Examining inconsistencies in student reasoning approaches

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Abstract. Student-centered instruction can lead to strong gains in physics learning. However, even after targeted instruction, many students still struggle to systematically analyze unfamiliar situations. We have been identifying sequences of questions that allow for an in-depth examination of inconsistencies in student reasoning approaches. On these sequences, many students demonstrate that they possess the abilities to perform the required reasoning, yet they fail to apply this reasoning to arrive at a correct answer. In certain contexts, students tend to "abandon" suitable formal reasoning in favor of reasoning that was (perhaps) more intuitively appealing at that moment. In other cases, erroneous student reasoning approaches can be attributed to the relative salience of specific features of the problem. We present results from one sequence revealing inconsistencies in student reasoning in the context of capacitors. This sequence was administered in an introductory course in which *Tutorials in Introductory Physics* were implemented as interactive lectures.

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INTRODUCTION

Implementation of research-based instructional materials specifically designed to target conceptual and reasoning difficulties often produces significant gains in student understanding of many topics [1]. However, recent research findings on certain topics have revealed that, even after targeted instruction, many students are not able to systematically and productively analyze related situations that are complex and/or unfamiliar [2,3,4]. Often, further attempts to refine instruction on these particularly challenging topics do not yield significant improvements in student abilities to solve problems that demand multi-step reasoning [5]. It appears that when students are presented with a fairly challenging problem for which a solution is not immediately known, some students do not apply the more formal reasoning approaches developed in class. Instead, they tend to look for shortcuts in their reasoning which seem to satisfy the demands of the problem.

In this study, we build on an emerging body of research in order to develop sequences of questions intended to probe the nature of erroneous student reasoning. A specific sequence in the context of capacitors connected in series is used to investigate the extent to which incorrect student reasoning approaches may be attributed to (1) a lack of relevant knowledge and skills necessary to analyze an unfamiliar situation correctly and (2) an inability to recognize *when* and *how* to apply relevant knowledge and skills acquired during formal instruction. It is hoped that the findings from this research will help enhance ongoing efforts to further improve existing research-based and research-validated instructional materials, particularly those that foster the concurrent development of both conceptual understanding and reasoning skills.

CONTEXT FOR RESEARCH

This study was conducted in an introductory calculus-based physics course offered at a large Midwestern University. Approximately 180 students, primarily engineering majors, were enrolled in the course. Lectures were held four times a week. Enrollment in an accompanying laboratory course was required. No small-group discussion sessions were associated with the course. However, *Tutorials in Introductory Physics* [6] were frequently implemented in the lecture component using an interactive lecture format. The sequence of questions presented below was included on the third course examination [7].

QUESTION SEQUENCE: OVERVIEW

Students were shown two identically charged capacitors connected in series. (See Fig. 1.) Students were then told that the setup was subsequently modified by increasing the distance between the plates of the left capacitor. Students were asked to determine how, if at all, this modification would affect: (1) the charge Q_{left} on the left capacitor, (2) the potential difference ΔV_{left} across the left capacitor, and (3) the potential difference ΔV_{right} across the right capacitor.

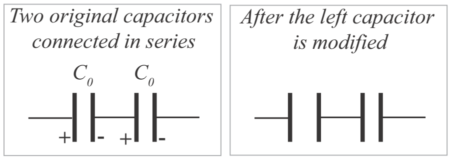

FIGURE 1. Diagram illustrating the experimental setup for the set of questions designed to probe inconsistencies in student reasoning.

QUESTION SEQUENCE: RESULTS

Question 1: Effect of modification on Q_{left}. Most students (~65%) correctly recognized that Q_{left} will remain the same since neither a source of additional charge nor a conducting path was available to allow charge to enter or leave the system. (See Table 1.) Approximately half of the students who did not answer this question correctly attempted to reason by examining the mathematical relationships between quantities that could be used to describe the left capacitor: $C_{left} = Q_{left}/\Delta V_{left}$ and $C_{left} = \varepsilon_0 A_{left}/d_{left}$. Most of these students correctly recognized that C_{left} decreases as a result of the increase in the distance d_{left} between the plates, but then incorrectly concluded that Q_{left} must decrease as well due to the direct relationship between charge and capacitance.

TABLE 1. Results from question 1: The effect of the modification on the charge Q_{left} on the left capacitor.

Student responses	Prevalence
Correct with correct reasoning	65%
Incorrect interpretation of mathematical relationships	15%
Q_{left} changes for other reasons	10%
Other responses	5%

Student difficulties with the interpretation and application of multi-variable expressions have been documented previously in many different contexts (*e.g.,* traveling waves and the ideal gas law) [2,3,8,9]. Students tend to manipulate the mathematical relationships between various quantities without first considering how specific aspects of an experimental setup affect these quantities. Such results suggest that students are not *analyzing the applicability of a particular reasoning strategy to a specific situation.* In the context of a math course, for example, it may be appropriate to reason that, for the given relationship $y = x/a$, if x increases, y must also increase; in such cases, it is commonly assumed that variables (*e.g.,* x and y) and constants (*e.g.,* positive a) have been clearly established. However, a direct mapping of the same reasoning in the context of physics (namely, for the given relationship $Q = C/\Delta V$, if C increases, Q must also increase) leads to an erroneous conclusion.

Question 2: Effect of modification on ΔV_{left}. Approximately half of the students reasoned correctly that Q_{left} remains the same, C_{left} decreases, and therefore $\Delta V_{left} = Q_{left}/C_{left}$ must increase. (See Table 2.) A significant fraction of the students (~10% of total) appeared to use an intuitive idea that *closer means stronger* [10]. These students reasoned that the increased separation between the plates weakens the effect of the interaction between the plates, thus decreasing the potential difference between the plates. The intuitive idea that *closer means stronger* is very appealing since it has numerous applications both in everyday life and in science classrooms. Therefore, we speculate that, in many cases, students *first* commit to their intuitive answers and *then* they tailor their formal reasoning to fit their intuitive idea [11]. In the presented context, this inverted problem solving approach may have led to errors with basic mathematical reasoning, as illustrated by the following response: *"V decreases,*

$$\uparrow C = \frac{Q}{V\downarrow} \qquad \downarrow V = Ed \uparrow$$

voltage is dependent on the electric field and distance and the electric field is dependent on charge. Since charge does not change, E-field does not change. The distance increases, so to have the same E, V must decrease by the same amount." This student demonstrated an understanding of many important ideas necessary to analyze this question correctly. However, it appears that the student's "raw intuition" or "gut feeling" may have prevented him from arriving at a correct answer. Given the prevalence of this type of student reasoning, it is unlikely that student deficiencies with basic mathematical skills are the primary source of this error.

TABLE 2. Results from question 2: The effect of modification on the potential difference ΔV_{left} across the left capacitor.

Student responses	Prevalence
Correct with correct reasoning	55%
Correct with correct, incomplete or no reasoning	15%
Decreases, *"Closer means stronger"*	10%
Decreases, with other incorrect reasoning	10%
Remains the same	15%

Question 3: Effect of modification on ΔV_{right}. Less than half of the students recognized that since neither Q_{right} nor C_{right} were affected by the change in distance between the left plates, the potential difference across the right plates must remain the same. (See Table 3.) Most students who answered this question incorrectly used an argument commonly referred to as *compensation reasoning* (~40% of total). Many students articulated responses similar to the following: *"There is so much total potential across the two*

capacitors. Since the potential across the left capacitor increases, the potential across the right capacitor must decrease." Others translated analogous arguments into a primarily symbolic form: $\uparrow \Delta V_{left} + \downarrow \Delta V_{right} = \Delta V_{total}$.

Much like those student responses to question 2 that invoked *closer means stronger* reasoning, many student responses in the *compensation reasoning* category also contained elements of correct understanding. For example, one student wrote that ΔV_{right} *"would increase, as the potential of the left capacitor drops, the right capacitor gains because it is not connected to the battery. If it was connected it would stay the same due to the constant potential being supplied by the battery."* This student (and others) explicitly articulated the correct idea that the potential difference across a capacitor may change unless a battery is directly connected across the capacitor. When applied to the two-capacitor series network, this argument can be used to conclude that there is no reason why the potential difference across the network needs to be constant. However, it seems likely that intuitive notions of "equilibrium" or "conservation," which are ubiquitous in introductory physics courses, may have cued the use of compensation reasoning and prevented the student from applying this correct idea about potential difference appropriately. In this case, the correct idea was used to justify an erroneous conclusion. It appears that the student applied the correct idea in a selective manner (*i.e.,* to the single capacitor) that supported a specific and likely anticipated conclusion while neglecting to employ the same idea in a slightly different manner (*i.e.,* to the two-capacitor network) that would refute this conclusion. Therefore, as in our analysis of the reasoning approaches employed in question 2, we speculate that many students again committed to their answers *first* (perhaps cued by the intuitive notion of equilibrium) and then constructed an argument in support of these answers.

TABLE 3. Results from question 3: The effect of modification on the potential difference ΔV_{right} across the right capacitor.

Student responses	Prevalence
Correct with correct reasoning	40%
Correct with incorrect, incomplete or no reasoning	10%
Compensation reasoning	40%
Other	10%

Analysis of inconsistencies in student reasoning

The examples of student reasoning approaches discussed above illustrate how students' intuitive ideas may dominate student thinking, influence their reasoning approaches, and make it more challenging for students to draw on other (often correct) elements in their understanding. One possible explanation is that student understanding is fragmented. Namely, students have a collection of correct ideas that do not form a coherent conceptual framework; as a result, students often struggle to form a logical argument and therefore frequently make inappropriate assumptions (*e.g.,* ΔV_{total} remains the same). In order to probe this hypothesis, we examined the responses of those students who answered the first pair of questions (regarding the left capacitor) correctly with correct and complete reasoning. Approximately 50% of all students reasoned correctly that Q_{left} remains the same, C_{left} decreases, and therefore $\Delta V_{left} = Q_{left}/C_{left}$ must increase. However, only half of these students (~25% of the total) were able to apply the *same line of reasoning* to the right capacitor in order to recognize that, since Q_{right} and C_{right} are unchanged, $\Delta V_{right} = Q_{right}/C_{right}$ must remain the same. Almost all of the students who analyzed the changes to the left and right capacitors inconsistently used compensation reasoning.

Our findings therefore suggest that, after instruction, half of the students demonstrated the ability to do the type of reasoning required for this problem (on the first pair of questions). However, half of these students failed to apply that same reasoning later (on the last question). These students went on to "abandon" the appropriate formal reasoning in favor of an alternative line of reasoning that was (perhaps) more intuitively appealing at that moment [12, 13].

DISCUSSION

Research suggests that inconsistencies such as those examined in this study are not unique to the context of capacitance. Certain contexts tend to elicit intuitive rather than formal reasoning. In many other cases, erroneous student reasoning approaches can be attributed to the relative salience of certain features of the problem. Students often over-generalize specific simple cases into broadly applicable rules [5]. Our results indicate that even those students who do possess the knowledge and skills necessary to analyze many challenging situations correctly often fail to utilize relevant ideas and skills productively; students tend to "abandon" their correct reasoning in favor of more intuitive solutions. We hypothesize that even when guided through the steps in reasoning necessary to arrive at a correct answer, many students do not examine their own reasoning and thought processes in order to recognize *what* they are doing and *why* they are doing it. As a result, students do not learn to analyze the *applicability of particular reasoning* *approaches to specific situations* (*e.g.,* the arbitrary manipulation of mathematical symbols in the expression for capacitance). It is therefore not surprising that instruction that targets student reasoning difficulties within a specific context often fails to facilitate either (1) improved student performance on more challenging questions in the original context or (2) student transfer of correct reasoning approaches to new situations. Evidence suggests that students must also be guided to change their *general* approaches to reasoning about physics [13]; otherwise, they may apply their original erroneous reasoning approaches in new situations. For example, results from our ongoing investigation suggest that explicit instruction that targets student difficulties with multivariable expressions in the context of waves (*i.e.,* $\lambda=v/f$) does not lead to significant improvement in student reasoning approaches in the context of capacitance.

We also hypothesize that if students who answered the presented set of questions inconsistently attempted to *reflect* on their solutions and *evaluate* their conclusions, they might be more successful at identifying and correcting their mistakes. It is worth investigating whether or not students who apply what appears to be an inverted problem solving strategy are aware that they are employing this approach.

Students' awareness of their own thinking together with (1) an ability to recognize the applicability of a specific line of reasoning in a given situation and (2) an ability to evaluate their results productively (*e.g.,* by checking for consistency) are all crucial elements of student metacognition [14]. As such, we anticipate a link between student performance on the present set of questions and students' metacognitive abilities.

CONCLUSIONS

As researchers, curriculum developers, and physics instructors, we feel that one of the weaknesses in our current understanding of student learning is an inability to differentiate clearly between those student reasoning difficulties that stem from inadequate conceptual understanding and those that stem from poor metacognitive skills. This distinction is critical to efforts to improve student learning of physics. Therefore, our future efforts will focus on developing research techniques for probing: (1) student conceptual understanding of physics, (2) student metacognitive knowledge and practices, and (3) the interplay between student understanding of physics and metacognition.

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