A qualitative investigation of college students' conceptions of electric fields

Brian W. Adrian and Robert G. Fuller
Department of Physics and Astronomy, University of Nebraska, Lincoln, Nebraska 68588

This paper describes a qualitative investigation of introductory college physics students' understandings of the electric field prior to and following instruction. Four aspects of electric fields are examined: existence/definition (or mental construct), source, vector representation, and superposition. Results are discussed as a means for improving the development of the physics curriculum.

I. INTRODUCTION

The Research in Physics Education Group at the University of Nebraska-Lincoln has been interested in using research in student understanding to develop effective instructional materials. The first step in creating these materials is to investigate student understanding of the relevant concepts. This paper discusses such an investigation into college physics students' understandings of the electric field concept. Investigation of students' preconceptions helps design instructional materials that may be effective in overcoming students' difficulties. Investigation of students' misconceptions after instruction provides insight for making additional changes to the instruction.

It has been suggested that students who can successfully solve quantitative physics problems may not qualitatively grasp the underlying concepts.[i] This investigation examines how college physics students understand electric fields before and after instruction, and the student difficulties that need to be addressed by formal physics instruction in order to improve qualitative comprehension.

II. DESCRIPTION OF THE RESEARCH

A. Reasons for the study

After formal physics instruction, many students who can successfully perform mathematical algorithms to arrive at numerically correct answers seem to lack a conceptual understanding.[ii] To overcome this failure to understand, instruction must address the specific difficulties students have with the relevant concepts.[iii] An appropriate qualitative research study can explore these student difficulties.

The concept of a field is fundamental to the study of physics. It is through the field concept that action-at-a-distance is described.[iv] Fields are used in the study of gravity and electricity and magnetism, topics studied in virtually all introductory college physics courses. While there have been several studies on conceptual understanding of simple circuits,[v] other topics in electricity and magnetism have not received similar attention.[vi] By investigating how college physics students think about electric fields, a foundation for future studies and a guide to physics curriculum development can be prepared.

B. The field concept

The concept of a field has been very powerful for modern physicists. The field construct has led to the development of such useful entities as photons and gluons. Even with his work on gravity, however, Isaac Newton was not comfortable with the idea of action-at-a-distance.[vii] Despite the observation of electric and magnetic phenomena by the ancient Greeks, the concept of a *field* to explain action-at-a-distance was not introduced until the early 1800s by Michael Faraday. The pinnacle of classical field theory was reached upon publication of Maxwell's equations for electromagnetic fields in 1873. Although Maxwell's equations summarize and describe all classical electromagnetic phenomena, students in introductory physics courses may not use such advanced methods. Their experiences with and perceptions of electric fields may not have provided them with the understanding or language to offer an adequate description of an electric field. We attempted to determine their understandings of various aspects of electric fields by using oral interviews and written examinations.

C. Methods of investigation

For instruction to address specific student difficulties, it is necessary to understand student conceptions. Such knowledge was obtained in this study using written tests and individual oral interviews, similar to the interviews developed by Piaget.[viii] Careful analysis of the interviews and written answers provided some understanding of student difficulties. This study focused on a fundamental concept in electricity and magnetism, the concept of the electric field. Table I shows an account of the Test Instruments (TI) used in data collection.

Many of the subjects were both pre- and post-tested; however, the pretests and posttests had different forms, and it is not believed that the pretests had a measurable effect on the posttest responses. For example, Test Instruments 3 and 5 asked subjects to sketch electric fields for various charge distributions. On pretests (Test Instruments 2 and 4), the same subjects were not presented with charges, but were asked only to sketch their idea of an electric field.

TABLE I. Test Instruments (TI) used in data collection.

TI #	Instrument	Pre- or post-instruction	Date	Number of subjects
1	oral interview	pre	summer '96	8
2	written test	pre	fall '96	39
3	written test	post	fall '96	78
4	written test	pre	spring '97	177
5	written test	post	spring '97	176
6	written test	post	spring '97	93
7	oral interview	post	summer and fall '96	32

1. The oral interviews

Individual oral interviews were conducted with common laboratory objects for electrostatics experiments, such as pith balls, plastic rulers, and wool, which could be used by the subject or the interviewer. Subjects were encouraged to use the provided objects to explore electrostatic phenomena. They were asked to predict what would happen and explain their reasoning. Paper and pen were provided for sketching. The interviews were designed to encourage the subjects to discuss electrostatic interactions. The subjects were free to discuss any aspect of their observations of electrostatic phenomena. Guidance was provided from the interviewer through probing questions. Words such as "fields" were not used unless used by the subject. The format for the interview was constant throughout the investigation. The interviews were videotaped and professionally transcribed for later analysis. The transcripts were compared to the original videotapes for accuracy by the author. Every interview subject in this study was asked to explain an example of electrostatic attraction or repulsion. If the subject did not suggest using electric charges to cause action-at-a-distance, the interviewer would demonstrate the effect of a charged ruler on a pith ball and ask the subject to explain the interaction. A typical interview began as follows (the letter "I" represents the interviewer and the letter "S" the subject):

I: The system in question consists of a string suspended from the ceiling, from which we can hang any of several objects, including a pith ball, a Styrofoam sphere, a steel marble, and a hollow aluminum sphere. The first question I would like to ask you is this: can you think of a way we can affect this system without touching it?

S: Yes, you can induce a charge on [the pith ball] and with that charge you could move it in just about any direction depending on how much charge you used.

I: How would you go about inducing a charge on this?

S: What we did in lab was use the plastic ruler which was, I believe, a negative charge and held it underneath the pith ball to make the positive charge on the lower half of the ball and the negative charge on the top half of the ball. We touched the top of the ball to remove the negative charge and left positive charge on the ball.

I: You used something just like this [plastic ruler]?

S: Right.

I: If you were just to charge this up and bring it close what would happen first?

S: ... I would say that it would attract.

I: Why do you think it would attract? Draw pictures if you want.

S: Because of what I said ... that [ruler] is charged where this [pith ball] isn't.

I: Ok let's see what it does.
S: They attract.
I: Now that you know it attracts, can you explain why it attracts?
S: ... back to the old saying that opposites attract

After this subject also observed electrostatic repulsion, the interviewer asked why they now repelled.

S: I don't know why they repel each other, other than 'like charges' ... I guess I don't know how to put it in words.

It was important that the interviewer used the same vocabulary as the subject. For example, once the subject used the word "field," the interviewer would ask for clarification, and thereafter use the word in the same manner as the subject. When possible, the subjects were asked to provide sketches.

2. The written tests

The written tests used in this study consisted of open-ended questions. Students were asked to answer the questions and to provide an explanation of their reasoning, including sketches when possible. This was an evolving process in which knowledge obtained from the early written tests and the oral interviews guided the development of the later written tests. The written answers were evaluated by two independent professionals, the author and another physicist.

With the exception of some sketches, explanations were required for all answers, written and oral. The oral interviews gave a more thorough understanding of students' difficulties, while the written answers provided an indication of the prevalence of the difficulties.

D. Student population and instruction

All student subjects used in this study were enrolled in the algebra-based introductory physics course at a typical midwestern university. This was a two-semester, five credit-hour course, with three fifty-minute lectures, a single fifty-minute recitation, and one two-hour-and-fifty-minute laboratory per week. The recitations and laboratories were led by graduate teaching assistants who were not part of this research study. The laboratories included multimedia lessons designed in parallel with this study as part of an ongoing process of curriculum development, described elsewhere.[ix]

Approximately 50% of the students indicated taking a high school physics course. Approximately 44% of the students used in this study were female, and 56% male. The written test instruments were administered to the entire class, therefore the entire range of grades and abilities was represented. The oral interview subjects represented approximately 90% of their class, and none of the interview subjects received a grade lower than C.

III. SPECIFIC STUDENT DIFFICULTIES BEFORE INSTRUCTION

From analyses of the test instruments, the student difficulties with electric fields were grouped into four categories which are explained below: existence/definition, source, vector representation, and superposition. These categories are not mutually exclusive – difficulty with the vector representation may cause difficulty in demonstrating an understanding of superposition, for example – and may not be exhaustive. In addition, there may be uncertainty in categorizing an individual student's difficulties because of the imprecise use of language on the part of the subject, the researcher, or both.

A. Difficulties with existence/definition of the electric field

Of the pre-instruction oral interviews considered in this study, one-fourth (2 of 8) were judged to have an adequate mental construct for an electric field. For this study, an "adequate" mental construct allowed for action-at-a-distance requiring no physical contact, having a direction, and depending on distance, without being described as outlined below. The 6 oral interview subjects who did not meet the criteria for an adequate mental construct used one or more of the five misconceptions of an electric field discussed below.

The written pretests for this category were analyzed independently by two investigators as described above. The analyses were compared, and differences discussed between the investigators. Cases of simple misinterpretation of students' answers were resolved. The prevalence of the various conceptions are summarized in Table II.

TABLE II. Prevalence of misconceptions about the existence or definition of an electric field prior to instruction

TI #	Number of subjects	Correct	Misconceptions					
			Area	Area of charge	Charg e	Force	Energ y	Blank/ no info.
2	39 100%	0 0%	8 21%	9 23%	10 26%	3 8%	0 0%	9 23%
4	177 100%	12 7%	36 20%	56 31%	27 15%	1 1%	4 2%	42 24%
total	216 100%	12 6%	44 20%	64 30%	37 17%	4 2%	4 2%	51 24%

1. Description of an electric field as an area

When asked to explain their idea of an electric field, some students indicated that a field is an area where something exists or occurs. For example, "a corn field is an area with corn." Below is a typical statement from an oral interview that expresses this.

I: ...What could you tell me about a field?

S: ...I thought an electric field was just the area around this charge or something ... just area around it [in which] the force could be exerted.

The same description was also use on written pretests. (The letter "Q" designates the question and the letter "S" the response.)

Q: What does the term "electric field" mean to you? Please be as specific as you can.

S: The area around a charged object.

Approximately 20% of the 216 students who were asked this question on written pretests responded in a similar manner. When analyzing these responses, we were careful to make certain that the response emphasized the *area* and not what was in that area. What was important for this study was what the students *said*, since we cannot probe any further than the words they used in their written answers.

2. Description of electric fields as charges

Many students think of an electric field as a collection of charges. This often was equated with a group of ions or electrons. Below is a typical statement of such belief from the oral interviews.

I: How do you suppose this [ruler] is affecting that [aluminum sphere] without even touching it?

S: There's a flow of electrons around the ruler and a flow of electrons around this aluminum ball, so when you get them close enough to one another then they exchange electrons and that probably causes them to stick to one another but if you get really far away then their clouds are not close enough to exchange their electrons.

Approximately 17% of the students in this study described an electric field as a collection of charges, ions or electrons, as in this answer from the written pretests:

Q: What does the term "electric field" mean to you? Please be as specific as you can.

S: Electric charge surrounding a (sic) object.

2.5. Description of electric fields as areas of charge

About 30% of the written student responses were not clearly area *or* charge, but seemed to emphasize both the area *and* the charge. For example,

Q: What does the term "electric field" mean to you? Please be as specific as you can.

S: An electric field is an area of charge surrounding an object.

This description did not appear in the oral interviews. Answers such as "An electric field is the area around the nucleus of an atom that contains electrons," or "The cloud of electrons around a nucleus," were difficult to categorize as area, charge, or both, contributing to uncertainty in some categories.

3. Description of an electric field as a force

This category is used for students who can only talk about forces, not fields. This was often the case for students who only understand "likes repel and opposites attract." This category is also used for students who have heard of "force fields" in science fiction movies. This description was often combined with an area, as below:

S: Force fields.

I: Where have you heard of those?

S: Scientific TV shows, SciFi movies.

I: How would you describe the force field?

S: The area around the object in which the force is present.

Note that if force is discussed with charge, and the discussion is close to the formal definition of the electric field ($E \equiv \frac{F}{q}$), the description is accepted in this study as correct.

Only about 2% of the written responses on the pretests were limited to describing fields by "likes repel and opposites attract." When students offered more information, we classified their answer elsewhere.

4. Description of electric fields as energy

Some students provided very clear descriptions of their conception of an electric field:

S: A field is energy. You can't create energy so you cannot create a field.

This student also indicated that all objects must come with their own fields, and all we can do it make them larger or smaller, since we cannot create them.

Only about 2% of the students described electric fields as energy on the written pretests, perhaps indicating that this is also not a significant preconception.

B. Difficulties with source of the electric field

If students affirm the existence of a field, then it is logical to ask them about its source. It was not expected at this introductory level that students would suggest that changing magnetic fields give rise to electric fields; our expectations were correct, and no student in this study made such a suggestion. Many students in this study did state that electric charges create electric fields.

Confusion between electric and magnetic phenomena was evident from some interviews. The terms "positive" and "negative" were used to discuss electric charges and magnetic poles. This made it difficult to ascertain whether some subjects understood that electric charges produce electric fields.

With few students (6%) demonstrating a mental model for an electric field prior to instruction, it was difficult to measure student understanding of the source. It was clear from the oral interviews as well as written tests that students knew "likes repel," but it was not clear if they knew *what* repels. Confusion of electric with magnetic phenomena was evident, but not quantifiable given the imprecise use of language.

C. Difficulty with representation of the electric field

The improper use of the vector representation for electric fields has been suggested as a possible source of confusion,[x] and we included it in our investigation. Of the 8 pre-instruction interview subjects, seven could not attempt a sketch of an electric field. The single interview subject that did sketch a field did not use vectors.

On the 216 written pretests, only five students drew sketches that resemble a vector representation of an electric field. All five students had taken a high school physics course. Of these five, one drew a circle around the vector arrows and said the field is "the area where an electric charge can be measured," a second drew field vectors but called them "the flow of electrons," and the third drew lines but no vectors. The fourth sketch had only three vectors, but they were well-drawn. The fifth sketch contained an accurate representation, and it included the following caption: "The electric field of an object is an area around a charged object where electric force is 'felt.' The force decreases w/ increasing distance." Despite the indication that an electric field is an area, this student understood that the field is the vehicle for action-at-a-distance, and that the force decreases with distance. This fifth student demonstrated that the language needed to describe the concept may be as problematic as the concept itself. If a student cannot accurately describe a concept, it may be impossible to accurately assess that student's understanding.

D. Difficulty with superposition of the electric field

Superposition of electric fields was not discussed in the oral interviews. These oral interviews were part of a larger investigation,[xi] in which superposition of magnetic fields was found to be a problem for students. It has been suggested in the literature[xii] that students do not understand superposition of electric fields. Superposition is used throughout physics; we frequently linearize systems so we can simply add. Given the importance of the concept, we created this category for investigation.

Attempts to measure students' understandings of superposition of electric fields in written pretests were inconclusive. To demonstrate an understanding of superposition in this study, it was necessary for students to know that electric charges create electric fields and be able to sketch the fields thus produced. As discussed above, only 6% of the students used in this study demonstrated an understanding of the field concept. With such small numbers, it was not possible to reach a conclusion about the pre-instruction level of understanding of superposition.

IV. INSTRUCTION

All student subjects used in this study were enrolled in the algebra-based introductory physics course at a typical midwestern university. This was a two-semester, five credit-hour course, with three fifty-minute lectures, a single fifty-minute recitation, and one two-hour-and-fifty-minute laboratory per week. The written test instruments were administered during the second semester of this course, the semester in which electricity and magnetism were introduced. The textbook was a popular text[xiii] that covers topics in a traditional manner. Mechanics, thermodynamics, and waves are covered in the first fourteen chapters. The second semester of this course began with electric forces and electric fields, followed by energy and capacitance, electric current and resistance, direct current circuits, magnetism, inductance, alternating current and electromagnetic waves, then optics and modern physics.

Hour exams were offered approximately once per month and were at least 80% quantitative. Some of the instructors used multiple choice for as much as 40% of each exam and rarely used short-answer or essay questions.

The recitations and laboratories were led by graduate teaching assistants who were not part of this research study. Recitations were used primarily for practice at solving quantitative homework problems. The laboratories included multimedia lessons, including the following: Batteries and bulbs; Analyzing circuits with graphs; Electric charges, fields, and forces; Electric fields and potentials; Magnetic fields and electric current; Magnetic fields due to coils; Electric and magnetic fields; and Ratio of electric charge-to-mass for electrons. These multimedia lessons[xiv] were designed based on Karplus' Learning Cycle.[xv] Specific activities were developed to address student difficulties found in the early parts of this study, so the effects of the instruction may not be typical.

V. PREVALENCE OF SPECIFIC STUDENT CONCEPTIONS AFTER INSTRUCTION

This study investigates the impact of instruction on students' understandings of electric fields. Individual oral interviews and written pretests gave a measure of students' preconceptions of the electric field. Using written posttests, we investigated the persistence of these preconceptions after instruction. The posttest questions used in this study were not identical to the pretests, having been modified by the pretest results. The posttests also used the appropriate vocabulary, which was not always possible on the pretests.

A. Difficulty with existence/definition of the electric field

Analysis of the written posttests was performed in the same manner as with the written pretests. Results from Test Instrument 6 are shown in Table III.

TABLE III. Prevalence of misconceptions about the existence or definition of an electric field after to instruction.

TI #	Number of subjects	Correct	Misconceptions					
			Area	Area of charge	Charg e	Forc e	Energy	Blank/ no info.
6	93 100%	58 62%	7 8%	0 0%	6 6%	0 0%	1 1%	11 12%

Ten students provided only a specific example of an electric field, usually a sketch of the electric field between charged parallel plates.

Accurate descriptions of electric fields were much improved, with 62% of these students providing essentially correct definitions. No students described electric fields as areas of charge on the posttest.

B. Difficulties with source of the electric field

When asked after instruction "What is the fundamental source for electric fields?", 65% (60 of 93) of the students said "charges." Five of these answers were "moving charges," which is correct, but may indicate a confusion with magnetic fields.

The next most common answer involved potential difference, and was used by 14% (13 of 93 students) of the students. This misconception was not present in the pre-instruction difficulties and was added by the instruction process. Students may believe that terms on the left-hand side of a formula are caused by the terms on the right-hand side; this is called *cause in the formula*.[xvi] This may have been responsible for some students claiming that electric potential difference is the source for electric fields and may have come from the equation $E_x = -\frac{\Delta V}{\Delta x}$. Students expressing this belief often said that a single, isolated electric charge could not produce an electric field.

C. Difficulty with representation of the electric field

The post-instruction written tests showed that a majority of students could sketch the vector representation for the electric fields for simple arrangements of charged objects. These results are summarized in Table IV.

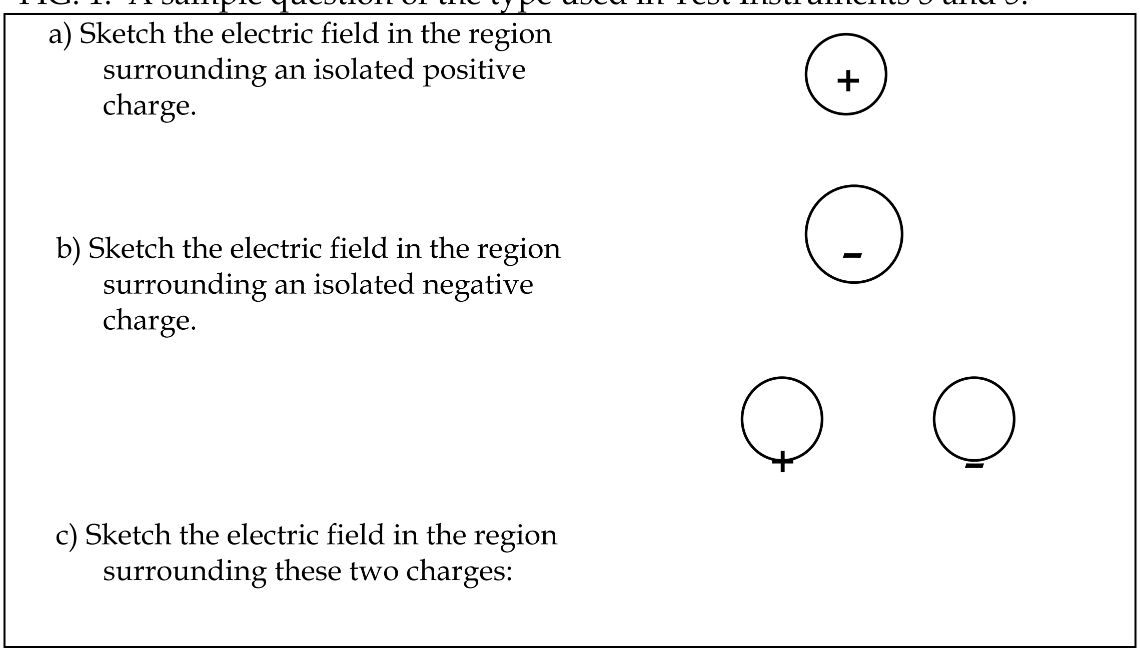
TABLE IV. Sketches of the electric fields for various configurations of charges or charged objects.

TI #	Number of subjects	correct	Alternate representations			No information
			opposite vectors	equipotentials	cloud of charges	
3	78	49 63%	3 4%	6 8%	0 0%	20 26%
5	176	114 65%	5 3%	28 16%	10 6%	19 11%
6	93	77 83%	4 4%	3 3%	1 1%	8 9%

More than 60% of the students drew proper vector representations, while some students drew vectors pointing in the wrong direction, sketched equipotentials rather than field lines or field vectors, or sketched a cloud of charges near the charged objects.

Test Instruments 3 and 5 were similar; both posttests asked questions such as the sample in Fig. 1, taken from Test Instrument 3. Test Instrument 5 used macroscopic objects instead of charges

FIG. 1. A sample question of the type used in Test Instruments 3 and 5.



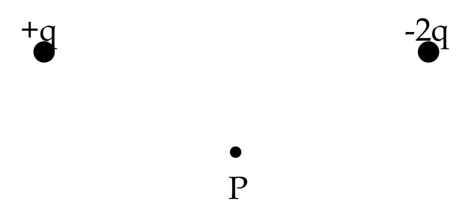
Test Instrument 6 asked for a sketch of the electric field due to an isolated negative charge. Eighty-three percent (77 of 93) of the students drew essentially correct diagrams. About 4% (4 of 93) drew arrows pointing in the wrong direction, and 3% (3 of 93) drew equipotentials. Only one student drew a cloud of charges surrounding the isolated charge. The remaining sketches were unclear and were not classifiable. The higher success rate on this posttest is most likely due to the relative ease of the task; rather than requiring a sketch for two separated charges, this test only ask for the field due to a single charge.

D. Difficulty with superposition of the electric field

The question in Fig. 2 was asked on Test Instrument 6. Nearly 70% (65 of 93) of the answers were essentially correct in showing superposition, with minor errors. Acceptable answers showed field vectors due to each of the given charges, and an attempt to add them vectorially. The sketches that were not correct contained a wide variety of errors, including treating point P like a charge, and drawing only a net electric field due to the charges.

FIG. 2. A question on superposition used on Test Instrument 6.

Sketch the electric field at point P due to each of the two given charges, then sketch the NET electric field at point P.



VI. CONCLUSIONS

This study shows for the first time how students' descriptions of four aspects of electric fields studied here improved with instruction. Pretests showed that only about 6% of the students entering our algebra-based introductory college physics courses could describe electric fields in a manner that we found acceptable. This increased to more than 60% after instruction, 65% of the students named charge as the source for electric fields, more than 60% could sketch a vector representation of the electric fields produced by simple charge configurations, and nearly 70% of these students could demonstrate superposition. On pretests, only five out of more than 200 students could sketch an electric field, and all five had taken high school physics. While five is not a large number, *none* of the students who did not take high school physics were able to sketch anything resembling a vector representation for electric fields. Our present modes of instruction enable two-thirds of our students to demonstrate an understanding of an electric field. For the other one-third of the students, some specific misconceptions persisted.

A. Persistent misconceptions about existence/definition of electric fields

The description of an electric field as an area remained after instruction. Knowing this, we can design instructional materials that address this specific problem. Also persistent was the description of electric fields as charges or clouds of charge. To do away with such persistent misconceptions, instruction must address them directly. Further study is needed to discover why 12% of these students could not provide a useful description of electric fields on posttests.

None of the post-instruction oral interview subjects who received an *A* for this class lacked the mental construct for an electric field. The presence of an adequate mental construct did not guarantee a good grade; the students who had the mental construct spanned the entire grade spectrum from the highest grade in the class to the lowest. The average grade for those with the mental construct was 2.95 (on a 4.0-point scale), while the average grade for those without the mental construct was 2.69. No final grade information was obtained for the written posttest students.

B. Instruction-induced misconception about source of electric fields

No pretest students believed that a potential difference is the source for electric fields, but 14% of the posttest students did. A post-instruction interview subject (who earned an A in the class) was convinced that charges do not create electric fields:

> I: ... What's the source for this [electric] field?
>
> S: ... In general it's going to be a difference in electrical potential.
>
> I: Would you say that a charge creates a field?
>
> S: I know it senses fields. I guess no, an individual charge could not create a field, because there's no [potential] difference. It's just this one charge and it's positive or a negative. And so I think that it's influenced by fields or electrical forces. But I don't believe it generates electrical fields.

One other post-instruction interview subject shared this misconception, earning a *C* for the course.

C. Persistent difficulty with representation of electric fields

More than 80% of these students could sketch the electric fields due to single charges, but one-third of the students could not sketch the electric fields for multiple charges. The most frequent identifiable error was to sketch the equipotentials, and as many as one-fourth of the students drew sketches that did not resemble vector fields, potentials, or any other representation used in their physics class. Additional research may find one or more specific student difficulties that prevented such a large portion of the students from producing clear sketches.

As the instruction in this study was based partially on pretest results, these results may not be typical. Similar studies should be done to obtain results that may be more widely gerneralizeable.

While only a few persistent difficulties were identified in this study, we feel that similar studies need to be conducted with other important concepts in physics. For instruction to produce substantive improvement in students' understandings, curriculum development must take into account the specific student difficulties and devise strategies to overcome them. The strategies should directly confront and challenge these difficulties. Only when students directly observe that their understandings do not explain their observations will they begin to alter those understandings. Instruction that ignores the results of such research may not successfully overcome some of the persistent difficulties students have in learning the concepts of physics.

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