APPENDIX B: TUTORIAL SEQUENCE *DYNAMICS OF RIGID BODIES*

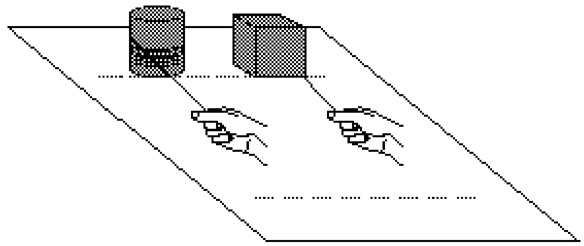
- Pretest
- Tutorial
- Homework

Pretest for tutorial sequence *Dynamics of Rigid Bodies* (page 1 of 2)

PRETEST: DYNAMICS OF RIGID BODIES Name:_______________

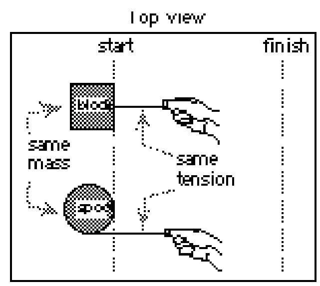
1. Two objects, a block and a spool, are each pulled across a level, frictionless surface by a string.

The string pulling the block is tied to a small hook at the center of the front face of the block.



The block and the spool have the same mass. The strings are pulled with the same constant tension and start pulling at the same time.

(Make the approximation that the strings and the hook are massless.)



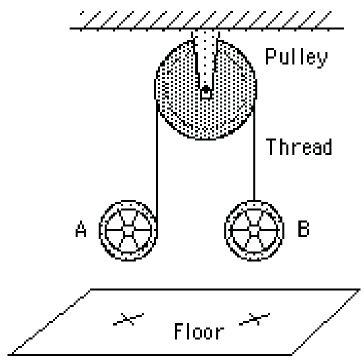
a. Will the spool begin to rotate? Explain.

b. Will the spool cross the finish line *before*, *after*, or *at the same instant as* the block? If the center of the spool will not cross the finish line at all, state that explicitly. Explain.

2. Two identical spools are held the same height above the floor. A thread is wrapped many times around spool A. The same thread passes over a pulley, and is attached to a fixed point on spool B, so that spool B will not rotate. An "X" is marked on the floor directly below each spool.

   Both spools are released from rest at the same instant.

   (Assume that the pulley and thread are massless and that the axle of the pulley is frictionless.)



   a. Is the tension in the part of the thread just above spool A *greater than*, *less than*, or *equal to* the tension in the part of the thread just above spool B (after the spools are released but before either spool hits the floor)? Explain.

   b. Is the magnitude of the acceleration of the center of mass of spool A *greater than*, *less than*, or *equal to* the magnitude of the acceleration of the center of mass of spool B? Explain.

   c. Will spool A hit the floor *before*, *after*, or *at the same instant as* spool B? Explain.

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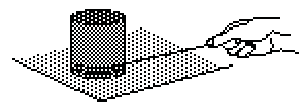
DYNAMICS OF RIGID BODIES

I. Spool on a variety of surfaces



A. A spool sits on a piece of sandpaper. Thread has been wrapped many times around the bottom of the spool. A hand pulls on the thread, moving horizontally, in a straight line away from the spool. (The sandpaper is fixed in place.)

Predict:

- Whether the spool will rotate.
- Whether the center of the spool will move across the sandpaper. If it will move, describe the direction it will move.

Explain the reasoning you used to make each prediction.

B. Imagine repeating the above experiment *without* the sandpaper, on a smooth tabletop.

Predict:

- Whether the spool will rotate.
- Whether the center of the spool will move across the table. If it will move, describe the direction it will move.

Explain the reasoning you used to make each prediction.

C. Ask a tutorial instructor to provide you with the equipment you need to check your predictions. (Make sure that the thread is wrapped *very close to the bottom of the spool*, so that the spool does not start to tip over when the thread is pulled.)

1. When the spool is <u>on the sandpaper</u>, and you pull the thread:

   a. Does the spool rotate?

   b. Does the center of the spool move? If so, in what general direction does it move?

2. When the spool is <u>on the table</u>, and you pull the thread:

   a. Does the spool rotate?

   b. Does the center of the spool move? If so, in what general direction does it move?

D. Suppose that you could repeat the experiment again, using an even smoother surface. Describe how (or whether) you think the resulting motion of the spool would be different from the motion you observed in the experiments above.

⇨Discuss your answers with a tutorial instructor before continuing.

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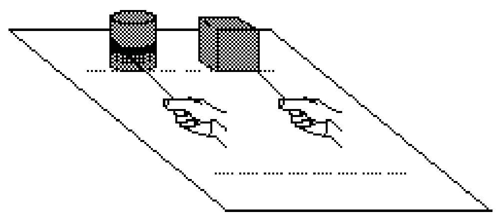
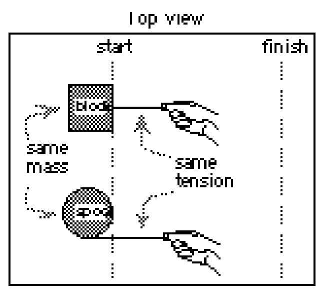
II. The "block and spool" problem

Two objects, a block and a spool, are each pulled across a level, frictionless surface by a string.



The string pulling the block is tied to a small hook at the center of the front face of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.

The block and the spool have the same mass. The strings are pulled with the same constant tension and start pulling at the same time.



(Make the approximation that the strings and the hook are massless.)

A. Will the spool cross the finish line *before*, *after*, or *at the same instant that* the block crosses the finish line? Explain.

B. Three students discuss the block and spool problem:

Student 1: ***"The spool will rotate, and it will cross the finish line at the same time as the block. They have the same mass and the same net force, so their centers of mass will have the same acceleration. It doesn't matter whether the spool starts rotating- the tension force will still have the same effect on the spool's translational motion."***

Student 2: ***"The spool will rotate, and it will cross after the block. This is because some of the tension force on the spool is being used to rotate the spool. When a force causes an object to rotate, it has less of an effect on the object's translational motion."***

Student 3: ***"The spool will rotate, and it will cross after the block. I was thinking about energy. The spool and block must each have the same total kinetic energy when they get to the finish line. Since the spool will have some rotational kinetic energy, it must have less translational kinetic energy than the block. Therefore, it will be slower, and it will arrive at the finish line later."***

With which student(s), if any, do you agree? Explain your reasoning.

In the next section, you will consider a way to test which of the above ideas is a correct solution to the "block and spool" problem.

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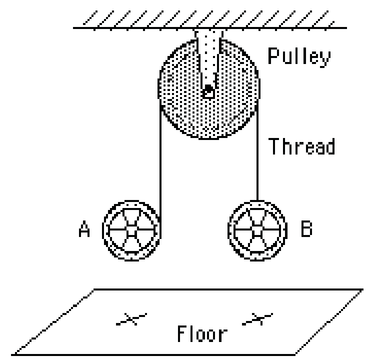
III. Testing ideas about combined translation and rotation with an experiment

Two identical spools are held the same height above the floor. A thread is wrapped many times around spool A. The same thread passes over a pulley, and is attached to a fixed point on spool B, so that spool B will not rotate. An "x" is marked on the floor directly below each spool.

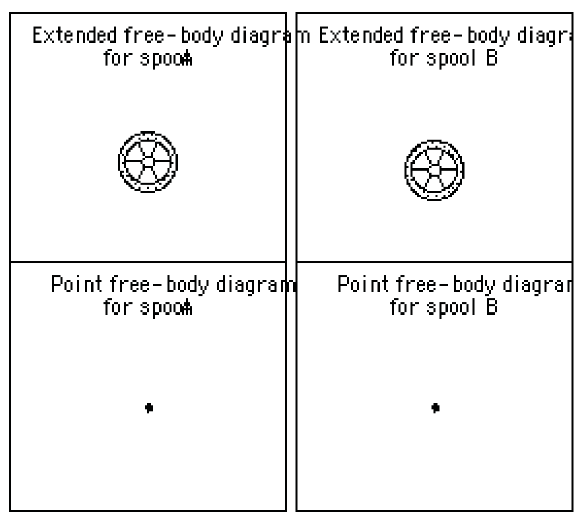


Both spools are released from rest at the same instant. (Assume that the pulley and thread are massless and that the axle of the pulley is frictionless.)

A. *Predict* whether spool A will hit the floor *before*, *after*, or *at the same instant as* spool B. If either spool starts to move only after the other spool hits the floor, state that explicitly. Explain.

B. Draw an *extended* free-body diagram for each spool after they are released. (An extended free-body diagram shows where on the object each force is exerted.)

C. Draw a *point* free-body diagram for each spool after they are released. (A point free-body diagram shows the forces on an object as if the object were located at a single point.)



D. Recall the student discussion from the previous page. For each of the three students, state the prediction you think that student would make if s/he were to use the same reasoning in this experiment that s/he used when thinking about the "block and spool" problem. (Try to agree with your partners about what you think each of the students *would* predict, and why.)

Student 1: Student 2: Student 3:

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1. Which student's reasoning, if any, is the same as the reasoning you used to make your prediction?

2. Can you use the outcome of this experiment to decide which reasoning about the "block and spool" problem is correct? If so, explain how. If not, explain why not.

⇨Discuss your answers with a tutorial instructor before continuing.

E. Ask a tutorial instructor to provide you with the equipment you need to check your predictions. (Ignore *small* differences in the motion of the spools.)

How does the acceleration of the center of mass of spool A compare to that of spool B:

- in magnitude?
- in direction?

If necessary, describe how you would revise your free-body diagrams to be consistent with your observations.

F. Do the results of the experiment support any of the ideas presented by Student 1, Student 2, or Student 3? Explain.

G. Generalize from your observations to answer the following questions:

If you want to determine how a force affects the motion of the center of mass of an object, should you consider:

- where on the object the force is exerted?
- how the force is affecting the rotational motion of the object?

H. Energy analysis

You have analyzed the "block and spool" problem and the falling spools experiment in terms of forces. You will be guided to describe the energy in these situations in the homework.

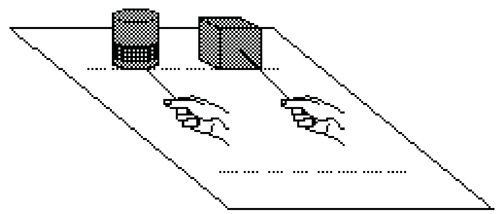
Tutorial homework for *Dynamics of Rigid Bodies* (page 1 of 4)

HOMEWORK:
DYNAMICS OF RIGID BODIES Name________________________

1. In tutorial, you considered the situation described below:

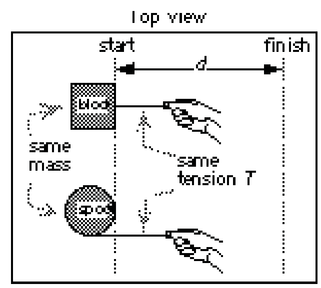
Two objects, a block and a spool, are each pulled across a level, frictionless surface by a string.

The string pulling the block is tied to a small hook at the center of the front face of the block. The string pulling the spool is wrapped many times around the spool and may unwind as it is pulled.



The block and the spool have the same mass. The strings are pulled with the same constant tension T and start pulling at the same time. (Make the approximation that the strings and the hook are massless.)

The distance between the start and finish lines is d.



As the spool moves from the start to the finish line:

a. Is the distance traveled by the hand that is pulling the spool's thread *greater than*, *less than*, or *equal to* d? Explain.

b. Is the work done by the hand that is pulling the spool's thread *greater than*, *less than*, or *equal to* the product Td? Explain.

c. Is the work done by the hand that is pulling the spool's thread *greater than*, *less than*, or *equal to* the work done by the hand that is pulling the block's thread? Explain.

d. Consider the following statement:

"It's not possible for the spool to rotate, AND cross the finish line at the same time as the block. If that were true, then the spool would have more total kinetic energy than the block. This is because they would have the same translational kinetic energy, and the spool would have additional rotational kinetic energy. But they must have the same total kinetic energy because the work done on them is the same."

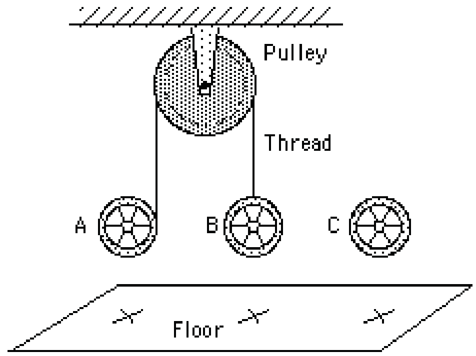
Explain, using the work-energy principle, why the spool has more total kinetic energy than the block when they cross the finish line together.

DYNAMICS OF RIGID BODIES

2. In tutorial, you considered the situation described below:

Two identical spools are held the same height above the floor. A thread is wrapped many times around spool A. The same thread passes over a pulley, and is attached to a fixed point on spool B, so that spool B will not rotate. An "x" is marked on the floor directly below each spool.

Both spools are released from rest at the same instant. (Assume that the pulley and thread are massless and that the axle of the pulley is frictionless.)



a. You observed the motion of spools A and B after they were released. Describe the motion of each spool (*i.e.*, In which direction did each spool move? In what order did the spools hit the floor?) Ignore small differences.

A third identical spool, C, is added to the experiment. All three spools are released from the same height at the same time. Spool C is not in contact with any other objects as it falls.

b. Rank spools A, B, and C according to the magnitude of the acceleration of the center of mass of each spool. Explain.

c. Rank spools A, B, and C according to the *translational* kinetic energy that each spool has just before it hits the floor. Explain. (Use the definition $K_{trans} = \frac{1}{2}mv_{cm}^2$.)

Consider the system consisting of all of these objects: spool A, spool B, thread, pulley, and Earth. Since the net work done on this system is zero, the sum $U_{grav\,A} + U_{grav\,B} + K_{trans\,A} + K_{trans\,B} + K_{rot\,A} + K_{rot\,B}$ is constant as spools A and B fall.

d. Suppose that this system starts with $U_{grav\,A} = U_{grav\,B} = 9$ J. Just before the spools hit the ground (*i.e.*, when $U_{grav\,A} = U_{grav\,B} = 0$ J), spool A has translational kinetic energy $K_{trans\,A} = 4$ J. Determine the value of the rotational kinetic energy of spool A. Show your work.

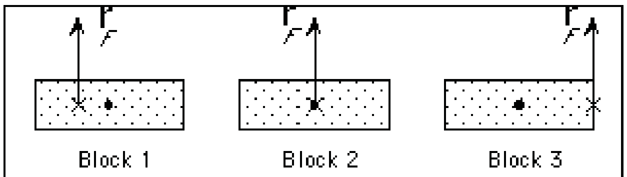
e. Rank spools A, B, and C according to the *total* kinetic energy that each spool has just before it hits the floor. Explain.

Tutorial homework for *Dynamics of Rigid Bodies* (page 3 of 4)

HOMEWORK:
DYNAMICS OF RIGID BODIES Name________________________

3. Three identical rectangular blocks are at rest on a horizontal, frictionless ice rink. Forces of equal magnitude and direction are exerted on each of the three blocks. Each force is exerted at a different point on the block (indicated by the symbol "X"), as shown in the top view diagram below. (Each small circle in the diagram indicates the location of the block's center of mass.)

For the instant shown in the diagram below:



Top view diagram

a. On the diagram above, sketch a vector on each block to indicate the direction of the acceleration of the center of mass ($\vec{a}_{cm}$) of that block. If for any block $\vec{a}_{cm} = 0$, state that explicitly. Explain.

b. Rank the magnitudes of the accelerations of the centers of mass of the blocks ($a_{cm\,1}$, $a_{cm\,2}$, $a_{cm\,3}$). Support your ranking by drawing a *point* free-body diagram for each block.

4. A rigid rod sits on a frictionless surface. The boxes below indicate four different combinations of net force on the rod and net torque on the rod about its own center. In each box, draw arrows that show where you can strike the rod (in one place or in two places at the same time) in order to achieve each combination. If any combination is not possible, write that explicitly.

(*Example:* In the second box, show where you could strike the rod so that the net force on it is zero, and the net torque on it is *not* zero, while you are striking it.)

$\vec{F}_{net} = 0$, $\vec{\tau}_{net} = 0$	$\vec{F}_{net} = 0$, $\vec{\tau}_{net} \neq 0$	$\vec{F}_{net} \neq 0$, $\vec{\tau}_{net} = 0$	$\vec{F}_{net} \neq 0$, $\vec{\tau}_{net} \neq 0$

Top view

DYNAMICS OF RIGID BODIES

5. Summarize your results thus far by answering the questions below:

   a. If you want to determine how a force affects the *motion of the center of mass* of an object, should you *ignore* or *account for* the rotational motion of the object?

   b. If you want to determine how a force affects the *total energy* of the object, should you *ignore* or *account for* the rotational motion of the object?

6. Three objects, A, B, and C, of equal mass, are released from rest at the same time from the same height on identical ramps.

   Objects A and B are both blocks, and they slide down the ramps without rotating. Object C rolls down the ramp without slipping. Its moment of inertia is unknown.

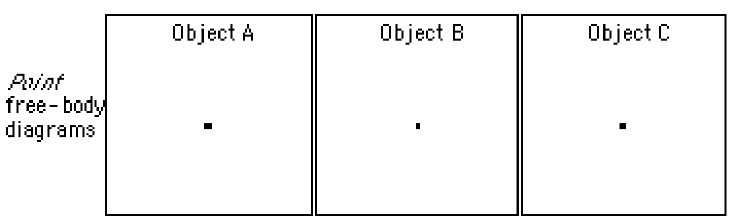
   Objects A, B, and C are made of different materials. (*i.e.*, Do not assume that the coefficient of friction between any object and its ramp is the same as that for any other object.)

   Object A reaches the bottom of its ramp first, followed by objects B and C, which reach the bottom at the same time.

   a. Rank the objects according to the magnitude of the acceleration of the center of mass of each object. Explain.

   b. Rank the objects according to the magnitude of the *net force* on each object. Explain.

   c. In the boxes below, draw and label a *point* free-body diagram for each object.



   d. Rank the objects according to the magnitude of the frictional force exerted on each object by its ramp. Explain your reasoning.

   Make sure that your ranking is consistent with your answer to part a of question 5.