

# Mapping university students' epistemic framing of computational physics using network analysis

Madelen Bodin\*

*Department of Physics, Umeå University, Umeå, Sweden*  
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Solving physics problem in university physics education using a computational approach requires knowledge and skills in several domains, for example, physics, mathematics, programming, and modeling. These competences are in turn related to students' beliefs about the domains as well as about learning. These knowledge and beliefs components are referred to here as epistemic elements, which together represent the students' epistemic framing of the situation. The purpose of this study was to investigate university physics students' epistemic framing when solving and visualizing a physics problem using a particle-spring model system. Students' epistemic framings are analyzed before and after the task using a network analysis approach on interview transcripts, producing visual representations as epistemic networks. The results show that students change their epistemic framing from a modeling task, with expectancies about learning programming, to a physics task, in which they are challenged to use physics principles and conservation laws in order to troubleshoot and understand their simulations. This implies that the task, even though it is not introducing any new physics, helps the students to develop a more coherent view of the importance of using physics principles in problem solving. The network analysis method used in this study is shown to give intelligible representations of the students' epistemic framing and is proposed as a useful method of analysis of textual data.

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## I. INTRODUCTION

Numerical problem solving in university physics education, including interactive visualizations and simulations, has been a subject of increasing attention during the past years [1,2]. Increasing computer capacity together with newly developed modeling and problem-solving environments have cleared the way for new approaches in physics education, for physics majors, engineering students, as well as for other physics students. By using a problem-solving environment such as, for example, MAPLE or MATLAB, students can numerically solve physics problems, create visual simulations, practice mathematical and physical modeling, and investigate physics phenomena, rather than just calculating an answer [3–7]. This approach differs from the traditional way of teaching and learning physics where problem solving is generally limited to analytical problems that can be solved with pencil and paper. Using the computer as a tool offers possibilities to investigate more complex problems, which opens the way for a deeper comprehension of how physics is used to explain phenomena. However, a problem-solving and simulation environment that requires extensive programming can also be expected to draw

attention from and interfere with elaboration of the conceptual content of a physics task. The effects of these types of tasks on students' cognition therefore need further attention in order to design tasks according to their purposes. This study focuses on investigating university physics students' cognitive representations, in terms of epistemic framing, associated with an assignment about solving a computational physics problem. Epistemic framing is here referred to as the cognitive patterns that are made up of descriptive elements, which I call epistemic elements, such as knowledge, skills, beliefs, and strategies, of how the students experience the learning situation. Previous studies have approached representations of cognitive structures using concept mapping [8], mental models, [9], and schemata [10]. Recent studies have shown that network analysis can also be a useful tool in order to measure and visualize mental representations [11,12] since dynamic characteristics and flow of information within these representations can be investigated. In this study a network analysis approach is used in order to build and investigate representations of students' epistemic framing before and after a task in numerical physics problem-solving. In what follows, the underlying conceptual framework for this study is discussed from the aspects of (a) simulations and modeling in physics education and the autonomous characters of these tasks, (b) cognitive representations within the context of physics education as epistemic framing, and (c) network analysis as a method of visualizing and analyzing the complexity of a cognitive representation.

\*[madelen.bodin@physics.umu.se](mailto:madelen.bodin@physics.umu.se)

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## II. CONCEPTUAL FRAMEWORK

### A. Simulation and modeling in physics education

Previous studies show that students often leave the classroom with the same novicelike physics knowledge and beliefs that they had when they entered the classroom [13]. One reason is that the students are not challenged with problems and other learning activities that encourage them to use and develop their physics knowledge towards being expertlike [14,15]. Hestenes [16] suggests that students must practice even more to develop and use models for specific physics systems and processes in order to learn the structure of physics. Indeed, computer environments for simulation, modeling, and problem solving have shown to be powerful tools in physics education for developing conceptual understanding [17] as well as for modeling and problem-solving skills, e.g., PhET simulation [18–20], easy JAVA simulations [4], and VPython [6]. However, these environments may differ significantly with regard to the learning conditions they provide, representing different levels of autonomy and requiring more or less knowledge and skills in modeling, programming, mathematics, or physics. The level of autonomy in a learning situation constitutes whether a student can choose how, when, and where the learning should take place but is also indicated by the student's perceived competence and control of the situation [21]. The possibility to autonomous behavior can thus be considered as being an important entity in the constructionist learning framework [22] where tasks are designed to motivate students to use their own knowledge in order to build things and thus create new knowledge. Other researchers argue that the level of autonomy provided by a task must match the students' epistemological beliefs for the situation to produce positive effects with respect to cognition and motivation [15,23]. Autonomous behavior can be encouraged by guidance from teachers as well as the feedback from the results of their exercises, e.g., computer simulations and visualizations or a physical product such as a building or a construction. The problem-solving environment used in this study, MATLAB, requires programming skills but also provides support concerning functions and graphics in order to numerically solve a physics problem and visualize the result in a simulation [7]. The particle-spring model system together with the MATLAB environment used in this study is therefore considered to represent a situation which provides autonomy in the problem-solving process as well as feedback in terms of visualizations and compilation messages.

### B. Cognitive representation in physics education

Physics is a unique subject field since it involves many levels of abstractions in different forms of representations [24], e.g., conceptual (laws and principles), mathematical formalism, experimental (equipment), and descriptive

(tables, graphs). Hence, what it actually means to understand physics is a challenging question. As argued earlier, solving computational problems in physics is not only about physics knowledge but also about skills and knowledge associated with math, programming, and modeling. There are few studies about how these knowledge and skills interact and what the students actually learn, and an important issue is whether students mainly train programming skills or if they actually learn physics.

The organization of knowledge with a cognitive approach is described in several ways in the literature about physics education research. Schema [10,25], scripts [26], mental models [9,27–29], and frames [30,31] are some of the constructs that are used in order to describe how physics knowledge is organized in the human mind. These constructs have distinctions in meaning and how they are used, but they have in common an underlying idea of knowledge as represented by elements that are connected in some pattern. These elements can have different meanings and represent different knowledge or beliefs about knowledge. Schemas and scripts are rather run schedules for a particular situation [26] while mental models can be seen as working models for conceptual comprehension of a situation [32]. Since frames have been described to represent a wider representation of how a learning situation is experienced including knowledge as well as beliefs [33,34], this construct is chosen to be a part of the framework.

#### 1. Beliefs

The choice of including students' beliefs as elements contributing to the cognitive structure in the present study was based on previous results showing beliefs as an important actor in the learning process [35,36]. Previous studies have also shown that beliefs were important predictors of performance in the context of computational physics [37,38]. When characterizing student's epistemological beliefs in introductory physics, Hammer [36] used a framework consisting of three dimensions: structure of physics, content of physics knowledge, and learning physics. Student beliefs were found to be involved in their work in the course and were consistent across physics content. Hammer found, for example, that if students believed that physics knowledge consisted of facts rather than general principles, it was reflected in how these students solved problems and explained phenomena, i.e., relating to isolated facts rather than using physics laws and principles.

Previous research on motivation in learning has also put emphasis on the importance of student beliefs. The expectancy-value framework is an important contribution to research on motivated learning behavior in order to predict academic achievement [39] and holds expectancy beliefs, describing self-perceptions of competence, and value beliefs, referring to the reasons the student may have for engaging in a task, as the two most important

variables in achievement behavior. Students may also have beliefs about how to attribute their achievement outcomes, e.g., ability, effort, task difficulty, or luck, which has shown to be connected to beliefs about ability as well as value [40].

Epistemological beliefs as well as efficacy, value, and attribution beliefs are therefore expected to provide important information to students' framing of this particular learning situation where the computational physics context provides scope for autonomous behavior as well as cognitive challenges.

## 2. Epistemic framing

The construct of framing as a representation of knowledge has previously been used in linguistics, cognitive psychology, and anthropology. Tannen and Wallat [41] defined framing in terms of individual reasoning and summarized the concept of frame as the set of expectations, based on previous experience, an individual has about a given situation or, widely spoken, a community of practice. Minsky [31] used frame when describing the cognitive structure that a person recalled (remembered) from a similar situation when entering a new situation. If the frame did not fit, it was replaced or revised until it fit the situation. In an educational setting, a student's framing used for interpreting a learning situation could be expected to be based on cognitive and context-specific experiences concerning, e.g., prior knowledge, skills, and beliefs but also social aspects, such as relations to other people, and on what external tools are available for learning, such as literature and teachers [26,42]. In previous research epistemological framing [30,34,43] has been used to describe the network of activation of epistemological resources, or cognitive elements, that is present in a learning situation.

In this paper the notation *epistemic elements* has been chosen to represent knowledge, skills, and beliefs, and the notation *epistemic framing* has been chosen to describe their organization. A detailed description of how epistemic framing is represented is found in Sec. III. The notation epistemic framing has previously been used in several studies. In order to represent knowledge structure in mathematics and physics problem solving, Bing and Redish [44] used students' argumentation in terms of warrants as indicators of epistemic framing. Shaffer [45] described epistemic frames as not only containing information about knowing what and how but also about knowing with, forming an organizing principle for practices, which was found useful in interactive learning environments where students could adapt epistemic frames in educational role playing in order to help students deal more efficiently with real-life situations. In another study, epistemological mindsets, similar to epistemic framing, of students' understanding of physics equations were identified in six different context-dependent epistemological components which could be present in a student's frame about physics problem

solving [46]. An important aspect of the epistemic frame is thus, as argued earlier, not only the collection of epistemic elements but also how these are interrelated. In the next section I describe how these interrelations can be investigated using the features of network analysis.

## C. Network modeling

Network modeling makes it possible to visualize and understand the complexity of a system with many interacting elements. Common applications of network modeling are, for example, social interactions, biological systems, information flows, and semantic patterns.

Epistemic framing, as described in the previous section on cognitive representation, is proposed to correspond to a knowledge network which is here referred to as an *epistemic network*. Epistemic framing can thus be visualized using a network analysis approach where organization of epistemic elements as well as flow of information can be revealed using a mapping method. Knowledge networks within a particular context have previously been represented as semantic networks where the relations, or links, between concepts relevant for the context, nodes, form cognitive patterns as mental models [32,47]. In the epistemic network the nodes are represented by the epistemic elements that express conceptual knowledge, skills, beliefs, and other personal characteristics of a learning situation. These elements have relations that besides adjacency can be dependent on, for example, meaning and causality. In a study by Shaffer *et al.* [12] epistemic network analysis was used for assessing epistemic frames during an activity of a digital learning system. Shaffer *et al.* showed how the interaction between epistemic elements changed during the learning activity, from showing a loose frame of few interacting elements towards a denser, more complex frame with more interacting elements representing knowledge and skills. Network analysis as a method of analysis of knowledge structures is an exciting and still a new approach in education research and there is a need for further investigation of its properties in this context.

## D. Purpose of the study and research questions

The purpose of this study was twofold. The primary purpose was to investigate the character of physics students' epistemic frames, when solving a computational physics problem. The research questions are as follows:

- What are the students focusing on, in terms of knowledge and beliefs, when describing a numerical problem-solving task, before and after doing the task?
- What role does physics knowledge take when students describe a computational physics problem-solving situation?

A second purpose was to investigate the use of a network analysis approach in order to visualize students' epistemic framing. Analysis of qualitative data such as interviews often generates results that could be difficult to survey. A

concept map or a network could therefore be a useful tool for presenting results from a qualitative analysis. A third research question is therefore:

- Are the proposed epistemic networks of the learning situation in this study useful for describing epistemic framing?

### III. METHOD

#### A. Participants and context

Students participating in this study were second year university students at a five-year study program in engineering physics. The assignment that was the subject of this study consisted of a one-week project as a part of a seven-week full time course in mathematical modeling of physics. The students were initially informed about the study by Email and participation was voluntary. The author did not have any teaching role. The course had 36 registered participants and 6 of these were chosen to participate in the study and contribute to the data set. The selection was done in order to get representatives with different beliefs profiles. The Colorado Learning Attitudes about Science Survey instrument, which was designed in order to distinguish between expert- and novicelike beliefs about physics and learning physics, was used for this selection [48]. It had previously been used in a similar context [38] and was considered as appropriate for this situation. The questionnaire was administered to the students right before the project start and statistically analyzed in order to project the students' profiles on an expert-novice beliefs map. Of the chosen students, two showed novicelike beliefs, two intermediate expertlike beliefs, and two expertlike beliefs. The mean age of the participating students was 23 years.

The context of this study was represented by a numerical problem using particle-spring simulations. At completion of the project assignment, the students would have built a simulation of a macroscopic elastic object sliding over a rough surface in two dimensions, Fig. 1. The particle-spring model is a fundamental model system in science

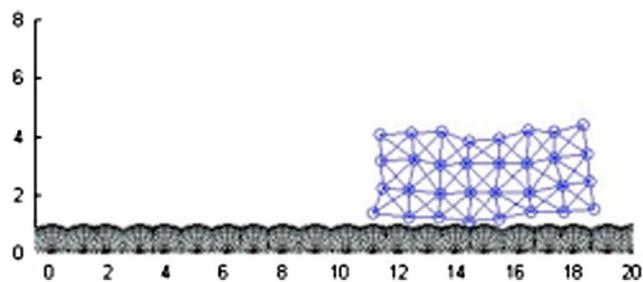


FIG. 1 (color online). Screen shot of the simulation students were building by using particle-spring systems to model an elastic object sliding over a rough surface. This model consists of  $4 \times 8$  mass particles connected with springs, up to eight springs for each particle. The springs are assigned damping in order to model the object with different elastic properties.

and can be used for any science phenomena where vibrations and oscillations are modeled. To understand how to model a physics problem using a particle-spring approximation therefore contributes to fundamental knowledge in science. The purpose of this modeling task was to train students to build mathematical models of a physics situation and to create a visual simulation of that situation. The project consisted of four successive steps. In the first step the students were introduced to the model system, starting with a numerical solution of two mass particles connected by a spring. In the following steps more particles and springs were added in order to model a larger system until a macroscopic elastic object sliding over a rough surface was simulated. The problem-solving environment used for the project was MATLAB. Prior to this course the students have had several occasions to practice MATLAB in other math courses in their study program. The physics that was needed in order to understand and perform the assignment was classical mechanics, a course which all students had completed. An introductory lecture was held before the MATLAB project started. The lecturer gave a short summary of the physics that was used in the simulation and made a short comparison between two time integration methods, Euler and leapfrog, which would be suitable for this task. All students were, in order to pass the assignment, required to submit a written report including answers to a set of questions, a description of how the simulation was tested, and the MATLAB code for the simulation.

#### B. Data collection

Data consisted of individual interviews with the six students on two occasions. The first set of interviews took place right after the introductory lecture prior to the start of the assignment and the second set of interviews was performed when the assignment was completed. The interviews were semistructured and the questions from the first occasion were followed up at the second occasion. The interviews in the first round lasted about 15–20 min and the second round about 25–35 min. The questions were designed with the framework of epistemic framing in mind in order to attain information about what knowledge, beliefs, and resources the student used in the problem-solving process and how they were used. In the first interview the students were asked to conceptually describe the task and what problems they were about to solve. They were also asked about what strategies they planned to use in order to solve the problem. Another question concerned if the students could foresee any difficulties and problems with the task and what they would do to deal with that. An important question dealt with what expectations the students had concerning gaining new knowledge and skills. In the second interview these questions were followed up in order to find discrepancies between their expectations and their actual experience. Special emphasis was put on how the students troubleshooted their solutions and how they had

overcome the difficulties that arose in the problem-solving situation. All students experienced errors in their numerical solutions and the troubleshooting process was expected to reveal information about what the students focused on, e.g., debugging the MATLAB code or investigating the physics principles, such as energy conservation. In addition to the follow-up questions, the students were in the second interview asked to demonstrate parts of their simulations on a computer. This stimulated recall-inspired part of the interview was used in order to find additional comments on the students' knowledge, beliefs, and resources they had already expressed. The interviews were transcribed verbatim.

### C. Analysis

The information revealed in the interviews and transcribed to verbal protocols was expected to correspond to a sample of the information in the student's mind at the time of the interview. The structure of that information was assumed to reveal the organization of the individual's knowledge and beliefs in terms of how different epistemic elements connected to each other. In order to identify which epistemic elements were present in the verbal protocol and their relation to each other, a thematic method of analysis according to Carley's method of formalizing the social expert's knowledge [49] was used. This method is based on the assumption that information in a particular context can be separated into discrete units (corresponding to epistemic elements) and the corresponding knowledge structure (corresponding to epistemic framing) can be represented by the relational phenomena connecting these units of information. The information provided by the interviews was therefore assumed to be represented as a network of epistemic elements and their relationships.

The analysis of the transcripts were done in two steps. In the first step the transcripts were broken down in statements and manually coded into epistemic elements. In the second step, semantic networks were generated from the coded transcripts. These steps are described in more detail below, followed by an illustrative example of how this process was performed.

#### 1. Coding of the transcripts

The epistemic elements that were used in order to build the vocabulary were based on the focus of the research and the educational situation, resulting in a coding scheme that was unique for this particular context. This coding scheme was therefore expected to reflect the expert's knowledge base [49]. This coding scheme is provided in the Appendix in Table III. The epistemic elements were defined by coding the transcripts using a thematic approach where the interview transcripts were broken down into statements which were analyzed as knowledge or beliefs. The epistemic elements chosen to represent the knowledge base of the context of this study were categorized into the subject fields of *physics*, *math*, *programming*, and *modeling*.

Epistemic elements describing beliefs were categorized into *expectancy or efficacy*, *value*, and *attributional* as argued in the conceptual framework. In addition to the knowledge and beliefs categories, there was also a need for adding a category which represented the resources that students used when performing this assignment, e.g., literature, other students, teachers, or feedback from the visualization. The coding procedure followed an exploratory, iterative process where new epistemic elements within these main categories could be added as they were found to show relevance in the data set. A total of 77 unique epistemic elements were identified from the transcripts from the two interview occasions; see the Appendix, Table III.

#### 2. Epistemic networks

From the coded transcripts epistemic networks were created by using the semantic adjacency between epistemic elements. The presence of a link between two epistemic elements was thus determined by closeness in the coded transcripts, and the value of one link was always equal to one. All links were bidirectional and the matrices representing relations between epistemic elements were therefore symmetric. If the same elements were connected several times in a coded transcript, the links were added. The weight of a link between two concepts was therefore determined by how many times the two epistemic elements appeared connected in the transcripts. Epistemic elements that did not appear in conjunction with another element were removed from the node set. While creating the epistemic networks the original transcripts were used as constant references in order to not lose any important links between epistemic elements. Each of the interviewed students generated two epistemic networks, one representing the epistemic framing before the task and one after the task. These individual networks were merged into two networks by adding the matrices that represented each student's epistemic framing before and after the task, respectively. These two networks, one representing the students' common framing before the assignment and one representing the students' common epistemic framing after the assignment, were used to compare how students epistemically framed this learning situation before and after the assignment.

The comparison was thus done between the students' common epistemic framings and not between the individual students' framing.

#### 3. Example of the coding process

What follows is an example of how an excerpt from an interview was coded and how the coded transcript was transferred to and visualized in an epistemic network. The excerpt was chosen from one of the students after the task.

*What I've been checking constantly is the energy level. It should be constant because if there is no damping no energy will be lost from the system. So that has been the*

constant check, that is what I've been dealing with most. Then the simulation has to show a fairly reasonable view of reality, sort of. It is suppose to look good. But, here I probably have some error in some computation, so there are icicles on the energy.

The following codes are applied for different sections in the excerpt. For a list of all codes used in coding the transcripts, see the Appendix, Table III:

- *What I've been checking constantly*—troubleshooting (mo\_t)
- *the energy level. It should be constant*—energy conservation (ph\_ec)
- *because if there is no damping*—modeling spring-damper (mo\_sd)
- *no energy will be lost from the system* (ph\_ec)
- *So that has been the constant check*—troubleshooting (mo\_t)
- *simulation has to show a fairly reasonable view of reality*—modeling a real situation (mo\_rs)
- *It is suppose to look reasonable*—visualization as a resource (r\_v)
- *here I probably have some error*—troubleshooting (mo\_t)
- *in some computation*—numerical solution (ma\_ns)
- *there are icicles on the energy*—energy conservation (ph\_ec)

The coded excerpt is transformed into:

mo\_t ph\_ec mo\_sd ph\_ec mo\_t mo\_rs r\_v mo\_t ma\_ns ph\_ec

This text can be visualized as a semantic network based on the adjacency between the codes representing epistemic elements, i.e., codes that are close are linked. The semantic network thus describes how these codes are related, giving a map of the epistemic frame, or an epistemic network, given by the interview excerpt. For the interview excerpt used in the example above, the resulting epistemic network is given by the adjacency matrix below where the number indicates whether there is a link between the two concepts, i.e., if they are adjacent in the coded excerpt. The matrix is symmetric, i.e., the direction of the relation is not considered.

	mo_t	ph_ec	mo_sd	mo_rs	r_v	ma_ns
mo_t		2		1	1	1
ph_ec	2		2			1
mo_sd		2				
mo_rs	1				1	
r_v	1			1		
ma_ns	1	1				

The visualization of this sample network is shown in Fig. 2. The size of each node represents its importance in the network and the width of each link represents its weight. From this small epistemic network it can be interpreted that the student had troubleshooting (mo\_t) in mind and connected that with different aspect of which energy conservation (ph\_ec) seems most important. In order to

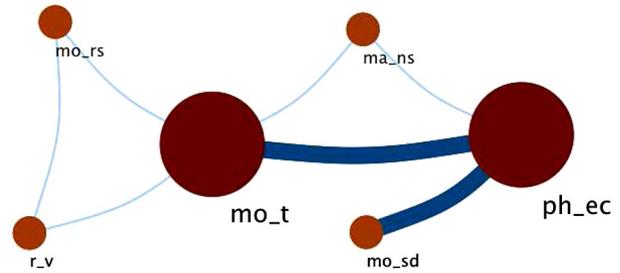


FIG. 2 (color online). Sample epistemic network visualizing the epistemic frame given by an example of an interview excerpt.

understand the epistemic network in detail, it is necessary to return to the original text for a more thorough analysis.

#### D. Visualization of networks

If a system is small, with only a few nodes and links, it can generally be visualized as is, e.g., as in Fig. 2. Many systems, however, are very large and need to be described in a simplified manner. Depending on what information we want to draw from a network there are different methods of analysis. Investigating the formation of a network might require other methods than investigating the dynamics of a network. The formation of a network is often studied by a modularity approach where modules are formed according to higher density within the modules and a sparser structure between modules [50]. For a system's behavior, e.g., how local interactions between few nodes induce a flow through the whole system, it is interesting to understand the network's dynamic structure and how the links regulate the flow. The map equation [51] is an information-theoretic algorithm which can optimize the path of a random walker in a network and describe how information flows in the network. Groups of nodes cluster into modules where information is exchanged quickly. For each module a PageRank [52] and a flow can be calculated. The PageRank is a measure of the relative importance of a node or module for a network's dynamic structure and corresponds to a eigenvalue centrality of that node or module, i.e., how many other nodes that node is linked to weighted by the other nodes' importance. The PageRank shows how much of the total flow in the network that is kept within that particular module. The links between modules represent information paths which connect the clusters and reveal the whole network's dynamic structure. In previous research the map equation has been used in citation networks to show the flow of information within the scientific community by studying cross citations of scientific journals [53]. The results revealed a bidirectional structure between basic sciences and a directed flow from applied sciences back to basic sciences giving information of how knowledge was flowing among scientists from different subject fields. The visualization of the epistemic networks in this study was done using this map equation algorithm [51] and the corresponding visualization tools: a

map generator [53] showing the structure of the epistemic network and an alluvial generator [54] showing changes between networks. The epistemic networks from the two occasions, before and after the task, were clustered according to the map algorithm trying to minimize a random walk, which in this case corresponded to a random association path, through the network, finding a pattern of nodes where a random walker was likely to spend a certain amount of time before moving to another module. The modules thus represent groups of nodes that are linked to each other to a higher degree and where information flow is concentrated. Changes in structures between the two networks representing before and after the task were visualized in an alluvial diagram, a diagram often used for visualizing Earth layers in geological contexts, where individual epistemic elements within modules as well as whole modules could be followed between the two occasions. In order to investigate the stability of the networks' module structure a significance analysis was performed by resampling bootstrap networks using the original network link weights as resampling units [54]. The bootstrap confidence interval was set to 85%, giving that of 100 resampled bootstrap networks, significant nodes are clustered together in at least 85 of the bootstrap networks.

**IV. RESULTS**

The results are here presented as two compound epistemic networks that were generated from the interview

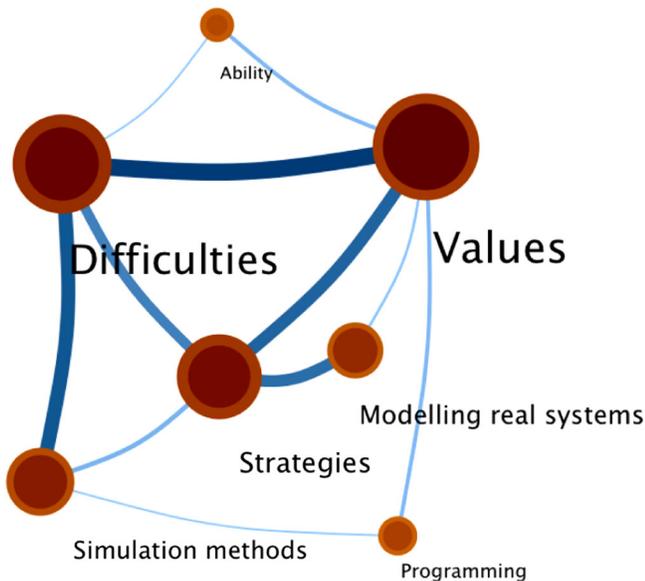


FIG. 3 (color online). Simplified epistemic networks before the problem-solving task. The size of each module corresponds to the importance of the module in the network, i.e., how much focus students put on associating between epistemic elements within that module, and the width of each link represents the information flow between modules. This network captures 97% of the node flow and 82% of the link flow.

data, shown in Figs. 3 and 4. Each network map constitute interview data from all six students before and after the task, respectively. The network data were calculated into simplified networks according to the map equation [51] and visualized as maps of modules of epistemic elements. Each module can be expanded into subnetworks revealing the structure of individual nodes. The modules have subjectively been assigned with labels in order to indicate what kind of epistemic elements each module represents.

Tables I and II summarize statistics from the networks. Table I shows information about which epistemic elements significantly make up the most important modules in the two networks, respectively. The size of each module is proportional to the amount of focus that students put on the epistemic elements within that module and is indicated by a PageRank measure [52]. The width of the links between modules is proportional to how students relate information between the modules and is represented by a flow measure.

The network generated from the second interview contained more nodes and more links, but the density, i.e., the number of links divided by the total number of possible links, is about the same for both networks. Table II shows data about the frequency of unique epistemic elements belonging to each of the main categories. Frequency analysis shows that the number of unique epistemic elements increases with about 25%. This increase is mostly due to physics elements which increase with more than 100%.

**A. Before the task**

In the first interview the students focused on descriptive aspects of the assignment. The epistemic elements capturing most of the flow were clustered in five major modules. As seen in Fig. 3 the flow between the largest modules was

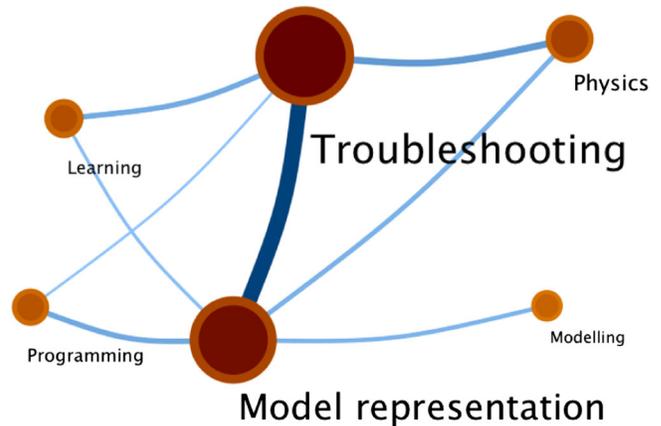


FIG. 4 (color online). Simplified networks after the problem-solving task. The size of each module corresponds to the importance of the module in the network, i.e., how much focus students put on associating between epistemic elements within that module, and the width of each link represents the information flow between modules. This network captures 95% of the node flow and 83% of the link flow.

TABLE I. Clustering of selected number of epistemic element into the modules with the highest flow for each network. For a description of epistemic elements, see the Appendix, Table III.

Network	PageRank (%)	In/Out Flow (%)	Significant <sup>a</sup>	Epistemic elements <sup>b</sup>
<i>Before</i>				
Values	23	7.9	yes	pr_m be_co bv_l be_s
Difficulties	21	9.4	yes	ba_d r_gs r_li r_t
Strategies	18	8.1	no	ph_o mo_ls
Simulation methods	13	5.1	yes	mo_sm mo_ts
Modeling real systems	11	3.9	no	mo_rs ph_sf mo_sps mo_cg ph_f
<i>After</i>				
Troubleshooting	34	11	yes	r_v mo_t ph_ec mo_sd
Data representation	29	11	no	pr_rd ba_d mo_cg mo_cd ba_e
Physics	12	4.7	yes	ph_k ph_f ph_c

<sup>a</sup>Modules significantly stand alone, 85% confidence interval.

<sup>b</sup>Epistemic elements significant within the module, 85% confidence interval.

strong (indicated by the thickness of the link). The meaning of each module is described below and data are also found in Table I.

- *Values* clustered epistemic elements that described what students expected and valued. The main epistemic elements were beliefs about expectancies of learning MATLAB. Students also expressed confidence that this will indeed happen and ascribed this to knowledge acquired in previous courses. Students also expressed value beliefs, such as interest, profession, and real world connection.
- *Difficulties* represented the second largest module and clustered how students expected the difficulties with how to get started with the assignment, i.e., understanding how to formulate the numerics. Within this module were also the students' suggestions of how to cope with the difficulties. They were aware that they needed to put out a lot of effort and they also would take help from each other as well as from teachers. Students ranked reading the lab instruction as important in order to get started.
- *Strategies* mainly described how students planned to start with modeling a small system before moving on to the more complex larger systems that were part of the assignment. Within this module students described the physics involved as oscillations and kinematics. Students also, to a certain degree, expressed the possibilities to visualize the systems, an epistemic element which was very common after the task.
- *Simulation methods*. This module clustered the epistemic elements the students used when describing how to select a simulation method and how it worked. A qualitative comparison between the Euler method and the leapfrog method were the main content of this module. Students explained these methods in terms of errors related to time steps issues.
- *Modeling real systems* described the relation between using the particle-spring system to model real

TABLE II. Comparison of the before and after epistemic networks.

	Before	After			
Density	0.102	0.114			
Number of links	350	600			
Number of nodes	59	73			
		PageRank (%)	PageRank (%)		
Number of nodes belonging to each main category		Before	After		
	Physics	9	17	19	24
	Math	7	11	7	4
	Programming	10	12	11	19
	Modeling	13	28	15	24
	Beliefs	14	19	14	18
	Resources	6	13	7	11

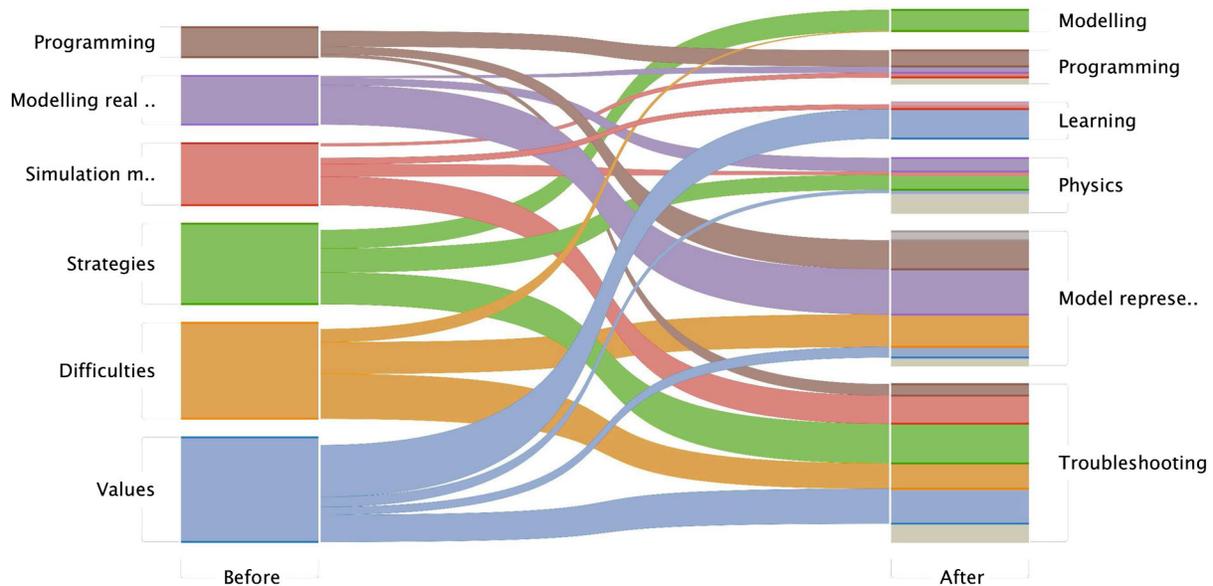


FIG. 5 (color online). Alluvial diagram showing how the modules in the epistemic networks rearrange between before and after the task.

systems, such as collisions and friction, and the physics that was needed to describe this in terms of forces associated with springs and friction.

### B. After the task

The second interview revealed a different epistemic structure than the first interview. More nodes and links were added. The network, however, was more evenly distributed and the clustering into modules was not as apparent as in the first interview. The flow was concentrated to two modules: one where the epistemic elements relating to troubleshooting of the simulation could be distinguished, and one where issues concerning the representation of model data in the MATLAB environment were interacting. A third module clustering many physics elements also appeared which was considered of special interest due to the increase of unique physics elements in the second interview.

- *Troubleshooting*. When students described their experiences of the task, the troubleshooting process caught most of the attention. Students associated troubleshooting with visualization of energy levels, and the main flow within this module was concentrated to these three epistemic elements expressing troubleshooting, energy conservation, and visualization. Within this module were also physics about oscillations and the effects on energy levels from damping the system interacting.
- *Model representation* captured students' experiences about representing the model systems in programming code, something that students experienced as difficult and demanding a lot of effort. Modeling the systems' particles and springs and finding expressions for forces and positions in order to numerically

solve the dynamics were the major epistemic elements interacting within this module.

- *Physics* captured many of the epistemic elements describing conceptual physics and interacts with both of the major modules. If the module *troubleshooting* mainly described energy conservation, *physics* captures physics concepts related to collisions, kinematics, friction, and associated forces, but also physics principles such as conservation of momentum.

### C. Comparison between before and after

The structures of the epistemic networks before and after the assignment were indeed different. These changes between the epistemic networks are visualized in an alluvial diagram shown in Fig. 5. The modules shown in this diagram represent exactly the same modules as those in the networks, Figs. 3 and 4, and their PageRank is represented by their height. However, in the alluvial diagram no links between the modules in a network are shown. Each module is color coded in order to follow what happens with the structure of each module, i.e., whether the modules are intact or whether the interaction between epistemic elements changes between the two occasions. It is clear that the modules giving the structure of the first network are not intact. The epistemic elements rearrange into other modules, showing that the pattern of interaction between epistemic elements changes between before and after the task. This represents a change in how students frame the situation based on the experiences they learn from the task. The largest module in the *after* network is represented by epistemic elements from every module in the *before* network, indicating a more complex interaction between the epistemic elements, possibly showing a more coherent view of the task.

It is also interesting to see what actually happens to the epistemic elements describing physics, math, modeling, and programming. The most noticeable result is that students use more physics concepts after the task. The increase of unique physics concept is over 100%, as shown in Table II. Also the importance of physics concepts in the network, indicated by the PageRank measure, increases from 17% to 24%. Programming aspects do increase as well even though students do not start using new programming concepts. Students frame the task in modeling aspects almost equally before and after the task. Math, however, is used in students' framing before the task but more sparsely after.

In the following section these results are interpreted and discussed in order to present some concluding remarks.

## V. DISCUSSION

Computational problem solving in university physics education is a challenging field in teaching and learning as well as in research about physics education. The complex mix of knowledge and skills needed from the subject fields of physics, math, programming, and modeling, together with what beliefs and expectations the student has, puts limitations on how to investigate and interpret data in order to understand how students frame this field in epistemic aspects. In this study a thematic analysis of interview transcripts has generated epistemic elements of how students express knowledge, expectations, and experiences concerning a computational physics problem. The interaction between these epistemic elements has been visualized using a network analysis approach in order to get a picture of the students' epistemic frames of this task.

The network analysis approach reveals patterns of interaction between the elements that are involved in describing a particular learning situation, similar to concept maps [8] and mental models [29]. The epistemic networks presented here should, however, not be seen as equivalent to previous mental models but could rather function as a complement to further understand how students experience a particular learning situation. The map equation method adds information to these patterns by calculating clusters of elements and the corresponding flow between the clusters. The maps generated from the data in this study have potentials to visualize how ideas concerning a specific task interact within students' minds and how these ideas and the corresponding flow change when personal experiences from the actual task are added.

The interaction between epistemic elements indicates how prior knowledge is used in order to create new solutions and thus new knowledge. According to the first network, students do think they possess the conceptual knowledge needed for solving this task. However, they have vague ideas about how to use this knowledge and tend to array concepts without considering how the different subject fields are interacting. They fail to present a coherent view of the task. In the second network, the

rearrangement of epistemic elements into other clusters of modules shows that this actually has happened. Students integrate concepts from all fields in order to describe how they have solved the problem. The results indicate that students are thus able to use physics, math, and programming in a new context where the modeling of the problem drives towards the problem's solution. Learning is indeed to figure out how to put prior knowledge in new contexts and this is suggested to have happened here. That students troubleshoot their solution by checking energy conservation rather than debugging program code shows that students epistemically frame the task as a physics problem, not just a MATLAB problem. This is also revealed in the increased number of used physics concepts. It is thus proposed from this study that students are confronted with a deeper meaning of physics concepts and thus develop a more coherent understanding of how to use physics in new contexts. The epistemic network after the task shows how students emphasize the visual feedback from the simulation. Previous research has shown that students who receive their simulated results as visualizations rather than numerical values tend to use mental imaginary to solve problems rather than algorithmic thinking [55]. This implies that students who solve computational problems and get feedback from a visual, interactive simulation during the solution process have more opportunities to develop a visual mental representation where physics aspects are interacting rather than an algorithmic mental representation where the coding aspects are dominant, which is supported by this study. In numerical approaches to physics problems students need to use physics in order to get intelligible results, to just focus on the math and the programming is not enough.

Students have expectations about this assignment. They do see difficulties but they still express expectancy of success. They ascribe this success to their own ability in math and physics in general but also to the possibility to cooperate with other students. However, the expectations of learning are attributed to autonomy, i.e., having the responsibility to solve the problem themselves. What they will learn is generally expressed in terms of MATLAB. Few students express knowledge in terms of memorizing and learning, except in relation to remembering specific expressions in physics, and do generally strive for a coherent view of physics as well as problem solving. In general, students express the same beliefs before and after the task. Beliefs about expectancies and efficacy, attributions, as well as about values are present at both occasions and to the same amount. However, these beliefs' interactions with other epistemic elements are changed and difficulties and effort associated with the numerical algorithms are shifted towards being associated with how the forces and particles are represented in the programming code. The fact that the module with the largest PageRank after the task, indicating troubleshooting, hardly contains

any programming elements indicates that students did have focus on the underlying aspects on the assignment.

### VI. CONCLUSIONS

The findings from this study imply the potential in giving university students majoring in physics challenging assignments to work with. An assignment that mixes competencies from different disciplines is suggested to encourage students to actually use prior knowledge in order to create new knowledge, that is, to gain a more coherent view of a compound problem situation. In this case where competencies in physics, math, programming, and modeling were applied in a computational physics problem where the output solution was a visualization of the simulation, students did seem to have acquired a more focused view of the disciplines involved in this task subject, shown by the changes in network structures. In particular, the number of physics concepts used in the interviews in relation to other concepts reveals a deepened focus on the physics aspects of this task. This is also shown by the focus on physics aspects when troubleshooting and debugging their numerical solution.

The beliefs and knowledge structures presented here as epistemic frames and visualized as epistemic networks are suggested to open up deeper understanding of the complex relation between beliefs and attitudes and the more

context-dependent knowledge and skills aspects. Epistemic networks could give useful information about the beliefs and knowledge that are available for other learning situations, for example, when developing teaching and teaching material where it is important to know how the students' learning is affected and if students' focus coincides with the learning intentions. The network method gives a novel way of identifying and visualizing how students conceptualize a task. The work presented in this study is limited to a special case, but the method is suggested to complement other methods of measuring beliefs and knowledge structures in order to understand students' behavior in learning situations.

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### APPENDIX: CODING SCHEME

Table III lists the coding scheme used for analyzing the interview transcripts.

TABLE III. Coding scheme used for coding transcribed interviews with students into epistemic elements.

Main category	Epistemic element	Code
Physics	ph_physics	ph
	ph_angular_momentum	ph_am
	ph_angular_momentum_conservation	ph_amc
	ph_collision	ph_c
	ph_classical_mechanics	ph_cm
	ph_center_of_mass	ph_com
	ph_energy_conservation	ph_ec
	ph_friction	ph_f
	ph_gravity	ph_g
	ph_kinematics	ph_k
	ph_kinetic_energy	ph_ke
	ph_momentum_conservation	ph_mc
	ph_moment_of_inertia	ph_mi
	ph_normal_force	ph_nf
	ph_newtons_laws	ph_nl
	ph_potential_energy	ph_pe
	ph_oscillation	ph_o
	ph_spring_force	ph_sf
	ph_torque	ph_t
	Math	ma_math
ma_algebra		ma_a
ma_analytical_solution		ma_as
ma_derivative		ma_d
ma_differential_equation		ma_de

TABLE III. (Continued)

Main category	Epistemic element	Code
	ma_linear_algebra	ma_la
	ma_numerical_solution	ma_ns
	ma_statistics	ma_s
Programming	pr_programming	pr
	pr_algorithm	pr_a
	pr_code	pr_c
	pr_code_previous	pr_cp
	pr_datastructure	pr_d
	pr_error_message	pr_em
	pr_function	pr_f
	pr_indexing	pr_i
	pr_loop	pr_l
	pr_matlab	pr_m
	pr_representation_of_data	pr_rd
	pr_real_time_computing	pr_rtc
Modeling	mo_constraint	mo_c
	mo_change_dimension	mo_cd
	mo_collision_with_ground	mo_cg
	mo_collision_with_net	mo_cn
	mo_evaluate	mo_e
	mo_euler	mo_eu
	mo_leap_frog	mo_lf
	mo_large_system	mo_ls
	mo_making_simplifying_assumption	mo_msa
	mo_real_situation	mo_rs
	mo_spring_damper	mo_sd
	mo_select_model	mo_sm
	mo_spring_particle_system	mo_sps
	mo_small_system	mo_ss
	mo_troubleshoot	mo_t
	mo_time_step	mo_ts
Resource	r_get_started	r_gs
	r_lab_instruction	r_li
	r_previous_course	r_pc
	r_paper_and_pen	r_pp
	r_problem_solving_reflection	r_psr
	r_students	r_s
	r_teachers	r_t
	r_visualization	r_v
Expectancy beliefs	be_autonomy	be_a
	be_collaboration	be_c
	be_coherence	be_co
	be_equations_and_formulas	be_ef
	be_failure	be_f
	be_memorizing_and_learning	be_ml
	be_success	be_s
Value beliefs	bv_identity	bv_i
	bv_learning	bv_l
	bv_personal_interest	bv_pi
	bv_real_world_connection	bv_rwc
Attribution beliefs	ba_ability	ba_a
	ba_difficulty	ba_d
	ba_effort	ba_e

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