much easier to identify these correct answers than the correct answers for the abstract case. Typically, people find the abstract Wason task difficult the first time they encounter it but not as difficult in subsequent encounters. One implication for teaching abstract concepts of physics is that once the instructor has built an intuition about a set of related concepts, the concepts may not appear difficult to him/her even if they are abstract. In such situations, the instructor may overlook the cognitive complexity of the problem for a beginning student unless the instructor puts himself/herself in the students’ shoes. Tutorials provide scaffolding to help students develop a robust knowledge structure so that there is less possibility for students reasoning in a convoluted manner.

5.5 PERFORMANCE OF TUTORIAL AND CONTROL GROUPS

Table 5.1 shows the student performance (on each question and also overall) on the pre-test and post-test in each of the three tutorials (III-V) in percentage for each class. The classes utilizing each tutorial may differ either because additional pre-/post-test questions were added or the pre-test for tutorial V was not administered to some of the classes. The differences in the performance of different classes may also be due to the differences in student samples, instructor/TA differences or the manner in which the tutorials were administered.

Table 5.1 : Average percentage scores obtained on individual questions on the pre-/post-tests for tutorial classes

Tutorial	Class	n	Pre-test							Post-test						
			1	2	3	4	5	6	Pre-total	1	2	3	4	5	6	Post-total
III	1	86	48	-	-	-	-	-	48	79	86	73	93	83	-	83
	2	62	49	-	-	-	-	-	49	78	90	83	98	-	-	86
	3	63	55	-	-	-	-	-	55	83	80	85	96	88	-	86
IV	1	64	41	19	53	54	43	3	37	86	84	92	95	96	86	90
	2	51	30	8	57	29	-	-	31	88	91	88	54	-	-	87
	3	65	29	22	58	58	41	18	40	83	91	96	92	91	92	91
V	1	87	-	-	-	-	-	-	-	69	59	68	69	96	73	70
	2	57	20	26	35	-	-	-	27	82	76	85	-	-	-	81
	3	65	-	-	-	-	-	-	-	93	81	90	91	98	92	90

Table 5.2 shows the pre-/post-test data from a comparison group which consists of a class in which tutorials III-V were not used. The total class time devoted to these topics was equivalent to the classes in which the tutorials were administered. In the tutorial classes, instructors did not do some of the examples but let the students do them as part of the tutorials to save time. The pre-tests were given to the students in the comparison group immediately after relevant instruction but post-tests were given in the following week as part of the weekly

recitation quizzes after the students had the opportunity to complete all the homework problems from those topics.

Table 5.2 : Average percentage scores obtained on individual questions on the pre-/post-tests for the non-tutorial class.

Tutorial	n	Pre-test							Post-test						
		1	2	3	4	5	6	Pre-total	1	2	3	4	5	6	Post-total
III	59	43	-	-	-	-	-	43	36	70	72	72	28	-	56
IV	69	33	13	52	36	46	12	34	45	59	51	51	45	26	43
V	57	18	11	49	-	-	-	24	18	19	44	27	69	34	35

Table 5.3 shows the p values obtained by comparing the performance of tutorial classes and non-tutorial class on the pre-/post-tests. The results of Table 5.1-5.3 indicate that regardless of whether the students belonged to the tutorial or comparison groups, their performance on the pretest was poor after traditional instruction. However, students in the comparison group did significantly worse on the post-test than the classes in which students worked on the tutorials.

Table 5.3 : P value for t-tests comparing performance of tutorial classes and non-tutorial classes on the pre-/post-tests.

P value	Tutorial III	Tutorial IV	Tutorial V
pre-test	0.32	0.52	0.60
post-test	0.00	0.00	0.00

Table 5.4 shows the performance of students on the pre-/post-tests for each tutorial partitioned into three separate groups based upon the pre-test performance (see the Range column). As can be seen from Table 5.4, tutorials generally helped all students including those who performed poorly on the pre-test.

Table 5.4 : Percentage average pre-/post-test scores (matched pairs) in tutorial classes for each of the three tutorials (III-V), divided into three groups according to the pre-test performance.

N	Tutorial	Range (%)	n1 (class 1)	pre	post	n2 (class 2)	pre	post	n3 (class 3)	pre	post
211	III	All	86	48	83	62	49	86	63	55	86
		0-34	30	18	80	23	21	84	21	16	85
		34-67	39	52	83	26	55	87	19	53	85
		67-100	17	92	86	13	86	89	23	92	89
180	IV	All	64	37	90	51	31	87	65	40	91
		0-34	31	17	86	30	16	83	26	15	88
		34-67	29	51	94	14	44	92	31	49	91
		67-100	4	78	94	7	64	93	8	85	97
57	V	All				57	27	81			
		0-34				43	14	78			
		34-67				9	48	90			
		67-100				5	96	92			

Table 5.5 shows the performance of students in the comparison group on the pre-/post-tests for each tutorial partitioned into three separate groups based upon the pre-test performance. As can be seen from comparing Tables 5.4 and 5.5, students in the comparison group for none of the pre-test range performed on par with the tutorial groups on the post-tests.

Table 5.5 : Percentage average pre-/post-test scores (matched pairs) in non-tutorial class related to each of the three tutorials (III-V), divided into three groups according to the pre-test performance.

Tutorial	Range (%)	n	pre	post
III	All	59	43	56
	0-34	28	20	52
	34-67	23	55	57
	67-100	8	91	63
IV	All	69	34	43
	0-34	40	18	36
	34-67	23	51	50
	67-100	6	77	65
V	All	57	24	35
	0-34	49	20	31
	34-67	7	49	48
	67-100	1	100	70

The post-test scores for the tutorial group are unusually high and can be attributed to a variety of reasons including the Hawthorne effect (Parsons 1974, Franke & Kaul 1978). Another possibility is that the tutorials were “teaching to the test” and, immediately after working on a tutorial, the concepts were fresh in students’ minds. We therefore administered a cumulative test at the end of the semester which includes concepts from all of the tutorials on Coulomb’s law (discussed in last chapter) and Gauss’s law (Singh 2006). Table 5.6 shows the average percentage scores from the cumulative test administered to different student populations. Although the performance of the tutorial group is not as impressive on the cumulative test as on the pre-/post-tests administered with the tutorials, students who worked through the tutorials significantly outperformed both the Honors students and those in upper-level undergraduate courses, but not first year physics graduate students.

Table 5.6 : The average percentage of correct responses to each of the 25 questions on the cumulative test (Singh 2006) for different student population.

Q	Without Tutorial N=135	Honors Students N=182	Upper-level Undergrad		With Tutorial N=278	Graduate Students N=33
			Pre N=33	Post N=28		
1	56	44	42	53	60	73
2	45	64	52	57	84	64
3	23	57	61	61	70	79
4	22	31	33	39	67	73
5	82	77	70	68	84	88
6	60	55	82	82	69	76
7	69	73	64	68	79	91
8	37	38	52	32	68	88
9	61	53	67	68	66	91
10	41	46	45	50	56	70
11	46	25	24	68	72	94
12	45	35	43	53	69	79
13	3	19	18	14	26	36
14	54	49	55	64	72	85
15	23	26	36	25	52	61
16	13	14	9	36	45	52
17	15	25	27	7	45	45
18	40	28	58	39	63	73
19	42	66	64	53	58	94
20	52	45	55	53	47	94
21	20	43	36	53	34	88
22	21	30	27	50	46	73
23	34	55	42	53	62	82
24	22	32	18	61	38	55
25	22	31	27	28	40	55
Avg	38	42	44	49	59	75
p	0.00	0.00	0.00	0.053		0.00

5.6 CONCLUSION

We have developed and evaluated tutorials to help calculus-based introductory students learn about symmetry and Gauss's law. Pre-/post-tests for each tutorial suggest that the tutorials can be effective in improving student understanding of these concepts. Students in four different classes who used the tutorials performed significantly better on a cumulative test related to these topics than even the students in the upper-level electricity and magnetism undergraduate course.

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6.0 DEVELOPING A MAGNETISM CONCEPTUAL SURVEY

6.1 ABSTRACT

We discuss the development and evaluation of a research-based conceptual multiple-choice survey related to magnetism, students' difficulties related to magnetism concepts and the performance of introductory students on the test before and after traditional instructions. We also discuss the performance of the upper level undergraduates and graduate students. We compare the performance of students from algebra-based classes and calculus-based classes and find that calculus-based students outperformed algebra-based students on the conceptual test both before and after traditional instruction. We also discuss the use of the survey to investigate gender differences in students' difficulties with concepts related to magnetism. We find that while there was no gender difference on the pre-test, female students performed significantly worse than male students when the survey was given as a post-test in traditionally taught calculus-based introductory physics courses (similar results in both the regular and honors versions of the course). In the algebra-based courses, the performance of the female students and the male students has no statistical difference on the pre-test or the post-test. We compare algebra-based and calculus-based students' performance on MCS both as a pre-test and a post-test. Significant difference appeared on the pre-test and persisted on the post-test. We also compare students' performance on MCS with their performance on CSEM. We find that students performed slightly better on CSEM.

6.2 INTRODUCTION

Research-based multiple-choice tests can be useful tools for surveying student learning in physics courses. They are easy and economical to administer and to grade, have objective scoring, and are amenable to statistical analysis that can be used to compare student populations or instructional methods. A major drawback is that the thought processes are not revealed by the answers alone. However, when combined with student interviews, well-designed tests are powerful tools for educational assessment. A number of multiple-choice tests have been developed and widely used by physics instructors to measure students' conceptual learning in physics courses. A commonly used research-based multiple-choice test for mechanics is the Force Concept Inventory (FCI) (Hestenes, et al. 1992). In Electricity and Magnetism (E&M), the CSEM and BEMA surveys have been developed which cover E&M concepts discussed in introductory courses (Maloney et al. 2001, Ding et al. 2006).

Magnetism is an important topic in introductory physics. We developed a research-based 30 item multiple-choice test on magnetism (called the Magnetism Conceptual Survey or MCS) to explore the difficulties students have in interpreting magnetism concepts and in correctly identifying and applying them in different situations. The identification of student difficulties with magnetism for these various groups can help in designing instructional tools to address the difficulties. We also wish to know the extent to which the difficulties are universal, and if there is a correlation with instructor or student preparation and background, e.g., whether they are in the calculus-based or algebra-based courses or whether they are females or males.

Previous research shows that there is often a gender difference in student performance in mathematics and other disciplines (Hyde et al. 1990, Kahle & Meece 1994) as well as in physics (Lorenzo et al. 2006, Pollock et al. 2007, Kost et al. 2009, Kohl & Kuo 2009, Kost et al. 2009)

which can sometimes be reduced by carefully designed curricula. Here, we explore gender difference in student understanding of magnetism concepts covered in introductory physics courses by surveying students in the calculus- and algebra-based courses using the MCS as a pre-test and a post-test (before and after instruction in relevant concepts).

6.3 MAGNETISM CONCEPTUAL SURVEY DESIGN

The Magnetism Conceptual Survey (MCS) covers topics in magnetism discussed in a traditional calculus- or algebra-based introductory physics curriculum up to Faraday's law. During the test design, we paid particular attention to the important issues of reliability and validity (Ding et al. 2006). Reliability refers to the relative degree of consistency in scores between testing if the test procedures are repeated in immediate succession for an individual or group. On a reliable survey, students with different levels of knowledge of the topic covered should perform according to their mastery. In our research, we use the data collected to perform statistical tests to ensure that the survey is reliable within the classical test theory. For example, the reliability index measures the internal consistency of the whole test (Ding et al. 2006). One commonly used index of reliability is KR-20 which is calculated for the survey as a whole (Ding et al. 2006).

Validity refers to the appropriateness of the test score interpretation (Ding et al. 2006). A test must be reliable for it to be valid for particular use. The design of the MCS test began with the development of a test blueprint that provided a framework for planning decisions about the desired survey attributes. We tabulated the scope and extent of the content covered and the level of cognitive complexity desired. During this process, we consulted with several faculty members

who teach introductory E&M courses routinely about concepts they believed their students should know about magnetism.

We classified the cognitive complexity using a simplified version of Bloom's taxonomy: specification of knowledge, interpretation of knowledge and drawing inferences, and applying knowledge to different situations. Then, we outlined a description of conditions/contexts within which the various concepts would be tested and a criterion for good performance in each case. The tables of content and cognitive complexity along with the criteria for good performance were shown to three physics faculty members at the University of Pittsburgh (Pitt) for review. Modifications were made to the weights assigned to various concepts and to the performance criteria based upon the feedback from the faculty about their appropriateness. The performance criteria were used to convert the description of conditions/contexts within which the concepts would be tested to make free-response questions. These questions required students to provide their reasoning with the responses.

The multiple-choice questions were then designed. The responses to the free-response questions and accompanying student reasoning along with individual interviews with a subset of students guided us in the design of good distracter choices for the multiple-choice questions. In particular, we used the most frequent incorrect responses in the free-response questions and interviews as a guide for making the alternative distracter choices. Four alternative choices have typically been found to be optimal, and we chose the four distracters to conform to the common difficulties to increase the discriminating properties of the items. Three physics faculty members were asked to review the multiple-choice questions and comment on their appropriateness and relevance for introductory physics courses and to detect ambiguity in item wording. They went over several versions of the survey to ensure that the wording was not ambiguous. Moreover,

several introductory students were asked to answer the survey questions individually in interviews to ensure that the questions were not misinterpreted.

6.4 MCS ADMINISTRATION

The final version of the MCS was administered both as a pre-test and a post-test to a large number of students at Pitt. These students were from three traditionally taught algebra-based classes, and eight regular (in contrast to the honors) calculus-based introductory classes. In our analysis presented here for the reliability index KR-20, the item difficulty and discrimination indices, and point biserial coefficient of the items, we kept only those students who took the survey both as a pre-test and a post-test except in one algebra-based class. In that class, most of the students who worked on the survey did not provide their names and seven more students participated in the post-test than the pre-test. Thus, in the algebra-based course, 267 students took the pre-test, and 273 students took the post-test. In the regular calculus-based courses, 575 students took both the pre-test and the post-test.

Pre-tests were administered in the first lecture or recitation at the beginning of the semester in which students took introductory second semester physics with E&M as a major component. The students were not allowed to keep the survey. Post-tests were administered in the recitations after instruction in all relevant concepts on magnetism covered in the MCS. Students were typically asked to work on the survey for a full class period (40-50 minutes).

The KR-20 for the combined algebra-based and calculus-based data is 0.83, which is reasonably good by the standards of test design (Ding et al. 2006). The MCS was also administered to 42 physics graduate students enrolled in a first year course for teaching assistants to bench mark the performance that can be expected of the undergraduate students. The average score for the graduate students is 83% with a KR-20 of 0.87.

The item difficulty is a measure of the difficulty of a single test question (Ding et al. 2006). It is calculated by taking the ratio of the number of correct responses on the question to the total number of students who attempted to answer the question. Figure 6.1 shows the difficulty index for each item in the survey for the sample of 848 students obtained by combining the algebra-based and calculus-based classes. The average difficulty index is 0.46 which falls within the desired criterion range (Ding et al. 2006). The average difficulty index for the algebra-based class is 0.45 which is lower than 0.53 for the calculus-based class.

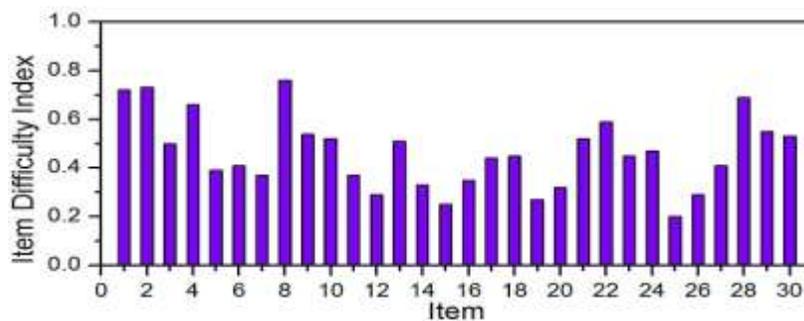


Figure 6.1 : Difficulty index for various items in the MCS

The item discrimination index measures the discriminatory power of each item in a test (Ding et al. 2006). A majority of the items in a test should have relatively high discrimination indices to ensure that the test is capable of distinguishing between strong and weak mastery of the material. A large discrimination index for an item indicates that students who performed well on the test overall performed well on that item. The average item discrimination index for the combined 848 students sample including all items on the MCS is 0.33 which is reasonable from the standards of test design (Ding et al. 2006). Figure 6.2 shows that for this sample the item discrimination indices for 22 items are above 0.3. The average discrimination index for the algebra-based class is 0.29 and for the calculus-based class it is 0.33.

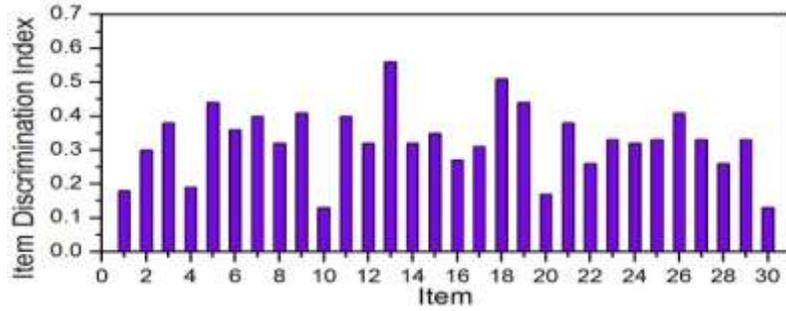


Figure 6.2 : Discrimination index for the MCS items

The point biserial coefficient is a measure of consistency of a single test item with the whole test (Ding et al. 2006). It is a form of a correlation coefficient which reflects the correlation between students' scores on an individual item and their scores on the entire test. The widely adopted criterion for a reasonable point biserial index is 0.2 or above (Ding et al. 2006). The average point biserial index for the MCS is 0.42. Figure 6.3 shows that all items have a point biserial index equal to or above 0.2.

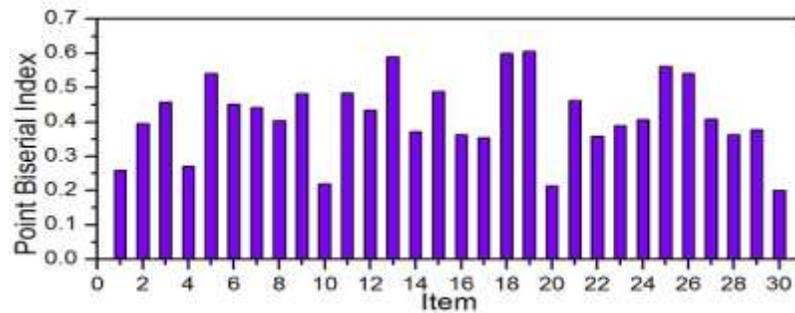


Figure 6.3 : Point biserial coefficient for the MCS items

6.5 DISCUSSION OF STUDENT DIFFICULTIES

The magnetism topics covered in the test include magnitude and direction of the magnetic field produced by current carrying wires, forces on current carrying wires in an external magnetic field, force and trajectory of a charged particle in an external magnetic field, work done by the

external magnetic field on a charged particle, magnetic field produced by bar magnets, force between bar magnets and static charges. Table 6.1 shows the concepts that were addressed by the various questions in the test. This is one of the several ways of classifying questions. This classification does not necessarily reflect the way physics experts would classify the questions. For example, one of the concept categories in which we placed Problem 9, 13, and 17 is “motion doesn’t necessarily cause force.” As shown in Table 6.2 and 6.3, one of the common misconceptions in Problem 9 was due to the assumption that there was force due to the particle’s initial velocity. In Problem 13, the most common incorrect response was choice (c) because students believed that since the magnetic field points opposite to the particle’s direction of motion, there must be a force due to the magnetic field to decrease the speed of particle. The most common incorrect response for Problem 17 was choice (a), which also arose due to the incorrect belief that there must be a force or a component of force in the direction of motion. Similarly, we classified Problem 22, 23, and 24 in the same concept category “distinguishing between current carrying wires pointing into/out of the page and static point charges” because many students misinterpreted the representation of current carrying wires pointing into or out of the page as positive or negative charges.

Table 6.1 : Concepts covered and the questions that addressed them in the test

Concepts	Question Numbers
Magnetic force on bar magnets	1, 2, 3
Distinguishing between charges and magnetic poles	4, 5, 6
Direction of magnetic field inside/outside a bar magnet	7,8
Forces on a charged particle in a magnetic field and other fields	15, 18, 19, 26
Motion doesn't necessarily imply force	9, 13, 17
Direction of motion or magnetic force on a moving charged particle in a magnetic field	10, 14, 16, 17
Work done by magnetic force	11, 12
Forces on current carrying wires in a magnetic field	21, 25, 27
Distinguishing between current carrying wires pointing into/out of the page and static point charges	22, 23, 24
Magnetic field generated by current loops	28, 29, 30

The average score of the algebra-based courses was 24% as a pre-test and 41% as a post-test. In calculus-based courses, the average pre-test score was 28% and post-test score was 49%. Table 6.2 and 6.3 shows the percentage of students who selected the choices (a)–(e) on Problems 1–30. The correct response for each question has been underlined. Table 6.2 includes the results from algebra-based courses on both the pre- and post-test. Table 6.3 contains the pre- and post-test results for calculus-based courses. The correct responses are italicized and bolded. Although some questions have a strong single distractor, others have several distractor choices that are equally popular.

Table 6.2 : Percentage of introductory algebra- based physics students who selected choices (a)-(e) on Problems (1)-(30) on the test.

item #	Pre-test A					Post-test A				
	a	b	c	d	e	a	b	c	d	e
1	3	5	11	<u>69</u>	13	23	2	6	<u>59</u>	10
2	23	17	<u>59</u>	1	0	28	11	<u>59</u>	0	1
3	9	70	<u>18</u>	3	0	19	40	<u>39</u>	2	1
4	<u>71</u>	2	2	20	6	<u>55</u>	3	4	13	25
5	<u>5</u>	5	2	34	54	<u>36</u>	6	6	10	43
6	4	<u>29</u>	10	5	51	4	<u>46</u>	6	7	38
7	14	11	6	<u>7</u>	62	16	8	10	<u>40</u>	26
8	19	13	15	<u>47</u>	6	7	5	10	<u>75</u>	4
9	3	<u>21</u>	42	33	0	2	<u>40</u>	36	22	1
10	19	23	4	<u>46</u>	9	4	3	3	<u>47</u>	43
11	21	2	<u>8</u>	27	42	13	4	<u>31</u>	20	32
12	27	18	22	<u>12</u>	21	19	27	19	<u>21</u>	15
13	<u>3</u>	6	64	6	20	<u>34</u>	12	26	12	16
14	14	29	6	18	<u>33</u>	9	50	6	12	<u>24</u>
15	32	24	18	7	<u>19</u>	24	10	28	15	<u>24</u>
16	<u>25</u>	22	39	11	3	<u>15</u>	31	42	4	9
17	29	22	24	<u>11</u>	14	32	11	9	<u>43</u>	6
18	8	11	45	23	<u>14</u>	5	13	35	14	<u>33</u>
19	16	40	29	7	<u>9</u>	29	18	8	20	<u>26</u>
20	26	<u>26</u>	18	21	10	21	<u>28</u>	17	24	10
21	30	9	<u>7</u>	42	12	11	16	<u>49</u>	20	4
22	28	14	<u>23</u>	23	13	4	9	<u>59</u>	22	7
23	5	6	<u>8</u>	5	76	2	4	<u>36</u>	7	50
24	13	<u>26</u>	39	13	10	6	<u>49</u>	25	11	9
25	35	18	31	8	<u>8</u>	10	36	28	10	<u>17</u>
26	11	42	27	9	<u>10</u>	28	21	8	16	<u>28</u>
27	<u>19</u>	18	23	9	30	<u>35</u>	24	17	11	12
28	<u>27</u>	18	37	10	8	<u>69</u>	17	8	2	4
29	12	<u>17</u>	24	36	11	14	<u>62</u>	8	4	13
30	22	<u>48</u>	14	12	3	28	<u>52</u>	10	9	0

Table 6.3 : Percentage of introductory calculus- based physics students who selected choices (a)-(e) on Problems (1)-(30) on the test.

item #	Pre-test C					Post-test C				
	a	b	c	d	e	a	b	c	d	e
1	11	2	8	<u>67</u>	12	2	2	5	<u>78</u>	13
2	23	12	<u>64</u>	1	1	12	8	<u>80</u>	1	0
3	10	52	<u>36</u>	1	0	5	40	<u>55</u>	0	0
4	<u>53</u>	8	4	24	12	<u>71</u>	3	2	17	6
5	<u>10</u>	6	5	24	54	<u>40</u>	4	4	11	42
6	5	<u>22</u>	8	5	59	3	<u>38</u>	5	5	49
7	11	16	11	<u>11</u>	51	3	3	19	<u>35</u>	40
8	14	20	8	<u>49</u>	8	7	8	6	<u>77</u>	2
9	2	<u>36</u>	25	35	2	2	<u>61</u>	20	16	2
10	18	9	3	<u>49</u>	21	6	3	3	<u>55</u>	32
11	23	5	<u>13</u>	27	32	17	3	<u>40</u>	19	21
12	24	21	20	<u>17</u>	19	17	24	16	<u>33</u>	11
13	<u>13</u>	15	56	8	9	<u>59</u>	3	24	9	6
14	11	29	4	15	<u>41</u>	5	46	3	9	<u>37</u>
15	32	24	17	11	<u>16</u>	19	13	24	17	<u>27</u>
16	<u>29</u>	25	35	7	4	<u>45</u>	19	31	3	3
17	35	18	23	<u>14</u>	10	28	11	14	<u>44</u>	3
18	11	8	50	14	<u>17</u>	4	9	26	10	<u>51</u>
19	16	33	28	11	<u>12</u>	19	24	10	20	<u>27</u>
20	23	<u>24</u>	18	25	<u>11</u>	16	<u>34</u>	17	26	7
21	22	19	<u>17</u>	29	13	11	22	<u>54</u>	8	6
22	17	11	<u>34</u>	30	8	4	4	<u>60</u>	29	4
23	8	9	<u>16</u>	6	61	2	2	<u>50</u>	6	41
24	10	<u>30</u>	41	13	7	6	<u>47</u>	33	10	5
25	26	20	38	9	<u>7</u>	7	25	35	12	<u>21</u>
26	15	39	25	8	<u>12</u>	20	27	7	16	<u>30</u>
27	<u>22</u>	23	21	15	20	<u>44</u>	21	12	10	14
28	<u>36</u>	16	20	9	19	<u>70</u>	9	5	2	14
29	19	<u>26</u>	23	22	11	17	<u>52</u>	6	5	21
30	22	<u>40</u>	20	15	4	28	<u>54</u>	10	8	1

Below, we discuss students' difficulties observed in understanding concepts on magnetism. The magnetism conceptual survey (MCS) is included in the Appendix for reference.

6.5.1 Magnetic force between bar magnets

Problems 1-3 are related to magnetic force on bar magnets. The most common distracter in Problem 1 was (e). Some students claimed that the net force on the middle bar magnet should be zero because the forces due to the bar magnets on the sides should cancel out. When students saw that there are two magnets on each side of a magnet they immediately concluded that the forces due to the magnets are opposite to each other and therefore there is no net force on the bar magnet. Students often did not bother to draw a free body diagram to determine the directions for forces due to the individual bar magnets. From the students' responses it appears that many students did not consider Newton's Third Law when they answered the questions. In Problem 2, whether there is a third bar magnet or not, due to Newton's Third Law, the force magnet 1 exerts on magnet 2 should be equal in magnitude to the force magnet 2 exerts on magnet 1. However, the appearance of the third bar magnet made many students think that the force with which magnet 1 repels magnet 2 is half or twice the force with which magnet 2 repels magnet 1. In Problem 3, the most common incorrect choice was (b). Because it is mentioned in the problem that Magnet 1 is twice as strong as magnet 2, some students intuitively thought that the stronger the bar magnet, the larger the force due to that bar magnet.

6.5.2 Distinguishing between magnetic poles and charges

Problems 4-6 exemplified students' difficulties in distinguishing between the north and south poles and localized positive and negative charges or between an electric dipole and a magnetic dipole. When considering these types of questions, many students claimed that a bar magnet is an electric dipole with positive and negative charges located at the north and south poles. Thus,

they incorrectly believed that a bar magnet can exert force on static charges or conductors. In Problem 4, some students considered the north pole of the bar magnet as consisting of positive charges which can induce negative charges on the conductor. Hence, they claimed that the bar magnet exerts a force on the conductor toward the magnet. In Problem 5, students had a similar difficulty. In the post-test, more than 40% of the students chose distracter (e), because they considered the north pole as a positive charge which repels another positive charge. This type of confusion was even more prevalent in Problem 6. On the post-test, 38% of the calculus-based students answered problem 6 correctly which is not significantly different from the pre-test performance of 22% (before instruction). The most common incorrect response was option (e), i.e., there is no net force on charge q_0 . Both written explanations and interviews suggest that students often considered a bar magnet as an electric dipole. When the interviewer asked them whether a magnet will retain its magnetic properties when broken into two pieces by cutting it at the center, most of them claimed that the magnetic properties will be retained, and each of the two smaller pieces will become a magnet with opposite poles at the ends. When the interviewer asked how that is possible if they had earlier claimed that the north and south poles are essentially localized opposite charges, some students admitted they were unsure about how to explain the development of opposite poles at each end of the smaller magnets but others provided creative explanations about how charges will move around while the magnet is being cut into two pieces to ensure that each of the cut pieces has a north and south pole. During the interviews, we probed how students developed the misconception that a bar magnet must have opposite charges localized at the two poles because when a child plays with a bar magnet, he/she does not necessarily think about localized electric charges at the end. Some students claimed that they learned it from an adult while he/she tried to explain why a bar magnet behaves the way it

does or they had heard it on a television program. More research is needed to understand how and at what age college students developed such incorrect notions.

6.5.3 Magnetic field of a bar magnet

Problems 7 and 8 assess students' understanding of magnetic field inside or outside a bar magnet. In Problem 7, the most common distractor was option (e). Students often asserted that the magnetic field at the midpoint between the north and south poles should definitely be zero. Some of them even believed that there should be no magnetic field inside of the bar magnet. In addition, students' responses in Problem 8 revealed another misconception. They claimed that the magnetic field should be zero not only at the midpoint between the poles, but also at any points on the perpendicular bisector of the bar magnet including those points outside of the magnet.

6.5.4 Directions of magnetic force, velocity and magnetic field

Students' answers to some problems suggest that when they were given the directions of two of the quantities magnetic force, magnetic field and velocity, they have difficulty in determining the direction of the third quantity. Tables 6.2 and 6.3 show that student performance on the post-test was worse than that on the pre-test on Problem 14. The most common incorrect choice for Problem 14 was option (b) because students used the redundant information about angle provided and had difficulty visualizing the problem in three dimensions. The correct answer was option (e) because the velocity of all of the three charged particles is perpendicular to the magnetic field. Written explanations and interviews suggest that some students incorrectly used

the superfluous information provided about the angles that the charged particles (1) and (3) make with the horizontal. During interviews, only when the students choosing option (b) were asked explicit questions about the direction of the magnetic field, velocity and the angle between them for particles (1) and (3), did they realize that the angle between them is 90° in all three cases. The fact that these students initially claimed that the angle between the magnetic field and velocity vector is 45° for particles (1) and (3) is similar to our finding in mechanics. In the context of kinematics, when introductory physics students were asked a question about a rabbit's motion whose ears were at 45° angle to the horizontal, approximately one third of the introductory students used this redundant information about the angle of the rabbit's ears to solve the problem incorrectly. In Problem 17, the calculus-based student performance on this question changed from 14% on pre-test to 44% on post-test. It is similar to the performance of algebra-based students. The most common incorrect response to this question was option (a) (but options (b) and (c) were also chosen). In written explanations, students who chose option (a) often incorrectly claimed that the velocity of a charged particle and the magnetic force can be at any angle to each other, but some claimed that the magnetic field and force must be perpendicular while others said that the field and force must be parallel. In interviews, students who provided incorrect responses were asked to write down an expression for the magnetic force on the charged particle in an external magnetic field. Approximately half of the students were unable to write the correct expression. Those who wrote the correct expression were explicitly asked about the cross product between the velocity and magnetic field and what it implies about the angle between the magnetic force and the velocity or the magnetic field vector. Approximately half of these students corrected their initial error and said that the cross product implies that the magnetic force must be perpendicular to both the velocity and the magnetic field. Others did not

know that the cross product of two vectors must be perpendicular to each of the vectors. In Problem 10, the most common distracter was option (e). It suggests that students have difficulty in realizing that for electrons the direction of motion while applying the right hand rule should be opposite to the direction for positive charges. This is also one of the difficulties students display on Problem 20. On Problem 20, students must first use the relation between the radius of the trajectory and the mass of a particle to determine whether this particle is an electron or proton. Then, based on this information, they can determine the direction of motion. Students' chose options (a) – (d) with almost equal probability. Interview and written explanations suggest that students have difficulty in deducing the relation between the radius of trajectory and the mass of a particle. Some students never thought of using this relation to solve the problem. There were also some students who correctly found that the particle is an electron. However, they incorrectly chose option (d) because of a difficulty similar to that discussed with Problem 10.

6.5.5 Work done by magnetic force

Problem 11 and 12 are related to the work done by the magnetic field. They also probe the extent to which students understand that magnetic force is perpendicular to velocity and only change the direction of motion of a charge. In Problem 11 and 12, the distracters were equally popular. Some students claimed that the work done by the magnetic force is always positive while other students believed that it can be positive or negative depending on the charge or the orientation of the initial velocity. Some students referred to the equation relating work to the inner product of force and displacement to solve for work. However, they did not realize that the magnetic force is always perpendicular to the direction of motion. As a result, they often believed that the directions of force and velocity can form any angle and the magnetic force can do work.

6.5.6 Movement doesn't necessarily imply force

Previous research in mechanics shows that students often believe that motion always implies a force in the direction of motion. Our findings are consistent with previous research. In Problem 9, the most common incorrect option was (c), i.e., there is a force due to the electron's initial velocity. Another common incorrect option was (d). In response to the Problem 13, students performed significantly better on the post-test compared to pre-test. The most common incorrect response to Problem 13 was option (c) followed by options (d) and (e). Interviewed students were asked such questions in the context of a lecture-demonstration related to the effect of bringing a powerful bar magnet from different angles towards an electron beam (including the case where the magnetic field and velocity vectors are collinear as in Problem 13). Students were explicitly told to predict the outcome for both the electron beam and a beam of positive charges. Students who incorrectly chose option (c) in Problem 13 explained that the particle will slow down because the magnitude of the magnetic field is opposite to the direction of velocity and this implies that the force on the charged particle must be opposite to the velocity. Of course, this prediction could not be verified by performing the experiment. Students who chose options (d) or (e) often incorrectly remembered the right hand rule about magnetic force being perpendicular to magnetic field and provided animated explanations such as "Oh, the electron wants to get out of the way of the magnetic field and that's why it bends". They were surprised to observe that the deflection of the electron beam is negligible when the strong magnet has its north pole or south pole pointing straight at the beam.

6.5.7 Net force on a charged particle or a current carrying wire in a magnetic field

Problem 15, 19 and 26 are related to the net force on charges in a magnetic field and another field. In these problems, students need to be able to figure out the directions of magnetic force and electric force on a charge and then to see if the forces are comparable based on the information provided to them. Problem 15 and 19 are similar types of problems and the differences are in the directions of the magnetic field and electric field in each problem. Problem 26 is similar to Problem 19 in which the electric field it is replaced with gravitational field. Students' response distributions for these problems were similar. As Table 6.2 and 6.3 show, students' choices are almost equally distributed. The percentages of correct responses on these problems ranged from 27% to 30% in calculus-based classes and from 24% to 28% in algebra-based classes. Interviews and written explanations suggest that there are two main reasons that students have difficulty in answering these types of questions. One reason is that students only consider the effect of one force, magnetic force or electric force (or gravitational force). Therefore, they often conclude that the direction of net force is in the direction of magnetic force or electric force. The other reason is that students don't know how to determine the direction of net force with two forces in opposite directions. To answer the questions correctly, they should notice that they were not provided any information to evaluate the magnitude of the forces. Therefore, the direction of the net force cannot be determined. However, written explanations show that some students believe that the force has to be in one direction, either in the direction of magnetic force or the direction of electric force (or gravitational force). When they have difficulty in determining which of those two directions to choose, they often chose one randomly. Those students who claimed that the net force is zero often claimed that because the directions of the two forces are opposite to each other, the net force should be zero.

In Problem 18, the electric and magnetic fields are parallel. Students' answers to this problem reveal another aspect of their difficulties. Table 6.3 shows that 51% of calculus-based students provided the correct response on post-test, three times the number on the pre-test. The most common incorrect option in response to Problem 18 was option (c). Written explanations and interviews suggest that students had learned about velocity selectors in which the electric and magnetic fields are perpendicular to each other (not the situation shown here) and to the velocity of the particles and there is a particular speed $v=E/B$ for which the net force on the particle is zero. These students had over-generalized this situation and had simply memorized that whenever the velocity is perpendicular to both the electric and magnetic fields, the net force on the charged particle is zero. They neglected to account for the fact that the magnitudes of electric and magnetic fields must satisfy $v=E/B$ and the two fields must be perpendicular to each other and oriented appropriately for the net force on the particle to be zero. During interviews, students were often not systematic in their approach and talked about the net effect of the electric and magnetic fields simultaneously rather than drawing a free body diagram and considering the contributions of each field separately first. A systematic approach to analyzing this problem involves considering the direction and magnitude of the electric force $\vec{F}_E = q\vec{E}$ and magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ individually and then taking their vector sum to find the net force. Some interviewed students who made guesses based upon their recollection of the velocity selector example discussed in the class claimed that the net force on the particle is zero for this situation. They were asked by the interviewer to draw a free body diagram for the case in which the charged particle is launched perpendicular to both fields. Some of them who knew the right hand rule for the magnetic force and the fact that the electric field and force are collinear ($\vec{F}_E = q\vec{E}$) were able to draw correct diagrams showing that the electric force and magnetic force are not

even collinear. One of these students exclaimed: “I don't know what I was thinking when I said that the net force is zero in this case. These two are (pointing at the electric and magnetic forces in the free body diagram he drew) perpendicular and can never cancel out”. Such discussions with students suggest the need for students to understand systematic approach to solve a conceptual problem instead of a guessing task. Many students resist making the effort to understand the systematic approach of solving problems and instead, they just walk away with a set of simplified assumptions which lead to the misconceptions.

Problem 25 and 27 are related to the net force of one of two parallel current carrying wires in a magnetic field. Students' responses to these two problems reflect the difficulties that are similar to those shown in Problem 15, 19 and 26. Problem 25 was the most difficult question in the test. 21% of the students responded correctly. Options (b) and (c) were the most common incorrect distracters. Students answered this problem incorrectly either because they only considered the effect of one force or because they randomly chose a direction for the force since they were unable to compare the magnitude of the forces. Unlike previous problems asking for the direction of net force, Problem 27 asks for the possible direction of the magnetic field to make the net force zero. Student performance on Problem 27 was slightly better than the previous problems. Distracters were almost equally popular. This revealed another students' difficulty that is mentioned in the next section. In particular, many students were unable to systemically solve for the direction of force on a current carrying wire due to another current carrying wire.

6.5.8 Distinguishing between current carrying wires pointing into/out of the page and static charges

We observed that when students were presented with current carrying wires with current flowing into or out of the page, they often failed to figure out the magnetic field or magnetic force due to the wires by using the right hand rule. Students' responses to Problems 22-24 reflect this difficulty. Problem 22 is about the direction of the force on a current carrying wire due to a parallel wire carrying current in the opposite direction. The most common incorrect response (option (d)) was chosen based on the assumption that the wires would attract each other. Even in the individual lecture-demonstration based interviews, many students explicitly predicted that the wires would attract and come closer to each other. When asked to explain their reasoning, students often cited the maxim "opposites attract" and frequently made an explicit analogy between two opposite charges attracting each other and two wires carrying current in opposite directions attracting each other. In lecture-demonstration based interviews, when students performed the experiment and observed the repulsion, none of them could reconcile the differences between their initial prediction and observation. One difficulty with Problem 22 is that there are several distinct steps (as opposed to simply one step) involved in arriving at the correct response. In particular, understanding the repulsion between two wires carrying current in opposite directions requires comprehension of the following issues: (I) There is a magnetic field produced by one wire at the location of the other wire. (II) The direction of the magnetic field produced by each current carrying wire is given by a right hand rule. (III) The magnetic field produced by one wire will act as an external magnetic field for the moving charges in the other wire and will lead to a force on the other wire whose direction is given by a right hand rule. None of the interviewed students was able to explain the reasoning systematically. What is

equally interesting is the fact that none of the interviewed students who correctly predicted that the wires carrying current in opposite directions would repel could explain this observation based upon the force on a current carrying wire in a magnetic field even when explicitly asked to do so. A majority of them appeared to have memorized that wires carrying current in opposite directions repel. They had difficulty applying both right hand rules systematically to explain their reasoning: the one about the magnetic field produced by wire 2 at the location of wire 1 and the other one about the force on wire 1 due to the magnetic field. This difficulty in explaining their prediction points to the importance of asking students to explain their reasoning. In Problem 24, the most common incorrect response was option (c). Similar to Problem 22, students did not follow the steps mentioned above to figure out the directions of magnetic forces and simply used “opposite attract”. As can be seen from Tables 6.2 and 6.3, for Problem 23, option (e) was the most common incorrect response on both the pre- and post-tests. These students claimed that the magnetic field is zero at point P between the wires. In written explanations and individual interviews, students who claimed that the magnetic field is zero at point P between the wires carrying opposite currents argued that at point P, the two wires will produce equal magnitude magnetic fields pointing in opposite directions, which will cancel out. During interviews, when students were explicitly asked to explain how to find the direction of the magnetic field at point P, they had difficulty figuring out the direction of the magnetic field produced by a current carrying wire using the right hand rule and in using the superposition principle to conclude correctly that the magnetic field at point P is upward. Interviews suggest that for many students, confusion about similar concepts in electrostatics were never cleared up and had propagated to difficulties related to magnetism. For example, several students drew explicit analogy with the electric field between two charges of equal magnitude but opposite sign, incorrectly claiming

that the electric field at the midpoint should be zero because the contributions to the electric field due to the two charges cancel out similar to the magnetic field canceling out in Problem 23. Some interviewed students who were asked to justify their claim that the electric field at midpoint between two equal magnitude charges with opposite sign is zero were reluctant claiming the result was obvious. Then, the interviewer told them that the influence of equal and opposite charges at midpoint between them must cancel out is not a good explanation and they must explicitly show the direction of electric field due to each charge and then find the net field. This process turned out to be impossible for some of them but others who drew the electric field due to each charge in the same direction at midpoint were surprised. One of them who realized that the electric field cannot be zero at midpoint between the two equal and opposite charges smiled and said that this fact is so amazing that he will think carefully later on about why the field did not cancel out at midpoint. There is a question similar to Problem 23 on the broad survey of electricity and magnetism CSEM (Maloney et al. 2001). However, students performed much worse on Problem 23 than on the corresponding question on CSEM (50% calculus-based courses on post-test for Problem 23 vs. 63% for the corresponding question on CSEM). One major difference is that we asked students for the direction of magnetic field at the midpoint between current carrying wires carrying current in opposite directions and many students had the misconception that the magnetic field is zero at the midpoint. Thus, they gravitated to option (e). In CSEM, all incorrect choices were quite popular because the misconception specifically targeted by Problem 23 about the magnetic field being zero at the midpoint between the wires was not targeted there.

6.5.9 Magnetic field due to current loops

Problems 28-30 explored student difficulties in determining the magnetic field generated by a current loop. Although these problems were not very challenging for students, common misconceptions were observed. The most common misconception for Problem 28 was that the magnetic field inside a loop should be zero. Interviews suggest that students confused the magnetic field inside a 2D loop with the electric field inside a 3D sphere with uniform charges. In Problem 29, the most common incorrect option was option (e) followed by option (a). Students believed that the magnetic field outside of a loop should be zero or in the same direction of the field inside the loop. In Problem 30, the most common distracter was (a). Interviews and written explanations suggest that, similar to determining the direction of magnetic field due to two current carrying wires pointing into/out of the page, students were likely to use “opposites attract” to solve the problem. They believed that two current loops with current flowing in opposite directions should attract each other, therefore, the magnetic force in between should be added up. Students should be encouraged to follow systematic steps to solve problems instead of use their gut feelings.

6.5.10 3D visualization and right hand rule

The performance of many students was closely tied to their ability in applying the right hand rule and three-dimension visualization. In magnetism, there are many questions related to magnetic field and magnetic forces. The students are often given a current carrying wire or moving charge in a magnetic field and they have to determine the magnetic force. Therefore, students must know how to use the right hand rule. For example question 21 requires the application of the

right hand rule once. However, the results shown in Tables 2 and 3 suggest that many students have difficulty with it. In response to question 21, many students incorrectly chose either the direction opposite to the correct direction or the direction of the current.

The requirement of 3D visualization is also high in magnetism because the directions of magnetic field, magnetic force and the direction of motion are always perpendicular to each other. In the MCS test, questions 10 – 30 all require the ability to visualize in three dimensions. Question 16 is a good example involving 3D visualization. In this problem, students need to understand that the proton will undergo circular motion in the plane that is perpendicular to the page and simultaneously move along the direction of magnetic field with a constant speed making its overall path helical. Failure to decompose the initial velocity into two components and use the correct sub-velocity to figure out the magnetic force results in incorrect responses.

6.5.11 Performance of upper-level undergraduates

We also administered the 30 item multiple-choice test as a pre- and post-test to the upper level physics students enrolled in an E&M course which used Griffiths' E&M textbook (Griffiths 1999). 26 students took the pretest and 25 students took the post-test. Tables 6.4 and 6.5 show the percentage of students who made the various choices on various questions on the pre-test and post-test, respectively. The correct response for each question has been underlined. The average pre- and post-test scores are 51% and 59%, respectively. This difference is not statistically significant.

Table 6.4 : Percentage of students in the upper-level undergraduate E&M course who selected choices (a) – (e) on Problems (1)-(30) on the pre-test.

		Upper Level Undergraduate							Upper Level Undergraduate				
Item #	a	b	c	d	e	Item #	a	b	c	d	e		
1	0	0	0	<u>73</u>	27	16	<u>50</u>	23	23	4	0		
2	4	4	<u>92</u>	0	0	17	42	8	8	<u>35</u>	8		
3	4	46	<u>50</u>	0	0	18	8	8	15	4	<u>65</u>		
4	<u>58</u>	4	0	35	4	19	4	35	8	8	<u>46</u>		
5	<u>42</u>	0	8	8	42	20	15	<u>54</u>	12	15	4		
6	8	<u>46</u>	0	0	46	21	12	39	<u>39</u>	8	4		
7	0	4	39	<u>27</u>	31	22	0	4	<u>69</u>	23	4		
8	4	4	0	<u>89</u>	4	23	0	0	<u>19</u>	0	81		
9	4	<u>50</u>	27	19	0	24	4	<u>42</u>	39	8	8		
10	8	0	4	<u>77</u>	12	25	8	39	27	4	<u>23</u>		
11	15	4	<u>39</u>	27	15	26	4	19	12	8	<u>58</u>		
12	8	12	27	<u>27</u>	27	27	<u>47</u>	27	12	0	15		
13	<u>50</u>	0	35	15	0	28	<u>85</u>	15	0	0	0		
14	4	31	0	12	<u>54</u>	29	23	<u>58</u>	0	0	19		
15	8	23	15	12	<u>42</u>	30	35	<u>65</u>	0	0	0		

Table 6.5 : Percentage of students in the upper-level undergraduate E&M course who selected choices (a) – (e) on Problems (1)-(30) on the post-test.

		Upper Level Undergraduate							Upper Level Undergraduate				
Item #	a	b	c	d	e	Item #	a	b	c	d	e		
1	0	0	4	<u>76</u>	20	16	<u>52</u>	34	34	0	0		
2	0	4	<u>92</u>	4	0	17	33	12	4	<u>50</u>	0		
3	4	44	<u>52</u>	0	0	18	4	4	12	12	<u>68</u>		
4	<u>68</u>	0	0	32	0	19	20	16	8	12	<u>44</u>		
5	<u>56</u>	0	0	16	28	20	8	<u>40</u>	8	32	12		
6	0	<u>52</u>	0	4	44	21	8	16	<u>76</u>	0	0		
7	4	0	12	<u>72</u>	12	22	0	0	<u>68</u>	28	4		
8	0	4	4	<u>92</u>	0	23	0	0	<u>52</u>	0	48		
9	0	<u>60</u>	24	16	0	24	0	<u>36</u>	44	20	0		
10	4	0	0	<u>52</u>	44	25	0	16	32	8	<u>44</u>		
11	8	0	<u>80</u>	4	8	26	12	28	0	8	<u>52</u>		
12	4	8	16	<u>28</u>	44	27	<u>56</u>	12	12	0	20		
13	<u>60</u>	4	24	4	8	28	<u>84</u>	16	0	0	0		
14	0	36	0	8	<u>56</u>	29	4	<u>76</u>	0	0	20		
15	24	8	24	12	<u>32</u>	30	36	<u>56</u>	0	8	0		

6.5.12 Performance of graduate students

To calibrate the test we also administered it over two consecutive years to a total of 42 physics graduate students who were enrolled in a seminar course for teaching assistants (TAs). Most of them were first year graduate students who were simultaneously enrolled in the first semester of the graduate E&M course. Students were told ahead of time that they would be taking a test related to electrostatics concepts. They were asked to take the test seriously, but it did not count for their course grade. The average test score for the graduate students was approximately 83% with the reliability coefficient 0.87. The better performance of graduate students compared to the undergraduates is statistically significant. The minimum score obtained by a graduate student was 33% and the maximum score obtained was 100%. Table 6.6 shows the percentage of graduate students who selected the various choices on the test.

Table 6.6 : Percentage of physics graduate students enrolled in a course for teaching assistants who selected choices (a) – (e) on Problems (1)-(30) on the post-test.

Item	Graduate					Item	Graduate				
	a	b	c	d	e		a	b	c	d	e
1	0	2	5	<u>83</u>	10	16	<u>74</u>	12	14	0	0
2	2	0	<u>98</u>	0	0	17	<u>12</u>	0	10	<u>79</u>	0
3	0	5	<u>95</u>	0	0	18	0	2	2	2	<u>93</u>
4	<u>81</u>	0	0	17	2	19	2	5	2	0	<u>91</u>
5	<u>93</u>	0	2	0	5	20	5	<u>60</u>	10	21	5
6	0	<u>91</u>	5	2	2	21	0	24	<u>74</u>	0	2
7	0	0	14	<u>71</u>	14	22	0	0	<u>74</u>	26	0
8	0	2	2	<u>95</u>	0	23	2	0	<u>74</u>	5	19
9	0	<u>93</u>	5	2	0	24	0	<u>86</u>	10	5	0
10	0	2	2	<u>81</u>	14	25	2	12	14	2	<u>69</u>
11	7	0	<u>88</u>	5	0	26	2	10	0	2	<u>86</u>
12	2	2	2	<u>88</u>	5	27	<u>74</u>	16	0	7	2
13	<u>95</u>	0	0	0	5	28	<u>91</u>	2	0	0	7
14	0	14	0	0	<u>86</u>	29	5	<u>76</u>	0	2	17
15	19	2	2	0	<u>76</u>	30	10	<u>83</u>	2	5	0

6.6 PERFORMANCE BY GENDER

For analyzing gender difference in students' performance on the MCS, we separate our data into male and female groups. Only the students who provided this gender related information were kept in this analysis. The gender comparison in the algebra-based classes includes 121 females and 110 males (total 231 students) on the pre-test and 106 females and 91 males (total 197) on the post-test. There were 168 females and 403 males (total 571 students) from the regular (not honors) calculus-based classes who took both the pre-test and the post-test and are included in the analysis below. In addition to comparing the results from the algebra-based and regular calculus-based classes, we also analyzed the gender data for the post-test of 95 students enrolled in the honors calculus-based introductory physics course. The honors students were not administered the MCS as a pre-test.

We perform analysis of variance (ANOVA) to investigate the gender differences from the pre-test and the post-test MCS data. Our null hypothesis is that there is no significant gender difference on MCS. If the p-value is less than the significance level 0.05, the rule of thumb is to conclude that the assumption is false (here it will imply that there is a significant difference between the male and female performance).

Tables 6.7-6.8 show the results for the algebra-based students on the pre-test and the post-test. Table 6.7 shows that in the pre-test, the means are 7.3 and 7.1 for the males and the females respectively. The p-value, 0.94, which is larger than 0.05, suggests no significant difference between the males and females on the pre-test in algebra-based classes. Table 6.8

shows the results for the post-test. It shows that the mean for the females is 12.3 compared to the mean for the males, 13.2. The p-value, 0.36, suggests that even on the post-test, the algebra-based students do not have a significant difference in performance based on gender.

Table 6.7 : Algebra-based course pre-test performance by gender

Gender	N	Mean	S.D.	P value
Male	91	7.3	2.5	0.94
Female	106	7.1	2.4	

Table 6.8 : Algebra-based course post-test performance by gender

Gender	N	Mean	S.D.	P value
Male	110	13.2	5.2	0.36
Female	121	12.3	5.6	

The results for the regular calculus-based classes are qualitatively different from the algebra-based classes for the post-test. The pre-test mean for males is 8.5 and for females is 7.8. The p-value for analysis of variance between these groups is 0.49 suggesting no significant difference based on gender on the pre-test. However, the results shown in Table 6.10 suggest that there is a significant difference on the post-test and males outperformed females. The mean for the males is 15.3 compared to the mean for the females which is 13.0 (p-value, 0.019).

Table 6.9 : Regular calculus-based course pre-test performance by gender

Gender	N	Mean	S.D.	P value
Male	403	8.5	3.4	0.49
Female	168	7.8	3.05	

Table 6.10 : Regular calculus-based course post-test performance by gender

Gender	N	Mean	S.D.	P value
Male	403	15.3	6.2	0.019
Female	168	13	5.4	

Table 6.11 : Honors calculus-based course post-test performance by gender

Gender	N	Mean	S.D.	P value
Male	75	17.4	5.9	0.030
Female	20	14.1	6.2	

The gender difference also exists on the post-test for the calculus-based honors introductory physics course. Table 6.11 shows that the mean for 75 males is 17.4 and for 20 females is 14.1 (p-value is 0.030).

To summarize the data presented in Tables 6.7-6.11, for both the algebra- and calculus-based classes, there is no significant difference between the males and females on the pre-test. After traditional instruction, there is still no gender difference in the algebra-based classes. However, a statistically significant difference appeared on the post-test for the calculus-based classes in which there are significantly fewer females in each class than males (both regular and honors).

Table 6.12 : Percentage of correct response on each item by gender in algebra- and regular calculus-based courses

Item	Alg-M	Alg-F	Calc-M	Calc-F
1	61	55	79	73
2	62	55	82	74
3	40	39	56	53
4	59	50	73	67
5	36	43	42	38
6	49	50	38	39
7	46	42	38	28
8	79	74	80	71
9	44	40	62	56
10	48	41	57	50
11	36	32	41	37
12	28	20	35	26
13	37	32	61	53
14	25	18	42	24
15	25	26	28	23
16	21	8	45	43
17	41	54	42	48
18	44	30	56	38
19	33	25	30	21
20	25	31	36	29
21	51	55	54	51
22	64	52	63	51
23	39	32	54	39
24	55	46	50	38
25	18	20	24	14
26	28	32	33	23
27	37	35	47	36
28	72	69	72	63
29	66	67	54	46
30	49	55	54	52

We looked at students’ responses to each MCS item individually to understand how males and females performed on each question. The results in Table 6.12 show that in the algebra-based classes, males outperformed females on 20 questions including 4 questions on

which the differences are larger than 10%. On the 10 questions on which females outperformed males, only one has a difference of more than 10%. In the calculus-based classes, males outperformed females on 28 questions and 9 of them have a difference larger than 10%. On the other two questions, females only performed slightly better than the males.

Answering many of the questions on the MCS correctly requires that students be able to visualize the situation in three dimensions (3D). For example, some questions require that students apply the right hand rule to figure out the directions of the magnetic field or the force on a moving charge or a current carrying wire. Some prior research suggests that females generally have a better verbal ability but worse spatial ability than males which can restrict their reasoning in 3D (Halpern 2000) and often there is a correlation between students' spatial ability and their self-confidence (Law et al. 1993, Casey et al. 2001). The reasons for gender differences are quite complex. In addition to the difference of spatial abilities, factors such as accumulated societal bias, value affirmation and self-efficacy issues all may play a role in gender difference.

6.7 ALGEBRA-BASED COURSES VS. CALCULUS-BASED COURSES

MCS investigates students' conceptual understanding of magnetism. It doesn't involve mathematical calculations when solving the conceptual problems. One issue we want to investigate is if students from calculus-based courses outperform those from algebra-based courses on MCS. Therefore, we compare algebra-based courses and calculus-based courses based on their pre-test and post-test scores. It includes 267 algebra-based students and 575 calculus-based students on the pre-test and 273 algebra-based students and 575 calculus-based students on the post-test.

Tables 6.13-6.14 show the results for the algebra-based students and calculus-based students on the pre-test and the post-test. Table 6.13 shows that on the pre-test, the means are 7.2 and 8.3 for the algebra-based courses and calculus-based courses respectively. The p-value, 0.00, which is smaller than 0.05, suggests significant difference between algebra-based courses and calculus-based courses on the pre-test. Table 6.14 shows the results for the post-test. It shows that the mean for algebra-based courses is 12.2 compared to the mean for calculus-based courses, 14.6. The p-value, 0.010, suggests that on the post-test, there is also significant difference between algebra-based courses and calculus-based courses.

Table 6.13 : Algebra-based course vs. Calculus-based course pre-test performance

	N	Mean	S.D.	P value
Algebra	267	7.2	2.5	0.00
Calculus	575	8.3	3.3	

Table 6.14 : Algebra-based course vs. Calculus-based course post-test performance

	N	Mean	S.D.	P value
Algebra	273	12.2	5.3	0.01
Calculus	575	14.6	6.1	

It is interesting to find that students in the algebra-based and calculus-based courses perform significantly different on conceptual questions before and after instruction. One possible reason may be that calculus-based students' better mathematic skills and scientific reasoning skills help them in understanding concepts better. Calculus-based students are more likely to build a robust knowledge structure and less likely to have cognitive overload during learning. These issues may contribute to the better performance of calculus-based students.

6.8 PERFORMANCE ON CSEM

The Conceptual Survey of Electricity and Magnetism (CSEM) developed by David Maloney et al. aims to assess students' knowledge of topics in electricity and magnetism covered at the introductory level (Maloney et al. 2001). This survey is a 32-question, multiple-choice test that can be used as both a pre-test and post-test. The average pre-test score for algebra-based students is 25% and for calculus-based students is 31%. The post-test scores are 44% and 47%, respectively. This includes more than 5000 introductory physics students at 30 different institutions.

We administered CSEM to students at the University of Pittsburgh (Pitt). One goal was to see if students' performance on CSEM at Pitt is comparable to the averages reported by Maloney et al. and also compare the performance on CSEM with the performance on MCS. The students at Pitt were from one traditionally taught algebra-based class and four calculus-based introductory classes. In our analysis, for calculus-based classes, we only kept students who took both CSEM and MCS as a pre-test and a post-test. For algebra-based class, we kept the students who took both the pre-test and the post-test with CSEM. These students might not be the same students who took the MCS as pre-test and post-test because many students who worked on MCS didn't provide their names. Thus, in the algebra-based course, 83 students took the MCS as pre-test and post-test, and 95 students took the CSEM as a pre-test and post-test. In the calculus-based course, 355 students took the CSEM and MCS as a pre-test and post-test. 26 first year graduate students who were enrolled in a semester long seminar course for teaching assistants were also administered the CSEM.

Tables 6.15-6.16 show the performance on CSEM and MCS for the algebra-based students and calculus-based students on the pre-test and the post-test. Table 6.15 shows that in

the pre-test, the average score for algebra-based class is 24% and for calculus-based class is 36%. The post-test average scores are 36% and 53%, respectively. The graduate students obtained 82%. In Maloney et al.'s previous research on CSEM, it shows that the average pre-test score for algebra-based class is 25%, for calculus-based class is 31%. The average post-test score for algebra-based class is 44% and for calculus-based class is 47%. The graduate students' scored 70% in that research (Maloney et al. 2001). So by comparing Pitt students' performance with students' performance at other institutions, we find that our algebra-based students performed slightly lower than those from other institutions on the pre-test and post-test. Our calculus-based students and graduate students performed better than students at other institutions. Table 6.16 shows that the average pre-test score on MCS for algebra-based students is 24% and for calculus-based students is 28%. The average post-test score on MCS for algebra-based students is 32% and for calculus-based students is 46%. The results suggest that algebra-based and calculus-based students performed better on CSEM than MCS as a pre-test and a post-test. Graduate students performance on both tests is about the same. Future research can investigate whether the better performance of introductory students on CSEM is due to the inclusion of the content of electricity on CSEM (but not on MCS) or due to other reasons (e.g., a wider variety of questions on magnetism in the MCS).

Table 6.15 : Overall results for CSEM pre-test and post-test

Course	Pre-test	(Standard deviation)	n	Post-test	(Standard Deviation)	n
Algebra	24%	8%	95	36%	13%	95
Calculus	36%	13%	355	53%	18%	355
Graduate				82%	13%	26

Table 6.16 : Overall results for MCS pre-test and post-test

Course	Pre-test	(Standard deviation)	n	Post-test	(Standard Deviation)	n
Algebra	24%	8%	83	32%	15%	83
Calculus	28%	10%	355	46%	19%	355
Graduate				83%	17%	42

We also compared students' performance in algebra-based class and calculus-based class. Table 6.17 shows that for the pre-test the mean for algebra-based courses is 7.6 compared to the mean of calculus-based courses, 11.7. The p-value, 0.00, suggests significant difference between algebra-based courses and calculus-based courses on the pre-test. Table 6.18 shows the results for the post-test. The means are 11.4 and 16.9 for algebra-based and calculus-based courses respectively. The p-value, 0.00, suggests that the significant difference between algebra-based courses and calculus-based courses persist on the post-test.

Table 6.17 : Algebra-based course vs. Calculus-based course pre-test performance

	N	Mean	S.D.	P value
Algebra	95	7.6	2.5	0.00
Calculus	355	11.7	4.2	

Table 6.18 : Algebra-based course vs. Calculus-based course post-test performance

	N	Mean	S.D.	P value
Algebra	95	11.4	4	0.00
Calculus	355	16.9	5.7	

The fact that students from algebra-based and calculus-based courses perform differently on CSEM is consistent with what we found on MCS. As noted earlier, one possible reason for this difference may be that calculus-based students' mathematical skills and scientific reasoning skills may help them to understand the physics concepts better.

6.9 SUMMARY

We developed a research-based survey on magnetism and administered the MCS as a pre-test and a post-test in introductory physics classes. We find no gender difference in the algebra-based courses but a significant gender difference in both the regular and honors calculus-based courses on the post-tests (but not on the pre-tests). Further research is needed to investigate the reasons for these differences. We also compared algebra-based students' performance with calculus-based students. A difference was observed on the pre-test which persists on the post-test.

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7.0 CONCLUSION AND FUTURE CONSIDERATIONS

In this thesis, we discuss investigations of students' common difficulties in learning electricity and magnetism and the development and evaluation of some research-based learning tools to improve students' understanding. We investigated students' conceptual understanding of circuit elements and of equations related to circuits. We developed research-based learning tutorials, and the corresponding pre- and post-tests on Coulomb's law, superposition principle and Gauss's law to help students build a robust knowledge structure of the relevant concepts. We also designed a research-based Magnetism Conceptual Survey (MCS) as an assessment tool to evaluate students' understanding of magnetism concepts.

7.1 INVESTIGATION OF STUDENTS' DIFFICULTIES

We investigated students' reasoning with circuit elements involving non-identical light bulbs and with equations related to circuit elements. By administering multiple-choice and free-response questions, students' common difficulties were observed.

We can further our study by collecting more data. For example, for the circuit element questions, we only collected data from graduate students in the multiple choice format. We can give them free-response format in the coming year to see if there is a difference in their performance. The multiple-choice version of the light bulb questions with a given resistance was

not given to introductory physics students. It would be interesting to collect data on introductory students' performance and compare it with that of the graduate students (whose data we already have). After collecting and analyzing data, it would be very useful to develop tutorials to help students with relevant concepts. For example, we can develop tutorials that help students better understand the relation between current, voltage, resistance and power. We can also design tutorials to help students make sense of equations better. These tutorials can teach students not only to reason about equations related to circuit problems, but other equations that involve defining a physical quality or the relation between physical qualities.

7.2 COULOMB'S LAW, SUPERPOSITION PRINCIPLE AND GAUSS'S LAW TUTORIALS

In traditionally taught E&M physics courses, students typically focus more on a plug and chug approach than on conceptual learning. Students don't have sufficient motivation to deepen their conceptual understanding because conceptual reasoning is not emphasized or rewarded. However, students need effective and systematic tools to help them improve their understanding of the basic concepts. Preliminary assessments indicate that our tutorials on Coulomb's law and Gauss's law improved students' understanding of the targeted concepts and principles.

In the future, we can improve the pre-test and post-test questions further to give us a better understanding of students' difficulties. In addition, since having students work in groups on the tutorials in the class takes time, some instructors prefer to make the tutorials self-study tools. In the future, it would be very useful to evaluate students' self-monitoring skills to see how much students can learn from the tutorials when working on them individually instead of in

groups. It will also be useful to explore whether students' learning can be transferred from one context to another. We can carry out research to investigate if students learning and reasoning skills can be transferred to upper-level E&M courses and whether this transfer is challenging.

7.3 MAGNETISM CONCEPTUAL SURVEY

Based on previous investigations of students' difficulties in magnetism, we designed a research-based multiple-choice test (MCS) to evaluate students' conceptual understanding of magnetism. MCS addresses many fundamental concepts and principles in magnetism up to Faraday's law. It was administered to algebra- and calculus-based classes as a pre-test and a post-test at Pitt. Many graduate students and upper level undergraduate students also participated. The statistical results proved that MCS is reliable and valid. We compared students from different groups and obtained interesting results. Our analysis indicates that there is gender difference in calculus-based courses on the post-test and males generally performed better than females on most questions. We also observed that algebra-based students significantly performed worse than calculus-based students on MCS on both the pre- and post-tests. Also, some students performed better in general on CSEM than MCS.

Future research on MCS can focus on detailed data analysis. We can perform Differential Item Function (DIF) to study if there is gender preference on each question. It would be useful to separate students into different levels based on their pre- or post-test scores and compare the gender difference at each level. It would be interesting to investigate if students' learning is related to their initial knowledge state. Normalized gains can be analyzed and correlation

analysis with students' pre-test scores can be performed to investigate the role of prior knowledge in students learning process.

APPENDIX A

LIGHT BULB QUESTIONS (CHAPTER 2)

A.1 MULTIPLE-CHOICE LIGHT BULB QUESTIONS

A.1.1 Multiple-choice questions in the wattage version

1) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 100W light bulb will be brighter.
- b) 25W light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
- e) None of the above

2) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 100W light bulb will be brighter.
- b) 25W light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
- e) None of the above

A.1.2 Multiple-choice questions in the resistance version

1) Two light bulbs have 200 ohm and 500 ohm resistances. They are connected in parallel to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 200 ohm light bulb will be brighter.
- b) 500 ohm light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 200 ohm bulb will be brighter.
- e) None of the above

2) Two light bulbs have 200 ohm and 500 ohm resistances. They are connected in series to each other and to a 20 V ideal power supply. Which one of the following statements is correct about the relative brightness of the light bulbs assuming their resistances to be ohmic?

- a) 200 ohm light bulb will be brighter.
- b) 500 ohm light bulb will be brighter.
- c) The light bulbs will be equally bright.
- d) Initially, they will be equally bright but after a few minutes the 100 W bulb will be brighter.
- e) None of the above

A.2 FREE-RESPONSE LIGHT BULB QUESTIONS

A.2.1 Free-response questions in the wattage version

1) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be ohmic? You must explain your reasoning.

2) Two light bulbs are rated 100 W and 25 W (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be ohmic? You must explain your reasoning.

A.2.2 Free-response questions in the resistance version

1) Two light bulbs are rated 200 ohm and 500 ohm (for a standard 120 V power supply). They are connected in parallel to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be ohmic? You must explain your reasoning.

2) Two light bulbs are rated 200 ohm and 500 ohm (for a standard 120 V power supply). They are connected in series to each other and to a 20 V ideal power supply. Which light bulb is brighter assuming their resistances to be ohmic? You must explain your reasoning.

APPENDIX B

CIRCUIT ELEMENTS QUESTIONS (CHAPTER 3)

B.1 MULTIPLE-CHOICE CIRCUIT ELEMENTS QUESTIONS

1) The resistance of a cylindrical ohmic resistor at a fixed temperature depends on:

- (I) the current;
- (II) the potential difference across it;
- (III) the cross-sectional area;
- (IV) the length of the resistor.

- Answers:
- A. (I) and (II) only;
 - B. (III) and (IV) only;
 - C. (I), (II) and (III) only;
 - D. (I), (II) and (IV) only;
 - E. All of the above

2) The capacitance of a parallel plate capacitor depends on:

- (I) the charge on the plates
- (II) the potential difference across the plates
- (III) the area of the plates
- (IV) the distance between the plates

- Answers:
- A. (I) and (II) only;
 - B. (III) and (IV) only;
 - C. (I), (II) and (III) only;
 - D. (I), (II) and (IV) only;
 - E. All of the above

3) The inductance of an inductor (long solenoid) depends on:

- (I) the current
- (II) the magnetic flux through the coil and number of turns of coil
- (III) the cross sectional area of the coil
- (IV) the number of turns per unit length

- Answers:
- A. (I) and (II) only;
 - B. (III) and (IV) only;
 - C. (I), (II) and (III) only;
 - D. (I), (II) and (IV) only;
 - E. All of the above

B.2 FREE-RESPONSE CIRCUIT ELEMENTS QUESTIONS

1) Choose all of the following quantities on which the resistance of a cylindrical ohmic resistor at a fixed temperature depends:

- (I) the current;
- (II) the potential difference across it;
- (III) the cross-sectional area;
- (IV) the length of the resistor.

You must explain your reasoning for your answer.

2) Choose all of the following quantities on which the capacitance of a parallel plate capacitor depends:

- (I) the charge on the plates
- (II) the potential difference across the plates
- (III) the area of the plates
- (IV) the distance between the plates

You must explain your reasoning for your answer.

3) Choose all of the following quantities on which the inductance of an inductor (long solenoid) depends:

- (I) the current
- (II) the magnetic flux through the coil and number of turns of coil
- (III) the cross sectional area of the coil

(IV) the number of turns per unit length

You must explain your reasoning for your answer.

APPENDIX C

TUTORIALS AND PRE-/POST-TESTS ON COULOMB'S LAW AND SUPERPOSITION

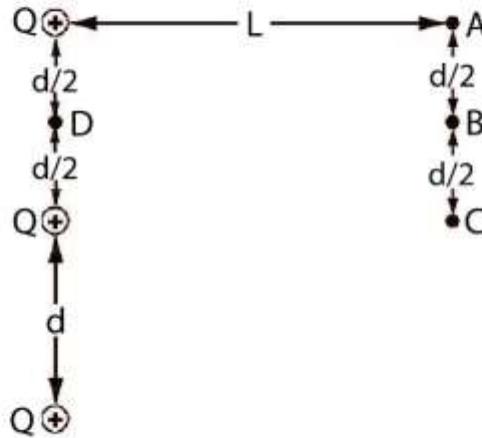
PRINCIPLE (CHAPTER 4)

C.1 TUTORIAL I

C.1.1 Pre-test

Pre-test: Coulomb's Law, Superposition Principle and Symmetry

Consider an isolated system of three identical point charges $+q$ arranged in a line such that adjacent ones are equidistant. We measure the net electric field at four points (three on a parallel line and point D equidistant from two of the charges on the straight line joining them) as shown:

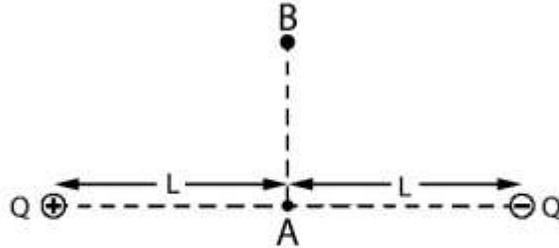


- (1) Draw arrows to show the approximate directions of the net electric field at each labeled point.
- (2) At which point (or points) can you *exactly* predict the angle the net electric field makes with the horizontal without knowing the numerical value of q ? Explain your reasoning.
- (3) Is the net electric field zero at any of the points shown? If so, where? Explain.
- (4) Is the magnitude of the net electric field at any of the points shown definitely the same as it is at point D? Which ones? Explain.

C.1.2 Post-test

Post-test: Coulomb's Law, Superposition Principle and Symmetry

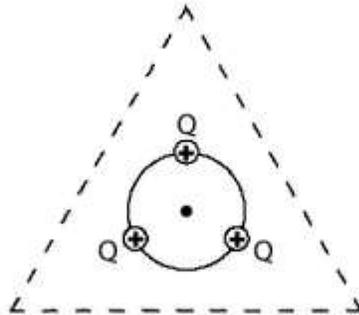
(1) Shown below are two charges which have the same magnitude but opposite signs. Points A and B are on the perpendicular bisector of the straight line joining the charges with point A being located on the straight line joining the two charges.



(a) Draw the directions of the net electric field at points A and B . Explain your reasoning.

(b) Consider the following statement from Emily: "The net electric field at point A is zero". Explain why you agree or disagree with her.

(2) A thin non-conducting ring has three identical equally spaced positive point charges, which define the vertices of an equilateral triangle. The dotted triangle shows an imaginary equilateral triangle concentric with the ring and with the same orientation as the equilateral triangle whose corners are the three point charges:

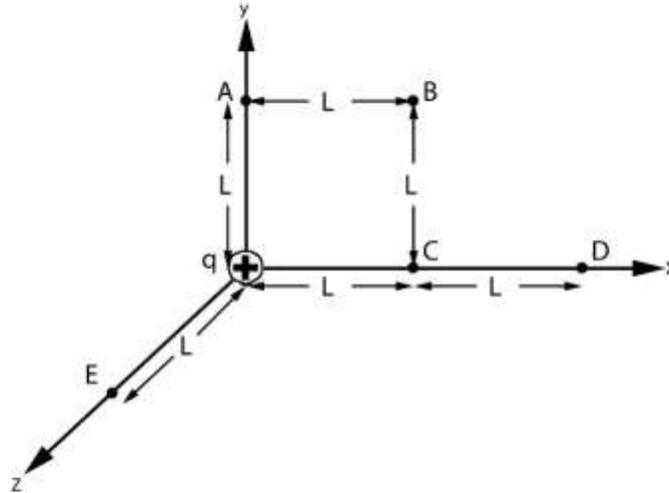


Consider the following statement from Sam: "The magnitude of the net electric field is the same everywhere on the dotted imaginary triangle because it has the same symmetry as that of the charges". Explain why you agree or disagree with him.

C.1.3 Tutorial

Coulomb's Law, Superposition Principle, and Symmetry

(I) Consider an isolated point charge $+q$ at the origin of a co-ordinate system. We measure the magnitude and direction of the electric field due to $+q$ at several points near it whose (x, y, z) co-ordinates are: A: $(0, L, 0)$, B: $(L, L, 0)$, C: $(L, 0, 0)$, D: $(2L, 0, 0)$, E: $(0,0,L)$, as shown below:

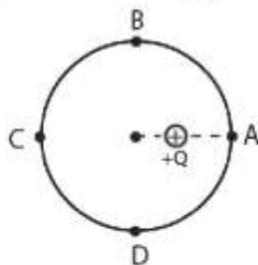


(1) Can you predict the EXACT direction of the electric field due to $+q$ at any or all of the points indicated? If so, use an arrow to indicate the direction at each such point on the drawing above. The tail of each arrow should always be drawn at the point where you are finding the electric field.

(2) At which other labeled points above does the electric field have the same magnitude $|\vec{E}|$ as that at point A? Write down $|\vec{E}|$ at those points in terms of q and L .

(3) Now imagine all points (not just those shown explicitly in the figure) that have the same electric field magnitude as that at point A. What is the shape of the surface formed by all these points? (Hint: Think in three dimensions.)

(II) (a) Consider the following situation in which a point charge $+Q$ is located inside an imaginary sphere. Shown below is a cross-section of the sphere. Note that $+Q$ is off center.



(1) Is the magnitude of the net electric field due to the charge $+Q$ the same everywhere on this spherical surface? Explain briefly.

(2) Draw arrows to indicate the approximate direction and relative magnitude of the net electric field at the four points shown above. (Hint: If the relative magnitude is larger, the arrow should be longer. Coulomb's law gives the magnitude and direction of the electric field at a point due to a point charge.)

(3) Draw arrows straight out from the center of the imaginary sphere (radially outward) at the four points on the spherical surface shown. Is the angle between the radial direction (the direction straight out from the center of the sphere) and the direction of the net electric field the same for all four points? Explain. If it is the same, what is the angle?

(II) (b)

The following analogy between a light bulb and a positive charge will help you reason about the electric field due to a point charge.

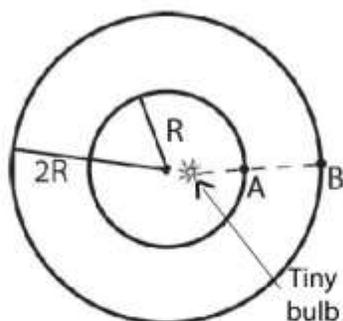
WARNING: The following analogy is only appropriate for a single point charge.

Consider the following statements from John and Susan about an idealized 60 W light bulb which can be considered to be a point source of light which radiates evenly in all directions.

- John: "If the bulb is anywhere inside a spherical surface, all points on the surface of the sphere MUST be equally lit."
- Susan: "I disagree. That will only be the case if the bulb is at the center of the sphere."

(1) With whom, if either, do you agree? Draw one situation with a tiny bulb inside a spherical surface of radius R that will show why you do not agree with the other person.

(2) Consider two concentric transparent imaginary spherical surfaces of radii R and $2R$ with an off-center light bulb as shown. Will the brightness at the two points A and B shown below on the two surfaces be the same or different (The bulb and points A and B are in a straight line)? Explain.



Use the following analogy as a guide to answering the questions below:

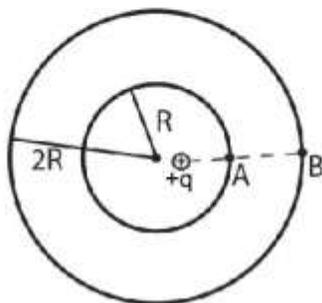
- brightness at a point \rightarrow electric field magnitude at a point

Consider the following statements from Emily and Susan:

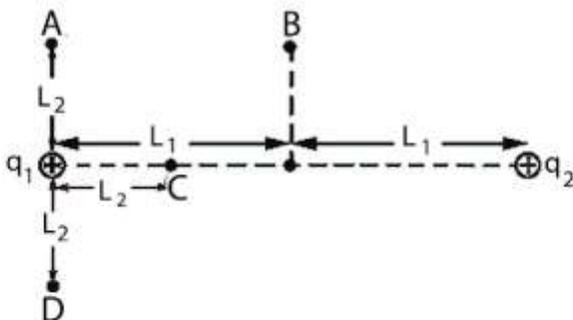
- Emily: "If you know that a point charge is anywhere inside a spherical surface, the magnitude of the electric field at all points on the surface of the sphere MUST be the same."
- Susan: "I disagree. That will only be the case if the point charge is at the center of the sphere."

(4) With whom, if either, do you agree? Draw one situation with a point charge $+q$ inside a spherical surface of radius R that will show why you do not agree with the other person.

(5) Consider two concentric imaginary spherical surfaces of radii R and $2R$ with an off-center point charge $+q$ as shown. Will the electric field at the two points A and B shown below on the two surfaces be the same or different? Explain.



(III) Consider two positive point charges q_1 and q_2 with $q_1 > q_2$.



(1) Consider the following statements from John and Mary:

- John: The net electric fields at points A, C, and D must have the same magnitude because only the nearest charge contributes to the electric field at a point. q_1 is equidistant from all these three points.
- Mary: I disagree. Both q_1 and q_2 will contribute to the net electric field at each point. At each point A, B, C and D, we must use the superposition principle and vectorially add the electric fields due to both charges.

With whom, if either, do you agree? Explain.

(2) At all of the labeled points above, draw two arrows each showing the contribution to the net electric field due to the individual charges q_1 and q_2 . At each of these points, label these arrows \vec{E}_1 , and \vec{E}_2 . The tail of each arrow should be at that point. Then, draw an approximate direction for the net electric field \vec{E}_{net} at each of the labeled points.

(3) Can you *exactly* predict the direction of the net electric field at any of the labeled points just by observation? If so, where? What if anything is special about that point or points?

(4) At which points shown is the *magnitude* of the electric field the same? Explain.

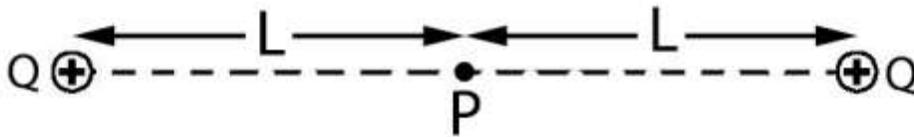
(5) If $q_1 = q_2$, can you now determine the exact direction of the net electric field at point B? Explain.

(6) Consider the following statements from Mira and Anita:

- Anita: If $q_1 = q_2$, the magnitudes of the net electric field at the four points above will be different than if $q_1 > q_2$ but not their directions. For example, the fields due to q_1 and q_2 at point A will each point radially away from those charges regardless of whether $q_1 = q_2$ or $q_1 > q_2$.
- Mira: I disagree. Both the magnitude and direction of the net electric field at point A will in general change if $q_1 = q_2$ as opposed to $q_1 > q_2$. You are getting confused between the direction of the electric field due to individual charges and the direction of the “net” electric field which is their vector sum. For example, the electric field at point B is vertically upward if $q_1 = q_2$ but not otherwise because the horizontal components of the fields due to individual charges q_1 and q_2 cancel out only if $q_1 = q_2$.

With whom, if either, do you agree? Explain.

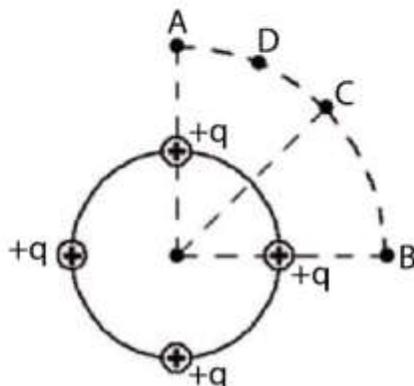
(7) Consider the following conversation between Pria and Mira about the electric field at point P which is midway between two identical charges $+Q$ on the straight line joining them:



- Pria: The net electric field at point P due to the two charges is NOT zero because both charges have the same sign. The net field is two times that if only one charge was present.
- Mira: I disagree. Since both point charges have the same magnitude and are equidistant from point P, the electric field due to each will have the same magnitude at point P. However, the directions of the field due to each charge will be radially away from *that* charge and will oppose each other. Therefore, the net field at point P, which is the vector sum of the individual fields, will be zero.

With whom, if either, do you agree? Explain.

(IV) Consider a thin nonpolarizable non-conducting ring on which four small regions, each 90° from the next, have equal amounts of positive charge as shown below. You can consider each of the localized charged regions to be a point charge. Points A, B, C and D are in the plane of the ring and equidistant from the center. Points A and B are straight out from two of the charges and point C is equidistant from points A and B. Point D is equidistant from points A and C as shown below.



(1) At all of the labeled points above, draw four arrows each showing the contribution to the net electric field due to individual charges. At each point, label these arrows \vec{E}_1 , \vec{E}_2 , \vec{E}_3 , and \vec{E}_4 . The tail of each arrow should be at that point.

(2) For each point, choose the radial direction (the direction straight out from the center) as the x axis and the direction perpendicular to it the y axis. Resolve all the four vectors \vec{E}_1 , \vec{E}_2 , \vec{E}_3 , and \vec{E}_4 into x and y components. Based upon your x and y components at each point, estimate the approximate directions of the net electric field due to the four charges at each of the points shown above. Then, draw arrows labeled \vec{E}_{net} to show the approximate direction at each point. The tail of each arrow should be at the point where you are finding the electric field.

(3) At which points above can you determine the exact direction of the electric field due to the four charges simply by symmetry consideration (without numerical calculation)?

(4) At which points shown is the magnitude of the electric field the same as it is at point A? Explain.

(5) Consider the following conversation between Mira and Anita:

- Anita: The magnitudes of the electric field at points A, B, and C due to the four points charges will be the same because all three points are symmetrically situated and at the same distance from the center of the ring.

- Mira: I disagree. The fields have the same magnitude only at points A and B. To determine the field at any of these points we can first use Coulomb's law to find the field due to the individual charges and then add them vectorially. Since point C does not have the same symmetry as the other two points with respect to the charge distribution, the field magnitude at point C will not in general be the same as that at the other two points.

Explain the flaw in Anita's argument and why Mira is correct.

(6) Consider the continuation of the above conversation:

- Anita: I don't get it! The field due to the individual charges q_1 and q_2 is stronger at point A than that at point C since those charges are closer to point A. On the other hand, the field due to the other two charges is stronger at point C than that at point A since those charges are closer to point C. Then, shouldn't the math just work out and the "net" fields at points A and C will be the same since both points are the same distance from the center?
- Mira: No...Just because two of the point charges are closer to point A and the other two are closer to C, the vector sum of the electric fields will NOT miraculously work out to be the same at both points. At any point, when you add the fields vectorially due to all charges, the details of the direction must come into play in determining both the magnitude and direction of the "net" field.

With whom, if either, do you agree? Explain.

(7) Consider the continuation of the above conversation:

- Anita: Hmm...I thought whenever you have charges on a circle, the electric field magnitude is the same at all points at a fixed distance away from the center in the plane of the circle.
- Mira: You are confusing the symmetry of the charge distribution with the symmetry of the non-conducting ring. If the charge was uniformly distributed on the ring, points A, B, C and D will all be equivalent with respect to the charge distribution. Think about removing the ring but still holding the four charges where they are. The electric field due to the four charges will be unchanged at A, B, C and D or any other point for that matter.

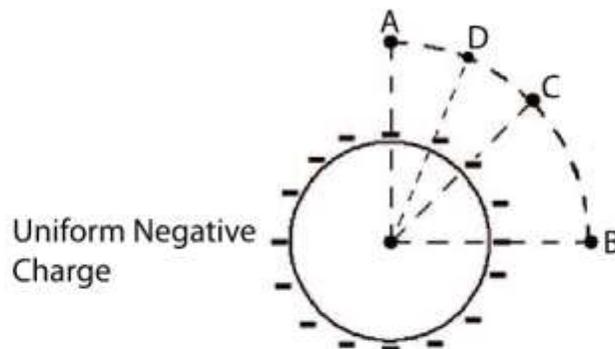
With whom, if either, do you agree? Explain.

(8) Consider the following statement from Anita:

- Anita: I get it now...isn't it true though that the direction of the field will be straight out from the center at points A, B and C but not at point D because we have to find the vector sum of the fields due to the individual charges to determine the "net" field at a point? Point D is not symmetric with respect to the charge distribution.

On the figure at the beginning of section (IV), draw the direction of the electric field at point D due to each of the four charges and label them \vec{E}_1 through \vec{E}_4 . Then discuss whether the net electric field at point D will be radial or not. Do you agree with Anita?

(V) Consider the case where charge $-Q$ is uniformly distributed on the entire nonpolarizable non-conducting ring (note that the charge is negative). Points A, B, C and D are in the plane of the figure and equidistant from the center.



(1) Draw arrows labeled \vec{E}_{net} to show the approximate direction of the electric field at each of the points shown above. The tail of each arrow should be at that point. At which points can you draw arrows showing the direction of \vec{E}_{net} just by observation?

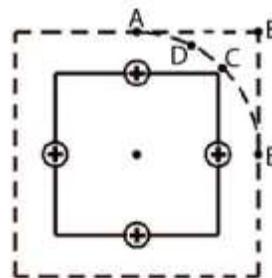
(2) At which points shown is the magnitude of the electric field the same as it is at point A? Explain.

(3) Consider the following statement from Anita:

- Anita: We can consider the ring to be made of a large number of point charges. We can use Coulomb's law to find the contribution of each of these point charges and then vectorially add their contributions to find the net field at each of the points shown. The magnitude of the electric field should be the same at all these points which are the same distance away from the center because all of them are equivalent with respect to the uniform charge distribution on the ring. The direction of the net electric field is radially inward since the charge is negative.

Do you agree with Anita? Explain.

(VI) Consider a thin nonpolarizable non-conducting square on which four identical positive point charges are symmetrically located as shown. Points A, B, C and D are in the plane of the figure and equidistant from the center of the square. Points A and B are straight out from two of the charges and point C is equidistant from both A and B. Point D is equidistant from both A and C. Point E is also in the plane of the figure and is located on an imaginary square (concentric with the non-conducting square) on which points A and B also lie.



(1) At which of the five points is the magnitude of the net electric field due to the four charges the same? Explain.

(2) Is the net electric field due to the four charges the same at all points on the dashed imaginary square? Explain.

(3) Draw arrows labeled \vec{E}_{net} to show the approximate directions of the net electric field at all the five points shown. The tail of each arrow should be at that point.

(4) Is the net electric field \vec{E}_{net} radially outward (straight out from the center) at all the five points? If your answer is "NO" for any of the points, draw four more arrows labeled \vec{E}_1 , \vec{E}_2 , \vec{E}_3 and \vec{E}_4 (with their tails at that point) showing the contributions to the net electric field due to each of the four point charges at one labeled point where the field is not radial. Then, explain in words, how you can tell that \vec{E}_{net} is not radial there.

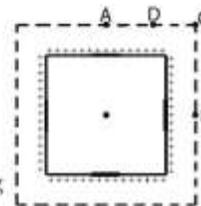
(5) Consider the following statement from Rita about the figure above:

- Rita: The net electric field is determined by the charge distribution and NOT by the shape of the nonpolarizable object in which the charges are embedded. Since the charge distribution in this problem is exactly the same as that in problem (IV), the electric field pattern in both cases will be the same everywhere.

Do you agree with Rita? Explain.

(6) Consider the following statement from Rita about the figure on the right:

- Rita: Although now the charge is uniformly distributed on the non-conducting square, the net electric field will NOT be the same at all points on the dashed imaginary square.



Using superposition principle explain why Rita is correct. Argue why the magnitude of the electric field cannot be the same at all four points shown. Which points shown have the same $|\vec{E}_{net}|$?

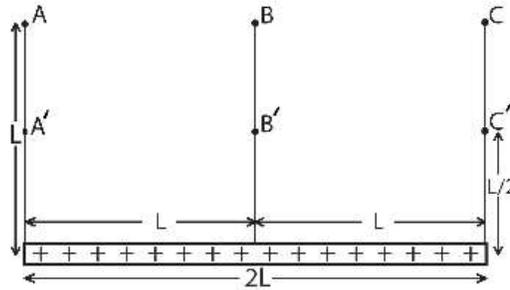
C.2 TUTORIAL II

C.2.1 Pre-test II

Pre-test II: Superposition Principle and Symmetry

- Assume all insulators (non-conductors) are non-polarizable.

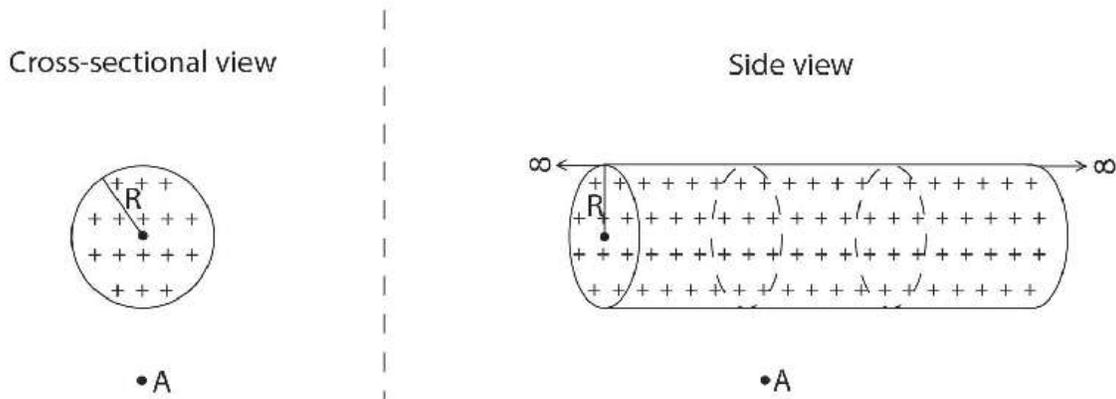
(1) Consider the three points A, B and C shown below, each at a distance L above a finite uniform line of positive charge of length $2L$. Draw arrows to show the direction of the net electric field at each of these three points. Explain.



(2) Consider the three points A' , B' , and C' shown above, each at a distance $L/2$ above the finite uniform line of charge. Draw arrows to show the direction of the net electric field at each of these three points. Explain how the direction at point A compares with A' and the direction at point B compares with B' .

(3) Shown below are two views of an infinitely long solid cylinder of radius R with uniform charge distributed throughout its volume with charge per unit length $+\lambda$. On the cross-sectional view below, draw all points in the plane of the paper at which the net electric field has the same *magnitude* as at point A. Explain.

Infinitely long uniformly charged non-conducting solid cylinder

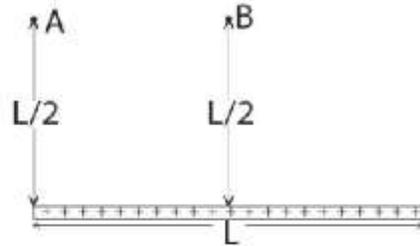


(4) On the cross-sectional view in question (4), draw arrows to show the direction of the net electric field at four of the points that have the same magnitude of the net electric field as point A.

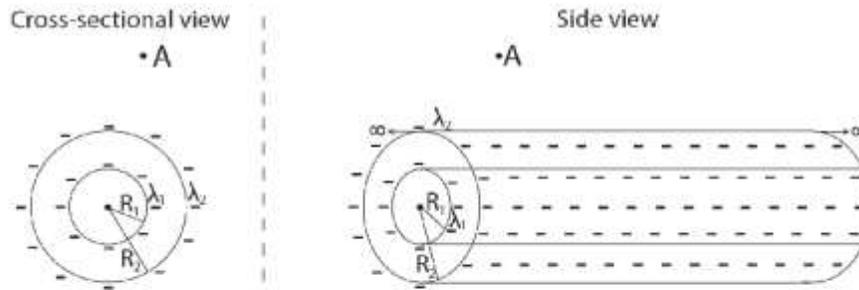
C.2.2 Post-test II

Post-test: Superposition Principle and Symmetry

(1) Consider the following statement from Susan about the electric field due to a *finite* line of length L with a uniform charge: "The field at a distance $L/2$ above the line at both points A and B (see figure) is directed perpendicular to the line of charge." Give a convincing argument that either supports her statement or refutes it.

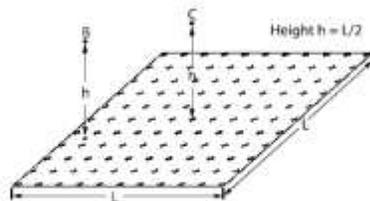


(2) Imagine two concentric infinitely long hollow plastic cylinders of radii R_1 and R_2 with uniform surface charge. The linear charge densities are $-\lambda_1$ and $-\lambda_2$ respectively. On the cross-sectional view below, draw all points in the plane of the paper at which the net electric field has the same *magnitude* as at point A. Explain. Draw the directions of the net electric field at four of those points.



(3) What is the shape of the three dimensional imaginary surface formed by all points that have the same magnitude of the net electric field as point A in question (2) above?

(4) Consider a square sheet of length L on each side on which positive charge is uniformly distributed with a charge per unit area σ (surface charge density). Consider two points, each at a height $h = L/2$ above the sheet: point C is directly above the center of the sheet and point B is off center. What if anything, can you say about the directions of the net electric field at points B and C? Explain. Are the magnitudes of the net electric field at points B and C equal? Explain.

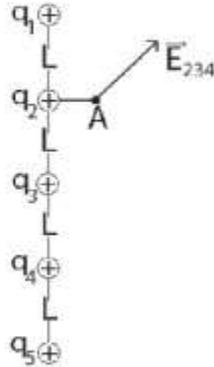


(5) Consider the limit as the length of each side goes to infinity ($L \rightarrow \infty$). What if anything, can you say about the direction of the net electric field at points B and C? Explain. Are the magnitudes of the net electric field at points B and C equal? Explain.

C.2.3 Tutorial II

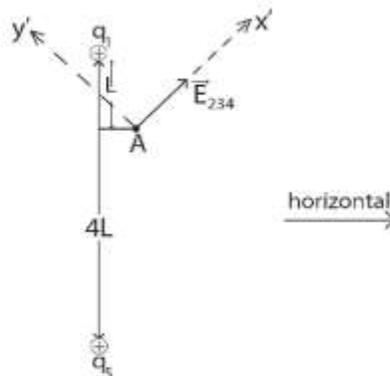
Superposition Principle and Symmetry

Five identical positive point charges are in a straight line separated by equal distance L from each other as shown below. The distance of point A from the straight line through the charges is less than L . The direction of the electric field at point A due to ONLY the three middle charges q_2, q_3, q_4 is shown by the arrow labeled \vec{E}_{234} . The perpendicular dropped from point A onto the straight line through the charges passes through q_2 .



(1) Does the net electric field at point A due to all of the five charges make a smaller or larger angle with the horizontal than \vec{E}_{234} ? Explain your reasoning.

To confirm your prediction above, let's vectorially add the contributions due to the charges q_1 and q_5 to the field \vec{E}_{234} (which is only due to the three middle charges). Since the contributions of q_2, q_3, q_4 to the electric field at point A is \vec{E}_{234} , we do not show those charges explicitly in the figure below so that you can focus on combining the contributions of q_1 and q_5 with \vec{E}_{234} using the superposition principle.

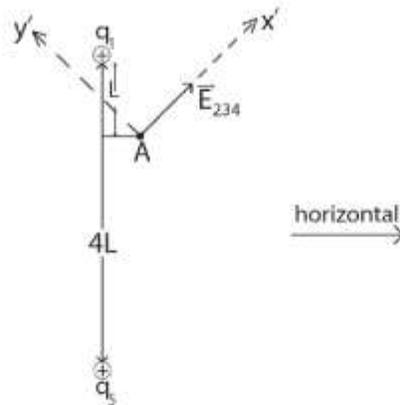


We will choose the coordinate axes along x' and y' which are parallel and perpendicular to \vec{E}_{234} :

(2) To understand why these coordinate axes are convenient, answer the following question: If the y' component of the net electric field due to ONLY the charges q_1 and q_5 is along the negative y' direction and \vec{E}_{234} does not have a y' component (it is completely along the x' direction), does the net electric field

due to all five charges (when you vectorially add the field due to q_1 and q_5 with \vec{E}_{234}) make a smaller or larger angle with the horizontal than the angle made by \vec{E}_{234} ONLY?

Let's work through the problem:



(3) In the figure above, draw separate arrows showing the direction of the electric fields \vec{E}_1 and \vec{E}_5 at point A due to the charges q_1 and q_5 , respectively. The tail of the arrows should be at point A. Does \vec{E}_1 or \vec{E}_5 have a larger magnitude? Explain. Make sure that the relative lengths of your arrows are qualitatively correct.

(4) In the figure above, draw the components of \vec{E}_1 and \vec{E}_5 along the coordinate directions x' and y' . Now label the x' and y' components of \vec{E}_1 and \vec{E}_5 using clear labels such as $E_{1x'}$.

(5) Is the combined contribution of \vec{E}_1 and \vec{E}_5 along x' positive or negative? Explain.

(6) Is the combined contribution of \vec{E}_1 and \vec{E}_5 along y' positive or negative? Explain.

(7) Will the component of the *net* electric field due to all five charges along x' be larger than \vec{E}_{234} (the field due to only the middle three charges, which is completely along x')? Explain.

(8) Will the y' component of the *net* electric field due to all five charges be positive or negative? (Hint: What is the y' component of \vec{E}_{234} ?)

(9) Based upon your previous responses should the net electric field make a smaller or larger angle with the horizontal than \vec{E}_{234} does? Explain.

(10) If we extend the line to seven identical equally spaced charges by adding q_0 at the top and q_6 at the bottom, will the net field at point A make a smaller or larger angle with the horizontal than when only five charges were present? Explain. (Hint: Is point A closer to q_0 ?)

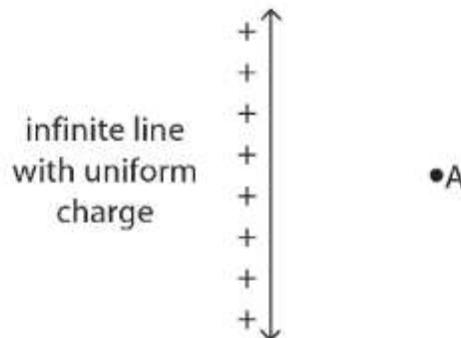
(11) Extend your findings in the previous parts to predict the direction of the net electric field at point A due to an effectively infinite number of identical positive charges in a straight line separated by equal

distance L . Explain.

(12) Suppose the discrete charge on the *effectively infinite* line of charge in part (11) is spread out evenly into a uniform continuous line of charge with charge per unit length (linear charge density) $+\lambda$. Answer the following about the net electric field due to the line of charge:

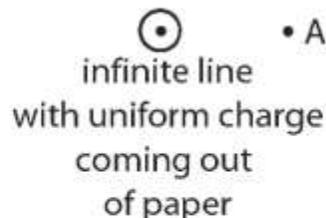
(I) Would evenly spreading the charge on the infinite line affect the *direction* of the net electric field at point A compared to the direction in part (11)? Explain.

(II) Are there other points that have the same net electric field as point A (same magnitude and direction)? In the figure below, draw a sketch showing those points and explain your reasoning.



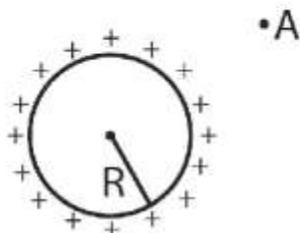
(III) Are there other points where the *magnitude* of the net electric field is the same as at point A but the direction may be different? If so, can you show *all* those points on the figure above? Explain. Draw a sketch showing the points you *can* show in the figure above. Draw the direction of the net electric field at four of those points.

(IV) In the figure below, the same line of charge is shown from a different perspective. In this view, imagine the line coming out of the paper. Draw a sketch showing all points in the plane of paper with the same *magnitude* of the net electric field as at point A. Explain. Draw arrows to indicate the direction of the electric field at four of those points.



(V) Combining your responses in parts (III) and (IV), can you predict the shape of the three dimensional surface on which the magnitude of the net electric field is the same everywhere but the direction may be different? Explain.

(13) Imagine an infinitely long hollow cylinder of radius R with uniform surface charge. The linear charge density is λ (the cylinder is perpendicular to the paper in the figure below). Draw a sketch showing all the points in the plane of the paper on which the *magnitude* of the net electric field is the same as at point A. Explain. Draw the direction of the net electric field at four of those points.



infinitely long hollow cylinder with uniform surface charge

(14) How would the magnitude and direction of the net electric field in part (13) change if the positive charge on the cylinder is replaced by uniform negative charge with a linear charge density $-\lambda$? Explain.

(15) Consider the following conversation between Pria and Mira:

- Pria: "An infinitely long uniform line of charge has the same symmetry properties as an infinitely long uniform cylinder of charge."
- Mira: "Yes, but all uniform lines of charge have the same symmetry property regardless of whether the line is finite or infinite."
- Pria: "I disagree. Imagine if the line is a uniform extended source of light instead of a line of charge which is a source of electric field. If this extended source of light is infinitely long, the surface of an imaginary tube with the light source as the axis will be equally bright. However, if the extended source of light is finite in length, the cylindrical regions around the source will be brightest at the points straight out from its center."

Explain why you agree or disagree with each of them.

(16) Consider the following conversation between Pria and Mira:

- Mira: "But even for the FINITE line of charge, shouldn't the net electric field magnitude be the same at all points which are the SAME distance away from the line regardless of whether those points are straight out from the center or not?"
- Pria: "No, you can think of the finite line as made of lots of point charges and the contribution to the net electric field at a point due to each of these point charges must be added vectorially to find the net field. Therefore, the magnitude and direction of the net field will vary depending upon whether the point is straight out from the center of the finite line or not."

Explain why you agree or disagree with each of them.

APPENDIX D

TUTORIALS AND PRE-/POST-TESTS ON GAUSS'S LAW AND SUPERPOSITION

PRINCIPLE (CHAPTER 5)

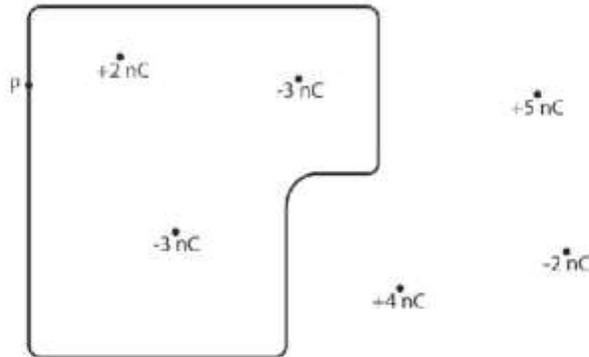
D.1 TUTORIAL III

D.1.1 Pre-test III

Pretest: Gauss' Law: Distinguishing between Electric field and flux

- For all problems below, leave your answers in terms of ϵ_0 .
- Outward electric flux through a closed surface should be considered positive.
- All physical objects in all questions are non-conducting.

(1) Consider the following sketch showing a cross-section of a closed surface with three point charges inside and three outside the surface at various locations. The surface is known to have an area of $0.4m^2$.



(a) If possible, find the net electric flux through the surface. If it is not possible to find the net flux from the information given, explain why it is impossible.

(b) If possible, find the net electric field at point P on the surface. If it is not possible to find the net field from the information given, explain why it is impossible.

(c) Can you draw a surface that contains some of the charges through which the net electric flux is zero? If so, draw its cross section on the diagram above; if not, explain why it is impossible.

D.1.2 Post-test III

Posttest: Gauss' Law: Distinguishing between Electric field and flux

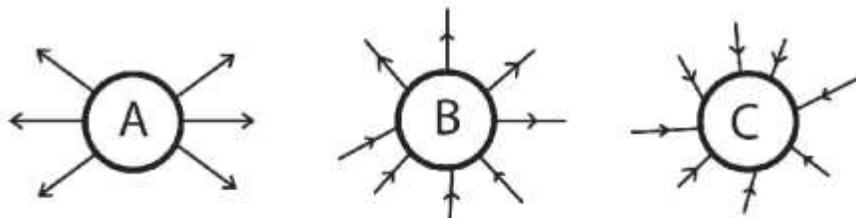
- For all problems below, leave your answers in terms of ϵ_0 .
- Outward electric flux through a closed surface should be considered positive.
- All physical objects in all questions are non-conducting.

(1) You are told that the net electric flux through the surface of a sphere (radius= $0.10m$, area= $0.13m^2$, volume= $0.004m^3$) is $-26Nm^2/C$. Use this information to answer the following:

(a) Either find the net charge inside the sphere or explain why it is impossible to do so.

(b) Either find the magnitude and direction of the net electric field at a point P somewhere on the surface of the sphere or explain why it is impossible to do so.

(2) The diagrams below give the pattern of the electric field lines near each of the three insulating balls that may or may not be charged. Sadly, we couldn't measure the electric field inside the balls, so you see only the pattern outside. The full pattern is three dimensional, of course, but these cross-sectional drawings are qualitatively correct.



(a) Which balls definitely carry a net positive charge? Explain.

(b) Which balls definitely carry a net negative charge?

(c) Which ball or balls carries a net charge of the greatest magnitude?

(d) Can we say for certain that any of the balls contain a single point charge? Explain.

D.1.3 Tutorial III

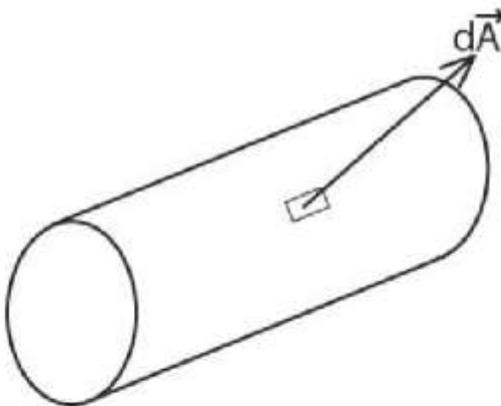
Reference Sheet

• Gauss' law lets us find the net electrical flux through any closed surface without doing an integral. It relates the net flux, Φ_E , to $Q_{enclosed}$, the total charge enclosed inside the surface:

$$\Phi_E = Q_{enclosed}/\epsilon_0 \quad (1)$$

where

- $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint |\vec{E}| \cos \theta |d\vec{A}|$ = net outward flux through the closed surface
 - Φ_E is a quantitative measure of the net number of field lines passing through the whole surface area.
 - Φ_E is a scalar which is *negative* when there are more field lines penetrating inward through the surface than outward.
- \vec{E} = electric field at a particular point on the surface
- $d\vec{A}$ = infinitesimal area vector, a vector that is normal to the infinitesimal patch of surface. For closed surfaces it points straight out (see figure). On such a tiny patch, the field \vec{E} is constant.
- θ = angle between \vec{E} and $d\vec{A}$
- \oint = circle represents an integral over a closed (Gaussian) surface
- ϵ_0 = a universal constant which gives the permittivity of free space



• Coulomb's law states that the electric field due to a point charge $+q$ at a distance r from the charge is radially outward from the charge and its magnitude is:

$$|\vec{E}| = k|q|/r^2 \quad k = 1/(4\pi\epsilon_0) \quad (2)$$

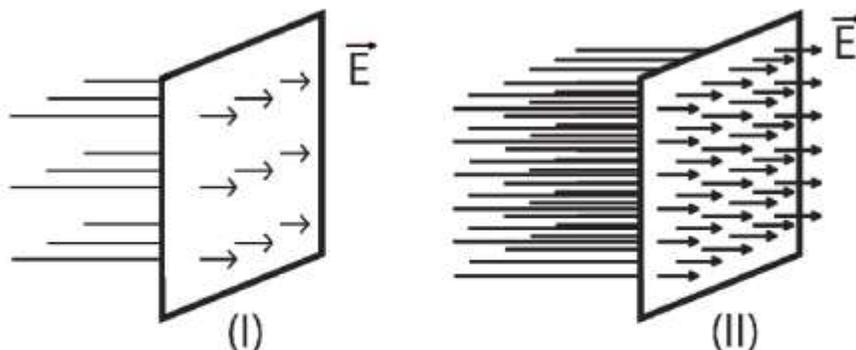
For a negative point charge $-q$, the field has the same magnitude, but its direction is radially inward toward the charge.

A guide to interpreting or drawing diagrams

- \vec{E} at a point due to a charge distribution, should be represented with a small arrow with its tail at that point. The length of the arrow qualitatively represents the field magnitude and its direction shows the field direction.
- Field \vec{E} is measured at a *single point* but flux Φ_E is measured through a *surface area*.

Gauss' law: Distinguishing between the electric field and electric flux

(I) The following diagrams show electric field lines penetrating through two identical square sheets. The electric field lines are perpendicular to the sheet (parallel to the area vector) in both diagrams. The diagrams are drawn to the same scale:



(1) Consider the following statements from Emily and Mary:

- Mary: The net flux through both squares is zero because there are equal numbers of field lines going in and out.
- Emily: I disagree. A square is not a closed surface like a cube. A square does not have an inside and an outside. For a cube, if the same number of field lines were going in through some faces and going out through the remaining faces, the net flux would be zero. In fact, in that case, Gauss' law tells us that there won't be any net charge enclosed inside the cube.

Explain why Emily is correct and Mary is not.

(2) Which one of the sheets (I) or (II) above has more electric flux through it? By what factor?

(3) Can you determine the exact magnitude of the electric flux through either of the surfaces above from the information provided? Explain. (Hint: Are the magnitude of the the electric field and the surface area given in the problem?)

Check: Your answers above should be consistent with the fact that the number of field lines penetrating a surface only gives a *QUALITATIVE* feel for the flux through that surface. The electric flux can be through open or closed surfaces but you can relate the net flux to the net charged enclosed within a surface using Gauss' law only for closed surfaces.

- (II) The following analogy will help you reason, *e.g.*, about why even if there is NO NET charge INSIDE a closed surface (which would imply ZERO net electric flux through the closed surface according to Gauss' law) there can be net electric field at points on the surface.

Assume that the net worth of a group of three people is the sum of the net worths of each individual, with debts counting as negative.

(1) If the net worth of a group of three people is \$3000, the net worth of each individual MUST be \$1000. Do you agree or disagree?

(2) If the net worth of a group of three people is zero, the net worth of each individual must be zero. Do you agree or disagree?

(3) If John is \$3,000 in debt and has no assets, Mary has \$5,000 in assets and carries no debts, and they have a child with \$2,000 in debt, what is the net worth of the family?

(4) Check to make sure your answers to questions (2) and (3) are consistent.

(IIIA) Distinguishing between \vec{E} and Φ_E : No NET charge inside the closed surface (zero net flux)

Use these analogies as a guide to answering the following questions:

- net worth of a group \rightarrow net charge enclosed
- net worth of an individual \rightarrow individual point charge enclosed

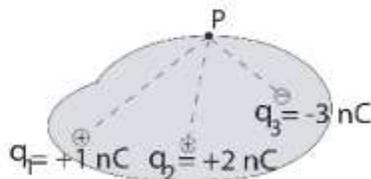
(1) If the *net* charge enclosed inside a Gaussian surface is zero, does it mean that there cannot be any charge inside the closed surface? Explain. (Hint: There are two types of charges, positive and negative.)

(2) Consider the following statements from John and Mary:

- John: If the net flux through a closed surface is zero, the space enclosed by it is empty of any charges.
- Mary: I disagree. According to Gauss' law, the net flux through a closed surface will be zero if there is zero NET charge INSIDE. For example, the net flux will be zero if there are equal amounts of positive and negative charges enclosed by the surface.

With whom, if either, do you agree? Explain.

(3) The *net* charge enclosed within the Gaussian surface shown below is zero. What is the *net* electric flux through the surface according to Gauss' law? (The NET flux through a closed surface is a measure of the NET number of field lines with field lines going out of the surface counting as positive and those going in counting as negative.)



(4) Based upon your response to questions (1) and (2) above, can there be a net electric field at point P on the Gaussian surface above even if the NET flux through the whole surface is zero?

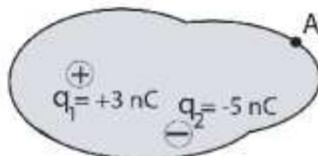
(5) Use Coulomb's law to draw three arrows labeled \vec{E}_1 , \vec{E}_2 and \vec{E}_3 to show the directions of the electric field at point P above due to each of the point charges. Draw the approximate direction of the resultant electric field (vector sum of \vec{E}_1 , \vec{E}_2 and \vec{E}_3) labeled \vec{E}_{net} at point P and explain in words how you will calculate the resultant field \vec{E}_{net} from the individual fields. Will it be useful to choose a coordinate axis to calculate \vec{E}_{net} ? Explain. The tails of \vec{E}_1 , \vec{E}_2 , \vec{E}_3 and \vec{E}_{net} should be at point P.

(6) Consider the following conversation between Pria and Mira:

- Mira: Gauss' law only applies to symmetrical closed surfaces like a sphere.
- Pria: I disagree. Gauss' law applies to ANY closed surface. If you know the net charge enclosed within a closed (Gaussian) surface such as the one above, you can easily find the NET electric flux through that closed surface using Gauss' law.

With whom, if either, do you agree? Explain.

(7) The figure below shows a Gaussian surface with two point charges inside. Is the net flux through the Gaussian surface zero according to Gauss' law? If your answer is "NO", suggest the simplest way to make the net flux through the surface zero and show your strategy on the figure below.

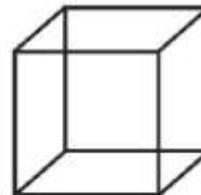


(8) Is the electric field zero at point A above after you made the net flux through the surface zero by adding or subtracting charges? If not, in the figure above, use arrows to show the directions of the electric field due to individual charges (labeled \vec{E}_1 , \vec{E}_2 , \vec{E}_3) and then the approximate direction of the net electric field \vec{E}_{net} at point A due to all the charges. Explain in words how you will calculate the resultant field \vec{E}_{net} from individual fields. Will it be useful to choose a coordinate axis to calculate \vec{E}_{net} ? Explain. The tails of \vec{E}_1 , \vec{E}_2 , \vec{E}_3 and \vec{E}_{net} should be at point A.

(9) If the net electric flux through a cube is zero, must the net electric field at all points on the surface of that cube be zero? If you disagree, draw one situation that refutes the statement.

(III.B) Distinguishing between \vec{E} and Φ_E : Net charge inside the closed surface (non-zero net flux)

(1) Consider the following statement from John: "If the net electric flux through a cube is $60 \text{ Nm}^2/\text{C}$, the flux through each of the six faces must be $10 \text{ Nm}^2/\text{C}$." Explain why John's statement is incorrect and sketch a simple counter-example by placing a point charge inside the Gaussian (imaginary) cube shown. Then, explain whether a point charge at the center of the cube would be a good counter-example.



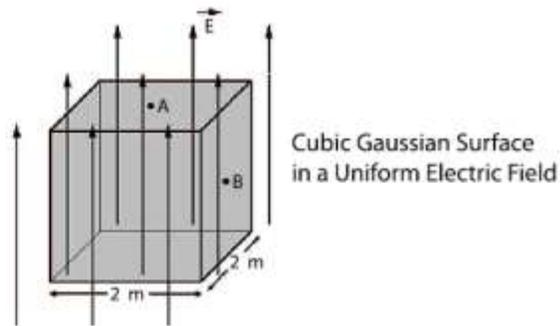
(2) Consider the following conversation between Harry and Susan:

- Harry: "If the net electric flux through a cube is $60 \text{ Nm}^2/\text{C}$, the flux through each of the six faces will be $10 \text{ Nm}^2/\text{C}$ if there is a point charge at the CENTER of the cube."
- Susan: "I agree. If the point charge enclosed is q , the flux through the whole cube will be q/ϵ_0 according to Gauss' law no matter where the point charge is located inside. But the net flux through each face will be $q/(6\epsilon_0)$ only if the point charge is at the center. Otherwise, there may be more flux through one face than another."

Do you agree with Harry and/or Susan? Explain.

(III.C) Distinguishing between \vec{E} and Φ_E : No charge inside but there are charges outside the closed surface producing electric field

A cubic Gaussian surface with $L = 2\text{m}$ on a side has two horizontal faces and is in a uniform electric field of 50N/C which is directed vertically upward.



(1) Find the net electric flux through the cube. (Hint: If there are equal numbers of field lines going into and out of a CLOSED surface, the net flux through the surface is zero. Is a cube a closed surface?)

(2) Consider the following conversation between John and Mary:

- John: "The uniform electric field lines which pass through the cube MAY be due to a parallel plate capacitor with plates which are effectively infinite in extent."
- Mary: "I agree. Since the charged capacitor plates producing the electric field are outside the cube and there is NO net charge enclosed in the cube, equal numbers of field lines are going in through one face and coming out through the other. Therefore, the net flux through the whole cube is zero."

Do you agree or disagree? Explain. If you agree, on the figure above in (III.C), draw how the parallel plates of the capacitor may be oriented clearly showing the positive and negative plates.

(3) Find the numerical value of the flux through the top and bottom faces of the cubes using $\Phi_E = |\vec{E}|A \cos(\theta)$ where A is the area of each face and θ is the angle between \vec{E} and the area vector. Explain why one of these contributions is positive and the other negative.

(4) What is the flux through each of the side faces of the cube? Explain.

(5) What is the net flux through the whole cube after you added the contributions from all faces of the cube? Make sure your answer is consistent with question (1) above.

(6) Consider the following conversation between John and Mary:

- John: "The magnitude of the electric field is 50N/C at point A but zero at point B because the electric field is perpendicular to the area vector at point B."
- Mary: "I disagree. Since the problem statement says that the field is uniform, it is 50N/C directed vertically upward at both points A and B. The angle between the field and the area vector matters only for determining the flux through each part of the surface, not for the field at a point on the surface."

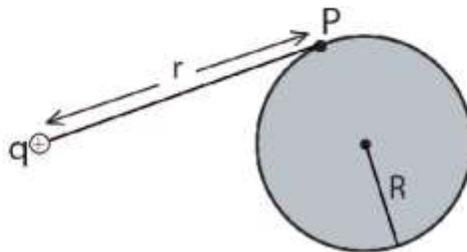
With whom, if either, do you agree? Explain.

(7) Is the electric field zero at all points on the cube? If not, what is its magnitude at points A and B shown? (Point A is on the top surface and point B on a side surface.)

(8) Using Gauss' law and your response to question (1) above for the net flux through the cube, what is the net charge enclosed within the cube?

(9) Consider the following statement from Mary: "If the net electric flux through a cube is zero, the electric flux through each face of the cube must be zero, because the electric field must be zero at all points on each face." Do you agree or disagree? Explain.

(V) Consider the following situation in which a point charge $+q$ is outside a Gaussian (imaginary) spherical surface. No other charges are around.



(1) According to Gauss' law, what is the net flux through the surface? (Hint: Gauss' law relates the net flux through a closed surface to the net charge enclosed.)

(2) Consider the following statement from John: "The electric field is zero everywhere on the sphere because there is no charge enclosed in the sphere." Explain why you agree or disagree.

(3) According to Coulomb's law, is there an electric field at a point P shown on the sphere? If so, what is the field magnitude in terms of the charge $+q$ and distance r shown? Draw the direction of the field at point P on the figure above.

(4) Make sure your answers to (2) and (3) are consistent.

(5) Based upon your response to questions (1)-(3), does zero net flux through this surface imply zero net field at every point on the surface? Explain.

(6) Consider the following statement from Harry: "The electric field lines due to the point charge $+q$ are radially away from that charge. Some of these field lines will penetrate the closed spherical surface shown above but the number of field lines going in through one part of the surface will equal the number going out through another part of the surface since the charge is outside the closed surface.

The net flux through a closed surface is the net number of field lines passing through the surface with lines going out counting as positive and those going in counting as negative.” Do you agree or disagree?

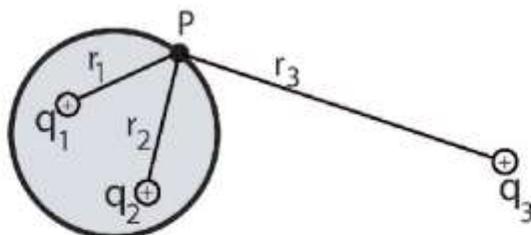
If you agree with Harry, draw two electric field lines on the figure above from the point charge that intercept the sphere twice to convince John in question (2) that even if the net flux through the whole surface is zero, the field at different points on the surface is not zero.

(7) Consider the following conversation between Pria and Mira:

- Pria: The electric flux Φ_E is a vector because it can be positive or negative.
- Mira: I disagree. The electric flux Φ_E is a scalar just like charge is a scalar even though it can be positive or negative.
- Pria: Then why is there an angle θ in $\Phi_E = \oint |\vec{E}| \cos \theta |d\vec{A}|$?
- Mira: The electric flux Φ_E depends on the SCALAR product of the electric field vector \vec{E} and the infinitesimal area vector $d\vec{A}$ and θ is the angle between those vectors.
- Pria: So should we use vector addition for electric field but scalar addition for electric flux?
- Mira: Yes, when you calculate the electric field at a point due to several charges, you have to resolve each vector into components and then add them vectorially. That’s not what we did for finding the net flux through the surfaces above. All we did was to count the field lines going out as positive and those going in as negative.

With whom, if either, do you agree? Explain.

(VI) Consider the following situation in which there are only three point charges $+q_1$, $+q_2$ and $+q_3$ near a spherical Gaussian (imaginary) surface as shown below. Point charges $+q_1$ and $+q_2$ are inside and $+q_3$ is outside the surface.



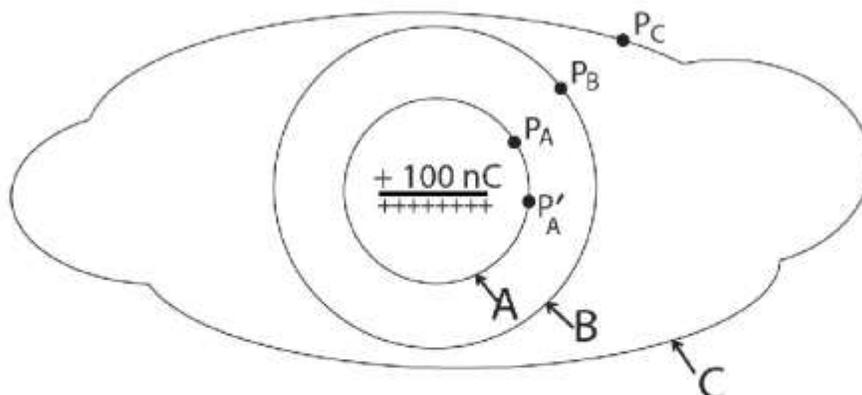
(1) Use Gauss’ law to find the net electric flux through the spherical surface in terms of the charges and the permittivity of free space ϵ_0 .

(2) Which of the charges above do NOT contribute to the net flux through the surface? In the figure above, draw two electric field lines each, starting only from those charges that will NOT contribute to the net flux. Make sure these field lines penetrate the spherical surface. Based upon your drawing explain why these charges should NOT contribute to the net flux (Hint: Are there equal number of field lines going in and out of the closed surface due to such charges?).

(3) Which of the charges above contribute to the net electric field at point P shown on the surface? Explain.

(4) Based upon your answers to (1)-(3), explain why charges that do not contribute to the net flux through a closed surface may contribute to the field at a particular point on the surface.

(VII) A thin non-conducting rod of length 1 meter, with a $+100 \text{ nC}$ (nano coulomb) positive charge uniformly distributed over it, is inside three nested Gaussian surfaces as shown below. Surfaces A and B are spherical with the non-conducting rod symmetrically placed inside and surface C is irregularly-shaped as shown:



(1) Draw four electric field lines from the $+100 \text{ nC}$ charge. (Your drawing should show only the field lines in the plane of the page, though your mental picture of the situation should include field lines radiating out in all directions, including out of and into the page.)

(2) Which surface, A, B, or C, has the most field lines going outward through it, or are they all the same?

(3) According to Gauss' law, which surface has the maximum net electric flux passing through it, or are they all the same? What is the flux through each surface? (Hint: Gauss' law relates the net flux through a closed surface to the charge enclosed.)

(4) Should your answers to parts (2) and (3) be consistent with each other? Explain. (Hint: Section (I) of this tutorial tells you how the flux relates to the number of field lines.)

(5) P_A , P_B and P_C are three points on surfaces A, B, and C, respectively, as shown above. At which of these points is the electric field magnitude greatest, or is it the same at all three points? (Hint: Are the field lines farthest apart on surface C and is the field at various points on surface C weaker than that on surface A or B?)

(6) Is the electric field magnitude the same at every point on surface A? Explain. (Hint: Imagine dividing the non-conducting rod into point charges and finding the electric field at two different points P_A and P'_A on surface A using the superposition principle.) Answer the same question for points on surface C. Explain.

(7) Can you calculate the electric field at any point on any of the surfaces without using complicated integration? Explain. Comment on why your answer makes sense even though you found the electric flux through all of the surfaces easily using Gauss' law in question (3) of this section.

(8) From your responses to the previous parts of this question, does equal flux through two surfaces imply equal field at each point on those surfaces?

(9) In the figure on the previous page, which of these can you determine without complicated integration using Gauss' law? If one of these is not easily determined, explain why not.

- Flux through a closed surface
- Field at any point on that surface

(10) Consider the following statements from Max and John:

- Max: We can use *Gauss' law* to find the electric field magnitude at a point on any of the closed surfaces due to the non-conducting rod with charge. All that matters is that the charge be inside the surface.
- John: I disagree. The net electric flux through the surface can be found easily using Gauss' law but finding the field at a point is not easy except in problems with a highly symmetric charge distribution such as a uniformly charged sphere, infinite cylinder and infinite sheet.

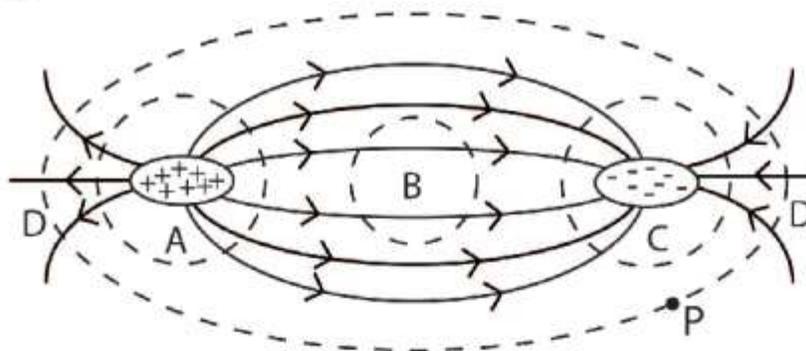
With whom, if either, do you agree? Explain.

(11) Suppose Sam measures the area of the spherical surface A above to be $2m^2$. Consider the following discussion:

- Sam: Now we can determine the electric field magnitude at any point on surface A since $|\vec{E}| = \text{Flux}/\text{Area} = +100nC/(\epsilon_0 \times 2m^2)$.
- Susan: I disagree! It is impossible to find $|\vec{E}|$ from this information. The net flux $\Phi_E = \oint |\vec{E}| |d\vec{A}| \cos \theta$ but the magnitude of the electric field $|\vec{E}|$ is not the same everywhere on the surface nor is $\theta = 0^\circ$ at every point on the surface. Therefore, we cannot pull $|\vec{E}|$ out of the integral and $\cos \theta \neq 1$ everywhere. So we cannot claim that the net flux $\Phi_E = |\vec{E}| \times \text{Area}$.

With whom, if either, do you agree? Explain. (Hint: According to Sam, the magnitude of the electric field due to the non-conducting rod with charge is the same everywhere on the spherical surface. Choose a few points on the spherical surface and comment on whether it is possible for the field magnitude to be the same at those points? Then, discuss the validity of Susan's statement.)

(VIII) The positive and negative extended charges (NOT point charges) shown here are each of magnitude 100nC :



The dotted 'bubbles' A-D shown above are cross-sections of Gaussian (closed imaginary) surfaces. P is a point on surface D. Consider outward flux to be positive.

(1) Count the "net" number of field lines crossing each Gaussian surface assuming that the contributions of the outward and inward field lines are positive and negative, respectively. Indicate which surfaces have positive, negative or zero "net" number of field lines through them?

(2) Based upon the net number of field lines in part (1), is the "net" flux positive, negative, or zero for each closed surface?

(3) Is the net charge enclosed positive, negative, or zero for each closed surface?

(4) Is there any correlation between your answers to parts (1)-(3) for each surface? Explain using the appropriate law.

(5) If the net flux through a closed surface is zero, can we definitely say that the space enclosed by it is empty of any charges? Explain your reasoning using one of the surfaces in the figure above.

(6) If the net flux through a closed surface is zero, can we say that the electric field at all points on that surface is zero? Explain. (Hint: Is the electric field at point P on surface D zero?)

(7) Can you use Coulomb's law or Gauss' law to determine the electric field magnitude at every point on any of the surfaces above without complicated integration? If so, which one(s)? If not, why not? (Hint: Is there a very high symmetry to advantageously use Gauss' law? We are NOT dealing with point charges.)

(8) Consider the following statement from John: "If more electric field lines are coming out of a closed surface than going into the surface, there must be a net positive charge inside that closed surface." Explain why you agree or disagree with John.

D.2 TUTORIAL IV

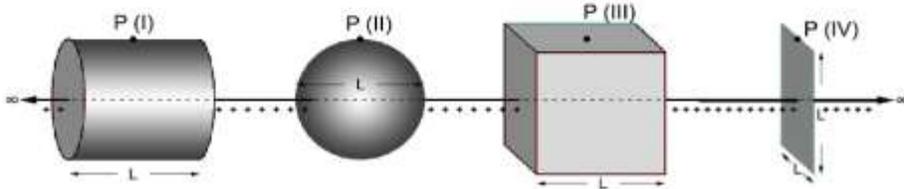
D.2.1 Pre-test IV

Pretest: Exploiting Gauss's law to find the magnitude of electric field

• Setup for the next two questions

Shown below are four hollow imaginary surfaces coaxial with an infinitely long line of charge (with uniform linear charge density λ):

- (I) a closed cylinder of length L .
- (II) a sphere of diameter L .
- (III) a closed cubic box with side L .
- (IV) a two dimensional square sheet with side L . The line of charge is perpendicular to the plane of the sheet.



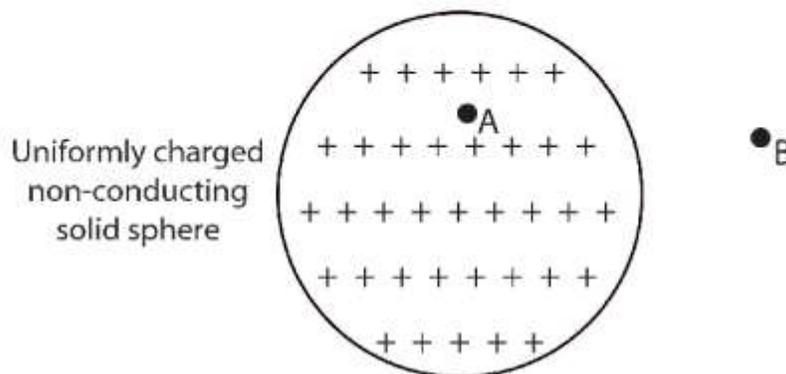
(1) List all of the above surfaces through which the electric flux is $\Phi = \lambda L / \epsilon_0$.

- (A) (I) only
- (B) (I) and (II) only
- (C) (I) and (III) only
- (D) (I), (II), and (III) only
- (E) (I), (II), (III), and (IV)

(2) List all of the above surfaces that can be used as Gaussian surfaces to easily find the electric field magnitude (due to the infinite line of charge) at a point P shown on the surface using Gauss' law:

- (A) (I) only
- (B) (I) and (II) only
- (C) (I) and (III) only
- (D) (I), (II), and (III) only
- (E) (I), (II), (III), and (IV)

(3) The following diagram shows the cross sectional view of a solid uniformly charged non-conducting sphere. To find the magnitudes of the electric field at points A (inside the sphere) and B (outside the sphere) using Gauss's law, what kind of Gaussian surface would you choose for each? Explain your reasoning. Can the same Gaussian surface be used for both points A and B? Explain.



(4) Draw cross sections of your Gaussian surfaces for each of the points A and B on the cross-sectional view above.

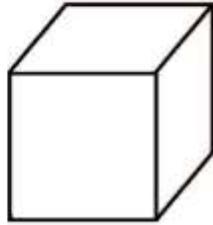
(5) Show the directions of the electric field at points A and B in the figure above due to the solid non-conducting sphere. Explain your reasoning.

(6) Consider the following statement from Harry: "To find the magnitude of the electric field at point P due to uniformly distributed charge on the surface of a finite cylinder of length L , we should imagine a Gaussian cylinder that passes through point P and is concentric with the charged cylinder. Then, Gauss' law can help us find the electric field without any complicated integrals." Explain why you agree or disagree with him.

D.2.2 Post-test IV

Posttest: Exploiting Gauss's law to find the magnitude of electric field

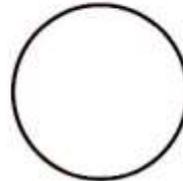
(1) Circle all of the following closed imaginary surfaces which can be used as Gaussian surfaces to determine the magnitude of the electric field due to an infinite uniform sheet of charge. For those surfaces you circled, indicate in words or drawing how you would orient the infinite sheet of charge with respect to each of the Gaussian surfaces.



Cube



Finite Closed
Cylinder



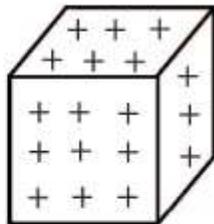
Sphere

(2) Consider hollow non-conducting objects with a uniform surface charge density. Circle *all* of the objects listed below for which we could easily use Gauss' law and symmetry considerations to find the electric field at any point due to the charge on the object:

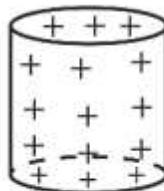
(a) cube

(b) closed cylinder of finite length L

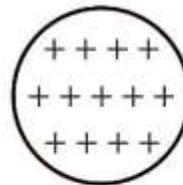
(c) sphere



Cube



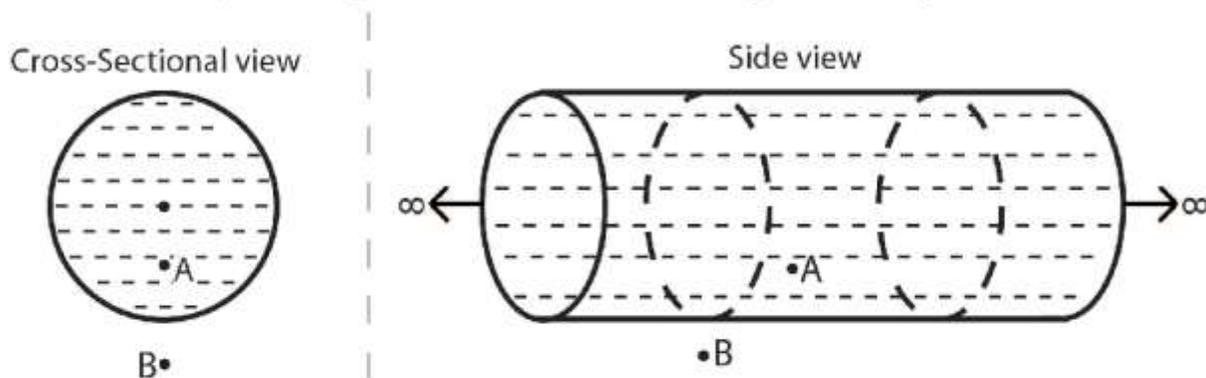
Finite Closed
Cylinder



Sphere

(3) The following diagram shows the cross sectional and side views of a solid non-conducting cylinder which is infinitely long and uniformly charged (with negative charge uniformly distributed throughout its volume). To find the magnitude of the electric field at points A (inside the cylinder) and B (outside the cylinder) using Gauss's law, what kind of Gaussian surface would you choose for each? Explain your reasoning. Can the same Gaussian surface be used for both points A and B? Explain.

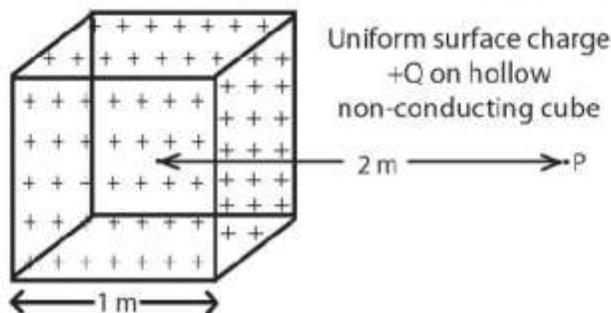
Uniformly Charged Non-Conducting Solid Cylinder



(4) Draw cross sections of your Gaussian surfaces for each point A and B both on the cross-sectional and side views below.

(5) Show the directions of the electric field at points A and B due to the solid non-conducting infinitely long cylinder with uniform charge in the figure above on both views. Explain your reasoning.

(6) Shown below is a hollow thin-walled non-conducting cube with a net charge $+Q$ uniformly distributed on its surface. The sides of the cube are $1m$ long. Point P is located outside the cube at a distance of $2m$ from its center. Can you use Gauss' law to easily find the magnitude of the electric field at point P? If so, explain what kind of Gaussian surface would you choose. If not, explain why it is not easy.



D.2.3 Tutorial IV

Exploiting Gauss's law to find the magnitude of the electric field

Now we want to connect what we have learned about symmetry to what we learned about Gauss's law in the previous tutorials.

Summary of the previous tutorial: Gauss's Law allows us to relate the net electric flux through a *closed* surface to the *net* charge inside the surface.

$$\Phi_E = Q_{enclosed}/\epsilon_0 \quad (1)$$

This means:

- If we know the net electric flux through a closed surface, we can easily find the net charge inside it.
- If we know the net charge inside a closed surface, we can easily find the net electric flux through it.

It does NOT mean that, in general, we can use Gauss's Law to easily find the magnitude of the electric field, $|\vec{E}|$, at a point. Only in situations where the charge distribution has very high symmetry can we find $|\vec{E}|$ from the net electric flux Φ_E . Let's inspect the relation between the net electric flux through a closed surface and the electric field magnitude at various points on the surface closely:

$$\Phi_E = \oint |\vec{E}| \cos \theta |d\vec{A}|. \quad (2)$$

Using the above equation, the net electric flux Φ_E over a closed (Gaussian) surface can be exploited to determine the electric field magnitude $|\vec{E}|$ at an arbitrary point P , on the surface easily only if the following conditions are met:

- We can determine the exact direction of \vec{E} relative to the area vector at *every* point on the closed surface by symmetry (only $\theta = 0^\circ, 180^\circ$ or $\pm 90^\circ$ are associated with sufficiently high symmetry).
- In some cases, we can divide the closed surface into sub-sections (for each sub-section the electric flux can be easily calculated e.g., side and two caps of a cylinder) such that one of the following is true:
 1. $|\vec{E}|$ is the same everywhere on the sub-section by the symmetry of the charge distribution.
 2. \vec{E} and the area vector (outward normal to the surface) are perpendicular ($\theta = 90^\circ$) so that there is no electric flux through that sub-section.

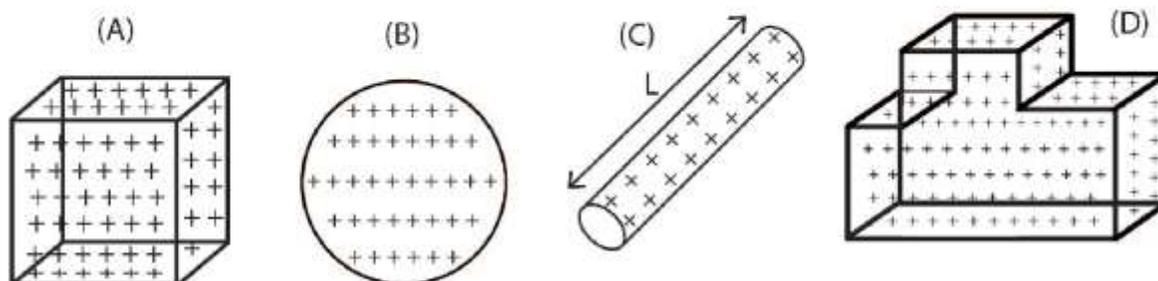
Thus, to determine if the information about the net electric flux through a closed surface can be exploited to determine $|\vec{E}|$ at a point P , we may cleverly *choose* a Gaussian (closed imaginary) surface such that:

- it contains the point P where we want to determine $|\vec{E}|$
- $|\vec{E}| \cos \theta$ is known (by symmetry) to have a constant value on *each* sub-section of the surface so that it can be pulled out of the flux integral in Eq.(2). Then, $\int |d\vec{A}|$ =total area of the sub-section of the surface.

(I) **Recognizing symmetry to calculate the magnitude of the electric field using Gauss's law**

Shown below are four hollow objects made of nonpolarizable non-conducting material. Each object is electrically isolated and has the same charge $+Q$ uniformly distributed over its outer surface. The objects are

- (A) a cube
- (B) a sphere
- (C) a cylinder of finite length L without caps
- (D) an irregularly shaped object



(1) Enclose each of the above objects with a uniform surface charge completely by a large imaginary spherical Gaussian surface (make it concentric with each object that has a well-defined center).

Consider the following conversation about whether it is possible to use Gauss's Law to find the net electric flux through the spherical Gaussian surface:

- John: "Whoah, this is easy."
- Mary: "But shouldn't the electric flux depend on whether the charges are distributed symmetrically and also on the shape of the Gaussian surface that encloses the charge?"
- John: "No Mary! The question asked about the net flux, not the electric field. And the imaginary surface is a closed surface. It is really easy to use Gauss's law to find the net flux through it if the charge enclosed is known."

With whom, if either, do you agree? Explain. What is the net electric flux through each surface? How do they compare with each other?

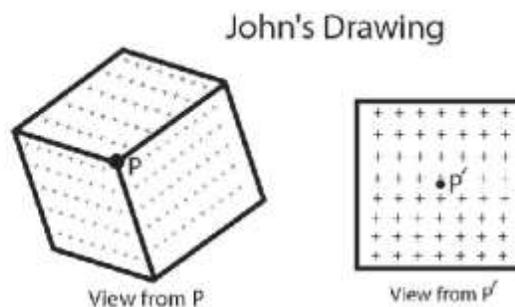
(2) For which objects above can you use Gauss's Law to easily find the magnitude of the electric field at a point on the spherical Gaussian surface? Explain.

(3) Consider the following conversation between John and Mary:

- John: "The electric field at a point on the spherical Gaussian surface can only be found for the charged sphere."

- Mary: "Since the sphere, the cube and the cylinder are symmetric shapes with uniform surface charge, shouldn't we be able to find the electric field at a point on the spherical Gaussian surface around them using Gauss's law?"
- John: "I disagree. For the cube and the cylinder, there just isn't enough symmetry! So the magnitude of the electric field, $|\vec{E}|$, or the angle θ between the electric field and the area vector on the spherical Gaussian surface will not be the same everywhere and we cannot pull them out of the integral relating net flux and $|\vec{E}|$, i.e., $\Phi_E = \oint |\vec{E}| \cos \theta |d\vec{A}|$. So even though the net flux can be found easily from $Q_{enclosed}$, we cannot find $|\vec{E}|$ at points on the surface easily."
- Mary: "I don't understand why the electric field magnitude is not the same everywhere on the spherical Gaussian surface due to the charged cube."
- John: "Look at points P and P' . If you imagine breaking the charge on the cube into infinitesimal point charges and use Coulomb's law and the superposition principle to find $|\vec{E}|$ at those two points, there is no reason why it should be the same at both points. Therefore, $|\vec{E}|$ cannot be pulled out of the integral and we cannot find $|\vec{E}|$ easily. In fact, if the charge distribution does not have the same symmetry for two points, don't assume that \vec{E} will miraculously work out to be the same at those two points."

With whom, if either, do you agree? Explain.



(4) Now enclose each of the objects on the previous page completely by a large imaginary cubic Gaussian surface (make it concentric with each object that has a well-defined center). What is the net electric flux through the Gaussian cubic surface in each case?

(5) For which objects on the previous page, if any, can the cubic Gaussian surface be exploited to easily find the magnitude of the *electric field* at a point on the surface using Gauss's Law? Explain.

(6) Your answer should be "NO" for all objects in question (5). Explain why we cannot employ Gaussian surface of any shape to easily find the magnitude of the electric field due to the cube with uniform surface charge using Gauss's law. (Hint: Think about the relationship between the flux and field $\Phi_E = \oint |\vec{E}| \cos \theta |d\vec{A}|$ and the consequences of not having $|\vec{E}|$ the same everywhere on the Gaussian surface.)

(7) Consider the following conversation about determining the magnitude of the electric field at a point nearby due to the uniform cube of charge:

- Mary: “The magnitude of the electric field due to the uniformly charged cube is NOT the same at all points on a concentric Gaussian cube. For example, on the cubic Gaussian surface, a point that is at the center of a face will not have the same electric field as another point that is off-center.
- John: “I agree! A cube with uniform surface charge is symmetric, but it is not symmetric enough to guarantee that $|\vec{E}|\cos(\theta)$ will be the same on any Gaussian surface. So we cannot find the electric field due to the charged cube easily from the information about the net electric flux through any closed surface.”

Do you agree with Mary and/or John? Make sure that your previous responses are consistent with your answer here.

(8) Consider the following conversation about the electric field at a point inside the non-conducting objects with uniform surface charge in the hollow region:

- Harry: The electric field at all points in the hollow region inside each of these charged insulating objects is zero. If we draw a Gaussian surface that is completely inside the charged object, there won't be any charge enclosed by the surface. So the net electric flux through our Gaussian surface will be zero which means the field at every point on the Gaussian surface is zero.
- Max: I disagree. Zero net flux through a closed surface does NOT in general imply zero net field at each point on the surface. The surface charges on the objects will in general produce a net *field* at points inside the objects. Mathematically, $\Phi_E = \oint |\vec{E}| \cos\theta |d\vec{A}| = 0$ does NOT mean $|\vec{E}| = 0$ because different terms in the integral (which is a continuous summation) can give positive and negative contributions to make the net flux $\Phi_E = 0$ even though $|\vec{E}| \neq 0$ at points on the surface.
- Harry: Then, why is $|\vec{E}| = 0$ everywhere inside the sphere?
- Max: For the uniformly charged spherical shell, there is enough symmetry to make $|\vec{E}| = 0$ everywhere inside. If we choose the Gaussian surface inside to be a concentric sphere, we can pull out $|\vec{E}|$ and $\cos\theta$ from $\Phi_E = \oint |\vec{E}| \cos\theta |d\vec{A}|$, because $|\vec{E}|$ and $\cos\theta$ are constant everywhere on the surface. Then, zero net flux $\Phi_E = |\vec{E}| \cos\theta \oint |d\vec{A}| = 0$ implies $|\vec{E}| = 0$ since $\cos\theta$ and the area $A = \oint |d\vec{A}|$ CANNOT be zero.

With whom, if either, do you agree? Explain.

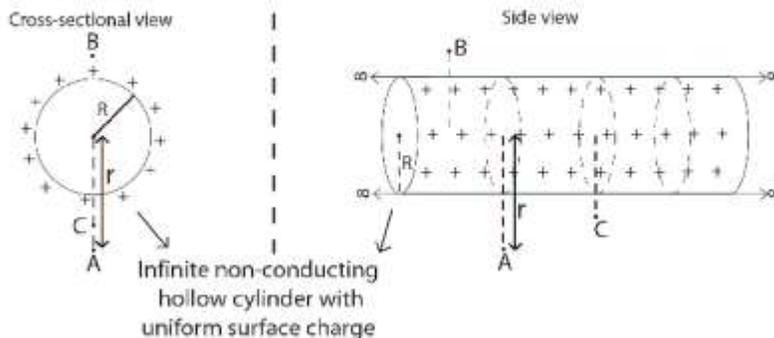
(9) Consider the following conversation about the electric field at a point inside in the hollow region of the non-conducting hollow objects with uniform surface charge shown at the beginning of section (I):

- Pria: The electric field at all points in the hollow region inside each of these charged insulating objects is zero because the inside is shielded from the charges on the surface.
- Mira: I disagree. Only a conductor is shielded from the charges on its surface because surface charges can redistribute to make the net electric field zero everywhere inside. For insulating objects, the electric field in the hollow region need NOT be zero.

With whom, if either, do you agree? Explain.

(II) **Electric field due to an infinite non-conducting hollow cylinder or line with uniform surface charge**

Let's apply Gauss's law to determine the magnitude of the electric field, $|\vec{E}|$, at a point A, a distance r away from the axis, due to an infinitely long hollow non-conducting cylinder of radius R with a uniform surface charge. The charge per unit length is λ (charge density). The cylinder is shown below, in cross-sectional and side views:



(1) On each view of the infinite cylinder above, indicate the direction of the electric field at the points marked A, B, and C. Explain your reasoning.

(2) On the cross sectional view above, draw the cross section of an imaginary cylindrical surface without caps which includes point A and on which the magnitude of the electric field, $|\vec{E}|$, is the same at EVERY point.

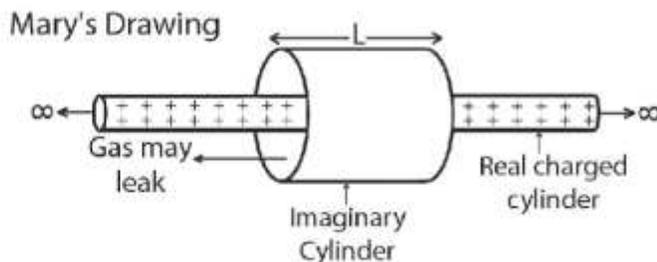
(3) Draw this imaginary cylindrical surface (without caps) on the side view above, giving it a length L .

(4) With a tiny square, show an infinitesimal patch of area (on which the direction of the electric field can be considered reasonably uniform) anywhere on the imaginary cylindrical surface you drew above. What is the angle θ between the area vector (outward normal) for this patch and the direction of \vec{E} ?

(5) Consider the following conversation:

- Susan: "We can use Gauss's law to find the net electric flux Φ_E through this imaginary cylindrical surface. Since $|\vec{E}|$ and θ are the same at each point on the cylindrical surface, we can write $\Phi_E = Q_{enclosed}/\epsilon_0 = |\vec{E}|\cos\theta \oint d\vec{A} = |\vec{E}|\cos\theta A$ where A is the area of the cylindrical surface."
- Mary: "I don't think this *open* cylinder *without caps* encloses the charge inside it in the manner required for Gauss's Law. Imagine enclosing some gas in this imaginary cylinder of length L , it would leak out...Didn't the professor say that if gas may leak out from the inside to the outside, it's not a Gaussian surface?"

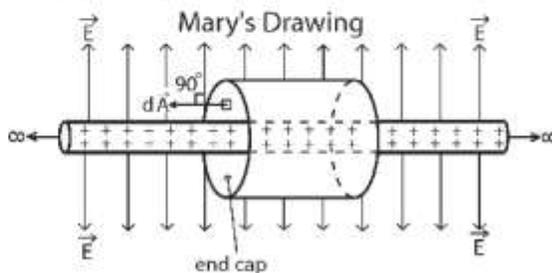
With whom, if either, do you agree? Explain.



(6) Consider the continuation of the above conversation:

- Susan: "I agree with you. We should have end caps on this imaginary cylinder. But the magnitude of the electric field isn't the same at all points on the end caps. For example, points on the cap closer to the charged cylinder (but outside of it) have larger electric field than points farther away. Then, on the caps, we can't pull $|\vec{E}|$ out of the integral relating the net flux and $|\vec{E}|$. It looks like we cannot find $|\vec{E}|$ easily from the information about the net flux through the closed cylinder..."
- Mary: "Oh, but the area vector on the caps is at 90° to the electric field everywhere. Since $\cos(90^\circ) = 0$, the electric flux Φ_E is zero through the caps. Actually, it is making sense visually since none of the electric field lines are penetrating through the caps."

With whom, if either, do you agree? Explain.



(7) Consider the continuation of the above conversation after they close the imaginary cylinder of length L with caps:

- Susan: "Since we want to calculate the electric field at a point due to an infinite cylinder of charge, shouldn't the Gaussian surface CERTAINLY be a concentric cylinder of INFINITE length? Shouldn't all the charge on the cylinder CERTAINLY be inside the Gaussian surface for it to correctly account for the electric field?"

- Mary: "I disagree. A Gaussian surface which is a concentric closed cylinder of FINITE length L will work too. The electric field only depends on the charge per unit length λ because the real cylinder which has charge on it is *infinite*. The length of the Gaussian cylinder, L , will cancel out in the end."

With whom, if either, do you agree? (Note: Don't get stuck. We will come back to this discussion later.)

(8) Given that the charge per unit length on the infinite uniformly charged cylinder is λ , find the Q_{enclosed} by your Gaussian cylinder of length L .

(9) Use Susan's comments in question (5) ($\Phi_E = Q_{\text{enclosed}}/\epsilon_0 = |\vec{E}|\cos\theta \int_{\text{side}} |d\vec{A}| = |\vec{E}|\cos\theta A$) where A is the area of the cylindrical side (curved part) and also the fact that the area of a cylindrical side of length L and radius r is $2\pi rL$ to determine the magnitude of the electric field in terms of $Q_{\text{enclosed}} = \lambda L$.

(10) Go back to the conversation between Susan and Mary in question (7) and make sure your response there is consistent with your response to question (9).

(11) Consider the continuation of the above conversation:

- Susan: "If we had an infinite uniform thin line of charge with charge density λ instead of a cylinder, the appropriate Gaussian surface would still be a cylinder and the $|\vec{E}|$ we found above would still be valid."
- Mary: "I agree. However, we cannot use Gauss's law to find $|\vec{E}|$ if the charge distribution on the infinite hollow cylinder is NOT uniform. This is because we could not claim that $|\vec{E}|$ is the same at every point on the side of our imaginary closed cylinder."

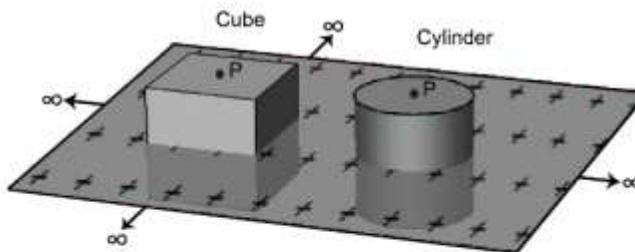
With whom, if either, do you agree? Explain.

(12) If you had to determine the electric field at a point inside at a distance r_{in} from the axis of the infinite uniform cylinder of charge, what kind of imaginary Gaussian surface would you choose? Does the length of this surface matter?

(13) Consider the following statement from Susan: "To determine $|\vec{E}|$ inside ($r_{in} < R$) the *hollow* infinite non-conducting cylinder with uniform surface charge, we can choose a Gaussian cylinder of radius r_{in} and of any length L concentric with the charged cylinder. We can proceed exactly as we did in question (9) above and find $|\vec{E}| = Q_{\text{enclosed}}/(2\pi\epsilon_0 r_{in}L)$. But $|\vec{E}| = 0$ for all r_{in} because $Q_{\text{enclosed}} = 0$." Explain why Susan is correct appropriately citing the symmetry considerations.

(III) Electric field due to an infinite non-conducting sheet of charge

Consider a horizontal infinite non-conducting sheet of charge with uniform charge density σ and two closed imaginary Gaussian surfaces: a cylinder and a rectangular box. Each Gaussian surface is symmetrically situated with half of each surface above and half below the infinite sheet of charge and with their axes of symmetry perpendicular to the sheet. P is a point on the top surface of each Gaussian surface, a distance d above the sheet.



(1) Consider the following conversation between John and Susan:

- John: "The electric field due to the infinite sheet of charge everywhere near the sheet is perpendicular to it and points away from the sheet. There is neither any electric flux through the cylindrical side (the curved part of the cylinder) nor through the vertical (side) faces of the box since these surfaces are perpendicular to the sheet of charge. The area vector on these surfaces is perpendicular to the field everywhere so that $\cos(90^\circ) = 0$."
- Susan: "I agree. Visually, there won't be any field lines penetrating those surfaces."
- John: "In fact, any Gaussian surface that is flat on the top and bottom, parallel to the sheet, with sides perpendicular to the sheet will work."

Do you agree with John and/or Susan? Explain.

Although both the Gaussian surfaces above are appropriate for calculating the electric field at point P due to the infinite sheet of charge, let's first calculate the magnitude of the electric field using the cylindrical Gaussian surface above:

(2) If the area of each cap (top and bottom) of the cylinder is A_{cap} , write down the charge enclosed by the Gaussian cylinder in terms of σ and the area A_{cap} . (Hint: On the figure above, draw a dashed line showing the part of the infinite sheet of charge that is inside the imaginary cylinder.)

(3) Use Gauss' law to write the net electric flux through the Gaussian cylinder in terms of the charge enclosed.

(4) What is the flux through the curved part of the cylinder? Make sure your answer is consistent with your response to the conversation in question (1) above.

(5) Is the magnitude of the electric field due to the infinite sheet of charge the same or varying at different points on the top and bottom caps of the Gaussian cylinder?

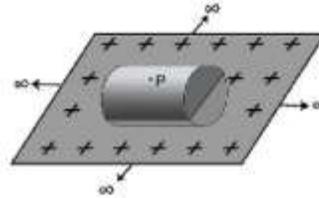
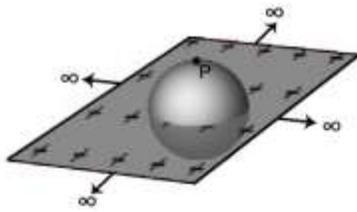
(6) Draw the area vector and the direction of the electric field at point P on the top cap. Do the same for any point on the bottom cap of the Gaussian cylinder. Are the area vector and the electric field in the same direction ($\theta = 0$) everywhere on both the top and bottom caps?

(7) Since there is no flux through the curved surface of the cylinder, the net flux through the Gaussian cylinder is only through its top and bottom caps. Since $|\vec{E}|$ is the same everywhere on the top and bottom caps, use the relation $\Phi_E = \oint |\vec{E}| \cos\theta |d\vec{A}|$ to determine the net flux through the closed cylinder in terms of the area of each cap A_{cap} and $|\vec{E}|$. (Note: \oint over the closed surface can be broken up into $\oint = \int_{top-cap} + \int_{bottom-cap} + \int_{curved-surface}$ and the contributions to the net flux from the top and bottom caps are the same.)

(8) Equate the net flux Φ_E obtained in questions (3) and (7) to find the electric field magnitude $|\vec{E}|$ at any point on the top or bottom cap of the Gaussian cylinder. (Question (3) gives Φ_E in terms of the charge enclosed and Question (7) in terms of $|\vec{E}|$, which you want.)

(9) If your friend chose a Gaussian cylinder with larger A_{cap} , would you two obtain different magnitudes of the electric field, $|\vec{E}|$, at point P? Explain. (Hint: Does the area of the cylindrical cap or the area of the top face of the box affect $|\vec{E}|$ that you found in question (8)?)

(10) Repeat all of the calculations from (2)-(8) above for the Gaussian rectangular box to find the magnitude of the electric field at point P on the top face of the cube in the figure at the beginning of section (III). Assume that the area of each face of the box is A_{box} . Make sure the answer for $|\vec{E}|$ is the same regardless of whether you choose a Gaussian cylinder or a rectangular box.



(11) Is the electric field due to the infinite sheet of charge equally strong regardless of how far away point P is from the sheet? (Hint: Does the distance of point P from the sheet appear in the expression for $|\vec{E}|$ in question (8)?)

(12) How would your answer to question (8) for $|\vec{E}|$ change if the surface charge density on the sheet were negative ($-\sigma$)? How would the direction of \vec{E} change?

(13) Why could you not use Gauss's law to find the electric field if the sheet of charge was finite in extent?

(14) Why won't a symmetrically situated Gaussian sphere which is half above and half below the infinite sheet of charge or a Gaussian cylinder with caps perpendicular to the sheet of charge as shown above be suitable for determining $|\vec{E}|$ at a point P shown? (Hint: Look at the integral relating the net flux Φ_E and $|\vec{E}|$ and whether the angle between \vec{E} and $d\vec{A}$ is the same for all $d\vec{A}$ on these surfaces so that it can be pulled out of the integral.)

SUMMARY: The only cases we have studied in which there is "enough" symmetry of the "right kind" to use the information about the net flux (obtained from Gauss' law) to find the magnitude of the electric field easily are:

- Spherical symmetry: (Gaussian surface is a concentric sphere) A single point charge or any SPHERICALLY SYMMETRIC charge distribution: uniformly charged sphere, single or concentric spherical shells, etc. The CHARGE DISTRIBUTION must be spherically symmetric, not the object on which charges are distributed. (For example, we CANNOT find the field at a point due to a sphere with non-uniform charge which lacks non-radial symmetry using Gauss's law.)
- Cylindrical symmetry: (Gaussian surface is a coaxial cylinder of any length L) *Infinite* line or infinite cylinder with charge distribution that is radially symmetric in cylindrical coordinates.
- Planar symmetry: (Any Gaussian surface that is flat on the top and bottom, parallel to the sheet, and sides perpendicular to the sheet will work.) *Infinite* sheet with uniform charge density.

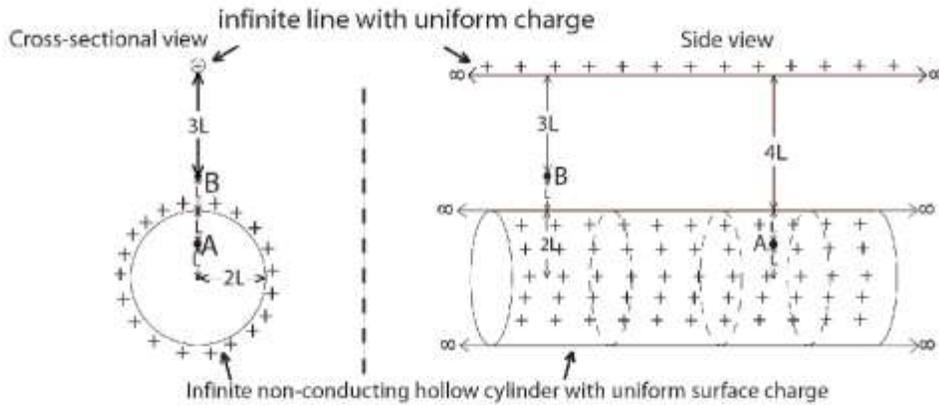
D.3 TUTORIAL V

D.3.1 Pre-test V

Pretest: Revisiting the Superposition Principle after Gauss' Law

- The magnitude of the electric field due to an infinite uniform line of charge with charge density $+\lambda$ at a distance r from the line is $+\lambda/(2\pi\epsilon_0 r)$.
- The magnitude of the electric field due to a uniform sphere of charge, with total charge $+Q$, outside the sphere at a distance r from the center is kQ/r^2 .
- The magnitude of the electric field due to a single uniformly charged infinite sheet is $\sigma/(2\epsilon_0)$.

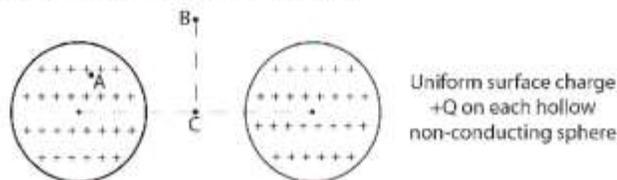
(1) Consider a thin line and a hollow non-conducting cylinder of radius $2L$, both infinite in extent with charge uniformly distributed on them (see the cross-sectional and side view below). Both have the same charge per unit length $+\lambda$. They are parallel to each other. The perpendicular distance (shortest distance) separating the infinite line of charge from the axis of the hollow charged cylinder is $6L$. Points A and B lie on radial straight lines between the line of charge and the axis of the charged cylinder as shown below. On the cross-sectional view below, draw arrows to show the directions of the net electric field at the two points A (inside the cylinder at a radial distance L from the axis) and B (outside the cylinder at a radial distance $3L$ from the axis).



(2) Write the *magnitude* of the net electric field at points A ($|\vec{E}_A|$) and B ($|\vec{E}_B|$) above in terms of λ and the distances shown.

(3) A cross section through the equators of two adjacent identical hollow non-conducting spheres, each with uniform surface charge $+Q$, is shown below. Find the direction of the net electric field at points A, B and C. Point A is in the hollow region of one of the spheres. Points B and C are on the perpendicular bisector of the straight line joining the centers of the two spheres. Explain your reasoning.

Hint: Use Gauss' law and the principle of superposition.

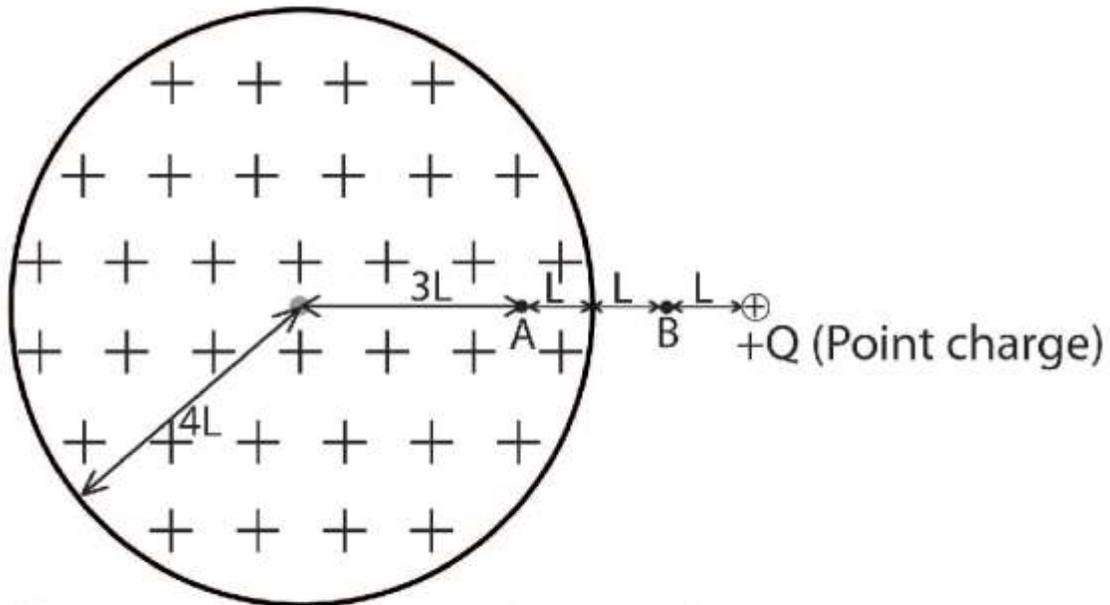


D.3.2 Post-test V

Posttest: Revisiting the Superposition Principle after Gauss' Law

- The magnitude of the electric field due to an infinite uniform line of charge with charge density $+\lambda$ at a distance r from the line is $+\lambda/(2\pi\epsilon_0 r)$.
- The magnitude of the electric field due to a uniform sphere of charge, with total charge $+Q$, outside the sphere at a distance r from the center is kQ/r^2 .
- The magnitude of the electric field due to one infinite sheet is $\sigma/(2\epsilon_0)$.

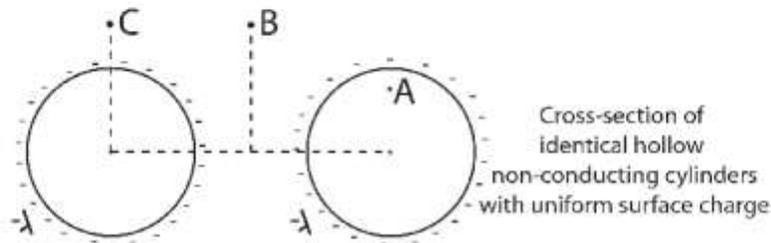
(1) In the figure below (cross-sectional view), a point charge $+Q$ is near a thin hollow non-conducting sphere of radius $4L$ that has an equal amount of charge $+Q$ uniformly distributed on its surface. No other charges are around. Draw arrows to show the direction of the net electric field at two points: A (inside the sphere) and B (outside the sphere) shown below.



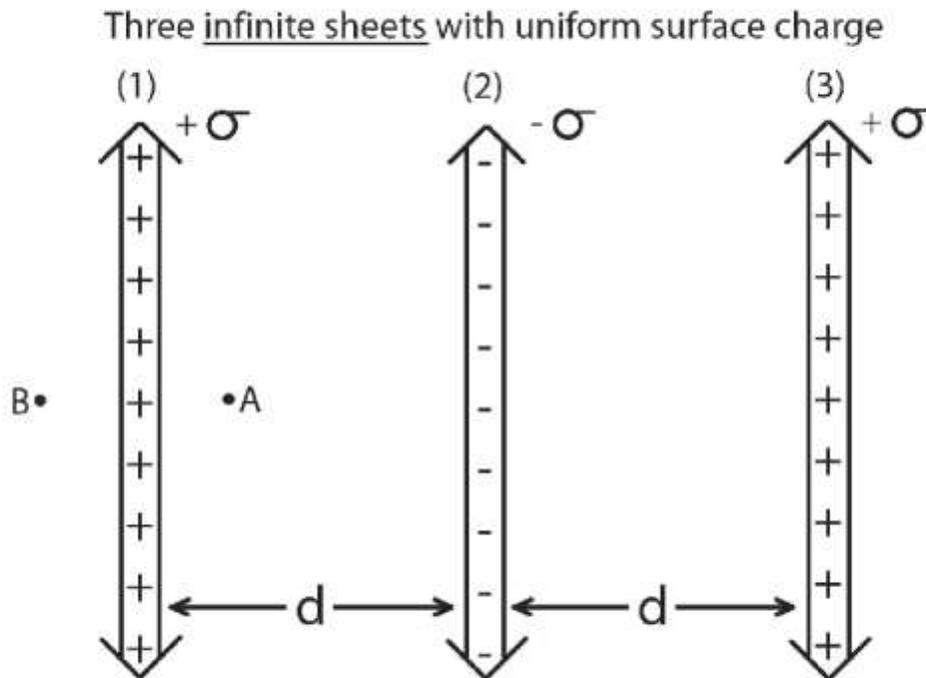
Hollow non-conducting sphere with uniform surface charge $+Q$

(2) Write the *magnitude* of the net electric field at points A ($|\vec{E}_A|$) and B ($|\vec{E}_B|$) above in terms of charge $+Q$ and the distances shown.

(3) Two adjacent identical hollow non-conducting cylinders, both infinite in extent and each with the same uniform negative charge per unit length $-\lambda$ are shown below. The axes of the two cylinders are parallel to each other. A cross section of the cylinders is shown below. Find the direction of the net electric field at points A, B and C. Point A is in the hollow region of one of the cylinders. Point B is on the perpendicular bisector of the straight line joining the center of the two cylinders in the plane of the paper. Explain your reasoning for each point. (Note: If you cannot find the exact direction at any of the three points shown, draw an approximate direction.)



(4) Three parallel thin infinite sheets with uniform charge are such that the adjacent ones are a distance d apart. The surface charge density (charge per unit area) on sheets (1) and (3) is $+\sigma$ and on sheet (2) is $-\sigma$. Find the magnitudes of the net electric field at points A and B shown below due to the charges on the sheets.



(5) Show with arrows the direction of the net electric field at points A and B due to the infinite sheets of charge in the figure above.

(6) How would the electric field at points A and B obtained in questions (4) and (5) be affected if sheets (2) and (3) are removed? Explain.

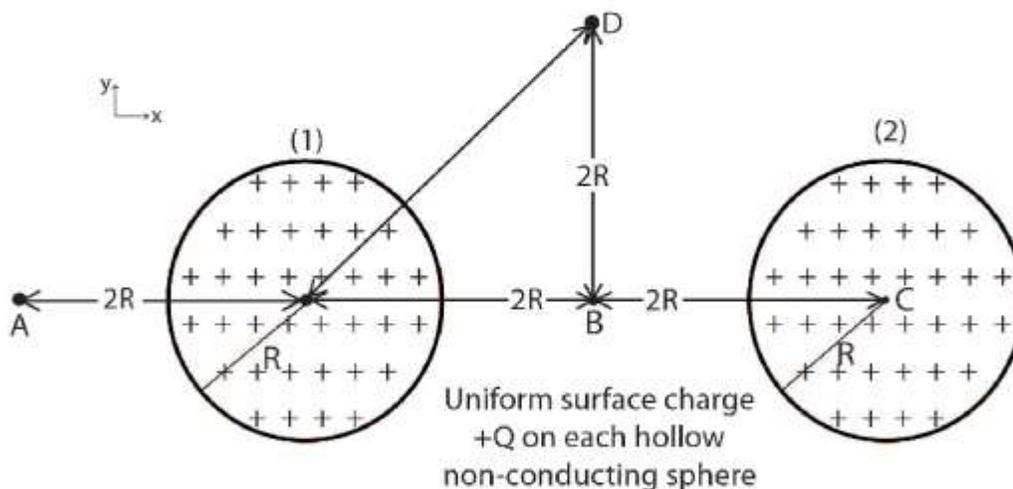
D.3.3 Tutorial V

Revisiting the Superposition Principle after Gauss' Law

In the following section, we will use the principle of superposition to find the electric field at a point due to multiple sources. Remember, the electric field at a point due to different sources of charges can be added vectorially to find the net field.

(I) Two identical hollow non-conducting spheres, each of radius R and the same total charge $+Q$ uniformly distributed on the surface, are a distance $4R$ apart measured center to center, as shown below. You are asked to find the electric field at points A, B, C, and D which are in the plane shown in the figure. Points A, B and C are on the straight line joining the centers of the two spheres and:

- A is to the left of sphere 1 at a distance $2R$ from its center.
- B is equidistant from the two spheres on the straight line joining their centers.
- C is at the center of sphere 2.
- D is on the perpendicular bisector of the straight line joining the centers of the spheres at a distance $3R$ from the center of each.



(1) Consider the following plan proposed by John:

- Step 1: First, think about each sphere one at a time, in isolation. Since the charge is uniformly distributed on the surface, use Gauss's Law to determine the electric field at each point due to the charges on each sphere.
- Step 2: Use your results from Step 1 and the principle of superposition to calculate the net field at each of the points A, B, C and D.

Is this a reasonable plan? Explain.

(2) Use the principle of superposition (the net electric field is the vector sum of the electric field due to various sources) proposed by John to calculate the magnitude and direction of the net electric field at points A, B, C and D assuming that the magnitude of the electric field due to an isolated sphere with charge $+q$ uniformly distributed on its surface is:

- $|\vec{E}| = 0$ at any point inside
- $|\vec{E}| = kq/r^2$ at any point outside a distance r away from the center

(3) Consider the following conversation between John and Susan:

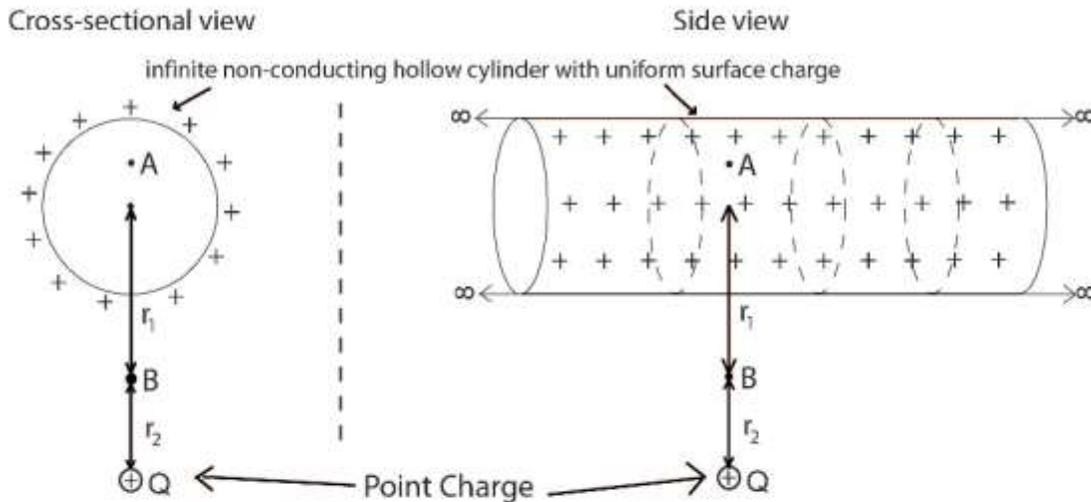
- John: For points outside a sphere, the distance r in $|\vec{E}| = kq/r^2$ should be measured from the surface of that sphere.
- Susan: I disagree. For a point outside the sphere, the distance r should be measured from the center of that sphere because the uniform sphere of charge behaves as a point charge at the center.

With whom, if either, do you agree? Explain.

(4) Is the net electric field zero at any of the points? Explain.

(5) Which points, if any, have net electric field that is along the x axis? Which points, if any, have electric fields that are along the y axis? Make sure to draw arrows to show the directions of the net electric field at each of the points shown in the figure above.

(II) Consider the following situation in which there is a point charge $+Q$ near an infinite non-conducting cylinder with uniform surface charge (charge per unit length λ). Points A , B and the point charge $+Q$, are all in the same cross-sectional plane of the cylinder and all are on a straight line joining the point charge $+Q$ to a point on the cylindrical axis.



(1) Consider the following conversation between Harry and Susan:

- Harry: "The net electric field should be zero everywhere inside the hollow region of the cylinder since there is no charge enclosed".
- Susan: "I disagree. The electric field due to the infinite cylinder of charge is zero inside it but not the electric field due to the point charge $+Q$. Therefore, the net electric field inside the cylinder cannot be zero!"

Explain why you agree or disagree with each.

(2) Consider the following conversation between Pria and Mira:

- Pria: "The point charge and the infinite cylinder of charge together do not have high enough symmetry so that we can construct one Gaussian surface and exploit Gauss's law to find the magnitude of the net electric field, $|\vec{E}|$, at a point due to both."
- Mira: "I agree. But we can use the principle of superposition here to find the electric field at a point. We know the field at each point due to a point charge using Coulomb's law and due to an infinite cylinder of charge using Gauss' law. All we need to do is to add them vectorially."

Explain why you agree or disagree with each.

(3) In the figure above, point A is inside the infinite non-conducting cylinder of charge. Draw separate

arrows to indicate the directions of the electric field at point A due to the point charge $+Q$ and due to the infinite cylinder of charge. If the field is zero due to either of these, say so explicitly.

(4) Is there a net electric field at point A ? Make sure your answer is consistent with your answer to question (1). If there is a net electric field at point A , draw an arrow in the cross-sectional view above to show its direction.

(5) In the figure above, point B is outside the infinite cylinder of charge. Draw separate arrows to indicate the directions of the electric field at point B due to the point charge $+Q$ and due to the infinite cylinder of charge. If the field is zero due to either of these, say so explicitly.

(6) Consider the following conversation between Pria and Mira:

- Pria: For point B outside the cylinder, the distance r in $|\vec{E}| = \lambda/(2\pi\epsilon_0 r)$ due to the cylinder should be measured from the surface of the cylinder.
- Mira: I disagree. For a point outside the cylinder, the distance r should be measured from the axis of the cylinder because the uniform cylinder of charge behaves as a line of charge at the axis.

With whom, if either, do you agree? Explain.

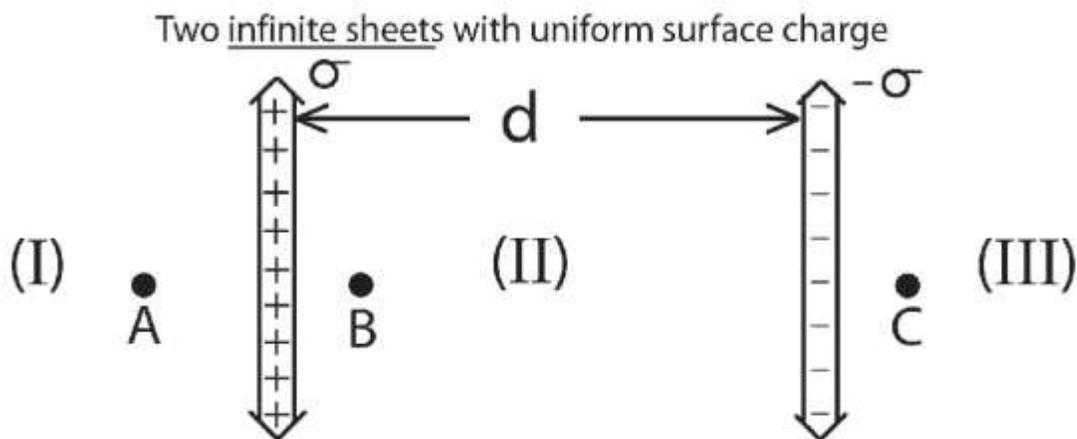
(7) If the radial distance of point B from the center of the cylinder is r_1 and from the point charge $+Q$ is r_2 , what is the magnitude of the electric field at point B due to the point charge and due to the cylinder of charge separately?

(8) Suppose the magnitude of the electric field at point B due to the point charge $+Q$ is greater than that due to the cylinder of charge. Using this assumption, draw an arrow to show the direction for the net electric field at point B above.

(9) Suppose the magnitude of the electric field at point B due to the point charge $+Q$ is exactly the same as that due to the cylinder of charge. What will be the direction of the net electric field at point B then?

(III) Two parallel thin infinite sheets with uniform charge are a distance d apart. The charge density (charge per unit area) on sheet (1) is $+\sigma$ and on sheet (2) is $-\sigma$. Find the magnitudes of the net electric field at points A, B, and C in regions (I), (II) and (III) shown below due to the charges on both infinite sheets.

(Note: The magnitude of the net electric field due to a single infinite uniformly charged sheet is $\sigma/(2\epsilon_0)$ and the direction of the field is perpendicular to the sheet everywhere.)



(1) Draw arrows labeled \vec{E}_1 above to show the direction of the electric field due ONLY to the infinite sheet with positive charge at each of the three points. (Hint: The electric field due to the positively charged sheet should point away from the sheet everywhere.)

(2) Draw arrows labeled \vec{E}_2 above to show the direction of the electric field due ONLY to the infinite sheet with negative charge at each of the three points. (Hint: The electric field due to the negatively charged sheet should point towards the sheet everywhere.)

(3) Consider the following conversation between Pria and Mira:

- Pria: "In region I, the length of the arrow for \vec{E}_1 should be longer than the length of the arrow for \vec{E}_2 because the electric field due to the positively charged sheet will be stronger than the field due to the negatively charged sheet."
- Mira: "I disagree. Since the sheets are infinite, the electric field everywhere due to each sheet is the same regardless of how far you are from its surface."

With whom, if either, do you agree? Explain and draw the lengths of the arrows above to reflect your answer.

(4) Use the superposition principle to find the magnitude of the net electric field at each of the three points.

(3) Consider the following conversation between Pria and Mira:

- Pria: “The net electric field at points A and C is zero because the field due to each sheet has equal magnitude but opposite direction. The net electric field at point B is σ/ϵ_0 and it points away from the positive sheet.”
- Mira: “Yes. In fact, the net electric field at any point in regions (I) and (III) will be zero and in region (II) will be σ/ϵ_0 .”

With whom, if either, do you agree? Explain.

APPENDIX E

MAGNETISM CONCEPTUAL SURVEY (CHAPTER 6)