Appendix A

I. Topics

- **A. Coulomb's Law**
- **B. Electric Field**
- **C. Gauss's Law**
- **D. Electric Flux**
    1. **Closed surface**
    2. **Open surface**
- **E. Field Lines**
    1. **Direction**
    2. **Magnitude**
    3. **Source**
- **F. Electric Potential Difference**
    1. **Definition**
    2. **Relationship to Energy**
    3. **Equipotential Lines**
- **G. Electric Potential Energy**
    1. **Work**
    2. **Assembling Charges**
- **H. Conductors**
    1. **Electric field inside**
    2. **Electric field at surface**
    3. **Equipotential**
    4. **Shielding**
- **I. Capacitance**
    1. **Definition**
    2. **Physical Capacitor**
    3. **Networks (contracting and expanding)**
    4. **Energy stored**

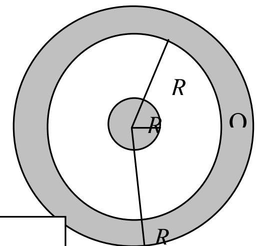
II. Mathematics

- **A. Calculus**
- **B. Vectors**
- **C. Complex Algebra**
- **D. Graphs**
- **E. Symbolic**

Appendix B

Paired problems that showed a large improvement. The algebraic answer to the two problems is identical, so it would be possible to plug the numbers from one problem into the answer for the other.

A charged sphere of radius R_1 and total charge Q is placed at the center of a hollow spherical conducting shell (inner radius R_2, outer radius R_3) which has a net charge Q_{shell}.

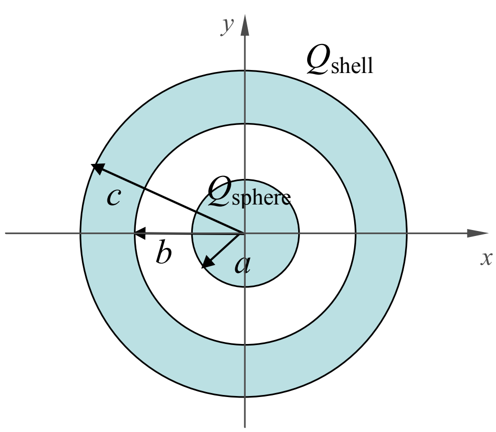


11. Find the magnitude of the electric field, E, outside of the spherical shell at a distance of R_4 = 5 cm from its center.

a. $E(R_4) = 0$
b. $E(R_4)$ = 7200 N/C
c. $E(R_4)$ = 14400 N/C

$Q = 6\times10^{-9}$C
$Q_{shell} = -4.0\times10^{-9}$C
R_1 = 0.1 cm
R_2 = 1.0 cm
R_3 = 2.0 cm
(Note: the figure is not drawn to scale)

A metal sphere of radius a is centered on the origin, and carries a total charge Q_{sphere}. Surrounding this sphere is a spherical metal shell of inner radius b and outer radius c. This shell is also centered on the origin, and carries a total charge Q_{shell}.



a = 1.5 m
b = 3.0 m
c = 4.5 m
Q_{sphere} = –5 μC
Q_{shell} = +2 μC

2. Find the magnitude $|E|$ of the electric field at a radius of 8 m from the origin.

a. $|E| = 2.8\times10^{2}$ N/C
b. $|E| = 4.2\times10^{2}$ N/C
c. $|E| = 7.0\times10^{2}$ N/C
d. $|E| = 2.3\times10^{3}$ N/C
e. $|E| = 3.4\times10^{3}$ N/C

Solution

To find the electric field in this spherically symmetric problem we can use:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q_{encl}}{r^2}$$

The enclosed charge includes both the sphere and shell:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{(-4\ \text{nC} + 6\ \text{nC})}{(0.05\ \text{m})^2} = 7200\ \text{N/C}$$

(B)

To find the electric field in this spherically symmetric problem we can use:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q_{encl}}{r^2}$$

The enclosed charge includes both the sphere and shell:

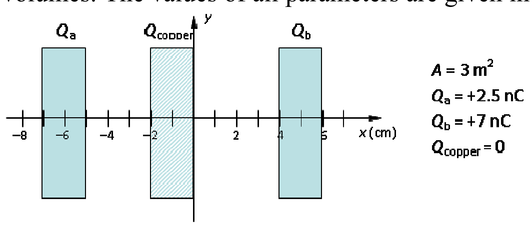
$$E = \frac{1}{4\pi\varepsilon_o}\frac{(-5\ \mu\text{C} + 2\ \mu\text{C})}{(8\ \text{m})^2} = 4.2\ \times\ 10^2\ \text{N/C}$$

(B)

Appendix C

Paired Problems that showed negative improvement. Perhaps because if one uses the algebraic solution for one problem, it does not give the correct answer for the second problem.

Consider three flat slabs of identical dimensions: their area $A = 3\ \text{m}^2$ is so large compared to their 2 cm thickness that they may be considered of infinite area for purposes of calculation. The figure below shows how they are positioned. Slabs *a* and *b* are made of <u>glass</u> (an excellent insulator), while the middle slab is made of <u>copper</u> (an excellent conductor). The copper slab is uncharged. However, the two glass slabs *a* and *b* are given total charges Q_a and Q_b respectively, distributed uniformly throughout their volumes. The values of all parameters are given in the figure.



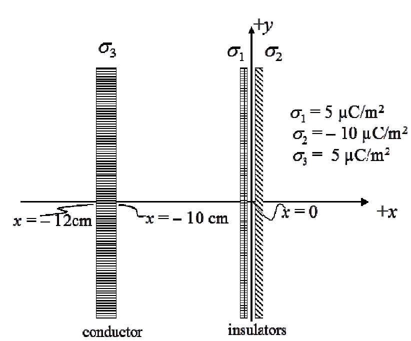
9. Calculate the total charge Q_R which resides on the right-hand face of the copper slab (i.e. at $x = 0$).

a. $Q_R = -2.25$ nC
b. $Q_R = -7.00$ nC
c. $Q_R = -9.50$ nC

Two thin infinite planes of <u>insulating</u> <u>material</u> with charge densities $\sigma_1 = 5\ \mu\text{C/m}^2$ and $\sigma_2 = -10\ \mu\text{C/m}^2$ are directly adjacent to each other at the origin. At $x = -12$ cm to -10 cm an infinite conducting plane of thickness 2 cm has a net charge density $\sigma_3 = +5\ \mu\text{C/m}^2$. All planes are perpendicular to the x axis.

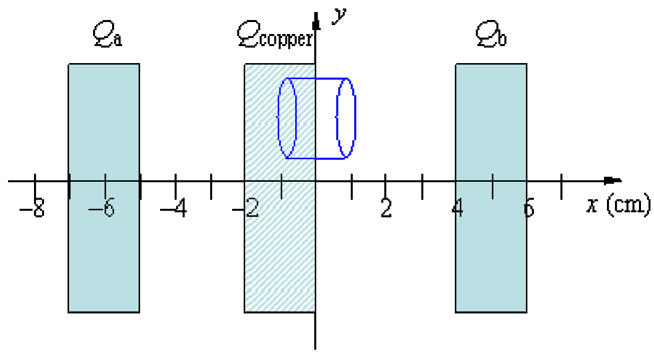
4. Calculate the charge density, σ_{3L}, on the <u>left surface</u> of the conducting plane.

a. $\sigma_{3L} = 2\ \mu\text{C/m}^2$
b. $\sigma_{3L} = -2\ \mu\text{C/m}^2$
c. $\sigma_{3L} = 0\ \mu\text{C/m}^2$
d. $\sigma_{3L} = -5\ \mu\text{C/m}^2$
e. $\sigma_{3L} = 5\ \mu\text{C/m}^2$



Solution

Draw a Gaussian surface around the right surface of the conductor with one side in the conductor:



The electric field in the conductor is zero, and the electric field to the right of the conductor is (same as above):

$$E = \frac{\sigma_a}{2\varepsilon_0} - \frac{\sigma_b}{2\varepsilon_0} = \frac{\left(\frac{2.5\text{ nC}}{3\text{ m}^2}\right)}{2\varepsilon_0} - \frac{\left(\frac{7\text{ nC}}{3\text{ m}^2}\right)}{2\varepsilon_0} = -84.7\text{ N/C}$$

Finally, from Gauss's Law we can find the surface charge density:

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_0}$$

$$0 + E \cdot A = \frac{\sigma_R \cdot A}{\varepsilon_0}$$

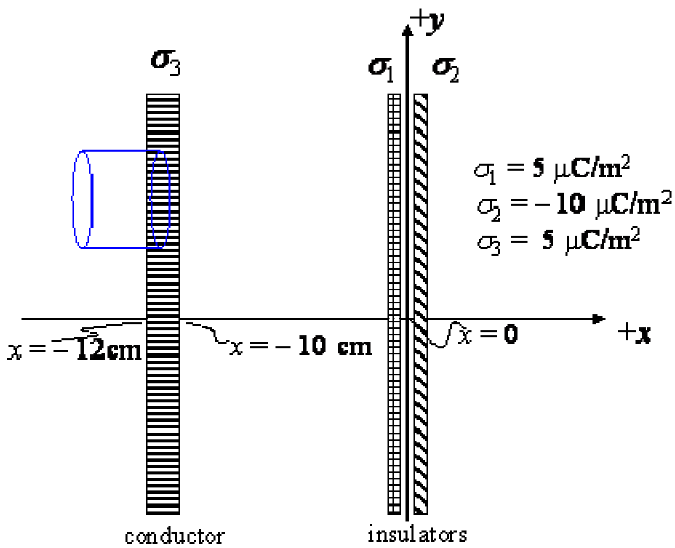
$$-84.7\text{ N/C} = \frac{\sigma_R}{\varepsilon_0}$$

$$\sigma_R = 0.75\text{ nC/m}^2$$

And then the charge is calculated by multiplying by the area.

(A)

Draw a Gaussian surface around the left surface of the conductor with one side in the conductor:



The electric field in the conductor is zero, and the electric field to the left of the conductor is:

$$E = \frac{-10\ \mu\text{C/m}^2\ +5\ \mu\text{C/m}^2\ +5\ \mu\text{C/m}^2}{2\varepsilon_o} = 0\ \text{N/C}$$

Finally, from Gauss's Law we can find the surface charge density:

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_o}$$

$$0 = \frac{\sigma_{3L}}{\varepsilon_o}$$

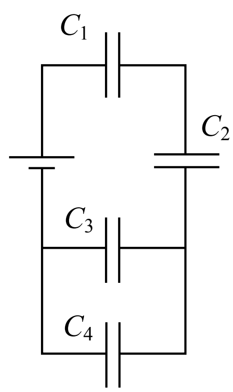
$$\sigma_{3L} = 0\ \mu\text{C/m}^2$$

(C)

Appendix D

Paired Problems that showed no improvement. Perhaps because the solution presented uses some symmetry in the particular problem. This is specific to the problem, and although applicable, not easy to transfer to the paired problem.

Four capacitors are connected to a battery in the manner shown at right. All capacitances are given in the table next to the figure, but the voltage $\mathcal{V}$ delivered by the battery is unknown. What *is* known is that, when connected to the network, the capacitor C_1 acquires a charge of magnitude $|Q_1| = 15$ nC $= 15 \times 10^{-9}$ C on each of its plates.



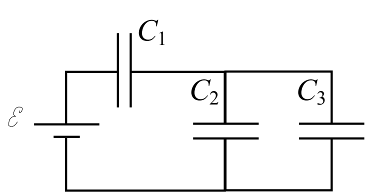
$Q_1 = 15$ nC

$C_1 = 8$ pF
$C_2 = 6$ pF
$C_3 = C_4 = 5$ pF

26. What is the charge Q_4 on capacitor C_4?

a. $Q_4 = 7.5$ nC
b. $Q_4 = 15$ nC
c. $Q_4 = 30$ nC

Three uncharged capacitors are connected to a battery as shown in the figure below. The battery voltage and capacitances are given on the right side of the circuit.



$\mathcal{E} = 12$ V
$C_1 = 6$ μF
$C_2 = 4$ μF
$C_3 = 2$ μF

25. Calculate the voltage drop, V_3, across the capacitor C_3.

a. $V_3 = 2$ V
b. $V_3 = 6$ V
c. $V_3 = 9$ V

Solution

First collapse the capacitors 3 and 4 into one capacitor:

$$C_{34} = C_1 + C_2 = 10\ \text{pF}$$

Since this combination C_{34} is in series with C_1, they have the same charge:

$$Q_{34} = Q_1 = 15\ \text{nC}$$

Now we can calculate the voltage across capacitor 4 (which is the same as the voltage across capacitor 3 since they are in parallel):

$$V_4 = V_3 = V_{34} = \frac{Q_{34}}{C_{34}} = 1500\ \text{V}$$

Finally we can find the charge on capacitor 4:

$$Q_4 = C_4 V_4 = 7.5\ \text{nC}$$

(A)

(Note since C_3 and C_4 are identical we know the charge has to split evenly between C_3 and C_4 so the charge should be half the total.)

First collapse all the capacitors into one equivalent capacitor:

$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2 + C_3}} = 3\ \mu\text{F}$$

Then find the total charge in the capacitor circuit:

$$Q = CV = 36\ \mu\text{C}$$

This total charge is on C_1 and the C_{23} combination since they are in series. Next we can find the voltage across capacitor 3:

$$V_3 = V_2 = V_{23} = \frac{Q_{23}}{C_{23}} = 6\ \text{V}$$

(B)

(Note since C_{23} and C_1 are equivalent in value, it makes sense that each has a voltage drop of half the total.)

Appendix E

	Number and percentage of students who correctly answered on each problem ID					
Problem ID	As the first problem **(Baseline)**		As paired problem when feedback is the **answer**		As paired problem when feedback is the **solution**	
1001	266	74.1%	152	84.4%	136	76.8%
1024	294	79.9%	138	80.2%	142	79.8%
1002	173	52.4%	90	58.4%	95	57.6%
1025	191	55.8%	105	66.5%	95	62.1%
1003*	109	36.3%	88	60.7%	113	77.4%
1026*	148	48.7%	69	50.4%	116	80.6%
1004	182	64.5%	105	72.9%	101	73.7%
1027	183	64.7%	103	70.5%	99	75.6%
1005	169	63.1%	95	76.6%	101	73.7%
1028	160	60.8%	81	65.9%	86	67.2%
1006*	249	98.4%	132	97.1%	123	99.2%
1029*	257	98.1%	128	97.7%	120	99.2%
12*	201	79.4%	128	94.1%	116	93.5%
30*	197	75.2%	113	86.3%	96	79.3%
1008*	172	61.0%	109	80.7%	114	83.8%
1031*	219	77.4%	113	89.0%	132	93.0%
1838	191	67.7%	102	75.6%	111	81.6%
1032	161	56.9%	75	59.1%	94	66.2%
1010*	134	52.1%	60	54.1%	68	58.1%
1033*	177	67.3%	88	73.3%	88	79.3%
14	93	36.2%	43	38.7%	48	41.0%
1034	61	23.5%	42	36.5%	46	44.2%
15	132	51.4%	58	52.3%	60	51.3%
32	146	56.2%	67	58.3%	57	54.8%
1013	100	50.3%	48	52.7%	49	56.3%
1036	119	60.4%	58	68.2%	57	58.2%
110	90	45.2%	33	36.3%	35	40.2%
36	74	37.6%	33	38.8%	30	30.6%
1016	147	57.9%	80	60.2%	83	69.2%
1039	208	78.8%	102	84.3%	104	81.3%
1017	55	23.3%	23	21.7%	22	19.3%
1040	86	36.1%	38	35.8%	53	47.7%
1018	107	54.9%	49	45.0%	49	47.6%
1041	151	71.2%	75	75.8%	79	82.3%
1019	123	65.1%	57	62.0%	66	66.0%
1042	115	57.2%	46	56.8%	68	73.1%
1020	73	42.7%	36	40.4%	36	43.9%
1043	91	49.5%	46	56.1%	38	52.8%
1255	124	63.9%	63	70.0%	64	67.4%
1044	98	50.8%	65	69.9%	70	77.8%
1022	72	39.6%	32	38.1%	32	38.6%
1045	90	53.9%	57	64.8%	50	56.2%
1023	111	68.9%	45	61.6%	57	70.4%
1046	126	79.7%	54	77.1%	64	78.0%
Average		**58.8%**		**63.5%**		**66.0%**

*Paired problems with exact final formula