

FRAMING DISCOURSE FOR OPTIMAL LEARNING IN SCIENCE AND
MATHEMATICS

by

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ABSTRACT

This study explored the collaborative thinking and learning that occurred in physics and mathematics classes where teachers practiced Modeling Instruction. Four different classes were videotaped — a middle school mathematics resource class, a 9th grade physical science class, a high school honors physics class and a community college engineering physics course. Videotapes and transcripts were analyzed to discover connections between the conceptual structures and spatial representations that shaped students' conversations about space and time.

Along the way, it became apparent that students' and teachers' cultural models of schooling were a significant influence, sometimes positive and sometimes negative, in students' engagement and metaphor selection.

A growing number of researchers are exploring the importance of semiotics in physics and mathematics, but typically their unit of analysis is the individual student. To examine the distributed cognition that occurred in this unique learning setting, not just among students but also in connection with their tools, artifacts and representations, I extended the unit of analysis for my research to include small groups and their collaborative work with whiteboarded representations of contextual problems and laboratory exercises.

My data revealed a number of interesting insights. Students who constructed spatial representations and used them to assist their reasoning, were more apt to demonstrate a coherent grasp of the elements, operations, relations and rules that govern the model under investigation than those who relied on propositional algebraic representations of the model. In classrooms where teachers permitted and encouraged students to take and hold the floor during whole-group discussions, students learned to probe one another more deeply and conceptually. Shared representations (whether spatial or propositional/algebraic), such as those that naturally occurred when students worked together in small groups to prepare collaborative displays of their thinking, were more apt to stimulate conceptually oriented conversations among students than individual work, i.e., what each student had written on his or her worksheet.

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This work is dedicated to my father, Frank M. Megowan, who always hoped
that one day, one of his children would become Dr. Megowan.
I know you're pleased, Dad. I only wish I could see your smile.

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TABLE OF CONTENTS

INTRODUCTION.....	1
Optimizing Discourse.....	2
LITERATURE REVIEW.....	6
Overview.....	6
Cognition is Situated – Semantic Frame v. Context.....	6
Cognition is Culturally Mediated.....	7
Cognition is Embodied.....	7
Cognition is Distributed.....	8
Cognition is Metaphorically Framed.....	8
The Culture of the Learning Environment.....	9
Modeling Physics.....	10
Classroom Culture.....	10
The Modeling Cycle.....	12
What is a model?.....	12
Doing things with models.....	14
Discourse in the Modeling Physics Classroom.....	16
A Question of Motivation - To Engage or not to Engage?.....	17
Cognition and Learning in Modeling Instruction.....	17
Learning as a Group Process.....	20
Communication and Learning.....	21
Why do we need a reference frame?.....	21
How many reference frames are there?.....	22
Doing things with reference frames.....	23
What can my students do with reference frames?.....	24
What is the Role of the Whiteboard in Modeling Physics Discourse?.....	25
Research questions.....	26
METHODOLOGY.....	27
Constraints.....	27
Choosing a Unit of Analysis.....	28
Factors Affecting Data Collection.....	29
Approaches to data analysis.....	31
The students.....	33
OBSERVATIONS.....	34
A tale of four classrooms.....	34
Middle school mathematics: teacher as scout leader.....	34
9 th grade physical science: teacher as stern but kindly parent.....	38
Honors Physics – teacher as coach.....	44
Community College Physics – teacher as general contractor.....	50
Summary.....	59
ANALYSIS.....	61
Keeping an Eye on the Model.....	62
What is a model again?.....	62
A delicate but critical shift of attention: zooming in and zooming out.....	62
If only I had a hammer.....	63
The Contextual Dimension of Whiteboard Mediated Cognition and Modeling.....	69

The Candy Problem.....	74
Clues to students’ mental models.....	92
“Big D” Discourse in physics.....	98
Distributed Dimension of Whiteboard Mediated Cognition and Modeling.....	101
Small group labs.....	101
Small group worksheets.....	104
Going Over Homework – The Power of the Marker.....	105
Practicing With the Model.....	107
Whole group whiteboard presentations.....	110
Whole group board meetings.....	112
Structuring Dimension of Whiteboard Mediated Cognition and Modeling.....	117
Frames, schemas and metaphors.....	117
What are the frames, schemas and metaphors that students use?.....	117
Whiteboards as containers.....	120
Whiteboards as road maps.....	120
How does language evoke spatial and temporal images?.....	125
How is it different for a diagram?.....	126
Summary.....	134
CONCLUSIONS.....	135
The Value of Spatial Representations.....	136
Implications for Instruction.....	137
APPENDIX A.....	141
REFERENCES.....	143
BIOGRAPHICAL SKETCH.....	147

LIST OF FIGURES

Figure	Page
1. Student presenting whiteboarded problem.....	11
2. A schematic of Jackendoff's Theory of Representational Modularity (1996, p.3).....	24
3. A schematic of the theoretical framework that served as my lens for this study.....	26
4. Kiki's whiteboard of the E field of a charge infinite plane.....	52
5. Professor Donnelly's meter stick "circuit".....	57
6. Jackendoff's Theory of Representational Modularity (reproduced from page 47, Chapter 2).....	62
7. Which direction is the force?.....	66
8. A journal prompt that makes use of both conceptual structure and spatial representation.....	68
9. Rigo's initial inscription for part one of the candy problem.....	75
10. Rigo's redrawn inscription for part one of the candy problem contains more explicit information after he has had to explain it to the TA.....	76
11. Rigo's inscription evolves as more detail becomes encoded in the representation.....	77
12. Hannah uses the picture of the cart and the hill to construct energy bar charts.....	84

Figure	Page
13. Jimmy points out an inconsistency in Hannah's interpretation of the diagram.....	86
14. Hannah translates what her bar charts showed her into equations.....	89
15. Hannah accounts for energy transfer reasoning from the drawing and from the equations she replaced her spatial representation with.....	90
16. Hannah reconstructs her energy bar chart diagram, this time quantitatively.....	91
17. EMCC college physics students conceptualize electric field.....	92
18. DHS physics students consider the case of an object moving backward and slowing down.....	93
19: CCHS science students grapple with the relationship between force and acceleration.....	93
20. WTMS mathematics students represent the relationship between time and position.....	94
21. No spatial representation is presented here by Jen and Bonnie, and none is asked for.....	95
22. In general, the scribe starts in the upper left or upper center quadrant of the whiteboard.....	96
23. Jose, who functioned as both Measurer and Data Manipulator, tells Sophia, The Operator and Scribe for this lab how to construct a graph of their data.....	102

24. Stephen is both Measurer and Data Manipulator for his lab group.....	103
25. ... but from there to there it wouldn't be "I"	104
26. Christina reasons about the problem space by constructing a diagram.....	109
Figure	Page
27. Gabe reasons about the same problem space as Christina above, but he uses equations.....	109
28: The typical structure of a whole-group formal whiteboard presentation.....	110
29: The typical activity structure of a whole-group board meeting.....	113
30: Whiteboards as road maps: problem space navigation pathways.....	121
31. Another charge shows up behind me and what happens?.....	123
32 Hannah's leads with a discussion of their free-body diagram.....	126
33. SRs were sometimes absent altogether while algebra was featured prominently on whiteboards.....	135

LIST OF TABLES

Table	Page
1. Names used to identify reference frames in systems that divide reference frames into two categories.....	22
2. Names used by researchs who favor three types of reference frame.....	23
3. Parallel dimensions of whiteboard-mediated cognition and modeling.....	61
4. Small group laboratory activity (Note: at times the same individual may play more than one role).....	102
5. Small group whiteboarding - going over homework.....	105
6. Small group whiteboarding - practicing with the model.....	108

INTRODUCTION

Teaching and learning physics are not easy.

They involve abstract ideas, sophisticated mathematics, and a several thousand-year history of often-flawed reasoning about how and why things work. Physics is not typically taught as a stand-alone course by a qualified teacher until late in high school, (if at all) by which time students have had an opportunity to hear many appealing, plausible but misconceived explanations for physical phenomena from people they trust and respect—some of them science teachers. Moreover, complicating the process is a lifetime of commonsense assumptions, metaphors and beliefs that reside in the unconscious mind and are continually referenced to inform the reasoning process.

It takes a great deal of time, effort and skill to uncover, displace and replace students' flawed notions about how the world works (Camp & Clement, 1994), and as over 15 years of Force Concept Inventory (FCI) post-test scores suggest, we physics teachers are still not doing the job nearly as well as we would like (Hestenes, Wells, & Swackhamer, 1992).

The Modeling Method of Instruction in Physics (Hestenes, 1996) is a pedagogical approach that works better than most, with students performing well above the FCI mean scores for courses employing traditional physics instructional approaches.

How does it work? And how can practitioners use it best?

I have employed modeling instruction in the high school setting for a number of years, and have noted two key advantages that it has over traditional methods: the use of models as a way of organizing students thinking and reasoning, and the prominent role of whiteboard-mediated student-to-student discourse in the classroom, which prompts conversations that exteriorize students' reasoning processes and opens these processes to scrutiny in dialogue with their peers. In this talk lies one key to answering the question about how modeling physics works. A careful unpacking of the contents of student discourse, both oral and written, reveals clues to students' thinking as they construct their understanding of what space, time and interactions mean from the perspective of physics and physicists.

This study examines what the structure of talk in classroom discourse surrounding whiteboarded representations reveals about the shifts in students' thinking and reasoning as they progress toward a model-centered understanding of the Newtonian force concept, which is central to understanding physics as a whole. Discourse analysis, in the traditions of microethnography and Pragmatics, is used to examine the change as it unfolds.

Reform based methods—constructivism—inquiry—student-centered learning—collaborative learning—cognitively guided instruction: these are influential ideas that have entered the pedagogical lexicon in the past two decades (Baker & Piburn, 1997). Many of us who currently teach physics know that these ideas can be important elements of effective practice, but most of us were not taught by educators for whom these ideas were a significant element of their own teaching practice. Many teachers lack an effective approach for incorporating such ideas into their own classroom routine. Some try anyway to build them into their teaching practices. Others play it safe and stick with the traditional didactic methods that their own teachers modeled for them.

Modeling Instruction has evolved from a single physics teacher's classroom teaching experiment 20 years ago into a 'movement' with close to 2000 trained practitioners who employ this method to teach physics, chemistry and mathematics. The modeling method incorporates all the ideas mentioned above, offering effective strategies for designing a more collaborative, inquiry-based, student-centered learning environment that supports the cognitive processes that take place as students construct a coherent understanding of science and mathematics concepts. FCI average gain scores for students who learn physics via modeling instruction are one to two standard deviations above those of students taught using traditional approaches ("Modeling Instruction in High School Physics," 2001). This much is known. Little is understood (empirically) about how and why Modeling Instruction works for these students. This study will attempt to shed some light on the answers to this question by studying two of the key features that make the approach unique: the centrality of whiteboard-mediated small group student discourse and the organizing principle of models.

These elements – student-centered discourse and the use of models – have broad applicability in science and mathematics education, and the design of curricula and learning environments (Doerr & Tripp, 1999; Hestenes, 1987; Lesh & Doerr, 2003). A clear guide to stimulating, listening to and evaluating student discourse would be a useful tool for teachers at all levels and in all disciplines.

A number of physics education researchers have weighed in on the importance of attending to the role of epistemology in thinking and learning (diSessa, 1993; Halloun & Hestenes, 1985a, 1985b; Hammer, 1989; Hammer, Elby, Scherr, & Redish, 2004; Hestenes, 1979, 1987, 1992, 1996; Redish, 1994). Much has been written about how students know what they know and what flawed ideas they possess as a result of these ways of knowing. My physics teacher colleagues and I dutifully endeavor to root out our students' misconceptions each year and help students construct more accurate conceptions to replace them. How can we know if we are succeeding without having to wait for the FCI post-test scores? A simple rubric to evaluate changes in student thinking "on the fly", by listening to their conversations with each other, would be extremely useful. It would allow for immediate course corrections when students' thoughts take an unproductive turn, before they have had a chance to reinforce newly acquired wrong ideas by building a plausible story line to support them.

Optimizing Discourse

Why is this question worth answering? Physics teachers are routinely confronted with the gap between what we think our students understand and what they can actually demonstrate that they understand. Every time we sit down with a stack of tests, assignments or lab reports to read, we are struck by the tenacity of students' misconceptions.

Unfortunately, however, by the time these misconceptions show up on a test, any feedback we might give students about them is unlikely to have a substantial impact on their thinking. We need a way to weed out misconceptions before they take root in students' knowledge structures.

We cannot pull weeds that we cannot see, however. We need a way to detect them before they become established.

Enter whiteboard mediated classroom discourse.

Listening to student conversations that are centered on tasks involving key concepts is an opportunity to attend to “cognition in the wild” (Hutchins, 1995). As students share their thinking with one another in an attempt to build a coherent model of the relationship they are exploring, the listener has an opportunity to identify the metaphors students are using to organize this information (Lakoff & Nunez, 2000). They can hear how students assemble these building blocks into knowledge structures, and they have an opportunity to interrupt the formation of inappropriate constructs before these constructs have a chance to be reinforced.

It is a fundamental tenet of discourse analysis that the smallest particles of “language-in-use” are the ones that are used most unconsciously. The seeds of students’ cognition-on-the fly lie within these unconscious utterances. These utterances are grounded in students’ inventories of fundamental conceptual and experience-based metaphors and the representations to which they are connected (Lakoff, 1987).

For this process to succeed several conditions are necessary:

- An opportunity to listen to students engaged in productive inscription-mediated discourse about some task that will lead to model construction;
- A clear picture of how the model of this phenomenon ought to look in the mind of the listening teacher (i.e., the key elements necessary for completeness and usefulness and how they are connected);
- A grasp of the catalog of metaphors and how they encode the language that students use, and an awareness of the points at which the inherent ambiguity of this language can lead to misunderstanding among students;
- An ability to steer discourse without giving answers (questioning strategies);
- An ability to connect student inscriptions to their thinking about the models they are constructing;
- A grasp of the cultural models of schooling that students *bring into* the physics classroom with them and the new cultural models that *can be brought about* as a result of skillful management of classroom discourse.

Fostering a learning environment that is rich in student discourse is a pedagogical choice that is becoming more popular. Cooperative or collaborative learning caught on as a movement in the 1980’s (“Timeline of the History of Cooperative Learning,” 2004), primarily in elementary schools. It was touted for not only promoting socialization, but also for enlisting students as peer tutors, allowing the age old practice of distributing a share of the teaching responsibilities to the student.

In most disciplines, however, high school teachers have held doggedly to the time-honored university model of instruction, becoming the sage on the stage, casting his or her ‘pearls of wisdom’ before orderly rows of silently scribbling students who made of the information whatever they could.

On the other hand, outside class students instinctively grasped the value of learning in collaboration with their peers. They sat together in groups or talked on the phone as they did their homework, or stayed up late and gathered to cram for exams,

working together to make sense—to organize and structure the body of knowledge their teachers had imparted to them.

Only in the last 10 years with the rise of constructivism have high school teachers in large numbers embraced collaborative learning as a pedagogical choice that makes sense for their students and their discipline. Many teachers continue to eschew its use, however, regarding it as an inefficient use of the ever-shrinking number of instructional minutes at their disposal.

This view is not ungrounded. Given the opportunity for unstructured activity, students will “waste” time. Thinking and reasoning are hard work, so they take breaks. In order for collaborative learning to be a good choice for a learning environment, the teacher must be able to coordinate it effectively, and use a curriculum that is based on tasks that elicit productive discourse—tasks that lie within students’ “zone of proximal development” (Vygotsky, 1962). This is a situation in which students have at their disposal all the necessary tools and concepts, but they have not yet connected them into a coherent knowledge structure.

Most high school physics curricula are not yet designed with this approach in mind, although many teachers who value an inquiry-based approach centered on small group discussion have learned to redesign the instructional materials at their disposal to conform to this model of instruction. However, the results of this practice are mixed. Even with a curriculum designed explicitly with collaborative learning in mind, optimal learning does not always result. Teachers must manage and guide the process, and this is a skill that develops only with reflective practice. It is the aim of this research to assist teachers in their acquisition of this skill.

One key lies in establishing classroom norms for student participation that emphasize engagement in discourse so that the teacher can nudge/coax/steer students’ reasoning in productive directions with well-chosen questions or comments. To some extent this can involve changing students’ beliefs about what they should value in the learning setting—away from points and right answers and toward deep and well-connected understanding.

This study does not deal explicitly with the establishment of broad social norms in the classroom but will treat this process peripherally insofar as it involves the design and construction of a discourse environment in the physics and mathematics classrooms.

The goal is to study students’ discourse as they attempt to acquire a coherent understanding of key physics and mathematics concepts in collaboration with their peers. Students typically arrive in their first physics course with a collection of loosely connected commonsense concepts about force—many of them incorrect (Halloun et al., 1985a, 1985b). A fine-grained examination of their discourse and inscriptions (Roth and McGinn 1998) can uncover these and the metaphors upon which they are based, and illuminate the reconstruction process that occurs as students transform these primitive notions into a more robust, coherent, model-based force concept.

How do the talk and the representations change as this shift in thinking takes place? How can the teacher move the process along in the right direction without actually dragging students “kicking and screaming”?

I will explore the coded language (Gee & Green, 1998) that students and teachers use when communicating with each other about physics ideas. Teachers need to know how to decrypt this code to gain access to the reasoning behind it. The results of this study offer teachers a tool to allow this decryption process to be accomplished “in real time” as students’ communications unfold.

LITERATURE REVIEW

“Every utterance is really an act.” (Austin, 1962)

In his influential work on pragmatics, **How to Do Things with Words**, (1962) philosopher and linguist J. L. Austin showed how the use of particular verbs, called performatives, make performing acts explicit. His ideas about the relationship of this class of verbs with the actions they embody provide a starting point for considering student talk and thinking about motion, actions and reactions as they endeavor to learn physics.

Another key to unlocking student thinking about spatial, temporal and interactional relationships is in examining their use of prepositions (Coventry & Garrod, 2004; Herskovits, 1986; Jackendoff, 1996; Johnson-Laird, 1996; Lindstromberg, 1998; O’Keefe, 1996; Peterson, Nadel, Bloom, & Garrett, 1996; Talmy, 1996; Tversky, 1996). A quick review of these reveals that the majority of them are spatial referents. Many of those that are used to denote temporality do so using a spatial analog:

We will eat *at* 6 o’clock.

Pat will come for dinner *on* Sunday.

The smaller a grammatical particle is (i.e., a verb, a preposition) is, the more unconsciously it is used; yet it is these particles that provide clues to the metaphors students select to make sense of the physical world.

In this chapter, I will outline the disciplinary grounding and theoretical underpinnings of my perspective on understanding the discourse surrounding whiteboard preparation and sharing in modeling physics and mathematics classrooms with respect to student reasoning, and I will review relevant literature that outlines and elaborates upon these theories. In the course of this review, I will attempt to assemble ideas from disparate disciplines (i.e., linguistics, cognitive science, philosophy, physics and mathematics education) into a coherent framework for understanding the cognitive processes that underlie student-to-student discourse in the physics classroom.

Overview

What is cognition? This seemingly simple idea is extensively explored and exhaustively debated in the research literature but rarely defined. Elements of a good definition follow:

Cognition is Situated – Semantic Frame v. Context

Cognitive semantics is a branch of cognitive linguistics that studies how we make sense of things with language. It divides meaning making into meaning construction and knowledge representation. It holds that the “real” situation about which we think is internal—it is in the head. These thoughts are grounded in preconceptual structures called semantic frames (Lakoff & Johnson, 1999; Minsky, 1995), that derive from experiences we have internalized to the extent that they have become prototypical. There is huge variability in the frames that individuals bring to the meaning making enterprise, and therefore meanings constructed from the same data can vary widely.

The context that one brings to a situation is external. This external context activates a semantic frame that organizes an existing knowledge representation or schema.

In their seminal work on the situated nature of cognition in learning, Brown, Collins and Duguid (1989) laid out some basic ideas that were intuitively obvious, but rarely mentioned: cognition is a product of everyday activity and as such, is situated both culturally and contextually. In examining how knowing and doing have become separated in the learning environment, they describe what happens when conventional classroom practices ignore cultural and contextual effects on the learning process and contrast school learning behavior with the typical learning behaviors of both experts and “just plain folks” (JPFs). The revelation that JPFs and experts approach learning situations in similar, contextually grounded ways while students approach them as abstractions is revealing, though not surprising.

The availability of tools and the ability to use them (and to know when to use them) is a vital component of cognition. Brown et al (1989) point out that cognitive tools resemble knowledge in that they are only understood through use, and they assert that using tools changes both the user and the “world” in which she uses them.

Cognition is Culturally Mediated

Tomasello (Tomasello, 2001) goes further in making the case that cognition is culturally embedded. He traces the evolution of cognition in humanoids over 6 million years and concludes that the only possible explanation for the dramatic emergence of the species-unique cognitive skills that human beings demonstrate is social or cultural transmission. He describes what he calls the “ratchet effect” to explain the evolution of complex social practices (i.e., communication and tool use) that were not invented all at once by a single person, but rather were introduced by some individual in rudimentary form and then adopted and adapted to local conditions by other individuals who improved and passed along the inventions to subsequent generations. These subsequent generations continued to make “improvements” over time. He observes *that humans pool their cognitive resources in ways that animals do not* and this has allowed people to learn, not just from others but through others, thus transcending and including what preceding generations have accomplished.

One reasoning tool that is ubiquitous in every culture is the use of metaphors (Lakoff, 1987; Lakoff & Johnson, 1980; Lakoff et al., 2000) as common sense models to help us structure and make sense of a complex world. Metaphors are not usually employed consciously but they are always operating in the background, helping us make choices in thinking and acting. They help us to decide “what counts”—what elements we should pay attention to in attempting to understand a situation or event. These play a central role in construction and application of models and they bring with them a representational system that can be not only appealing but at times coercive.

Cognition is Embodied

This fits neatly with the increasingly popular notion that cognition is “embodied”-grounded in the sensory-motor system (Hestenes, 2006). The way we think is a function

of our body, its physical and temporal location and interactions with the world around us. Metaphors arise from the body's experiences. Thus, they are embodied.

Cognition is Distributed

Cognition as it is defined here – the utilization of information that one understands to reason about or make sense of something – can involve cognitive resources that extend well beyond what goes on in internal mental space. Indeed, the tool use described by Brown et al (1989) and Tomasello (2001), requires that we enlarge our view to include tools, artifacts and other resources that are beyond the boundary of the individual. Hollan and Hutchins (2000) propose that cognitive activity is often distributed across tools, artifacts, representations and groups of people. I will elaborate further upon this later in the section entitled “Learning as a group process”.

Cognitive tools

The ability to make and use tools is thought to be a capability of only the most intelligent beings. Despite a small number of counter-examples, this is generally true. Tool use implies that we have the ability to see a problem in sufficiently abstract terms so that the solution space becomes large enough for us to perceive alternate pathways to our goal. Then we must be able to make a connection between the task that we have abstracted and some analogous phenomenon we have observed, i.e. heavy objects falling can break light objects, or long objects reach distant objects.

As tool users, we see an action as a separate entity (from ourselves), in and of itself, not just as an act that we do. Tools extend our capacity for action or perception, i.e., microscopes and telescopes extend our ability to see, rocks or hammers amplify our ability to exert force.

For tasks of given type we tend to reach for tools we have used before. This may obstruct us from seeing other (better?) possible alternatives. Sometimes we acquire new tools only when we reach an impasse with the tools we routinely use (Lehman, Laird, & Rosenbloom, 1996).

According to Brown, et al., (1989) the design of formal schooling, in large measure, is based on the assumption that we need to have a large cognitive ‘toolkit’, so that no matter what problem comes along, we will theoretically have already acquired the tools we need tools to work on it. However, a tool is not of much use if do not remember that you have it, or if you cannot find it when you need it. In terms of knowledge and tools for thinking, it is not what you have or where you have stored it, rather it is what you can retrieve and bring to bear on the problem at hand.

Cognition is Metaphorically Framed

In **Where Mathematics Comes From**, Lakoff and Nunez (2000) identify a number of metaphors that structure mathematical thinking. Four of these, that they call “grounding metaphors” for arithmetic thinking, form a basis for numerical abilities that underpin the conceptual apparatus we employ to make sense of mathematical ideas. They are the container metaphor (arithmetic as a collection of objects), the construction metaphor (arithmetic as object construction), the measuring stick metaphor and the path following metaphor (arithmetic as motion along a path). These metaphors (and a few

more) appear to structure students' thinking in physics class as well, and I will present examples of how they enable and constrain student thinking in the ways they are used.

This research is, at its heart, interdisciplinary. It is, first and foremost, a work of physics and mathematics education research, but it draws heavily from the fields of cognitive science, anthropology and linguistics: in particular, from the disciplines of cognitive anthropology and cognitive linguistics, and it employs pragmatics as its discourse analysis lens.

In the following sections, I will situate this research project both culturally and contextually. Then I will narrow the focus to the fine details of the theories I use as a basis for my methodological and analytic choices.

The Culture of the Learning Environment

Conventional schooling in the United States is a cultural paradigm that is well understood and can be described with great fidelity by children of all ages. An informal poll of the neighborhood children (who visited my house last Halloween) reveals that schooling consists of going to a place with lots of other kids, and spending most of one's time in a classroom with other children who are all about the same age, learning reading, writing, mathematics, history, and science from the teacher—usually a woman. Their classrooms contain tools, such as books, papers, pens, pencils, whiteboards, calculators, rulers and computers. Expectations in their learning environment include sitting, being quiet, listening to the teacher, following directions, filling in blanks, raising their hand, waiting their turn, getting the right answer, knowing their place, finishing in the allotted time. The teacher sets the agenda and calls the shots. The main thing about schooling for these children appears to be following the rules, getting the answers right and getting the teacher to give them points.

By the time most students reach high school they are experts at playing “the school game”. They know how to suss out a teacher's expectations and boundaries, and they are adept at identifying the classroom practices that will yield for them the biggest effect (translation: buy them the most points) in exchange for the least effort. In general, motivated students are those who, by the teacher's definition, are interested in succeeding in a class (translation: earning lots of points), and success is frequently defined in terms of a student's grade for a course. If a student gets an “A” in mathematics, she has achieved success. Many teachers measure their own success by how many of their students earn A's in their classes (Tschannen-Moran, Hoy, & Hoy, 1998).

It is a longstanding complaint of corporate America that the bulk of young people (products of the American educational system) who enter the nation's labor force lack the most basic of thinking skills, and yet they possess diplomas that certify that they have succeeded in the classroom (Ray & Mickelson, 1993). One inference that might easily be drawn from this corporate indictment of public education is that conventional schooling may not have taught students how to think in ways that are most valued in corporate interests, but rather how to think in ways that earn points. The meaning is clear in the message. Points are given for behaving according to classroom rules. Success is measured in points earned. If you want to succeed in school, find ways to get points. (Of

course, corporate culture rewards those who adhere to its norms as well, so they are not entirely innocent in this respect.)

As with all cultures that frame the many worlds in which we live, the culture of the classroom is co-constructed by its participants—teacher and students. There is a default culture, a conventional schooling ‘paradigm’, that exists if no alternative culture is deliberately negotiated between teacher and students (Gatto, 1992), but this default culture can be and is regularly replaced by some other set of social norms when teachers and students make the effort to do so.

Modeling Physics

Classroom Culture

One of the notions that the Modeling Method of instruction (Hestenes, 1997) is founded upon is that the routine social norms of conventional schooling can and should be rewritten. Modeling is an approach to helping students learn that involves the construction, validation and use of scientific models (coherent knowledge structures) by students in collaboration with their classmates. This is accomplished via the engagement of small groups, usually 3 or 4 students, in a series of tasks whose structure reflects the structure of the model under investigation. Students work together to identify, explore and elaborate fundamental physical relationships—models—and then generalize these relationships for use in solving novel problems with similar structure. They present and defend their findings to their classmates via whiteboarded presentations that involve multiple modes of representation of the data they have collected and interpreted (see photo). Central features of the culture of a modeling physics classroom are inquiry, observation, collaboration, communication, and reasoning. The teacher is not the giver of knowledge—rather he or she is the asker of questions.

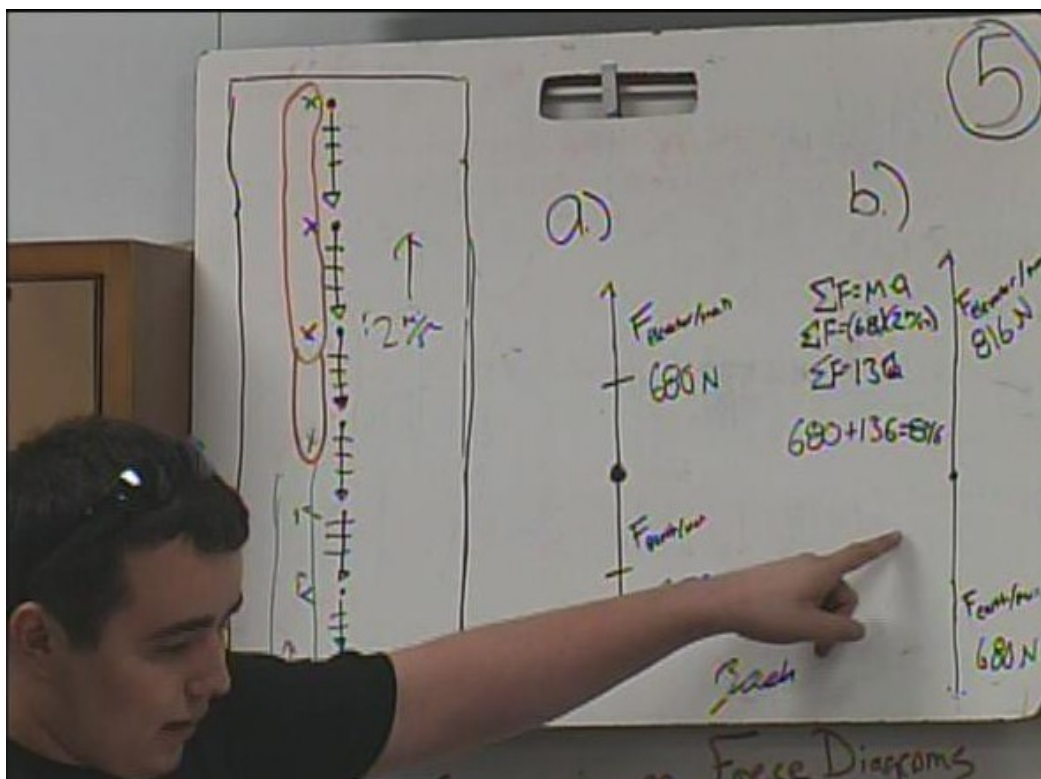


Figure 1. Student presenting whiteboarded problem.

Although a modeling instruction classroom has most of the tools that have become familiar to students in their years of schooling—pens, pencils, rulers, whiteboards, calculators and computers—they learn to use them in different ways. Computers and calculators become data acquisition and analysis tools. Papers provide problems or situations to think about and talk with their peers about. Whiteboards become an important cognitive and communication tool—the place they record what they have discovered or negotiated in collaboration with their peers—the processed data that maps the problem space as they have come to understand it and illustrates their assertions about what they know.

While success in a modeling physics classroom is measured in the conventional way—by the accumulation of points—these points are, ideally, earned by interacting with peers to successfully construct, validate and apply models rather than for simply giving correct answers. However, the reality at times falls short of the ideal, as my data illustrates. In the analysis and conclusion I discuss some of the possible reasons for this.

This study takes a close look at the tools, artifacts and cognitive processes at work in collaborative sense-making activities, and seeks to identify the characteristics of effectively distributed cognition as it occurs in modeling physics and mathematics classrooms. I also illustrate how and when the classroom culture keeps reasoning and sense-making from happening.

The Modeling Cycle

In a typical modeling cycle, students are shown some physical phenomenon, i.e., a toy car moving across the floor, a ball rolling down a ramp. Together they identify what physical relationship, i.e., the relationship between position and time for an object in motion, embodied in this occurrence that they can study with the tools at their disposal, and then they separate into small groups to take data and make sense of the phenomenon. Groups gather and analyze data in an effort to come to some conclusion about the relationship under investigation, and they record their findings and mutually agreed upon conclusions on a whiteboard that they subsequently share with the entire class: in other words, they construct a model. The class compares findings of all the student groups, questions assertions and comes to a consensus on what the experimental findings mean in terms of a generalizable physical relationship or model, and then they formalize their understanding of the model by representing it symbolically (i.e., $F = ma$). A model thus constructed is then available to students for use in solving novel problems with similar structure. Over the course of a semester students collaboratively construct a handful of models for understanding the way things work and all of these become conceptual tools for them to use in reasoning about the physical situations they confront.

A learning environment of this type employs carefully chosen and designed tasks that allow the students to probe one relationship at a time, tools that allow the student to collect accurate data in a straightforward and efficient way and process it simply and convincingly, communication media that allow them to convey their findings to their peers, and a teacher who encourages and expects active student engagement in the inquiry process, and, ideally, is willing to allow students some measure of arousal and control as the process unfolds.

What is a model?

The meanings of the terms ‘models’ and ‘modeling’ as they are used in this thesis must be clarified. The central feature of Modeling pedagogy is a collection of *models* (nouns), and the primary activity of students is *modeling* (verb). What, exactly is meant by these expressions?

A model (noun) is a representation of structure. Hestenes calls models conceptual representations of a real things (1987). Johnson-Laird says that mental (conceptual) models have a structure that corresponds to the structure of what they represent (1996). He defines perception as the transformation of sensory information into a mental model and defines thinking as the manipulation of these models.

Modeling (the verb) is an activity, much as Freudenthal (1983) thought of mathematics as an activity. It is the construction, validation and application of models.

The word model represents something somewhat different to physicists than it does to mathematicians. Mathematics helps us organize complex ideas. Mathematicians attend to patterns and relationships. Their models have a propositional algebraic structure which can be used to further define a geometric space, but the model is fully described by the algebraic structure. Physicists’ models typically have both geometric and algebraic structure because they are used to model real spatial and temporal phenomena.

According to Hestenes (1996) a model (in physics) has four types of structure: systemic structure, geometric structure, temporal structure and interaction structure.

According to Lesh and Doerr, (2003) a mathematical model is a conceptual system consisting of elements, relations, operations and rules. It is a construction that describes a mathematical experience that can have both internal (mental) and external (representational) component. Their notion of system differs from Hestenes, who defines a *system* as “a set of related objects” whose *structure* is the relations among these objects (2006). Mathematical models focus on the structural characteristics of a system rather than physical characteristics. Lehrer and Shauble call models mathematical descriptions that scientists and mathematicians write down to capture the essence of how the world behaves (2000).

In Realistic Mathematics Education, models are seen as representations of problem situations that reflect relevant mathematics concepts and structures. They must be rooted in realistic imaginable contexts and they must be flexible enough to be generalizable (Van den Heuvel-Panhuizen, 2003).

Minsky (1995) says that we construct models of the world with the type of structure that we need for them to have in order to answer the type of question we are asking. If we only need to answer superficial questions, we construct a superficial model.

In this thesis, when I use the terms *model*, *system* and *structure* I will use the Hestenes definition (2006), summarized as follows:

- System: a set of related objects
- Structure: a set of relations among the objects of a system
- Model: a representation of structure

The term model has been used loosely in the mathematics and science education literature, and this has led to some misunderstandings between mathematics and science teachers. In my experience, teachers tend to think of them as either “capital M” models—theoretical models (as in the punctuated equilibrium model of evolution)—or “small m” models—processes (as in the serial addition model of multiplication).

A model (physical or mathematical) can be used as a tool—it can be manipulated, it can be used for making predictions or for answering questions. A model is, thus, a tool for reasoning. Cognitive processes are geared to achieve the biggest bang for the buck. In order to accomplish this we must focus on the most relevant of the available inputs—a model helps us to decide what is relevant.

From a contextual perspective, every physical system has a unique system schema (way of representing variables). From a modeling perspective, system schemas are reusable. This idea corresponds to Realistic Mathematics Education’s transitions of progressive vertical mathematization where a student’s reasoning about a physical system moves from situational to referential to general to formal activity (Gravemeijer et al, 1997) at which time the reasoning is sufficiently abstracted to be useful for any system with the same underlying structure.

In their paper on defining, Zandieh and Rasmussen (2007) illustrate what this progressive vertical mathematization process looks like for students learning geometry, a subject at the heart of the study of physics. Students engage in situational activity when

they collaborate to abstract information from a particular physical situation or construct (in the case of Zandieh's students, a planar triangle), and attempt to construct a working definition with it, that they test by finding non-examples. They progress to referential activity when they extend their initial triangle definition so that it works for triangles on the surface of a sphere. From there, Zandieh's students move on to general activity as they attempt to generalize their newly acquired knowledge about the characteristics of different triangles to construct a description of all triangles, regardless of what prototypical image the word "triangle" might call to mind. Finally, they engage in formal activity when they take their definition of triangle use it as a tool in dealing with novel problems or situations.

A modeling physics analog to the progressive mathematization described above can be seen in the way students construct for themselves a model of constant velocity motion in one dimension. Situational activity occurs at the beginning of the modeling cycle when students are shown an example of constant velocity (a battery-powered car moving along a flat surface) that they then analyze by measuring the car's distance from a point of origin with respect to time. They progress to referential activity when they identify that there is a relationship between position and time (i.e., $x \propto t$). They move on to general activity when they are able to represent constant velocity symbolically as $\Delta x = v\Delta t$, and they ultimately arrive at formal activity when they succeed in constructing and successfully applying a general form of the above equation, $x(t) = v\Delta t + x_0$, to analyze other situations involving objects in constant velocity rectilinear motion. The first three types of activity represent the construction of (in the parlance of Realistic Mathematics Education) the "model-of" constant velocity. The final formal activity constitutes a move of the constant velocity model from "model of" to "model for" as students begin to use it as a tool for reasoning about novel situations. I do not see this process of vertical mathematization as simply a linear progression, however, but rather a spiral phenomenon. Students move from this formal model of constant velocity to consider situations (i.e., constantly changing velocity) in which this model is insufficient and the cycle begins anew with situational activity, which eventually progresses to a new more general formal model that adds an acceleration term to the symbolic representation above ($x(t) = \frac{1}{2}a(\Delta t)^2 + v_0\Delta t + x_0$).

Doing things with models

A model provides schematic structure for a concept space in a knowledge domain (how knowledge might be structured will be discussed in the section after next). It enables the chunking of a large amount of information, which is helpful in cognitive load management, placing fewer demands on working memory (Gerjets, 2003; Paas, 2003). A model can also serve as an organizing principle for instruction.

Modeling, as noted above, is the construction, validation and application of models (Hestenes, 1996). The process begins with a simple physical situation that the student seeks to understand. First, she must identify and represent the salient features of the system. Then she must identify the variables, their geometric properties, interactions

and how they vary. Once a model has been constructed, it must be validated. This is done by comparing what the model predicts about a physical system with the measurements of the behavior of that system. Model application is then used to solve problems: the student makes model-based inferences. In the parlance of the Theory of Conceptual Blending, which will be described below, this is called “elaboration” or “running the blend” (Fauconnier & Turner, 2002).

Modeling tools are both physical and mental, and they run the gamut from the standard verbal, (including analogic and metaphorical), symbolic, mathematical and graphical representations used in mathematics class to the more specialized representational tools of physics: system schemas, force diagrams, motion maps, energy diagrams and bar charts. In addition to these representations the computer can aid in visualization and simulations, and facilitate mathematical manipulation and calculation.

Language is also a powerful modeling tool. The words we choose to describe systems and interactions invoke various schemas – image schemas, aspect schemas and conceptual schemas – that carry with them ways of representing and structuring a problem space.

A simple communication tool used extensively in modeling physics instruction is the student whiteboard. As a tool for mediating discourse among both small groups and whole classes of students, it has proven extremely valuable. It provides a fixed external representation, or inscription, that illustrates student thinking at a particular moment, which can be explored and tested by members of the group that is working together.

In Modeling instruction classrooms, whiteboards are used by groups of three or four students working collaboratively to solve problems or represent the results of laboratory activities. They work together to come to a consensus on the meaning of their findings and then share their results with the entire class via presentations or “board meetings” where the students question each other and compare their thinking with one another via inscriptions that they have co-constructed.

In their theory of inscriptions, Roth and McGinn (1998) characterize inscriptions as representations or “signs materially embodied in some medium” (p. 37). Inscriptions are physical artifacts, not to be confused with mental or verbal (lexical) representations. Since they are visual, they are social objects that are transportable, portable representations and they do not change their properties or internal relations while in transit (think of a piece of paper or a whiteboard with a graph drawn on it that you can carry from one place in a classroom to another and show to different students or the teacher along the way). They can be incorporated into a variety of visual displays, scaled up or down without changing their meaning, layered with or superimposed upon other inscriptions to enhance or reinforce communication and meaning, and translated into other inscriptions to suit a variety of contexts.

Inscriptions, in the sense that Roth and McGinn use the word, represent a mapping of elements of the real world (i.e., nature) onto the mathematical world and as such, they function as boundary objects between these two worlds, revealing the student’s mental model of how the world is structured.

Discourse in the Modeling Physics Classroom

Student discourse is a vital element of modeling as it is practiced in modeling instruction. It is where sense-making about these boundary objects occurs. Students exteriorize their thought processes, compare them with one another, subject them to reasoned analysis, justify (or discard) them, and, ideally, identify the limits or boundary conditions of the model they are constructing.

There are many ways to look at discourse. James Gee (1999) identifies multiple discourses which he calls “big D” and “little d” discourses. Big D Discourses belong to language communities, (i.e., physicists, republicans, gang members, other ne’er-do-wells, etc.) and include non-language cultural “stuff” specific to particular identities and/or activities, while little d discourse is simply language in everyday use (p.7). We are all members of many different Discourses and these Discourses can influence each other positively or negatively and at times give rise to new “hybrid” Discourses. It may be this breeding of hybrid Discourses is a place where students are challenged in learning physics and mathematics. The data collected for this study suggest that this is certainly a factor affecting students’ willingness to engage in the learning process.

Another way to examine discourse is to consider the categories of speech acts that occur in language-in-use. This approach is called pragmatics and views language as “doing things with words” (Cameron, 2001). Examples of speech acts are apologizing, promising, reminding, etc. The speech act, modeling, is the subject of this research.

Critical discourse analysis looks at speakers’ and listeners’ position, identity, cultural or social capital, habitus (thoughts, values, worldview, skills, dispositions, ways of being on the world), and field (socially or culturally lived in worlds). These factors also influence the nature and extent of a student’s engagement in the discourse of the modeling instruction classroom and are related to the value they are able to extract from episodes of classroom discourse that accompany whiteboarding activities .

As with all other classroom activity, engagement in whiteboarding and the discourse surrounding it are goal driven activities. Goal theory (Ames, 1992; Nicholls, 1979) connects the level of student engagement in activities such as whiteboarding and discourse with their motivation. Goals can be divided into two broad categories: ego or performance goals, which for students are aimed at looking good in front of teachers or classmates or getting lots of points, and mastery or task goals, which involve doing a job well because the task is interesting or otherwise engaging. Ego/performance goals are seen as extrinsically motivating while mastery, task goals are seen as intrinsically motivating. Motivation research has found that intrinsically motivated students are more likely to persist in the face of difficulty and generally demonstrate a better self-concept than students who are extrinsically motivated (Middleton & Toluk, 1999).

Teachers sometimes settle on setting procedural performance goals for their students. Modeling Instruction moves goal setting more toward the conceptual (Thompson, Philipp, & Thompson, 1994), and I have found in my own classroom as well as in those classrooms I have observed, that this moves students naturally toward the setting of mastery and task goals necessary for self-regulation.

A Question of Motivation - To Engage or not to Engage?

For modeling to happen and models to be constructed, students must opt to engage in the classroom discourse enterprise. How does modeling instruction induce engagement? One key feature of this instructional approach is its emphasis on doing physics as physicists do. Students are encultured into the ways of physicists rather than just taking physics class. Sociolinguist James Gee likens traditional physics instruction to “reading a manual for a videogame that you will never play”. Modeling physics is more akin to encouraging students to play the game of physics before reading the manual.

Personal Constructs theory (Kelly, 1955) suggests that people hypothesize about the outcomes of situations, which they test and revise as they engage in activities connected with these situations and this guides decisions about the nature and extent of future engagement. Kelly’s theory explicitly links emotions and motives with cognition. This plays out in classrooms when students opt to engage in some learning experience (or not) based on their assessment of its value as “academic fun”, i.e., their likelihood of success in the context of this activity (Middleton, 1992; Middleton, Lesh, & Heger, 2003). According to Middleton, et al, students evaluate academic fun based on the levels of arousal and control that it affords them. Arousal is a function of whether or not an activity is stimulating and/or relevant to their interests or experiences. Control has to do with whether or not an activity is too easy or too hard, or whether there are multiple opportunities and/or ways to succeed.

Another area in which control is a factor in modeling instruction classroom discourse is in who has the floor (Edelsky, 1981). In traditional didactic classrooms, except for rare instances, the teacher always controls the floor. In modeling classrooms, the students may have the floor for extended periods during small group and whole group discourse.

The tasks that students are given in modeling physics afford some measure of both arousal and control. They are embedded in familiar contexts, and the modeling cycle that is utilized allows students to continually express, test and revise their model as they construct it. They play the game of physics as they learn its rules.

Modeling Instruction done well produces discourse that is consistent with an activity that is intrinsically motivating. Modeling Instruction done less well can produce the sort of engagement, and as a result, the sort of discourse, that is more characteristic of extrinsic motivation, where the object of the game is answers, points, and grades. There is a self-regulatory component to this outcome that seems to be related as much to teacher expectations as to perceived arousal and control. This merits further exploration.

Cognition and Learning in Modeling Instruction

Let us assume for a moment that students will elect to engage in the learning culture that characterizes their modeling physics or mathematics classroom. What might this mean in cognitive terms?

Students do not enter their learning environments as tabula rasa. New learning is overlaid upon a great deal of pre-existing ‘organized’ knowledge. In designing learning environments it is helpful to have a way of thinking about how existing knowledge is structured and accessed, how new information is assimilated into or coordinated with

existing models of the student's world, and how the interactions between students, their tools and artifacts affect the learning experience.

How is knowledge structured?

There are many theories of how knowledge is structured but most have, at their foundation, features that are common or at least analogous. All posit the existence of basic cognitive units or structures, although they carry a variety of names—elements, concepts, schemas, chunks, scripts—which can be constructed, modified, combined and/or elaborated. There are many parallels with Modeling Theory to be found in the theories of cognition that have been popular over the last seventy years.

Schema theories of cognition

F. C. Bartlett, in his seminal work entitled **A Theory of Remembering** (1932), proposed the term *schema* to identify the way knowledge is structured and stored in memory. His basic research revealed that on the whole, memories are reconstructed each time we call them up rather than being reproduced intact. In the decades since Bartlett first advanced these ideas (in the 1930's), numerous theorists have tried to discern the details of the mechanisms by which schematizing takes place.

In her Prototype Theory, Eleanor Rosch (1975) introduced a definition-based model of categories that corresponds well to the entities that relate to one another in schemas. A prototypical member of a category is one that possesses the most attributes that are characteristic of that category, i.e., if the category 'bird' has such features as beak, feather, wings, flight, a robin is a more of a prototypical bird than a penguin. Prototype theory holds that there are levels of categories and that so-called 'basic level' categories have a maximum number of attributes that are shared by category members and a minimum number that are shared by members of other categories. Prototype theory maps an entity's attributes onto a category and in doing so, infers the structure of the entity as homologous to the structure of that category.

Schank and Cleary extended Bartlett's assertion that memory is schematic with Script Theory (1995), in which knowledge structures (schemas) may be conceived of as subconscious scripts that determine how a person behaves in a particular situation, i.e., a "going to the movies" script might include standing in line at the box office, purchasing a ticket, entering the theater, buying popcorn, finding a seat in the theater, watching the movie, and exiting after the credits.

Information processing theories of cognition

Information processing theories of cognition are a particular class of schema theories that have employed a computer metaphor to model cognition. ACT-R Theory looked at remembering in a slightly different way from early schema theorists, (Anderson, 1996) subdividing memory into declarative memory, production memory and working memory. According to Anderson, declarative memory is a long-term repository of fairly stable knowledge structures (schemas), production memory is the place where these schemas are actively used and modified, and working memory is where incoming perceptions are encoded and outgoing actions are produced.

Newell has called his Soar architecture a “unified theory of cognition” (Lehman et al., 1996). It attempted to integrate the micro-theories (of which schema theory is one) contributed by disciplines such as psychology, anthropology, linguistics, and artificial intelligence to the field of cognitive science. Soar models thinking and learning by accessing the content of memory, depositing the information needed to think about something into a “problem space” that is created by goal directed activities, and structuring or constructing it there. Soar theorists claim that rehearsal aids learning by the creation of ‘chunks’ of knowledge, and suggests that chunking is the basis for memory organization. This symbol manipulation approach is characteristic of information processing theories of cognition. It is a useful model for artificial intelligence but ignores the neural network view of the brain as a massively parallel processing system that processes patterns (Hestenes, 2006).

Rumelhart and Norman (Rumelhart & Norman, 1978) identified three modes of learning: accretion—the collection of information, knowledge structuring—schema building, and tuning—the adjustment of knowledge to a specific task. Bruner’s take was similar (Bruner, 1996), arguing that learners construct new ideas relying on cognitive structures (schemas).

Physics Education Research on cognition

Physics education researchers’ literature has developed its’ own schema theories to describe the structure of knowledge. Hestenes Modeling Theory of Cognition (2006) holds that cognition in science, mathematics and everyday life is essentially about constructing and using mental models. This is aided by our capacity to represent things with language and symbol systems. Hammer has called knowledge structures ‘resources’, which are assembled in “frames” that then enable ‘transfer’ (Hammer et al., 2004). Vosniadou has said that her research subjects (1992) start with ‘initial conceptual structures’, which they integrate into ‘explanatory frameworks’ thus enabling conceptual change. DiSessa’s (1993) view was that students begin with a collection of ‘p-prims’—phenomenological primitives—that resolve into systematized coordination classes that result in conceptual development.

Embodied Theories of Cognition

Cognition is much more than symbol processing, and information-processing theories of cognition are gradually giving way to the notion of embodied cognition. These, too, are schema theories. One such theory, Conceptual Blending, has resemblances to all the above referenced theories, and many parallels with neural network modeling, which views cognition as grounded in the sensorimotor system. This accords well with current research in cognitive neuroscience, which is attempting to map activation patterns of neurons to particular functions in biological systems (Hestenes, 2006).

Fauconnier and Turner (Fauconnier et al., 2002) describe three fundamental representational processes in their Theory of Conceptual Blending. The first process is the selective mapping of structures (i.e., schemas, scripts, elements, resources, conceptual structures, p-prims, etc.) from input spaces (i.e., memory or perception) to a blended

space in the subconscious (similar to Soar’s problem space, ACT’s production memory, Hammer’s frame, diSessa’s coordination class). The second process is inference making, in order fill in the necessary details to complete the composite structure being assembled, stabilizing it as a larger emergent blended structure, (i.e., chunking). The last process, which they refer to as elaboration, entails manipulating the structure in the blend, or “running the blend” as they put it—doing cognitive work within the blend (like the work done by ACT’s working memory, Rumelhart and Norman’s tuning, Hammer’s transfer, and Vosniadou and DiSessa’s conceptual change) according to its own emergent logic to reason, solve a problem, answer a question, satisfy a need. Fauconnier and Turner assert that blending is an iterative process: “The existence of a good blend can make possible the development of a better blend.” They suggest that the human capacity to blend is at the root of the human ability to innovate and is responsible for such feats of cognitive legerdemain as the development of language, culture, religion, art, science and the production of sophisticated tools. Conceptual blending is an embodied theory of cognition—it views cognition as grounded in sensory and motor experience.

As we do not have truly objective knowledge of the world but only the subjective information provided by our senses, it follows that our thoughts about how the world works are grounded in our body’s sensation and motion.

Learning as a Group Process

Learning is a cultural process, situated in a context, mediated through social interaction, activity and representation. It is this social aspect of knowledge structuring that is explored next.

Paul Cobb (2002) and Helen Doerr (1999), among others, have suggested that a classroom community is a legitimate unit of analysis when examining student reasoning, and that in such a setting, content knowledge can be seen as an emergent property of the interactions of student groups. Tools, artifacts and inscriptions—written representations (Roth & McGinn, 1998)—are observed to be critical components of this relation, as is the social setting in which the interaction takes place.

Cognitive scientist Edwin Hutchins has dubbed this phenomenon “distributed cognition” (Hollan et al., 2000), and his theory of distributed cognition seeks to illuminate the organization of cognitive systems, which include not only groups of people, but also the resources and materials in their environment.

Hollan et al, (2000) cite four core principles of distributed cognition theory:

- “people establish and coordinate different types of structure in their environment
- it takes effort to maintain coordination
- people off-load cognitive effort to the environment whenever practical
- there are improved dynamics of cognitive load-balancing available in social organization.”

The theory of distributed cognition views cognition as embodied, that is, constrained and enabled by the unique ways that the human body functions in its environment. There is a relationship between internal and external processes that invites

coordination of resources—memory, attention executive function, objects, artifacts and whatever materials come to hand. In this event, the human body and the material world in which it resides take on central rather than peripheral roles.

The conceptual blending that is done by a small group of students as they study a physical system, and negotiate what they will put on a whiteboard to share with their classmates and agree that their inscription means, can be viewed an instance of distributed cognition in the classroom. The social organization of the group forms a cognitive architecture that determines how information flows through the group. Individual members of the group take on different (unassigned) roles as they progress through the task, enabling the team to optimize its performance by exploiting the unique strengths that each member brings to the partnership. Modeling instruction facilitates and frames this process by structuring the learning environment around a series of activities done by small groups—three to four students—working collaboratively.

Communication and Learning

In this section, I will discuss factors affecting the communication that shapes student thinking in physics class.

All utterances are communicated in some context, and it is this context, to some degree, that guides the listener in determining what it is about the utterance that is relevant (Sperber & Wilson, 1986). The speaker and listener have some common semantic frame (what Lakoff calls a “pre-conceptual structure”, such as an image schema or aspectual schema) that allows for mutual manifestness—the common understanding by speaker and listener of whatever the speaker is trying to communicate. According to Sperber and Wilson there is a tacit assumption on the part of speaker and hearer that whatever the speaker is attempting to communicate is worth knowing, and that she will communicate it as well as she knows how.

Where no preconceptual structure exists, metaphors provide a structure that unfamiliar things can be mapped onto, and specify the relationships among the elements of these things (Lakoff, 1987; Lakoff et al., 1980; Lakoff et al., 2000). Metaphors are so ubiquitous in our everyday thinking, acting and speaking that they go largely unnoticed. We must often look closely at the smallest particles of language in order to detect their presence. It is from this pursuit that cognitive linguistics has arisen, and the bulk of the analysis of that data collected for this study will ferret out students’ metaphors by examining their choices of prepositions and verbs.

Prepositions in particular are important for probing students thinking in physics as they encode information about spatial and often temporal reasoning (Herskovits, 1986; Jackendoff, 1996; Peterson et al., 1996).

Why do we need a reference frame?

In language as in physics, one must adopt a frame of reference to be able to establish a context for mutual understanding between speaker and listener (Levinson, 1996). Context serves as a cognitive environment for both speaker and listener—it contains a set of facts that are mutually manifest, that is, inferable or perceptible by both parties (Sperber et al., 1986).

A student's cognitive environment is a function of her physical environment and her cognitive abilities. Environments afford contextual effects, but accessing these entails a cost. There is effort associated with the activation of contextual effects brought about by mental processes—the more effort needed, the higher the cost.

The relevance of a concept is a function of the “effect/effort” ratio—a large contextual effect for a small effort results in high relevance (Sperber et al., 1986). Relevance has a direct bearing on the frame of reference one chooses to communicate about spatial and temporal information (Tversky, 1996). All other things being equal, a speaker will choose the reference frame that is most convenient and most likely to be well understood by the listener when communicating spatial or temporal information, perhaps out of politeness. In cases where the speaker cannot see the listener (talking by phone or written communication), the speaker is likely to choose the listener's perspective. However, as adopting the listener's perspective becomes more and more effortful, the speaker favors effort over politeness and often shifts perspective, often without signaling.

Reference frames help us organize things. They provide a syntax for ordering spatial inputs (Peterson et al., 1996). Levinson says they apply at three levels: conceptual, linguistic and perceptual (Jackendoff calls this last level spatial). Cognitive linguists refer to the features of interest in a reference frame as the ground (or environment, which typically carries with it an implicit coordinate system), the figure or reference object (sometimes called the *relatum*) or the *trajector* (if it is in motion) and sometimes the observer (called the *origo* in speaker centered reference frame).

How many reference frames are there?

There is not good agreement on how many reference frames people use in language, and it varies somewhat from culture to culture. O'Keefe's (1996) brain research offers evidence that the hippocampus employs a single geographic (north, south, east, west) or “Kantian” frame in constructing cognitive maps. In language however, adult English speakers tend to employ two or three distinct reference frames, depending on how you characterize them. For those who prefer to consider two reference frames, they seem to divide the frames along the following lines:

Table 1

Names used to identify reference frames in systems that divide reference frames into two categories

Egocentric	Allocentric (other centered)
Observer centered	Environment centered
Orientation bound – 2½D	Orientation free – 3D
Personal	Neutral

Albert Marr's (1982) research on vision likens the observer's visual image to a 2 ½ D sketch—it contains some depth information but it is incomplete. If one's frame of

reference implies that there is 3D information, then the perspective is orientation free of allocentric. Carlson, Radvansky and Irwin's research has determined that infants have only an egocentric or orientation bound reference frame until they reach about 16 months, at which time they are able to adopt an orientation free, Kantian environment centered reference frame.

Most linguists have identified three reference frames in language structures, but no one seems to agree on what they should be called. They fall into these basic classifications: ego centered, object centered and environment centered. Below are some of the names they are called by various researchers:

Table 2

Names used by researchers who favor three types of reference frame

Levinson	Tversky	Carlson, Radvansky and Irwin	Peterson, Nadel, Bloom and Garrett
relative	deictic	deictic	deictic
intrinsic	intrinsic	intrinsic	intrinsic
absolute	extrinsic	extrinsic	absolute

Levinson's relative perspective is a three way relationship, triangulating the speaker's (or viewer's) position with some feature of the environment in order to locate an object, i.e., the ball is to the left of the tree. This is a bit more clear cut than the deictic perspectives described by other researchers, who divide the perspective into primary and secondary deixis (Levelt, 1996), or speaker deixis, place deixis and time deixis (Tversky, 1996). Jackendoff, although he claims that there are only two reference frames, deictic and intrinsic, goes a step further, subdividing the intrinsic frame into four object centered sub-frames and four environment centered sub-frames.

What to do? Whose framework to choose? The answer is not obvious. My initial inclination was to keep it simple and opt for a two-reference-frame framework, but upon closer inspection these led to a need for subcategories, so I have chosen to adopt Levinson's framework of relative, intrinsic and absolute as simplest sufficiently complete framework.

Doing things with reference frames

Levinson's framework provides a means of establishing axes or coordinate systems (polar or rectilinear). The relative perspective, from the viewer's point of view triangulates an objects position with respect to some feature of the ground. The intrinsic perspective utilizes built in features of the ground to establish axes. In addition, the absolute perspective utilizes the cardinal compass directions. Since a great deal of the information content of the language of physics students is spatial, it will be important to have their spatial references well specified. And since research indicates that temporal information is coded "quasi-spatially" i.e., along a one dimensional time line (Johnson-Laird, 1996) Levinson's relative perspective, which centers on here and now should work well for this as well.

What can my students do with reference frames?

Jackendoff and others have spent some time studying the interface between language and space (Jackendoff, 1996; Peterson et al., 1996). He describes language representations, spatial representations and conceptual structure, and his theory of representational modularity describes how these fit together. The diagram below shows the relationships between elements of the language and vision modules.

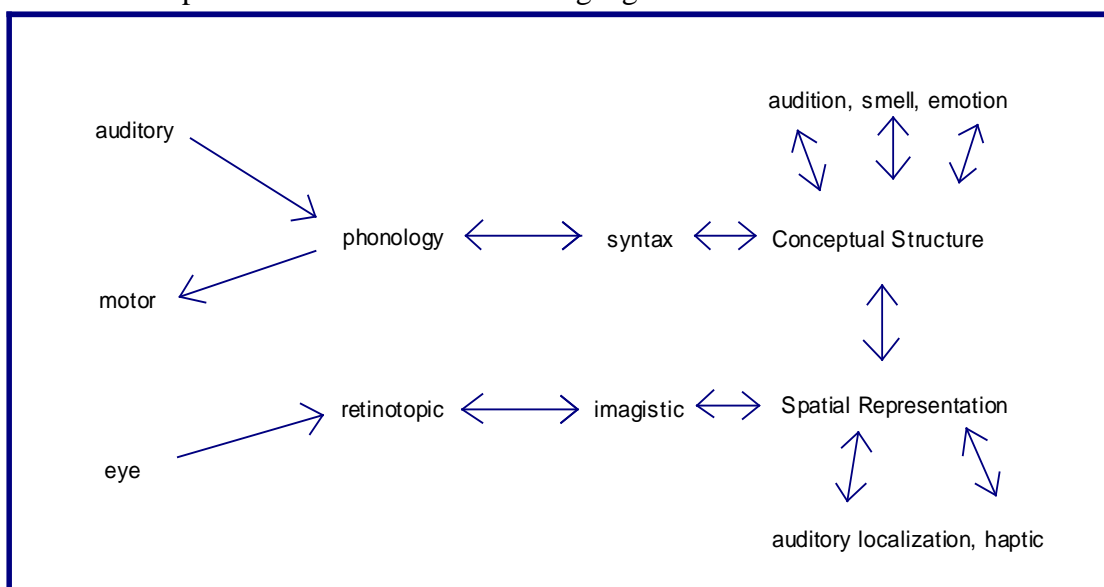


Figure 2. A schematic of Jackendoff's Theory of Representational Modularity (1996, p.3)

According to Jackendoff, language does not directly enter our thoughts as a spatial representation (SR) but must go through the conceptual structure (CS) module, which encodes propositional representations and interprets them spatially. The SR module encodes image schemas directly. Jackendoff therefore characterizes the CS as algebraic while the SR is geometric.

Jackendoff's research reveals that the CS and the SR share notions of object, place and path. CS encodes token (instance) vs. type (category), quantification, and taxonomic relations while SR encodes shape. A word *must* have a lexical CS, it *might* have a SR.

Understanding the process that language must undergo in order to evoke a spatial image, in comparison with the direct route that a diagram can take has important instructional implications. For axes and frames of reference to have an effect, they must be coded in CS.

The CS-SR interface is the boundary between language/motor activity and vision, and as such, has important ramifications for doing physics with whiteboards. Much more will be said about it later.

What is the Role of the Whiteboard in Modeling Physics Discourse?

The discourse of physics is about the physical world and as such, is necessarily spatial in nature. The whiteboard can be used by students working together to create a common picture of the physical situation they are attempting to understand. In a sense, it assigns an ‘absolute’ reference frame to the situation, eliminating one potential source of misunderstanding among group members and insuring that all the elements of the problem space are mutually manifest. As students construct white-boarded representations, the verbs and prepositions they choose as they negotiate these inscriptions with one another can offer insights into the metaphors that are shaping their reasoning processes.

Another source of insight into the distributed cognition that is taking place in the course of whiteboard preparation and sharing is the structure of the activity itself (Lemke, 1990). Who leads the conversation? Who writes (or erases what is being written) on the whiteboard? What information is deemed worthy of incorporation into the inscriptions being negotiated and what is ignored? When does the spatial representation occur and how is it used (or not) in reasoning about the problem? When the whiteboard is presented to the class after it is complete, what is mentioned and emphasized? What is ignored?

In a traditional lecture-style physics or mathematics class, the teacher has “the floor” (Edelsky, 1981) for the bulk of the class period. He decides who speaks, and he talks when and for however long he wants. If he wants to hear from students, he calls on them. If students want to talk, they must ‘bid’ for the privilege by raising their hand. In a modeling instruction classroom the floor holder may change frequently during small group whiteboard preparation, and there are two types of floor seen in this setting—the orderly, one at a time turn-taking characteristic of a teacher-controlled floor and the more informal overlapping poly-vocal conversations where multiple students seem to be holding the floor at the same time. Who has the floor and what type of floor they have has important implications for what eventually is written on and said about a whiteboard.

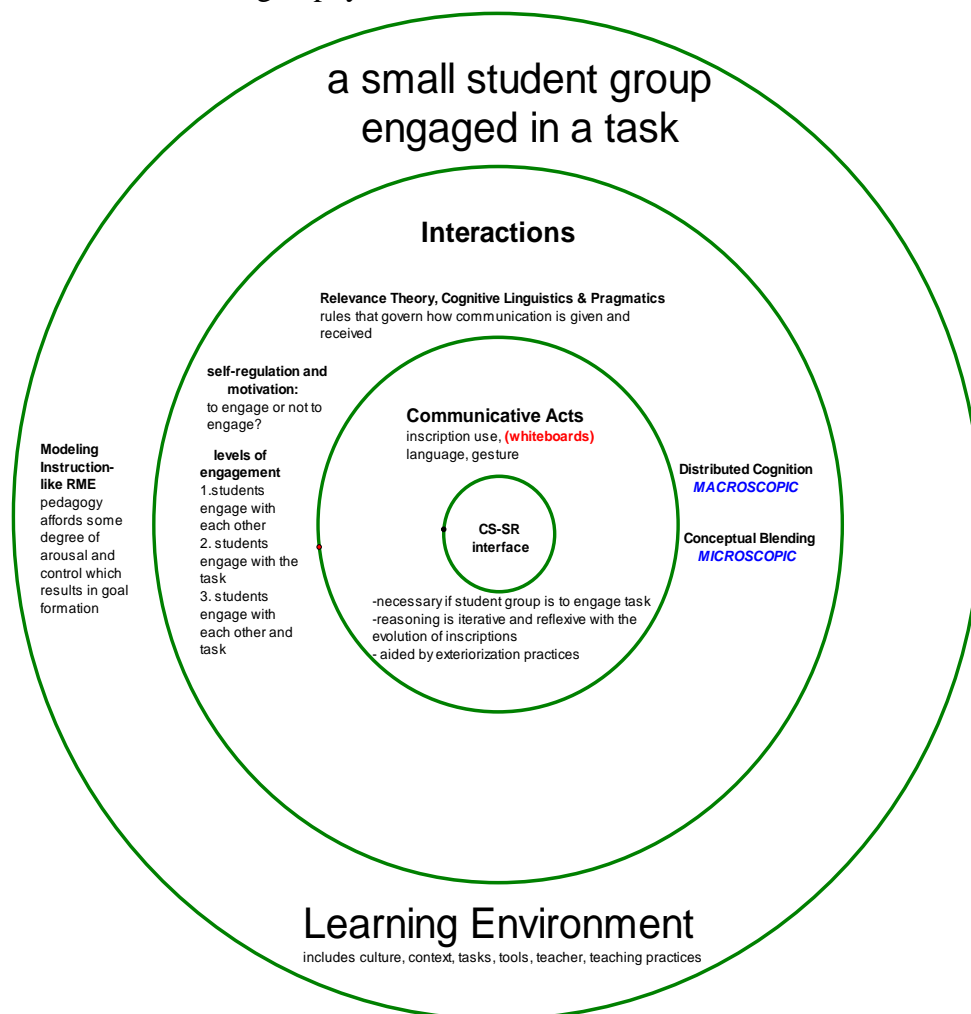
Whiteboarded inscriptions in modeling physics can either be ad hoc – dreamed up on the fly by one or more students – or introduced by the teacher. The latter is more often the case, and as students’ grasp of these introduced inscriptions grows, they become tools that advance reasoning as well as communication. This can be accompanied by a shift in metaphor use as well. It appears that the evolution of inscriptions and metaphors is reflexive. I will illustrate this with my data.

The metaphors that students choose can also constrain their ability to reason about a situation in certain ways. I will show instances of this. It often seemed that students’ cultural framing of schooling (a place where what counts are points, grades and answers) interfered with the advancement of their reasoning about physics problems. There was a broadly but tacitly agreed upon “good-enough-ness” to the solution of any problem that, once achieved, seemed to prevent further progress toward understanding. This has major implications for transfer in learning and is a subject I will be following up in my future research.

The figure on the following page is a schematic of the aspects of the learning environment that frame my theoretical perspective. My study will attempt to answer the following research questions with respect to this schema:

Research questions

- How do collaboratively constructed, shared visual representations aid students' collective meaning-making in physics and mathematics?
- What is the structure of the discourse that surrounds the preparing and sharing of collaboratively constructed visual representations?
- What can teachers do to optimize the way students use inscriptions and discourse to aid their reasoning in physics and mathematics?



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Figure 3. A schematic of the theoretical framework that served as my lens for this study.

METHODOLOGY

In this chapter, I will recount the story of my research process—where the initial ideas came from, the rationale for the many choices I made during the data collection and analysis process, and the constraints and affordances that shaped what has emerged from my data.

Constraints

In theory, classroom research directed toward understanding student reasoning should be designed to have unfettered access to students and their work. In practice however, the legal and logistical concerns of the district, the school administration, the classroom teacher, and the students often combine to constrain the nature and extent of the data that can be collected.

I set out to examine and describe how discourse shapes student reasoning in a particular instructional setting—the high school Modeling physics classroom because as a practitioner of the modeling method of instruction it is clear to me that students who learn physics in this way approach the subject differently and typically experience greater success. I originally hoped to study this in a 9th grade, “Physics First” classroom because Physics First is a curricular innovation that I have employed myself with great success for over 10 years, and one that I believe will eventually prevail as the standard model for high school science instruction in this country.

What I ultimately did, however, was somewhat different. Rather than looking at student discourse as a whole, which I found to be too large and diverse, a category of communication to assess meaningfully, I narrowed my focus to the discourse that surrounds the preparing and sharing of whiteboards. This activity is one of the unique hallmarks of Modeling Physics, and one that novice Modeling Physics practitioners spend a great deal of time trying to “get right”.

Another major difference in the existing study from what I had originally planned is that I was unable to restrict my data collection to a study of 9th grade Physics First students. As there are very few Physics First classrooms in the Phoenix area, and even fewer school districts that are open to the idea of classroom research that entails videotaping of students, in order to have a sufficiently large sample I ultimately collected four separate video data sets:

1. a four week twice weekly participant/observation in a middle school mathematics resource class at an urban K-8 school with Majority Hispanic student population;
2. a semester-long daily observation in an honors physics class for 11th and 12th graders at a large upper-middle class suburban public high school;
3. an eight week observation in a 9th grade physical science class at a large working class suburban public high school with majority Hispanic student population during their physics unit of instruction; and
4. a semester-long observation at a suburban community college in a 3rd semester calculus based physics course on electricity and magnetism.

When I conceived this study, in addition to classroom observation and videotaping, I envisioned think-aloud and retrospective analysis interviews, as well as

copies of student work and informal email conversations with willing student participants. However, in an abundance of caution, the districts and school administrations permitted me to gather only classroom video.

While this provides a somewhat smaller variety of data to consider than I had originally hoped for, it still represents a staggering volume of information, and affords a view of students' reasoning and communication practices that is comparable to what other members of the class—both students and teacher—are privy to on a daily basis. And since the goal of this study is to provide teachers with a way of viewing and evaluating how whiteboards shape student thinking and reasoning “in the wild”, it is appropriate to demonstrate that useful conclusions can, indeed, be drawn based on evidence of this type.

The data, then, are ‘emic’ in nature, that is, revealing an insider’s view of what transpires in the unique microculture of the model centered collaborative physics and mathematics classroom from the perspective of the students and teachers who populate it.

The practice of whiteboarding is responsible for the fact that a great deal of student reasoning is exteriorized in the typical modeling physics or mathematics classroom environment. The small-group work that this instructional method espouses, which is then presented and defended before the whole class, involves a careful negotiation of how to represent the group’s reasoning about a problem, and what the representations should mean. Videotaping is an ideal method for capturing the details of this negotiation process.

Choosing a Unit of Analysis

Learning and practicing science are inherently group activities, and modeling instruction exploits this fact by design rather than by accident. The modeling cycle involves construction, validation and use of scientific models (coherent knowledge structures) by students in a collaborative classroom setting. These activities are accomplished via the engagement of small groups (usually 3 or 4 students) in series of carefully designed and sequenced tasks whose structure reflects the structure of the model under investigation. Students work together to identify, explore and elaborate fundamental physical relationships, and then generalize these relationships to solve novel problems with similar structure. They present their findings to their classmates via whiteboarded presentations that involve multiple representations (i.e., graphs, diagrams and algebraic expressions) of the data they have collected and interpreted. Situated as it is in such an environment, the cognition that takes place in this setting is inherently a cultural process, mediated through social interaction, activity and representation.

Paul Cobb (2002) and Helen Doerr (1999) have maintained that a classroom community is a legitimate unit of analysis when examining student reasoning, and that in such a setting, content knowledge can be seen as an emergent property of the interactions of student groups. Tools, artifacts and inscriptions—written representations (Roth et al., 1998) are observed to be critical components of this relation, as is the social setting in which the interaction takes place.

Traditional cognitive research restricts the unit of analysis to the individual mind. However, if cognitive processes are mediated by group interactions, defining the unit of

analysis in this way can cause important contributing processes to be overlooked. By enlarging the unit of analysis from the individual to the group, the distributed nature of the process becomes apparent.

Cognitive scientist Edwin Hutchins has named this phenomenon “distributed cognition” (Hollan et al., 2000). He first began studying it in workplaces such as the bridges of ships and the cockpits of aircraft in the 1980’s.

Hutchins’ studies in these areas, which he calls “cognitive ethnographies”, use an event-centered approach to study the complex interactions among individuals, their tools, artifacts and inscriptions to obtain “prescriptive information on the design of work materials.” (Hollan et al., 2000) He conducts fine-grained analyses of the details of individual transactions among groups of individuals working jointly to perform complex tasks (i.e., the navigation of a crowded shipping lane in a harbor).

After considering these factors, I have chosen the lab group (usually 3 to 4 students) as my primary unit of analysis for this study, however mindful of the fact that ultimately, this type of learning environment results in ownership of the ideas it constructs by individual students who participate in their construction. These ideas join the collection of schemas, or knowledge structures, that a student routinely calls up and uses as building blocks for new ideas. Moreover, when enacted within a group, the process of co-construction by which this occurs (and which I will describe in ensuing chapters) aids in the synthesis of new ideas in ways that no single individual would be able to produce on her own.

Factors Affecting Data Collection

Cognitive ethnography, developed by Edwin Hutchins, is particularly suited to studying groups (Hutchins, 1995). Cognitive ethnography is an event-centered approach to observing group interactions that focuses on examining the various units of analysis pertinent to this study. This allows for reconfiguration of the cognitive models I develop in the course of data collection to suit the analysis process. The event that this study pays closest attention to is the activity of whiteboarding.

In addition to its advantages in studying social cognition, cognitive ethnography places great emphasis on the range of mechanisms, symbols, tools and inscriptions (diagrams, graphs, etc.) that mediate cognitive processes. Cognitive processes are not just distributed across individual members of a group but also across the physical and verbal representations they construct and use, and across time. This is particularly true in physics and mathematics. Both disciplines rely upon an extensive and exacting set of semiotic structures. Discerning the role of the semiotic structures that populate whiteboards in supporting discourse and in constructing schemas, scripts, frames and conceptual blends will be central to the process of arriving at useful models of the processes under study.

Classroom data collection in the 11th and 12th grade honors physics class began with an FCI pretest to ascertain students’ initial knowledge of the force concept. After parent and student consent were obtained in accordance with Institutional Review Board rules regarding experiments involving human subjects, a semester of classroom observation and videotaping was undertaken, followed by post-instruction FCI testing.

Video images of students' written work were captured whenever it was possible to do so without being intrusive. Newtonian mechanics was covered in this first semester course, and video data collection took place daily throughout the semester.

I selected a single group of three or four students to follow throughout the semester during small group work for several reasons: first, to attempt to track changes in the group's thinking and reasoning about mechanics; second, because there were only a couple of places in the classroom where I could conveniently plug in and place my videocamera without obstructing foot traffic, and third, because the student group at this table was agreeable to the increased level of scrutiny. The entire class was observed during whole group discussions.

The 9th grade physical science classroom was even more crowded, and there were few places that I could conveniently place a camera to observe small group whiteboard preparation. In this classroom, after securing the necessary consent forms, I videotaped whatever students sat at the table nearest the camera during small group work. Whole group discussions were recorded in much the same way as they were in the honors physics classroom. While I was unable to follow a single small group of student through the entire unit of instruction, I was able to observe the norms and routines by which discourse around whiteboards unfolded among students, the types of reasoning that it invoked, the processes that were common across all groups and all tasks, and the goals and motivations of the students and the groups that drove the reasoning processes forward.

The middle school mathematics resource class was crowded as well, limiting camera placement to a small spot in the rear of the classroom. As in the 9th grade class, the cameraperson was able to follow both whole group discussions and activities and the student group who sat adjacent to the camera.

The community college classroom was the roomiest and easiest in which to collect data (and the easiest group to obtain consent from, as all were over the age of 18). Although these students (15 in all, although attendance varied throughout the term) were not initially the primary focus of my inquiry, it was an interesting environment in which to do ad hoc hypothesis testing as my data collection progressed. When these students worked together in their small groups, I was able to place the camera near any table in the classroom and observe the group interaction that accompanied whiteboard preparation, and when the class engaged in whole group discussions, which they called "board meetings", I sat with the group who formed a 'circle' with their chairs, their whiteboards in front of them resting against their knees so that all whiteboards were visible to all members of the group as the discussion progressed.

The teachers whose students were observed for this study were selected based on the average FCI gain scores of their students, their experience as practitioners of Modeling Physics Instruction, and their (and their district's) willingness to allow data collection in their classrooms. I performed a Reform Teacher Observation Protocol (Sawada et al., 2002) on the teachers whose students I studied to document the degree to which their classroom environments are considered reformed (by AAAS, NSTA and NCTM standards). All teachers observed scored between 73 and 99 (out of 100) on repeated RTOP measures.

The video data collected as described above was digitized and stored on DVDs. The original tapes are archived in a fireproof filing cabinet in the event that they might prove useful in the future. Digital media and any transcripts of what they contain are stored on a password protected hard drive and in a locking file along with all other artifacts collected for analysis. In order to protect their identity, the student subjects in this study are identified by pseudonym, as are copies of whatever work they produced. Any record of students' actual identities will be destroyed when the study is complete.

Approaches to data analysis

Since the goal of this research is primarily interpretive, Erickson's (1986) approach to qualitative data analysis was employed in an "attempt to combine close analysis of fine details of behavior and meaning in everyday social interaction." I examine the specific structure of events rather than their general character, probe the semantic form and meaning perspectives implicit in students' language and inscriptions, and identify reflexive linkages between form and meaning as the process unfolds.

At the outset, I was advised to approach data analysis with Strauss and Corbin's Grounded Theory (Strauss & Corbin, 1994) perspective in mind. This approach provides guidelines for the advancing the analysis process. They suggest that individual observations and field notes be reviewed and compared with one another to see what patterns and categories emerge. These categories should then be tested, refined and augmented by applying them to different data sets. Definitions and boundaries can then be specified more explicitly. At the point when saturation occurs and no more new categories emerge, an attempt at theory building is made, and at this point, the literature is surveyed for relevant support.

The grounded theoretic approach seems a somewhat unrealistic approach for graduate research in that it requires that the researcher wait to enter the literature until a rudimentary theory is already in hand. Practically speaking, by the time a graduate student reaches the point of collecting and analyzing data, she is already extensively grounded in a great deal of relevant research literature, and it is bound to influence what is attended to in viewing the raw data as it is collected.

I am compelled to acknowledge that my bias as a high school physics teacher and practitioner of the modeling method of instruction had a significant impact on the program of study and the program of research that I chose for my PhD preparation. My interest in why modeling instruction works led me to a careful survey of the cognitive science literature, particularly with respect to schema theory, inscription theory, cognitive linguistics, distributed cognition and motivation. The time I have spent teaching integrated physics and mathematics to ninth graders (and to high school teachers) influenced me to delve into Mathematics education research where I discovered Realistic Mathematics Education and Cognitively Guided Instruction. My reading and thinking in these areas have certainly influenced the way I watch and listen to students communicating about physics and mathematics.

Although I made an effort to enter the data analysis process open to whatever new insights it might reveal to me, this study is not grounded theoretic. My initial question about how student discourse shaped thinking and reasoning in small groups was

motivated by interests in distributed and embodied cognition, and by extensive reading in cognitive linguistics. I hypothesized that these theories offered plausible explanations for the some of the successes of modeling instruction that I felt merited investigation. Nonetheless, my data revealed to me a many things that, in my years as a classroom teacher, I had never before noticed.

Classroom research is fiendishly complex. My theoretical framework is a testament to this. As the classroom videotaped observations progressed, it became apparent to me that I needed to focus my question more finely in order to produce results that would be useful to physics teachers. Whiteboards and the conversations that surrounded them were an obvious focal point to select, as this practice is the single most discourse intensive element that modeling instruction employs.

In this research, I attempt to examine what is taking place cognitively as students work together in collaborative groups to negotiate and create whiteboards, and how the group process is shaped by the motivations and the engagement of individual group members, by the metaphors students use to structure their reasoning, and also by their use of the tools, artifacts, representations and inscriptions that are typically thought of as beyond the boundary of the cognitive unit. It is my intention to test the usefulness of the somewhat elaborate theoretical lens I bring to the research process, and identify elements of this framework that might be of use to teachers in improving their teaching practice.

The theory of distributed cognition suggests a framework for observing the interactions between people and artifacts, highlighting their mutual interdependencies. The classroom is the setting, and the organization of the distributed system (of individuals, tools and artifacts) is an emergent structure. As the observer, I attended to both the interactions between the participants, and their individual and collective use of language, artifacts and representational tools.

This study attempts to uncover the implicit rules by which the students in modeling physics classrooms engage in discourse and reason around shared inscriptions and provide teachers with a rubric for evaluating student thinking in the wild, as it unfolds in real time in their classrooms.

Data analysis has proceeded on two levels. An ongoing review of classroom videotaped observations and field notes occurred daily during videotaping in each classroom. After data collection was complete, a finer-grained analysis and coding of transcribed data and video segments was undertaken to identify reasoning strategies and tactics that characterized students' whiteboard discourse.

Since distributed cognition is an emergent process, few predetermined codes for data analysis were developed, but these emerged naturally in the course of the review process. A list of the codes that eventually evolved, and their definitions, appears in Appendix A.

The coding schemes that have structured this analysis are loosely based on Hutchins' Theory of Distributed Cognition (1995), Hestenes' Modeling Theory (2006), Jackendoff's Theory of Representational Modularity (1996), Lemke's views on the role of language in science education (1990) and Edelsky's analysis of floor (1981).

Particular attention was given to how students share meaning via representation, gesture, the use of metaphorical language (Lakoff & Johnson, 1980; Lakoff & Nunez,

2000) and the use of spatial representations in reasoning. The code structure that emerged from this analysis process was tested and revised in examining and coding interview transcripts and videoclips. This step suggested additional strategies and categories. Whenever possible, shifts in student reasoning have been correlated with particular situations or events.

The research process, then, was a cyclic one, with data gathering via classroom videotaping, followed by data analysis in an effort to elaborate the theoretical framework for use in the next phase of classroom observation and analysis.

The students

Each student group under study brings to the learning situation a variety of cognitive tools, a basic vocabulary, a catalog of metaphors, and some primitive 'commonsense' schemas or knowledge structures relating to the concepts to be studied, and of overriding significance, a unique set of goals and motivations. During the learning process, members of the cooperative group negotiate, to some degree, a shared schema for concepts under consideration. They learn to characterize this schema with a common set of metaphors, tools, representations and inscriptions (Roth & McGinn, 1998). At the end of the unit of study, although each member of a group possesses a unique version of the group's model that is more or less functional than the group schema for the purposes of solving novel problems, they also possess a common set of communication and reasoning tools that have evolved via white-boarded inscriptions and the discourse that accompanies it.

OBSERVATIONS

A tale of four classrooms

In the next two chapters, I will paint a picture of students and learning environments in four separate classrooms that on the surface seem quite different but upon closer inspection share many characteristics.

The data are images and transcripts of videotaped mathematics and science classes where students are routinely engaged in small group collaborative problem solving activities followed by whole class presentations and discussions of their thinking.

It is a diverse data set: a 7th and 8th grade mathematics resource class, a 9th grade physical science class, an 11th and 12th grade honors physics class, and a community college engineering physics class.

In each of these classes, teachers employ Modeling Instruction and students routinely share their work with peers on 24" x 32" whiteboards. It will become apparent, however, that these teachers' interpretations of Modeling Instruction differ widely from one another and these differences affect how students engage in the discourse and visual representing that is central to cognition. In this chapter, I will try to capture the flavor each of these four discourse communities with an extended vignette that describes a typical day in each classroom. In chapter five, I will examine the structure of the whiteboard-mediated discourse as it unfolds and highlight how distributed cognition happens. I will illustrate how inscriptions and, at times, the metaphors that underlie them can shape and sometimes limit students' communication and reasoning about space, time and interactions. I will explore how power relations, among students, and between teachers and students, affect the nature and extent of classroom discourse. And I will highlight what is optimal in each of these settings.

Middle school mathematics: teacher as scout leader

Ms. Barnard's class consists of a mix of 7th and 8th grade students, mostly boys, who come to this resource classroom for mathematics class twice weekly during 3rd period. McKinley Elementary School, a K-8 public school (4% white, 93% Hispanic, 55% ELL, 93% subsidized lunch) is located in a working class, primarily Hispanic urban neighborhood in a large city in the southwestern US. Ms. Barnard indicates that the students assigned to this class are low performing eighth graders and high performing seventh graders. The class episode described below takes place in early April.

As the class opens, students' are seated at round or rectangular tables of three to five students each, and there is a 24"x32" whiteboard, markers and an eraser cloth on each table. I have been a guest teacher in this classroom for the previous two weeks and have introduced students to the use of whiteboards for small group work to motivate more student engagement and participation. The students are encouraged to be each other's teachers and told that they will be learning to connect with mathematics in ways that are new to them. I have told the students that I am a high school physics teacher and member of the ROLE grant research team that is studying the development of students' understanding of fractions.

As class opens, students' attention is directed to a question on the board: "In your notebook write about the following: "You are taking a trip to Tucson and you need to get some data from the trip to use in your mathematics class. What kind of data could you collect and what would your data collection plan be?" At the beginning, many students have trouble focusing on the task. After I remind them that there is limited time to finish the assignment, students write quietly in their notebooks, looking up and down from the board for the first minute or two as they copy the question. There is some looking around the table at others' notebooks and some erasing, but many students appear to be making an effort to get something on paper. There are still those who spend more time writing down the question than answering it. I move from table to table glancing at what is written in the notebooks but not engaging the students in conversation.

After three and half minutes, I say, "Okay. Let's talk." I indicate that I have seen students doing good thinking on paper about the question and that even those who had just written the question were smart to do so because they could continue to think about it even after class was over. I remind them that they are being graded on their efforts with respect to answering questions like this, and that class time is at a premium so we cannot wait indefinitely for students to get something down on paper. A discussion ensues when I ask for their suggestions about data sets to collect on the trip. One boy says he would collect and measure rocks to determine their weight. A girl then suggests collecting the time it takes to get there and the distance traveled. Another boy says he would find the probability of seeing red cars. Another boy reads from his notebook that he would find out how many miles it takes to get there and how much gas it would take. After each of their suggestions, I offer some affirmation and repeat aloud an abbreviated version of what the student has said so that everyone can hear the contribution. A few students appear to be writing down the suggestions their classmates are offering but most are just listening. In general, students look at whichever of their classmates is speaking during this exchange. The next boy calls out "how much money you spend," and then I ask, "what are you going to be spending money on?" to which he replies, "Gas, food," and I add, "Maybe even soda." I call on another student who raised his hand but then put it back down. This particular student has exhibited trouble with the new collaborative learning environment and is often seen conversing with his neighbor, looking at unrelated picture books, and fiddling with items at his desk. He offers that he would measure how boring the trip was, and I ask him how he would measure that. After some hesitation on the students' part, I ask if he might put it on a one to ten scale of "boringness?" Another student at his table says "eleven", and he says yes, and then changes his mind and says "no—a hundred." Laughter spreads across the classroom. I ask if he would get a boringness reading from every person in the car to which he responds "yes", and then I ask if he would get a reading from them every five or 10 minutes and he responds "every 10 hours". More laughter. I ask the class, "how long does it take to drive to Tucson? There is a chorus of different responses from one to three hours, to which I reply, "yes, unless you walk—then it might take a couple days at least." Next, we discuss counting cars and whether to count them on "your side of the road, the other side of the road or both." Another girl suggests that we measure change in temperature every 5 minutes. The discussion eventually progresses to the kind of measuring tools they would need to

measure the things they want to measure. Maps, rulers, measuring tapes, thermometers are mentioned by different students. I ask if the car has any measuring tools built into it. Various individuals suggest that a car can measure miles, gas consumption, speed, time and radio stations (at which point a student suggests that they count the number of songs that are played during the trip to Tucson.) Next, they discuss what they could do with their data when they get back to mathematics class. Suggestions are to table it, graph it or chart it. At this point 10 minutes of class time has passed.

I then point out two categories that are written on the board: graphing and questioning, and indicate that I want this day's learning objectives to be about these things, and I add that the word 'questioning' refers to student questioning—Ms. Barnard chimes in with "effective student questioning". I tell them that after they see what they will be doing this day, they will revisit these words and then they can propose actual objectives within these categories.

Next is a demonstration in which a battery powered car travels across the floor at the front of the room. Students discuss what they notice about it and eventually identify what they can measure with the rulers and stopwatches available to them—what they can "mathematize." There is brief instruction about whose responsibility it is to record the data they will gather (they volunteer that it is everybody's responsibility), the necessity of making sense of whatever data they gather, and finally another attempt at fleshing out the objectives that are on the board. Students are divided into two groups and head outdoors with cars to take data about time and position. One group has marks for position intervals on the ground and they record how long it takes the car to reach each of the marks, and the other group makes position marks on the pavement every 2 seconds and then transfers them to strips of paper that they can take back into the classroom and measure with a ruler.

After about 10 minutes, they return to the classroom to measure the spaces between marks and begin preparing whiteboard presentations of their data in groups of three or four students each.

A number of students are unsure about how to read a tape measure but they try to help each other. Eventually I demonstrate how to hold and read a tape measure for two of the groups. The groups who used pre-measured distances with the times that they have recorded have an easier time getting started on their whiteboard presentations. In the groups using tape measures, the task of measuring eventually falls to a single student—the one who was the most confident in his or her ability to read the tape measure. Data tables and scatter plots are sketched, discussed and re-sketched. At one table of two girls and two boys, one girl insists upon using the entire board for their scatter plot and erases the table that one of the boys is trying to draw on the one side of the board. The boys move to another table and begin making their own whiteboard of the data they have gathered. The room buzzes with conversations in both English and Spanish, as students work to achieve representations they can all agree upon. Once the general shape of the graph is apparent to them, they begin to discuss what colors to use, they redraw axes with rulers, and they title the graph, label axes and indicate units. I periodically tell them how much time they have left, and students can be seen watching the clock as they try to get finished. As the class ends and boards are collected for presentation during the next class

period, students hurry to add the last few touches. After the bell rings one group asks if they can come in and finish during lunch.

The expectations of both teachers and students are different in this classroom from the high school class observed for this study, and peripheral participation in the form of passive waiting is clearly not a norm, though there were a few students who still failed to participate in both large and small group activity. Fifteen of the twenty-one students participated in the whole group discussion at the beginning of class and virtually everyone participated in the conversations surrounding data taking and whiteboard preparation. For the most part, attention was given to whomever was speaking during the whole group discussion, and students raised their hands, waited politely and took turns without being reminded to do so. The atmosphere was relaxed and congenial. During small group work, students were cooperative rather than competitive. They conversed freely in both English and Spanish without evincing any concern over the amount of noise in the classroom. Students appeared aware that they needed to manage their time as they engaged in various tasks, and in spite of the fact that I periodically told them how much time they had left to get finished, a number of students could be seen glancing regularly at the clock as they worked to finish what they were doing.

Although the enrollment in this class is predominantly male, the girls were active participants. In several of the small groups, they took charge of the white board and set the standards for its content and appearance. Most groups had one student who took the lead in the discussion and whiteboard preparation, but in general, students listened attentively to each other as whiteboard preparation proceeded. They accepted responsibility as a group for seeing that tasks were completed in a timely fashion and they exhibited some signs of pride in their work as they negotiated their whiteboard presentations.

Their participation seemed motivated by a desire to hear their ideas and suggestions repeated and affirmed by the teacher and also by a desire to make a unique contribution to the list of things that the class was considering. They behaved as though they believed that knowledge resided in both their peers and in the teachers, and, to some degree, in the tools and representations they were using, and they spent a fair amount of time helping each other make sense of the activity they were engaged in. The incentives for engagement in this class were affirmation by the teacher, i.e., having their contributions to the discussion repeated and praised or written on the board, working outdoors, using tools, i.e., cars, stopwatches, tape-measures, whiteboards and markers, and having some sense that they knew what they were doing, i.e., they knew what features they should include in a graph or table in order to be able to justify their claims about the car's change in position with respect to time. Students were also rewarded with "Whiteboard of the Week" in which the group of students who created the best whiteboard each week would receive a small prize (a bag of M&Ms). The disincentives to engagement in this class were difficult to identify. There was some frustration with their inability to read a tape measure but they asked for help and once they received it, they moved ahead with their task.

There was very little guessing going on in this classroom and there was no obvious concern about knowing answers or earning points, but this may be due in part to

the nature of the task in which they were engaged. Reasoning, sense-making, collaboration, and a colorful, complete, neat whiteboard were valued by both students and teachers.

Motivational characteristics in this class differ from the other classes observed. Though the experimental class only spanned 5 weeks, each class was structured in a different way. Students were held individually and collectively accountable for work and were rewarded for creativity and participation. Students appeared to understand both Ms. Barnard's and my expectations. While most of the students adapted quickly to this new teaching style, a few were reluctant to participate. The transition, from a passive student role to an active role is not easy, and for some proved awkward. In general, however, for students in this class, work was a creative activity and class episodes bore more of a resemblance to extracurricular activities than to traditional mathematics classes. I had no sense that they were playing "the school game". By-and-large, students' goals appeared to be mastery rather than performance goals.

9th grade physical science: teacher as stern but kindly parent

Mr. Mendoza teaches at Carlos Cadena High School, a large public high school situated in a suburban part of a large city in the southwestern US. Like Ms. Barnard's, his students are largely Hispanic with middle or working class parents (8% white, 69% Hispanic, 13% ELL, 55%-subsidized lunch). Mr. Mendoza is an experienced teacher who has done post-graduate work in physics at a major research university.

Girls arrive in the classroom first; a few appear to be strutting for the benefit of the boys who slouch into the room behind them. The students settle themselves at black-topped lab tables, two to a table. Only one of the tables is co-ed. The rest are either all male or all female. There are a few more boys than there are girls in this class of 24 students. Physical science is a required course for ninth graders. The class episode described below takes place in early February.

The following statement is written on the board at the front of the room: "Draw a motion map for the car from the moment it is placed on the ground and released." A few seconds after the bell rings, Mr. Mendoza calls students' attention to a friction-powered toy car he is holding. He revs it up by dragging it across the table in front of him, and then he lets it go. It runs quickly across the table and over the edge, hits the floor and keeps going until it hits the leg of an adjacent table. He picks it up, asks one of the students to catch it before it goes over the far edge of the table and then lets the car go again. As the car travels quickly across the table, students crane their necks and stand up to see. A girl seated at the table catches the car before it reaches the edge, turns it around and lets it go back across the table to the teacher. Mr. Mendoza says, "Not that way—just one way," catches the car, reprises its one trip across the table and then asks the class, "Did everybody see get to see that?" A few students say they missed it so he moves to another table and says, "Okay, I'll try it over here." This time everyone watches. "Did everybody get a chance to see it this time?" A chorus of student voices answers "yes!" "Go ahead and sketch the motion map please...and I want you to write the description."

Within seconds, the sound of an electric pencil sharpener can be heard and conversations begin at many of the tables. Mr. Mendoza answers a few students'

questions as he moves toward his desk, and as they settle into the task, he seats himself at his desk and begins to take roll, entering it on his computer. The students work silently for the most part. A few are looking around but most are attempting to answer the question in their notebook. There are occasional roving eyes, brief whispered conversations and a couple more pencil sharpenings, but for the most part, they are focused on their notebooks. The teacher finishes taking roll and wanders the room looking at what students are writing. An alarm beeps after the students have been working for 5 minutes and the teacher immediately begins questioning the students. “Javier, what do you notice that the car does right at the moment that I put the car down...or right after?” “It goes fast.” Mr. Mendoza asks him for clarification about what he means and Javier elaborates a bit about the fact that car’s wheels are already moving fast when it is placed on the table. Mr. Mendoza then asks him how he should draw the first few dots of a motion map to represent that motion. Javier indicates they should be spaced apart evenly. Mendoza draws four evenly spaced dots in a row on the board and asks if anyone else has any different ideas about the car’s initial speed. Inez says she disagrees with Javier because the car changes direction as it crosses the table and then goes over the edge. The teacher says he wants her to think just about the motion of the car as it travels in a straight line across the table, and he revs up the car and lets it go across the table in front of her. She watches it, catches it when it reaches the edge, and says that she thinks that the car will eventually slow down. He allows that this is a possibility and asks if anybody else has other ideas. Randy raises his hand and say he thinks that the car is speeding up at first when it is placed on the table. Mr. Mendoza asks him how he would space the dots on his motion map to represent this and Randy tells him to place each dot a little further from the one before it. After Mr. Mendoza draws this row of dots on the board, he asks the class whether they agree with Randy or with Javier. Gino asks to see the car again and Mendoza comes to his table and lets the car run across it in front of him. Gino concludes that it speeds up. Two other tables of boys request that the car be run on their table. He complies. They conclude that the car moves at a constant speed. “Alright. Anybody else have any other suggestions there?” A few heads shake.

Next, he asks them to consider a longer trip for the friction-powered car. Would a motion map continue to look like one of these on the board? Most students think not. He revs up the car and lets it run about $\frac{2}{3}$ of the length of the classroom. It is clearly traveling more slowly than it was at the start by the time it reaches the wall. He fills in dots for the rest of the motion map and then says that if these dots represent the positions how would he represent the velocity, and Rosie and an unidentified male voice from off camera tell him to draw arrows of equal length at first, getting shorter toward the end. Then they ask him which map would be correct, and Mendoza tells them that the correct one is the one that represents what they have observed. He says that he hates to pick sides in a situation like this but he thinks he agrees with Randy—he sees it speed up a little when he first puts the car down but he’s not sure and tells the students that they can experiment a little with the car themselves if they want in a few minutes. “So initially the car accelerates to the right...?” Inez: “Yeah and then it slows down.” He agrees.

He then refers to a worksheet assignment given to them at the previous class. He says that he wants the class to whiteboard five of these problems. He will give students about seven minutes to prepare their boards; they will discuss them as a class and then move on. He previews for them briefly what they will be doing for the rest of the week: a lab tomorrow, Wednesday is a half day, and then there will be a “quiz-slash-test” on Friday. “So make sure you don’t miss any time between now and then.” He assigns students to groups of three or four, gives each group one of the problems, and then starts his timer.

Students grouped together move to sit around a single table, and a number of students walk to the front of the room to retrieve whiteboards and markers. The conversational volume rises. Mr. Mendoza intones “Okay. Eight minutes.” He erases the board and then begins to walk from table to table watching and asking and answering occasional questions and encouraging students to show their work.

The group of three boys nearest the videocamera, Juan, Jorge and Kevin, is working on question number four. Jorge gets a meter stick from the back of the classroom and he holds it steady as Juan begins making a carefully spaced row of dots on their whiteboard. Then after some deliberation, Juan writes on the board $\frac{4 \text{ m}}{1 \text{ s}}$. He considers what he has written for a few seconds. His partners do not comment. Kevin, a long black haired white boy garbed in black and sporting multiple piercings in his face watches silently but does not join in the conversation. Juan caps his marker. I am operating the video camera that records this scene. After a moment I ask him, “how do you know?” and he replies that he measured the distance from one dot to the next and in every case it was four centimeters and since there is a dot for each second this means that the object was going four centimeters per second. There is brief discussion about the fact that the fraction bar means per and then I note that the problem asks about average velocity. “Is there any difference between average velocity and the velocity at one of these points?” For the next few minutes, we talk about how they might find an average velocity for a trip to Los Angeles.

Mr. Mendoza stops by and asks if we are ready to begin and we say yes. He calls on Juan’s group to go first and he takes a seat at the back of the classroom to listen. The three boys bring their whiteboard to the front of the classroom. Mr. Mendoza tells the class that they might want to pay attention if they struggled with these questions on their homework and then tells them to go ahead. Kevin stands in the middle holding up the board and looking down over the top of it. The boys each look at one another for a few seconds to determine who will speak. Juan says softly, “Go ahead Jorge.” Jorge shakes his head ever so slightly and looks away, squaring his shoulders as he stands in front of the class and staring at a spot on the back wall. Juan smiles, points at the diagram and says that each dot represents a second and from one dot to the next dot is four centimeters and the distance between each of the dots is the same—four centimeters. So, the object must be traveling four centimeters per second. He finishes up with, “that’s all we got.”

An unidentified girl off camera asks what they are finding and Juan responds at first “average acceleration” but the quickly corrects himself saying, “average velocity”.

Mr. Mendoza asks, “Is it accelerating?” to which Juan quickly responds, “no.” Mendoza follows up, asking, “How do you know it’s not accelerating?” Again Juan responds, pointing to the dots and saying that each dot is four centimeters from the last and they are each one second apart so the speed must be the same all the way along. The teacher directs the next question to Jorge, asking him where the object would be ten seconds later if it was traveling at four centimeters per second. There is a long pause and Jorge finally says he does not know, nor does he really understand this stuff. A grin spreads slowly across his face. The teacher asks him what four centimeters per second means and he responds promptly, “I don’t know.” From the back of the room, Maria calls out “That means it’s going four centimeters per second, so if it goes for ten seconds all you’ve got to do is multiply that.” Jorge responds, “Yeah, well I knew that.” to which Maria fires back, “well then, why didn’t you just say it?” Mr. Mendoza talks about knowing what the expression *four centimeters per second* really means. He defines “per” as “for ever one of something” so four centimeters per second means four centimeters for every one second. Then he offers a money analogy: four dollars per hour. “Hopefully that’s not the case but if you were earning four dollars per hour and you worked for ten hours you’d be able to figure out how much you ought to be paid, right?” Jorge says “yeah.” Mendoza concludes, “It works the same way.”

He thanks the boys and then calls on the next group: Inez, Paula and Robert. Inez reads the problem and then begins explaining the information from the problem that they have written on the board. Maria asks Inez what the time is (time is the unknown the problem asks them to solve for, but this information was not on their board) and Inez responds “four centimeters per second.”

Mr. Mendoza intercedes at this point and says that they have v on their board but its value is not filled in. He asks them what the value of v should be. Inez responds haltingly that it is four centimeters per second. Mendoza prompts, “Robert can you write that up there please?” Robert picks up a marker in fills the information into the space on the board.

Maria redirects them to her earlier question:” so what was the time for 500 meters?” There is a hesitation while all three students look at their board uncertainly. Finally, Mr. Mendoza suggests, “How about the equation? Can you tell us the equation that would give us the time, given this information about position and velocity?” They look at the board and murmur among themselves but cannot answer his question. Finally, Mr. Mendoza asks the other group that did that problem to bring their board up to the front of the room. The four of them, two boys and two girls, make their way to the front of the room and Francisco hangs their whiteboard on the wall next to Inez’ group’s board. At Mr. Mendoza’s request, Anna starts to explain what they have written on their board. She explains where each piece of data comes from, how it fits into the computation and then gives the answer: 125 seconds. Mendoza notices that they have written their equation at the bottom of their board and asks Francisco to point to it. Francisco points to a long division problem. Mendoza points out that this is just a computation and prompts him again to point to the equation....with the equals sign. At first his finger hovers uncertainly over “1 m = 100 cm” but Maria calls from her seat at the back of the room,

“down...look down” and he eventually notices $t = \frac{d}{v}$ which is written on the bottom left hand corner of the board. After a few seconds, the teacher asks the class if there are any questions. They look at the boards and a few write in their notebooks but no one asks a question. Mendoza comments that the combination of the two boards makes a good solution for this problem. One board lays out the data and the other give the formula and the computations necessary to solve the problem. He asks again if there are any questions for these people and then thanks the two groups and they shuffle back to their seats.

A pair of boys carries their board to the front next. The shorter of the two, Gil begins describing the graph of position versus time they have drawn on their board. He says it shows that the object is slowing down because the curve is gradually flattening out to a horizontal line. There is a long pause and then the teacher asks if anyone has any questions. No one speaks. He asks again, and then says that on the quiz, he will be asking them to tell what is going on in graphs and this would be a good opportunity to ask if they are not sure or if they disagree with what Gil has said. He waits another 10 seconds and then asks the other boy, Luis, how the graph shows that the object is slowing down. Luis directs his attention to a series of dots on the bottom of the board Gil had not mentioned in his explanation. He said that the object was accelerating at first but then the dots were getting closer together and this meant that the object was coming to a stop and then the dots ended meaning that the object has stopped and this corresponded to the graph flattening out. Mendoza says he is glad that they put the motion map dots there. He repeats Luis' claim that the object was accelerating at first and then asked if it stopped accelerating. Luis says that it is accelerating the whole time. He points to the graph and says that initially it is going fast but then as time goes on it moves slower and slower. Mendoza asks what it is about the graph that shows this. Luis responds that the line is going diagonal and then further along it gets more horizontal. Maria wants to know if the graph should start at the top of the position axis and go down if it is supposed to represent that the object is slowing. Mr. Mendoza asks her if that is how she thinks the graph should be drawn and she says, “no, I'm just asking you if that's right? You know if you start out going fast...” (she traces a graph from left to right in the air, her hand descending as it describes a parabolic path). Mr. Mendoza asks her what that is a graph of as he points to Luis and Gil's graph. She replies “position.” He asks her, “why would you want to start at the top?” She hesitates. He asks Luis, “Where is the object starting at?” Luis points to the bottom left corner of the graph and responds, “the origin.” Mr. Mendoza adds. “It's moving quickly and then as you move away you're moving at a slower rate.” Maria nods her head in agreement. He asks if anyone else has questions and then compliments Luis and Gil on the good job they did and allows them sit down.

The next group of three girls and one boy bring their whiteboard to the front and hang it from one of the hooks above board. They mill around and look at each other, and Mr. Mendoza asks Linda to describe what is on the board. She shrugs and rolls her eyes. After a couple more questions from the teacher, her groupmate, Felicia whispers to her softly and Linda haltingly begins to repeat what Felicia is telling her, describing the

motion map that is drawn on their board. Mendoza presses each member of the group in turn to describe and explain what the various details of the motion map mean. Joey answers a bit uncertainly. Felicia is surer of herself, answering the teacher's questions of her patiently and at length. Celine answers readily also but requires interpretation of several of the teacher's questions, which Felicia supplies. When Mr. Mendoza is done, questioning there are no further questions from their classmates and they return to their seats.

The last to present is a group of three: Freddie, Rosa, and Gabe. They also have a motion map on their board which Freddie immediately begins explaining. As he finishes, Maria calls out a question from the back of the room. "I thought you said it was moving to the left." The arrows on the board were pointing to the right. The teacher clarified that it was their left, her right. There are no further questions and he allows them to take their seats.

Mr. Mendoza walks to the front of the room as students are erasing their whiteboards and talks to them for about a minute about making sure when they are working together to make a whiteboard that every member of the group contributes, even if it is just to ask their teammates questions about what it means. He says that it is the duty of everyone to have an idea of what is being written on the board even if they do not understand everything perfectly, and it is the job of the person who has taken the leadership role to make sure the other members of their group are engaged. There should never be a time that someone has to answer, "I just don't get this stuff," when they are presenting a whiteboard to the class.

As the class is packing up their books in anticipation of the bell, he hands out a worksheet that he describes, contrasting it with the assignment of the day before. He goes over the directions with them emphasizing words like "complete," giving an example of what a complete answer would consist of. In the couple of minutes remaining Jose, Jorge and Kevin begin working together on the homework problems.

The conversation level in the classroom begins to rise as students put their notebooks in their bags and a few seconds before the bell rings students begin to pick up their chairs and put them on the tables.

Teacher and student expectations in this classroom were a bit more consistent with those brought along by the 'culture of schooling' that most public students learn to navigate in elementary school, but there was a definite sense of kinship in the interactions between teacher and students. The social norms of student interaction in this classroom were similar to those in the middle school mathematics class although Spanish was not heard as often among students doing small group work. There were a few more disengaged students than there were in the middle school class but this may be due to the fact that this class was in session with the same teacher for four months prior to my arrival, whereas the middle school class was experiencing a new teacher (me) and a new tool (whiteboards), which may have temporarily piqued their curiosity. In general, most of the CCHS students appeared engaged in the lesson. Students enjoyed the demonstration with the toy car and asked for it to be repeated a number of times.

The atmosphere in this classroom was familial. Students seemed aware that they would be held accountable for their words and actions (or inactions). They appeared to

feel comfortable saying what they thought to each other and to the teacher, even to the extent of challenging the teacher's assertions (about physics) if it conflicted with their own commonsense concepts. There was no evidence that they were afraid of 'looking stupid' to one another or to the teacher. They behaved as though knowledge resided in their peers as well as their teacher, and, to some extent, in the tools and representations they used. Girls and boys, alike, were outspoken when they felt their classmates had made an error. There was an occasional boy who would put on a tough guy demeanor, challenging the teacher with a flip remark or shrugging off a question, but the teacher would warn or rebuke him, calmly remind him of the rules, and then smoothly resume his conversation with the class.

Students appeared to be motivated by a desire to live up to the teacher's expectations of them, and some by a genuine intellectual curiosity and desire to learn about the phenomenon under investigation. There were also social motivations in play, i.e., girls wanted to work with certain boys, boys wanted to lead or cop an attitude or entertain their classmates. The incentives in this class were a bit more subtle than they were in the middle school class. Students clearly enjoyed using laboratory tools and equipment, and those who spoke up (about half the students were regular contributors to whole class discussions) enjoyed having their answers discussed and validated by the teacher. Not once in the six weeks I observed this class did I hear a student ask about how many points something was worth or how an assignment would be graded. The teacher mentioned grading a few times, but usually in the context of when an assignment was due.

There was not much guessing going on in this classroom when students were preparing and presenting whiteboards, probably because there was no special emphasis given to "knowing the answer". The students who were engaged appeared most interested in being able to explain the reasoning behind what was written on their board. However, there was very little effort invested by students who took the lead in whiteboard preparation in making sure that their disengaged group-mates could make sense of the whiteboarded information. The teacher often put these disengaged students on the spot by directing questions to them in the whole-group discussion, and when this happened, their more engaged groupmates often rescued them with whispered cues and gestures.

There were two apparent cultural models at work in this classroom: the culture of schooling that the students brought along with them, and the culture of family that was brought about by the teacher's interactions with his students. For some students, class work was a creative activity. For others it was a chore. Work ethic varied widely across students but there was no sense that they were playing a game (i.e., "the school game"). Their goals with respect to the assigned tasks appeared to be mastery goals rather than performance goals.

Honors Physics – teacher as coach

Darnell High School is a large 10th through 12th grade high school that serves a suburban middle to upper middle class population (66% white, 22% Hispanic, 5% ELL, 23% subsidized lunch) in a large city in the southwestern United States. The students

enrolled in Mr. McEvoy's class are a select group of college bound students. Of the dozen or so students who chatted with me outside of class, all stated that they were enrolled in other advanced, honors or AP classes. All had college plans. None were planning to major in physics, but two indicated that they would major in engineering, two in pre-medicine, one in pre-dental, one in veterinary medicine, one in marine biology, two in pre-law and two in theater arts.

Mr. McEvoy is an experienced and respected high school physics teacher with a master's degree in physics teaching. He regularly teaches master's level professional development courses for teachers for the local state university.

One Friday morning late in September, as it invariably does, class opens with a polite good morning from the teacher. Students are seated at individual student desks arranged in neat rows in the front of a large classroom that has eight computer-equipped lab tables in the rear half of the room. Mr. McEvoy follows up his greeting with a request for last minute questions before they take their quiz. One off-camera male voice quips: "the answers." He rephrases his question: "Any *specific* questions before we begin our quiz?" A few hands go up and various female students ask whether or not they will need a calculator, if the equations on the board are the only ones they will need, and which equation is best to use when solving for "*a*". Then Hannah asks if he would give an example from real life of when she would need to use the equation for final velocity squared. McEvoy responds, "If you had the starting and ending velocity and the displacement, and you needed to find the acceleration." He waits a few seconds more and then says, "Okay, here we go," as he passes out the quiz. He tells them that when they are done they can pick up a copy of worksheet number 5 from the front desk and begin working on it while the rest of the class finishes up. The students work in silence for the remainder of the period.

The following Monday morning after Mr. McEvoy's usual greeting, he sends them immediately to the lab tables in back of the classroom to whiteboard the Worksheet 5 problems they did for homework. Jimmy, Hannah, Gui and Zane gather around their usual table and chat quietly while they wait for the teacher to tell them which question they will be whiteboarding. McEvoy stops by their table briefly and says, "number 6" and then moves on and Jimmy, Hannah and Zane turn their papers over and, almost in unison, chorus "eight point eight." Gui, a slight Asian girl who is not a native English speaker points to the answer she has on the worksheet in front of her, which is different, and Zane says, "well you probably did it wrong. Let's work it out."

Jimmy suggests they start by writing the equation while Zane picks up the whiteboard marker. He centers $\Delta x = \frac{1}{2}at^2$ in large characters near the top of the board and then writes date from the problem on the left side of the board while Hannah writes the date and each group member's name in the top right corner. Mr. McEvoy's voice can be heard in the background announcing that they have five minutes to finish putting their boards together, "starting...now!" He presses the start button on his watch timer. As the others look on, Zane then rewrites the equation with data plugged in for *x* and *t* and goes on to solve for *a*. Jimmy and Gui want him to write the rest of the equation and Zane

finally adds “ $+v_0t$ ” even though he maintains it’s not necessary in this instance. When he arrives at an answer of eight point eight meters per second squared he exclaims “boo-ya!” and then watches as Hannah traces over his writing to make it darker and more legible. There is still almost three minutes left before time will be called. Gui, who is standing opposite Zane and Hannah, studies Zane’s solution (which is upside down from her perspective) silently as Jimmy, Hannah and Zane talk about their and their parents’ handwriting. Finally, Mr. McEvoy’s watch alarm beeps and students are asked to return to their seats in the front of the room. A member of each group hangs each board from hooks across the side and front of the classroom.

“Alright. Let’s get going on number one.” Three boys and one girl make their way to their board and after a brief hesitation Dick begins by reading the question. “Alright, um, the golf ball rolls up the hill toward a miniature golf hole. It starts with an initial velocity of two meters per second and it accelerates at a rate of negative point five meters per second. A was what is its velocity after two seconds? And we just used the equation $v_f = at + v_0$ and we got velocity equals 1 meter per second after two seconds. And then after six seconds we used the same equation. We got negative one meter per second. And, uh, for the motion we said that the golf ball moves in a positive direction up a hill with a constant acceleration of negative five meters per second squared for four seconds. It then moves in the negative direction down the hill with a constant acceleration of negative five meters per second squared.”

Mr. McEvoy asks them to tell the class just one more thing about the ball’s acceleration. Lili answers that the velocity was getting closer to zero. McEvoy: “on the way up.” Lili: “On the way up.” McEvoy: “What about on the way down?” Lili: “It was getting farther and farther from zero.” The teacher asks, “Everybody okay on number one?” Anna asks why the acceleration was negative. Lili responds that it was constant. Mr. McEvoy guides her with a couple more questions to the conclusion that the velocity was increasingly negative as the ball rolled backward and then he adds that if it were rolling backward it would be speeding up. “Great question. Let’s give these guys a hand and go on to number two.

The next group, two girls, walks to the front of the room while Dick, Lili and the other two boys in group one return to their seats. The girls are soft spoken and shy. They take turns, first Jen and then Bonnie, quietly reading the two parts of the problem and then explaining how they plugged the information given into the equation and solved algebraically. There is an extended period of silence after they finish and they look to Mr. McEvoy for permission to sit down. Finally, he says that he got a different answer than they did. Jen keeps her eyes lowered while Bonnie looks at their board and then at her paper and says that she accidentally copied down the wrong answer from the problem above. Mr. McEvoy tells them that everything is correct except for the answer, which Bonnie quickly corrects, and the class claps politely as they make their way to their seats.

Three boys come up next. Two stand silently by while Nate quickly says, “For number four we used final velocity equals acceleration times time value plus initial velocity. Then we plugged in the given numbers and got nine point oh five meters per second. Anybody have any questions?” The class waits silently. Mr. McEvoy is seated behind his desk at the front of the room and he appears to be looking at something on his

computer screen. After about 10 seconds of silence he says, “No questions? Alright, let’s give these guys a hand.” The boys return to their seats.

Next up are Sean, Olivia and Mark. Olivia reads the questions and then identifies the givens and the unknowns, pointing to each on their whiteboard. Sean then gestures to the equation, and says, “It’s plug and chug.” Olivia reads the answer: 1350 meters. Sean asks, “Questions?” After about four seconds, the class begins to clap and the three head for their seats.

Jimmy, Hannah, Zane and Gui walk to the front of the room next and Jimmy reads question number 6. Hannah then gives a detailed description of the plugging in of information and algebraic manipulations—multiplying, squaring, dividing, etc—and finishes with their answer, “8.8.” Jimmy asks, “questions?” The class looks on silently. After about 4 seconds, Jimmy snaps his fingers, turns and walks to his seat with the rest of his group. As if on cue, the class begins to clap and the next group, three girls, comes forward. Mr. McEvoy breaks his silence and asks if anyone got the answer 4.4 instead.

Nate volunteers that he did but that he used a different equation: $\Delta v = \frac{\Delta x}{\Delta t}$. Mr. McEvoy rises from his desk and goes to the board. The three girls that were standing by to present their board return to their seats. He writes $v_f = v_i + at$ and underneath it $\Delta v = \frac{\Delta x}{\Delta t}$ and

then says, “Like that? Where did we develop that equation? Which lab? What’s that? The battery powered car lab. Is the motion being described in this problem consistent with the motion in the battery powered car lab? (waits for an answer but none is forthcoming) Not even close. So is it appropriate to use an equation from the battery powered car lab? The second problem is, this isn’t the change in velocity (he changes the formula he has written on the board—crosses out the delta in front of $v = \frac{\Delta x}{\Delta t}$ and writes the word average above the v)...it’s average velocity...watch out for that. That’s probably the most common mistake people make on these kinds of problems. They get to one—they’re in a rush—they see a distance, they see a time, they want to know acceleration. They take distance divided by time and get a velocity and then take that velocity divided by time and get acceleration. Does that work here? Because the velocity you’re finding is an average velocity, not a final velocity. Good job on thinking about that. Are you ready for number 7?”

Brenda, Anna and Carla come back up and stand near their board. Brenda describes the problem situation pointing out various pieces of information on their board, and then she indicates the equation they chose and the answer that they ended up with for part A: “you end up getting an acceleration of eleven meters per second squared.”

Anna continued, described part B of the problem, the equation they used and the answer it produced: “a displacement of negative twenty-two meters.” Then she asks “the usual”, “Any questions?” Silence reigns.

As Mr. McEvoy picks up his paper and walks toward the back of the room he says, “Really!” with obvious skepticism. After about 5 seconds, Hank raises his hand and softly asks an inaudible question. Brenda responds promptly, “Yes.” Mr. McEvoy

reminds them that the initial velocity is negative and Brenda replies that that's why she drew the velocity time graph, and pointed out that the area under the curve was below the t -axis so the displacement was negative. If it had been above it would have been positive. McEvoy asks her what the area of the triangle would be to which she replies, "negative twenty-two." He adds that this was an easy way to check your answer. He tells the class to give them a hand and they return to their seats.

The final group, Hank, Lindsey and Mike, goes to the front and Lindsey reads problem number 8. Mike then explains part A, another case of plug and chug. They got nine point eight meters per second for their final velocity. Then Hank recaps what part B is asking and says that the problem can be done in two ways. He indicates that they chose to do it algebraically and describes the process of substitution and computation to arrive at an answer of 88.8 meters. Once again, after a brief silence, students begin clapping and the group walks back to their seats.

Mr. McEvoy returns to the front of the room saying, "That went a lot quicker than I expected." Then after a brief pause, "One of the days I was gone to jury duty last week your assignment was to complete four problems. How many people got those problems?" Silence. "How many got them?" Three students raise their hands rather tentatively. He tells them to go work on them for the rest of the hour in their groups and if they get stuck to yell and he will help them out.

The students gather their notebooks and textbooks and head for the tables at the back of the room. Jimmy: "Did you get 62?" Hannah: "I didn't get 62c or 64. I didn't even get around to 64." Gui says she was absent and did not get the assignment so Hannah tells her which four problems were assigned from the textbook. Zane cannot find his paper. They work silently for a while, glancing at the textbook and then writing in their notebooks. Gui asks Jimmy to explain what one of the problems means. He explains to her what the problem is asking for and then quickly outlines the basic steps she needs to take to find the answer. Then he returns to his work.

Gui looks back and forth from the book to her paper and twirls her pen, evidently considering what he has told her but she does not write. Jimmy walks to another table to discuss a problem with Brenda. Zane is stepping Hannah through the procedures to solve problem 62. Gui waits until they are no longer talking and then asks Zane to explain the first problem to her. He tries to explain to her how to solve the problem graphically, guiding her with questions and asking her what she would do and why she would do that, and then correcting her when she makes a mistake. Finally, she seems to understand and starts to write in her notebook. Zane returns his attention to Hannah who is stuck again.

After he finishes reorienting Hannah, Gui asks him another question and he asks if she found out what he had told her to find earlier. She says she does not know the formula and he responds that there is no formula for this one. "You just have to think about it." She insists that there is missing information but he tells her that it does not matter. He tells her to convert the velocity to meters per second. She is unable to do this so after a quick check of how he had accomplished this in his notes, he leads her through it and she arrives at 20 meters per second. Then he asks her how many seconds it would take for the train to come to a stop if it was decelerating at 1 meter per second per second. She replies promptly, "20 seconds." He tells her that the best way to think about this

problem is to make a graph, which he sketches quickly. He continues to step her slowly and painstakingly through the procedures necessary to solve the problem, which she struggles to understand. Finally, they get to the answer and when she asks another question, Zane answers, “That’s just what you have to do to get the answer.”

Jimmy and Hannah are working on a different problem and when Hannah compares her procedure to Jimmy’s, Zane’s attention shifts back to her paper. When Jimmy and Hannah finish comparing notes, Zane asks, “Jimmy, what’d you get?” just as the bell rings. Jimmy’s answer is inaudible as they close their books and prepare to go to their next class.

There was a joint expectation on the part of both students and teacher that serious focused attention should be given to the activity in progress. Students in this class were courteous and deferential to the teacher and to each other. The atmosphere was businesslike and the general demeanor was professional. These were students who appeared accustomed to academic success. They had a good attitude, a good work ethic and were confident in their social interactions with one another in small groups. There was a sufficient level of trust in small groups that they did not appear to fear “looking stupid” if they asked questions about things they did not understand. This confidence did not seem to extend beyond small group collaborative interactions for most students, however, as only a few students asked or answered questions during whole group discussions and whiteboard sharing sessions. Small groups had obvious leaders—sometimes this leader was a girl but more often it was a boy. The role of leader appeared to fall to the person who understood the concept under study the best.

These students were influenced by the culture of schooling that they brought along with them into this class, to the extent that their primary motivations seemed to be a desire to obtain answers and earn points. Although the teacher did not explicitly emphasize grades and points, a great deal of class time was given to discussions of how to earn more points and get right answers. Unlike the 9th grade physical science class, quiet students were rarely put on the spot during whiteboard presentations in front of the group, nor were they singled out to answer questions in class discussions. There was no penalty for silence or non-participation and if the teacher asked a question and no one answered, he would rephrase and simplify it until the answer was obvious, and if this did not work, he would eventually answer it himself.

Guessing was common among these students during whole group discussions as well as teacher questioning following whiteboarding. Frequently the teacher was trying to elicit a particular fact with his questions and the students’ responses took the form of “target practice”. Students took turns guessing what the teacher was looking for until they hit the mark. Unlike in the middle school and 9th grade classes, this teacher rarely critiqued the group’s performance following a whole group whiteboard meeting. The vast majority of student generated questions and answers (~90%) during whole group activities came from just six members of this 23-student class, and although the rest appeared attentive, they rarely spoke.

Students’ predominant approach to problem solving was procedural and usually algebraic. When they were stuck, they resorted to looking for additive or multiplicative patterns. Units were seldom used as clues to understanding how quantities related to

each other, or in discussing solutions with one another. When presenting their solutions to the class, answers were often verbalized as numbers only. The units, although usually written on the whiteboard, were not mentioned. Sometimes the teacher made a point to mention units. Other times he did not. Occasionally he also omitted any mention of units when discussing a problem.

The teacher's expressed expectations of students centered on their use of language — the words they chose to describe motion, interaction, correlation, proportionality, graphical representation, etc. Emphasis was also placed on procedures used in graphical interpretation, i.e., linearization of data, the 10% rule, and so on. This carried with it its own set of “magic words” that the students were expected to use appropriately when called upon to explain something for the class. The students evinced an interest in public language use as well—often their use of language when presenting their whiteboard to class was markedly different from the language they used when preparing their whiteboard in small groups. In this class, unlike the other three I observed, the student who had the best command of Big D Discourse was generally the spokesperson for the group in whiteboard presentations, whether or not they had taken the lead in preparing the whiteboard.

It appeared that performance was important in this class, particularly in whole group activities. Students wanted to “look good” to the teacher and to a lesser but significant extent, to the group as a whole. When presenting their whiteboards, students spoke to the teacher. They made an obvious effort to do and say things the way he wanted them, perhaps because he was perceived as the ultimate arbiter of right and wrong, perhaps because he had the final say over their grade. Although students behaved as if knowledge resided in their peers as well as the teacher, and also in the tools, artifacts and representations that they used, they measured their understanding of concepts by whether or not their answers matched the ones the teacher posted in the back of the classroom. Students demonstrated both performance and mastery goals in small group work but the main goals on display in whole group discussions were performance goals.

In addition to the culture of schooling that these students brought along with them to this classroom, evidenced by their self-discipline and their concern for grades, points, answers and good performance in whole group situations, the interactions among students and teacher appeared to bring about a culture similar to that of team sports. The teacher, as coach, worked hard to help the students learn and practice good form, and expected them to work together to perform their best, while the students played to each others strengths in order to turn in a good performance.

Community College Physics – teacher as general contractor

Echo Mountain Community College is a medium sized (~6000 students) 2 year college on the outskirts of a large city in the southwestern US. Less than 1/4 of those enrolled there attend classes full time. Of these full time students, 60% are women, 31% are Hispanic and 47% are white. The class described below is a 3rd semester calculus based physics class on electricity and magnetism. The course enrollment at the beginning of the semester was 15 students but only 13 attended regularly. Most of the students in this class took their first two semesters of physics from this same teacher.

Professor Dave Donnelly is in his early 30s. He earned his PhD in Physics Education Research at a large state university. He is a respected educator and has served as president of the state section of the American Association of Physics Teachers (AAPT).

The episode described below takes place about a third of the way through the course in early October. The classroom is a large square room with lab tables that comfortably seat four students each, jutting out from three walls at the sides and back of the room. There is a large open area of about 20 feet by 20 feet between these tables. A small table at the front of the room holds a few papers and the teacher's laptop computer and there is a large whiteboard mounted on the wall behind this table. Students enter the room one or two at a time, retrieve whiteboards and begin working together in their small groups. As they finish and wait for the class discussion to begin on this Monday morning, they sit or stand at their lab tables looking at the whiteboards they have just prepared showing a homework problem they worked on over the weekend, that asks them to describe the electric field on a cube or cylinder embedded in an infinite plane of charge density σ .

The teacher, clad in shorts and sneakers and looking a great deal like his students, strolls around the room glancing at the boards that students have on the tables in front of them. He asks John, a 15-year-old white male and the youngest member of the class, if he would like to go first. John expresses reluctance. As Donnelly completes his circuit of the classroom he intones, "Alright ladies and gentlemen, bring yourselves on in." In an aside, as students scoot their chairs into a rough circle in the center of the classroom he tells John that he doesn't have to be first as long as he makes sure to bring up what is on his board during the discussion. He chides the students for making such a sloppy circle that it leaves one whole group out, and they move their chairs back so that everyone is accommodated. When everyone is settled, he gestures to a girl with long blond hair, Kiki, saying, "Thank you. Nice and loud." She looks down at her board and then begins, first talking about her diagram. "We chose a cube, because that was an electrical field that can be...like...equal...so because it's constant...it comes out at a right angle...we didn't pick a cylinder because I like straight lines...so...(she begins pointing out certain features on her diagram, a picture of a cube embedded in a plane with sides perpendicular to the plane)...we made the cube with sides of the length of l and all the four sides, the electrical field lines are perpendicular to the direction of the surface so the four sides don't really matter much." She goes on to describe how they constructed their equation: "Since the electrical field lines are constant we took that out of the integral, and the direction of the surface since we weren't really counting the height or the distance from the infinite plane—we just kind of left that out..." From there she proceeds to describe why they chose the various values that they substituted into their integral and that they multiplied by two to account for the top and the bottom of the surface.

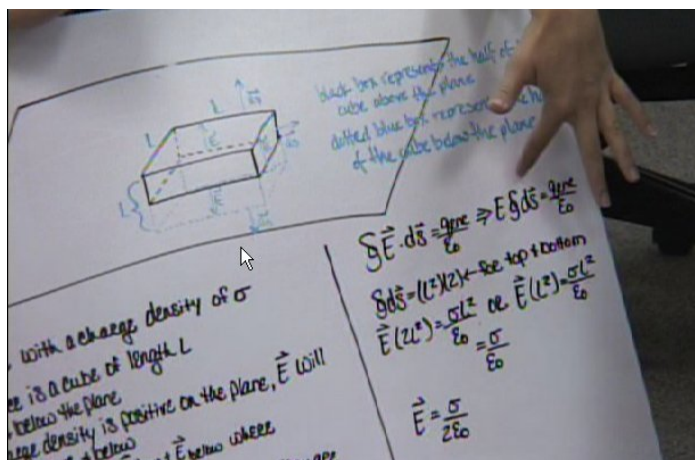


Figure 4. Kiki's whiteboard of the E field of a charge infinite plane

One of the students seated near her, Mark, an older (early 30s), slight, nearly bald white male suggests that it would be nice if she did separate equations for the top and the bottom and she responds that it would be the same on the top or the bottom—"one side would be equal to the other side." She finishes her explanation of the algebraic steps they

took, concluding that $\vec{E} = \frac{\sigma}{2\epsilon_0}$.

There is a pause while her classmates gaze thoughtfully at her board. Some are writing in notebooks. Kiki points to another group's whiteboard saying, "You guys got the same thing." Finally, a burly, pony-tailed, pierced-eared young Hispanic male, Ruben, asks why it is that the distance between the plane and the cube doesn't matter. Kiki responds that regardless of what the distance is the field lines will be perpendicular to the surface so "they cancel out." Gabe, a slight, bearded Hispanic male, says he was debating whether to use a cube or a cylinder and wondered if that would make a difference. Kiki responds that it makes no difference as long as the sides are perpendicular. He suggests that if you draw a diagonal from some point in the cube to one of its corners, "it wouldn't be l anymore." Other voices from the group chime in with "but electric field lines are perpendicular to the plane. There is some more discussion and Gabe finally capitulates, saying that he does not know how to describe what he is thinking but he will accept the fact that "it's the same" for now. As the talk subsides, Professor Donnelly (who has been sitting quietly outside the circle watching and listening) looks to Kiki and asks, "All done?" She nods. "Everybody got it?" They look around at each other, not quite willing to respond. "I see some quizzical looks. What about you John?"

"Makes sense," John responds cautiously.

"So, does it matter how high above your infinite plane you are?" A few heads shake. He presses them, "Does that make sense to everybody? Have you ever seen a situation where no matter where you are, something is always the same?" The follow-up with a discussion of the acceleration of gravity and the force of gravity, and they come around to the question of whether or not it is reasonable to pretend something is infinite

when it is not. The students think that it is. “When is it okay?” Silence. He scolds them gently for their passivity, and asks the question again. When is it okay to pretend that (gesturing to a whiteboard that he has tossed on the floor in the center of the group) is an infinite plane?” He reminds them about how we consider the earth to be flat at times—is it really? No. “Well then, when is it okay to pretend that (points to the whiteboard) is an infinite plane?” Students begin brainstorming: When you have to, to solve a problem? When it’s the whole universe?

Donnelly is relentless. “When can we consider that an infinite plane? What’s the definition of an infinite plane?” There is some discussion about the thickness of the plane. Once again, he brings them back to the question: “So when’s it okay to consider that an infinite plane?” The students are out of ideas. Donnelly picks up a marker, holds it at arms length above his head, and asks if they could use the electric field equation they have to calculate the field this high above the board. A few students shake their heads. “How about here?” He holds the marker a couple of centimeters from the surface of the board. Gabe quickly responds in the affirmative. “Why?”

“Because the distance is small relative to the size of the plane.” Donnelly looks around. “Does everybody understand what he’s saying?” He has Gabe stand up and demonstrate what he just said. Then Donnelly asks the group what they think. As the silence stretches out, he says to them, “Don’t look at me. I’ve asked the question ten times and nobody’s given me an answer yet. It’s a start. I’m not saying you’re wrong, I’m just saying what do the other 13 of you think?” A tall bespectacled dark haired white male, Rob, nods his head slowly and says, “That follows my train of thought. I think it’s close enough that the distance to the edges is so significantly far that we can...that it doesn’t matter how big it is. We can consider it infinity because it doesn’t matter...” his voice trails off as he gestures with his hands to indicate that the diameter is getting larger “...because it just goes so far that we don’t need to consider how big it is.” Students around the circle are beginning to nod their heads. Kiki adds, “When your distance in a straight line to the surface is less than your distance to the edge?” There is a long pause and then Donnelly asks John what he thinks and he responds that he is still a little confused. “What are you confused about?” John says he doesn’t see why the distance shouldn’t matter. Bill who is sitting across from him agrees. “Yeah. When we used that software that showed the electric field...” All eyes turn to the teacher. He smiles as he leaves the group and turns to walk back and sit down at his desk. “I don’t know. I’m going to let you guys answer that. Go ahead,” he gestures to Gabe who proposed the idea originally. Ruben jumps in, pointing to the equation. He says that this is different from what they did on the computer because it is an infinite plane. He asserts that no matter what it seems like, if the distance from the plane does not appear in the equation then it must not matter. At this point, several students talk about the counterintuitive nature of the situation. Kiki brings up the differences between how the field lines would look to a point in space above the plane if it was infinite versus how that would be different if it was bounded. They all seem to be coming around to an agreement with Gabe and Ruben. Kiki finally turns to the teacher and asks, “What was the question again?” Donnelly has been sitting at his table ostensibly looking at his computer screen but following the exchange out of the corner of his eye. “The question was, when can you consider that an

infinite plane, and John said that he didn't get it so I left everybody to get John's question answered. Did you get an answer?" After a pause John uttered a monotone, "Yes." So when can we consider this an infinite plane?" Silence. Donnelly prompts Ruben to repeat what he was saying to himself under his breath. "When you're a certain distance, close enough, that you consider it an infinite plane but if you go past that distance you have to...the distance is too great to consider it a plane. Like a point that if you cross it you've can't consider it an infinite plane but if you're below that you can."

Donnelly pushes on Ruben's idea a bit. "So, give me an idea. Where would you say that is?"

Ruben hesitates, "Well I don't know."

"I didn't ask you what you know—I want you to guess. Show me how high above it would be."

"I don't know. Probably really close." He leans in and hovers the marker in his hand a few inches off the board saying, "if you go too high you can't do it anymore but down here it would be okay."

"Does everybody follow that? So which is it? Where is this magical point? Does it exist? And the answer is...?"

No one is sure. More discussion ensues about whether or not it would be a function of the desired precision of the answer. In the end, they conclude that there is not an exact point. It all depends on how accurate you need to be.

"But why is that important?"

Donnelly orders everyone to their feet. They follow him out the door, down the corridor and out to the edge of the four-lane highway that runs along the edge of campus. He directs their attention to the power lines over their heads. "Above you are 4 wires. Are they infinite?" Heads shake. "No? Which way are they closer to infinity in? North or South?"

"North."

To the south about a quarter of a mile, the wires angle down to the ground. They agreed that if they wanted to use Gauss' law to determine the electric field where they were standing it would be okay because they were close to the wires in relation to their overall length and they were nowhere near the end. As they head back to the classroom the professor laments the fact that these wires are due to be buried in another 6 months and this is probably the last class he will be able to do this with.

"Alright. I heard some conversations on the way out there. Why is it that it doesn't depend on how far away we are from an infinite plane?"

"Because the density of the field lines is the same everywhere?" This is from Brad, a spiky haired bleached blond male who has been silent thus far.

Donnelly presses, "The density of the field lines is the same everywhere because the electric field is...the..."

"...same..." volunteers an unidentified male voice.

"...same everywhere. Alright? But as you get further above the plane, you just can see a little further out. Okay? How far is it to the end of the plane from this point if we've truly got an infinite plane here?"

"Infinite."

“Infinite. It doesn’t matter if I pick here, here, here...” he gestures at various heights above the board.

Brad interrupts, “Doesn’t the strength of the field change the further you get out?” Donnelly points at the equation with his foot. “The math says...”

“No.”

“Why is it hard for us to understand that? None of us have ever seen an infinite object. It’s hard for us to envision. An infinite object looks the same no matter how far you are above it.”

Brad finally appears to get it. “But that’s just for infinite objects. Okay.”

“So what I’m going to introduce you to today, with your new-found knowledge is...we’re going to play with these little things.” He holds aloft a large blue capacitor and gives them a brief rundown of safety rules and precautions. At the end, he says that any one who intentionally or unintentionally shocks anyone else OR themselves WILL fail the course. He explains what they look like on the inside and shows them the symbol used in circuit drawings. He tells them they will be building various circuits, gathering data and trying to make some conclusions about what happens, because what they expect to have happen is not what will happen. They will need computers, CASTLE kits, batteries, current probe, potential difference probe and a capacitor. He hands them a worksheet and they take their whiteboards and roll their chairs back to their tables.

Students wander around the room collecting the things they need and soon the classroom is filled with conversations at the five lab tables where students work together in groups of two or three. Donnelly circles the room, going from table to table, making sure that their computers work and they have what they need to get started. He is watching, listening, throwing out an occasional question or comment. Before they assemble the first circuit, they discuss their predictions and write them on their worksheet. As they ponder their answers to the questions on the paper, Kiki calls out, “Dave?” Donnelly strolls over to their table. “What does this thing do?”

“That’s what you’re trying to find out.”

“But we have to answer these questions.”

“They’re just predictions. The worst they can be is...”

“Wrong.” Kiki and her partner Ann look at him with resignation. “I have a question for you though. What’s in the middle here?” Donnelly points to the capacitor plates in the circuit diagram on the worksheet.

“An insulator.” Ann answers.

“So if you were a charge coming around what would happen when you get to that first plate?”

Ann again, “You wouldn’t be able to go anywhere.”

“So would that be a complete circuit?”

Kiki and Ann answer in unison, “no”.

“So would it light?”

“It would, like, to here...” begins Ann, but Kiki interrupts with, “No! Because it has to be a complete circuit for it to light.”

"I don't know. I'm just saying that those are the kind of questions you are going to have to ponder to answer that. The great thing about predictions is that the worst you can be is..."

"Wrong," intones Kiki with resignation.

"Because it's not the final answer that's important: it's your thinking..." This last bit he tosses over his shoulder as he walks on to the next table. Ann and Kiki continue to argue over whether or not the light would light. Ann thinks it will eventually light. Kiki thinks it will not. Ann leafs through her notes but does not find anything helpful. She watches Kiki draw her graph prediction, looks around the room to see what others are doing and then draws her own, making it look similar to Kiki's.

Before they begin collecting data Donnelly gives them one last instruction that he says he should have thought to tell them earlier. They need to increase the rate at which they collect data with the computer interface they are using. He gives them instructions for doing this using the experiment menu on their laptop computers.

Finally, they build the circuit shown on their worksheet, and the light glows for a moment and then fades out. "Hey we were right in the first place." Ann exclaims. Kiki is writing in her notebook. When she is done she holds it up for the camera: "DAVE'S PHYSICS CLASS where everything's made up and points don't matter".

They answer questions and draw a graph, studying intently the data on the computer screen. Then they discharge their capacitor and begin building another circuit, this time with two light bulbs in series. As Kiki connects the circuit, Ann exclaims "ooooo!" She is watching the computer screen. After a few seconds, she realizes that she did not watch the light bulbs. Kiki assures her that they both lit and then faded out. They discuss and write comparing the data from the first circuit with the second. They continue in this vein, building various circuits with one and two capacitors in parallel. Kiki gets up and walks around to other tables to see what sort of results they are getting.

As they work through the sequence of tasks, they try to make sense of the results they are getting. They look at how the results vary as to time in the different circuits and they cannot reconcile some of the differences they see with their model of what is going on at a microscopic level in the circuit. They rebuild two of the circuits and test them again to see if they get the same results. In one case, they find that the results are different. They decide that perhaps one of the clip leads they are using is bad. Periodically Donnelly checks in at their table and asks them what they are doing. He appears satisfied with what he sees because he rarely stays more than a few seconds. They continue to work—building, testing, sense-making and writing. Twenty minutes after beginning the lab, they are no longer handling the capacitors gingerly.

With only 10 minutes of class to go, Donnelly grabs a handful of meter sticks and lays them out on the floor in a pattern that resembles a circuit with a battery, a resistor, a light bulb and the two parallel plates of a capacitor.

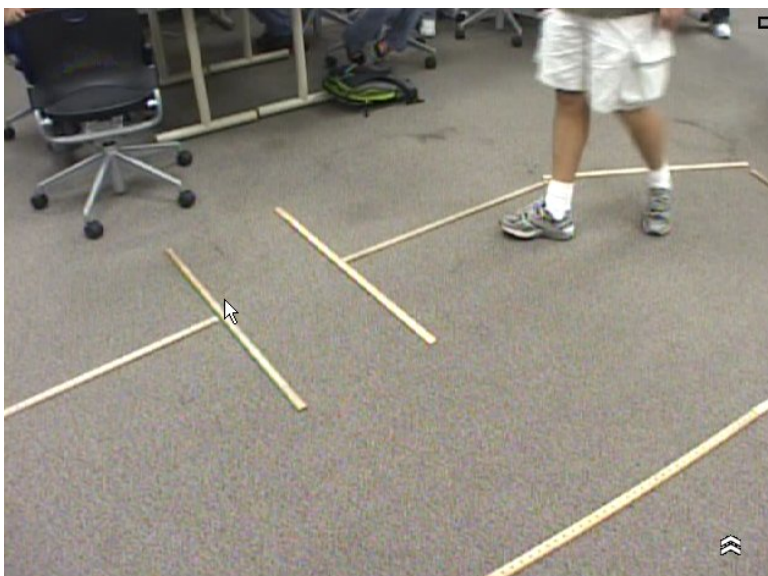


Figure 5. Professor Donnelly's meter stick "circuit"

He tells the students to find themselves a place to stand somewhere on this circuit. They are charges. He reviews with them the functions of all the circuit elements and then he moves one of the meter sticks, as if to close the circuit, and tells them to do what the charges would do. They all move toward one of the capacitor plates.

"So based on that theory, if I put the light bulb right here, will that light bulb light?"

Many voices answer, "Yes."

Yeah, but according to what you know...what? Does he ever get to cross there?"

Many voices: "No."

Donnelly: "Will he ever complete the circuit?"

Brad: "No."

Donnelly: "But wait a minute. The light bulb...lit."

"So what's the point?" asks Kiki.

Donnelly: "Of What?"

Kiki: "Of the capacitor?"

"That's a great question. Someone murmurs that it capacitates. "It capacitates. Okay. Everybody kind of rotate around to where you started. Now I got a question for everybody, and it's a good question: what's the purpose? Well, what's the purpose of a light bulb in a circuit? It's a resistor so what does it do?"

"Resist." Bill joins the conversation.

"Resist what?"

"Current."

"Current. And it's purpose is to do what to the energy of the system?"

"Transfer it out."

"Transfer it out. Okay. What's the purpose of a light bulb in a system? I mean a battery in a system?"

“Re-energize.”

“Re-energize. Storage of energy. The bigger question is: what’s the purpose of a capacitor? Okay. Well for one thing, you have an answer already. Um. What does it do to a light bulb when you hook it up like this?”

“It glows.” Bill again.

“It makes it stay on for a while and then it lets it go...” Donnelly’s voice trails off.

“Out.” Maria.

“Okay? So, why we might want that remains to be a mystery, okay? But at least it is something that does that. Now the real question is, normally why does this charge move?” He points to Maria.

Steve answers, “I pushed on her.”

“Because you got a little closer, right? He walks over to the boys standing at the ‘positive plate’ of the capacitor. “So, can you get any closer to him?”

Bob answers, “No.”

He points to Steve on the other side by the negative plate. “So should he ever move, according to what we know?” No. “Do we know he moves?” Yes. “Yes because the light bulb does...” Light. “It does light. Everybody see the problem here with the fact that this light bulb lit when this charge is permanently stuck right there.”

Brad quips, “You have a bad capacitor.”

Donnelly continues, “No, you have a good capacitor. That’s what capacitors are supposed to do. So how do...let’s say you start on this side of the battery...” He moves Kiki to the right. “What happens when she goes through the battery?”

“Energized.” John.

“She gets energy. Okay. So let’s say she comes over here and gets stuck. Boom.” He points to the positive plate of the capacitor. How’s her energy ever getting to the light bulb?”

“It doesn’t.”

“It doesn’t, but does the light bulb light?”

“For a second.”

“Yeah, well, the only ones that are going to get energy are the ones that star over...here...and they’re never going to get...”

“There.” Several students answer in unison.

“Changing electric fields are going to become really important idea for the rest of the semester. We’ll leave it there for today.” Students return to their seats and begin putting away the lab equipment and computers they had been using. Donnelly tells them that there will be a “celebration of knowledge” (his euphemism for exam) on Friday. Each student may make a 5 x 7 note card to use for this test but the information on it can only be taken from his or her journal. The note card and journal will be collected at the end of the test. Students’ air is one of calm resignation as they pack up to leave. They have been down this road before. If their journal is not up to date they will make it so by the coming Friday so that they can use a note card on the celebration.

The teacher and students in this classroom appeared to be engaged jointly in a construction project: they were constructing knowledge structures—models. There was an expectation on the part of students that they would be assigned tasks upon which they

would work jointly. The teachers expected them to help one another to build on what they already knew and to reason from spatial representations first and then from equations. In constructing their models, he expected them to be specific about parameters and boundary conditions. In discussing their models with the whole group he expected them to be thoughtful, attentive and forthcoming.

Students in this class were serious and focused. Very little student interaction was purely social. Their demeanor was business-like and they attended to the task of building models. They worked well together collaboratively no matter how they were grouped, and their major concern seemed to be to engage in the assigned tasks in such a way that the structure of the models they were building was clear to them and the models themselves were sound.

Their apparent motivations were to develop an understanding of the physical realities that the models under investigation represented, and to develop a sense of how these models could be used to answer questions. At no time during the semester did I hear a student asking about points or grades, and the teacher's only references to grades were about when things needed to be turned in to him in order to receive credit.

There was a certain amount of guessing among students in both small and whole group conversations, but this was more about conjecturing or hypothesizing than target practice. Generally, guesses were followed by some effort by the group to verify whether they were plausible explanations.

The cultural model of schooling that prevailed in classes described previously was barely evident in this classroom. Rather the prevalent model was that of a workplace. Students had a good work ethic, and worked together collaboratively and treated and regarded each other as equals. Their tasks were analogous to construction projects. They were competent tool users, and their tools ranged from physical tools such as batteries, wires, switches and capacitors, to technology tools such as computers and software, to intellectual tools such as models and mathematical symbols and operations. Work was definitely a creative activity for these students but it was also their job, and their goals appeared to be mastery goals.

Summary

In the vignettes above, we see students, from middle school through community college, engaged in a variety of modeling activities appropriate to their courses of study. The tools they used for communicating—whiteboards, language and gesture—were similar. Students' behavior toward each other varied somewhat. This was probably a function of their age to some extent. Their behavior toward the teacher and the tasks in which they were engaging varied as well, and this may have been related to the cultural models at work in these four very different learning environments.

The demeanor I brought to the middle school mathematics class was as much scoutmaster as teacher. My interest was in introducing these students to new experiences that I felt would result in learning. I was not concerned with points or grades. I wanted them to acquire new skills—modeling skills, communication skills, thinking skills. I took them on an extended field trip so that they could earn their whiteboarding “merit badge”. Students responded in kind. They joined in and followed along. Most bought

into the idea that it was a skill set worth acquiring and they practiced it together as well as they were able.

Mr. Mendoza's class resembled a family of which he was the father. There were routines, chores and house rules. Students were comfortable with themselves and each other. They were occasionally playful, competitive or eager to please and at times moody, demanding or contrary, but by-and-large they engaged in the classroom routines and did their "chores" without complaint. They could tell that their teacher cared about them and they usually made the effort to live up to his standards and expectations.

Mr. McEvoy was more like the coach of a team that is used to winning. From the beginning of the school year, he cultivated students' communication skills. As they practiced these, he critiqued their form. The students treated him with respect and worked hard to acquire and demonstrate the skills he valued. They knew that teamwork and performance counted, and when they presented their whiteboards, they tried hard to do a good job and look good doing it. They believed that his game plan would win them the game and they tried their best to follow it.

Professor Donnelly's demeanor was that of a foreman or general contractor. He had a set of blueprints to build something that the students needed (physics) and he broke it down into a series of tasks that allowed the students to build it for themselves. Working together was a given. Doing what was necessary was expected. Students were journeymen, accomplished in the use of the tools of their trade. They had the right stuff and they persisted until they got the job done.

All these groups were steeped in the culture of schooling. They could not help but bring it along. Nevertheless, teachers and students in each of these classrooms negotiated unique cultures that served to structure their interactions and frame the learning experience. These frames were not accidental—they were deliberately brought about in each of these settings. The next chapter will examine how they affected whiteboard mediated cognition and modeling in these classes.

ANALYSIS

In my data, I have identified three parallel dimensions of modeling and of whiteboard-mediated cognition that I use to categorize students' whiteboard mediated activities. In this chapter, I illustrate how these dimensions can be seen in the learning environment with excerpts of transcripts and screenshots of whiteboards that students are preparing and/or describing in the course of assigned classroom activities.

Table 3

Parallel dimensions of whiteboard mediated cognition and modeling

Dimensions	WB mediated cognition	Modeling
Contextual	Framing	Framing-identifying and selecting what's relevant
	Mapping	Analogical mapping-use of prototypes or examples
Distributed	Across individuals	Communication-activation of others' mental models
	Across representations	Symbolizing-abstraction and simplification Representing-composition of symbols and spatial representations
Structuring	Systemic (objects)	Manipulation – identify objects, how they fit together, interact; experimentation, exploration and elaboration of structure
	Geometric (space)	
	Temporal (time)	
	Interaction (forces)	
	Metaphoric (relationships)	

I see modeling activities and whiteboard-mediated cognition along three separate axes or dimensions: contextual, distributed and structuring. The contextual dimension functions to assist students in abstracting information from the problem statement or physical situation or activity from which the problem space arises. It involves choosing the spatial and semantic frames that allow students to find and choose the relevant information to map to their prototype or conceptual model. The distributed dimension is particularly visible in the architecture of the discourse surrounding whiteboard preparation. It involves the translation and coordination of inputs from various individuals' interpretations of what is significant in the problem space, along with the contributions they incorporate from their tools, inscriptions and artifacts. The structuring dimension can be seen in the coherence of the different types of structure that students attribute to the information that they abstract and represent in navigating the problem space they have constructed.

Before exploring these dimensions, however, I revisit the notion of models and modeling to frame the discussion that follows.

Keeping an Eye on the Model

What is a model again?

The definition of model that was laid out in chapter 2 is reprised here in condensed form: a model is a representation of structure. It is a conceptual representation of a system containing related objects, whose structure is the set of relations among these objects. In physics, models typically have four types of structure: systemic, spatial, temporal and interaction.

A delicate but critical shift of attention: zooming in and zooming out

It is all too easy for students to lose sight of “the model” in the course of routine classroom activity. The initial paradigm lab in which physics students engage in a modeling instruction sequence typically results in an equation, and instead of viewing this equation as *a mathematical representation that encodes the structure of the model* they are building, they sometimes mistake it for the model itself. When this happens, students may dispense with the task of **translating the conceptual structure (CS)** that the semantic frame brings along, **into its corresponding spatial representation (SR)**.

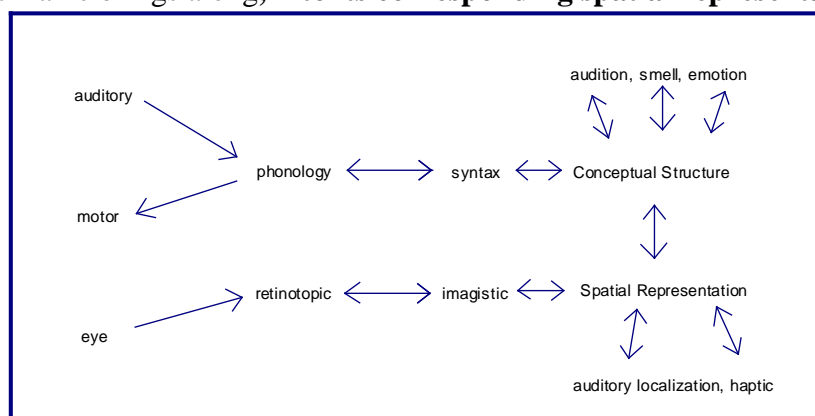


Figure 6. Jackendoff's Theory of Representational Modularity (reproduced from page 47, Chapter 2)

If symbolizing is the *extraction and reduction* of conceptual notes from a perceptual set, and the perceptual set that is being encoded symbolically is incomplete (i.e., lacking its SR analog), the meaning of the symbol system (i.e., the equation) is incomplete. While the acts of extraction and reduction necessarily result in some loss of information, which is an inevitable by-product of model construction, this step toward abstraction is one that must be taken mindfully.

If we take the microscope as a metaphor for the student's way of looking at a problem, it would be as if the student zooms to high power in order to see the details of the problem space. Once she has these details in focus, however, she must not forget to zoom back out in order to make sense of the problem by putting what she sees into some sort of 'big picture' context. At times, the student may overlook this critical 'zooming out' step, and this is a place where careful probing by the teacher can help the student refocus, and develop some skill in knowing when and how to zoom in and out.

Lili: The value of the velocity is getting closer and closer to zero.

TEACHER: on the way up.

Lili: On the way up.

TEACHER: What about on the way down?

Lili: it was getting farther and farther from zero.

TEACHER: Well, what do you mean by away from zero?

Lili: The value is more negative.

TEACHER: That's more consistent with what's on the board. [The object is] moving in the negative direction and faster. Dave, were you going to add something to that?

Dave: Oh yeah—I was going to say when it's rolling up the hill it's slowing down but when it starts rolling back it's speeding up.

TEACHER: perfect.

(DHS 9-30-05)

Here the teacher redirects the student's attention from the mathematical to the physical interpretation, and he is assisted by Dave, who translates the teacher's statement to something that everyone should be able to understand.

Knowing when and how to adjust her focus on a problem to a finer or coarser view requires considerable skill on the part of the student. This may be an issue related to framing (mentioned below) as every focal plane brings with it a slightly different frame.

If only I had a hammer

A whiteboard representation is a composition of symbols and therefore it requires the student to be able to call upon what she knows about the structure of what she perceives. An incomplete symbol set for use in representing a system's structure limits the ways that a student can compose and manipulate a representation.

One ubiquitous finding in my observations of classroom activities was the "equations are tools" metaphor, which was evident in the discourse of students and teachers alike. Here are three examples of "equations are tools" taken from two different students and from the teacher discussing whiteboards presented during the same class period:

Nancy: So we used the equation of the final velocity equals acceleration times time plus the initial velocity and plugged in negative five meters per second squared for acceleration, seven point oh seconds for time and an initial velocity of twenty meters per second, and we got twenty point five meters per second for the final velocity.

...

Peter: For number 4 we used final velocity equals acceleration times time value plus initial velocity. Then we plugged in the given numbers and got 9.05 m/s. Anybody have any questions?

....

TEACHER: So is it appropriate to use an equation from the battery powered car lab? The second problem is...this isn't the change in velocity (he changes the formula he has written on the board—crosses out the delta in front of $v = \Delta x / \Delta t$ and writes the word average above the v)...it's average velocity...watch out for that. That's probably the most common mistake people make on these kinds of problems. They get to one—they're in a rush—they see a distance, they see a time, they want to know acceleration...they use distance divided by time and get a velocity and then use that velocity divided by time and get an acceleration. Does that work here?

(DHS 10-3-05)

While the use of this metaphor to structure reasoning is certainly in accord with the “*model of-model for*” idea advanced by practitioners of RME, students’ uses of equations were limited by their lack of awareness of the SR that the symbols implied. It would be analogous to using a level to drive a nail into a board because the level is massive compared to the nail and has a flat side that makes striking the head of the nail easy. It works, but is limited in its effectiveness. In addition, it mars the tool, making it less effective for its intended use.

The tendency to want to think of equations as both models and tools is not surprising in light of many students’ preference to see learning in mathematics and science as a procedural activity. A great deal of both class time and textbook “page count” are given to cultivating procedural fluency. If students view the goal of performing a paradigm lab as finding an equation (rather than identifying a model) then the goals of engaging in the subsequent modeling tasks found on worksheets may become (1) finding the right equation to use, and (2) learning to use it correctly (rather than elaborating and applying the model).

Zane: You probably did it wrong...? Here. Let's work it out. Let's work it out. Let's do it on the board.

Jimmy: Start by writing the equation.

Zane: Yeah.

Jimmy: Delta x...

(Hannah is talking about food as she writes their names on top of the board. Zane is writing an equation and data on the board: $\Delta x = 1/2at^2$, $\Delta x = 110m$, $t = 1.5s$)

Jimmy: That's not the whole equation though.

Zane: But we don't need the rest though.

Jimmy: Why?

Zane: There's no...okay whatever...(adds + vot to the equation)

Jimmy: Then you just put a zero...

Gui: Just show all the work.

Hannah: This is supposed to be a five... (rubs the 1. off the 1.5 seconds)

Zane: Okay. Whatever. $110m = 1/2a(5s)^2$...[] squared...

Hannah: make it legible...

Zane: It's legible...I can read it... (pause while he continues to write)...no we'll get a slow...what's the answer?

(Board now shows

$$110m = 1/2a(5s)^2 + (0)(5)$$

$$\frac{110m}{25s^2} = \frac{1/2a(25s^2)}{25s^2}$$

$$\frac{110m}{1} \bullet \frac{1}{25s^2}$$

$$2 \bullet 4.4m/s^2 = 1/2a \bullet 2$$

$$8.8m/s^2$$

Hannah: 8.8

(DHS 9-30-05)

For Hannah, Jimmy and Gui, the whiteboard exercise recounted above was about choosing the correct equation, using it correctly and performing the necessary computations to find the correct answer.

If the teacher keeps in mind how the structure of the task we assign to a student maps to the model they are elaborating or applying, the teacher can listen for what sort of mapping (if any) is occurring, and prompt the student to bring additional elements into their model that might have been lacking originally.

TEACHER: Let's talk a little bit about this here. This whole issue of gravity whether it pushes or pulls. Whether, you know, does it do both at the same time and how does it act. Okay. This rubber band here... would you say that there's a force on this rubber band?

Ben: Your finger.

TEACHER: What's that?

Ben: Your finger.

TEACHER: Okay, what makes you say that there's a force on it?

Ben: Because your fingers are pulling it.

TEACHER: Let's say my fingers weren't here and all you did was saw this, what would that tell you?

Ben: Then there'd be no force.

Jorge: You're pulling it.

Ben: Because it's doing it like it's stretching behind it.

TEACHER: It's stretched, right? Okay, I think most of you realize that. I think most of you can kind of see that if it's just hanging there like that it's not stretched, it's not taut, right? But there is no force on it. (teacher pulling up and down) Right? Okay. Now, if I pull it in this direction like that, then which direction is the force?



Figure 7. Which direction is the force?

Ben: (points up and down) Coming this way.

TEACHER: It's coming from that direction. Right? And if I have it in this direction, it's from that direction. And if I have it out towards you then it's?

Ben: Coming out towards us.

TEACHER: So it's along the rubber bands. So, if I take this mass right here and I hang it from the rubber band there, okay, what would you say about the direction of the force on the rubber band?

Several students: It's going down.

TEACHER: It's going down? Okay. And, why is it going down?

Several students: The weight.

Ben: Because of the weight? The weight is attached to the rubber band.

(CCHS 3-1-06)

What teachers want is for students to work together to identify, explore and elaborate fundamental physical relationships, and ultimately generalize them so that they can be re-used. Mr. Mendoza's class is working at this in the previous transcript excerpt.

What teachers often get, however, is students working together to identify the necessary equation and use it as a tool to get an answer. They do this because, in their experience, this is what the culture of schooling values—what they can earn points for.

Teachers need to be explicit about valuing models over answers, and in order for these models to be complete, they must have both the conceptual structure and the spatial representation dimensions. Attention to task design, and the framing and scripting that necessarily accompanies this activity, can help them redirect students' attention so that they encode information spatially and well as propositionally.

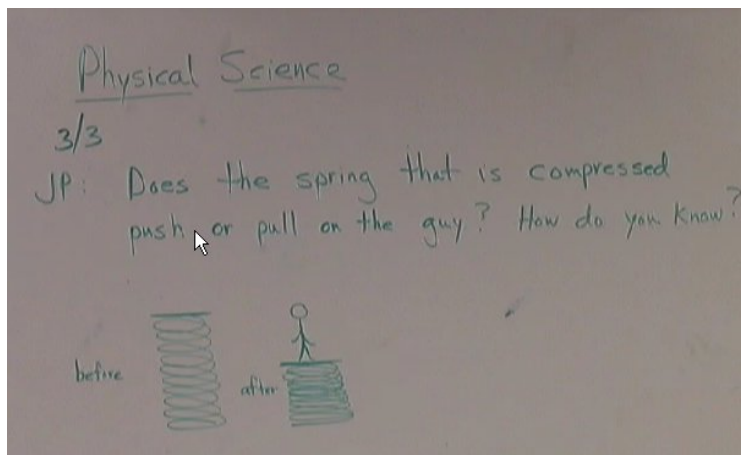


Figure 8. A journal prompt that makes use of both conceptual structure and spatial representation

One way to discover what a student's model contains is to watch and listen for 'what counts' as far as the students are concerned. When their whiteboard shows only an equation and their oral presentation describes only the procedures they used to solve the problem with the equation they chose, the metaphors students frequently use are the *object collection (container)* and *object construction* metaphors described by Lakoff (2000). Verbalizations that make use of these metaphors are often accompanied by the expression "plug and chug". When employing a *container* metaphor students describe the collection of pieces of information from the problem space. When employing the *object construction* metaphor, they talk about plugging things into an equation to get and answer (the thing they think they are supposed to construct).

Lili: We forgot to put a direction, but it says that...it says to assume the average mass of the riders is 75 kilograms so if you've got 20 people you have 1500 kilograms in the elevator...they say to make the mass of the elevator 500 kilograms so the total mass is 2000 kg...they said the max force that the cable can support is 30,000 Newtons, and so you just use that the[equation] net force is equal to "a" times "m". We said that the max force is the net force... [inaudible] acceleration...so you put in 30,000 Newtons equals acceleration times 2000 kilograms which gets you 15 meters per second squared.

(DHS 11-15-05)

Here Lili makes a valiant (but misguided) attempt to *construct* (note the verbs used: make, put, use, get) an answer to an elevator problem that asks her to determine how fast an elevator will accelerate using the equation $F_{net}=ma$ and substituting mass and force values given in the problem to find an acceleration.

Brought along v. Brought about

In cases such as the ones described above, it can be telling to hear what counts as "understanding" in the view of the students. Often "understanding = *getting* the right answer," i.e. demonstrating the correct *object construction* procedure. Again, the

tendency to define understanding as procedural fluency is something that students bring along with them as a part of the culture of schooling that they have experienced up to this point.

Zane: So what did you guys get for the acceleration on 7c

Jimmy: 7c?

Hannah: I got negative one.

Jimmy: negative 8.57

Hannah: Oh whoops (flips page). Yea I have that too! Ya see I am understanding this, I just get stressed out and people make me feel like I don't know what I'm doing.

Jimmy: (Clapping) Yea Hannah! You got the right answer on the last one.

*Zane: I'm just so happy I finally understand everything.
(DHS 10-5-05)*

The teacher must be alert for what is missing (this is not easy!) from students' model description in order to bring about an expanded definition of understanding that includes both conceptual structure and spatial representation.

The Contextual Dimension of Whiteboard Mediated Cognition and Modeling

The words that the teacher (and by extension, the textbook or worksheet), and the student use in discussing a problem space hint at how the elements within their conceptual systems relate to one another. Physics or mathematics word problems generally bring along with them a prototype or script that can help the student call up an image (i.e., a spatial representation) of the physical situation in which the action is taking place.

Choosing what is relevant

A physics problem asks about how much longer it will take a train to go from point A to point B when it must make a two minute stop at a station in between, than it would take with no stop. This invokes an image with a storyline that could go something like this: a diesel locomotive, pulling passenger cars, is traveling down a railroad track at some cruising speed. At some instant, it begins slowing until it comes to a stop. It remains at a standstill for two minutes while it discharges or takes on passengers. Finally, it pulls away from the station and accelerates back up to its cruising speed, continuing its journey from point A to point B. This scenario or "script" is compared to a train that just cruises on down its track at a constant speed without stopping at the station.

In a traditional problem solving approach, the student chooses and encodes the information she needs to extract from this contextual frame with identifiers (i.e., cruising speed, acceleration rates, and the length of time the train is stopped), selects formulas that will allow her to compute time in the two different cases, and maps her data onto the appropriate variables in the formulas. She performs the mathematical operations that the

formulas prescribe and presto! Out pops two answers. If she is adept, she will be able to compare the two answers she has computed and connect them back to the problem statement.

Recall that Jackendoff's Theory of Representational Modularity posits language enters our thoughts as a conceptual structure. If we examine this student's strategy in light of Jackendoff's theory (figure 5.1), it becomes apparent that she bypasses the step of translating her conceptual structure into a spatial representation and focuses her attention and effort on the propositional or algebraic structure of the problem space. It is easy to see in this instance how the student might mistake the equations for the model.

Students who adopt the procedural approach to this problem described above find it very difficult, but when the conceptual structure is mapped onto a spatial representation, even a mental one, it becomes a relatively straightforward task. If, in addition, the student can produce an inscription that encodes spatial information comparing the two journeys, they may be able to manipulate it to arrive at a solution without resorting to the multipart algebraic solution described above. In the next excerpt, Zane uses a graph, reinterpreting it at several points, to help Gui, who prefers to rely on a procedural approach to problem-solving, navigate through the problem space for the train problem described above.

Zane: First, you've got to find out how long the whole station thing takes. Did you figure that out?

Gui: What formula do you use?

Zane: There's no formula that you use. You have to think about this one. Let's think about it...it says... (looks back at the problem statement in the book)...the train decelerates at a uniform rate of one meter per second.

Gui: You need to know how long it takes.

Zane: It doesn't matter.

Gui: But...

Zane: It does not matter. Look. Get this number down to meters per second first. (Points to 72 km/h).

Gui: But...

Zane: Get it down to meters per second. (After some hesitation, Gui writes 72,000 m/h on her paper, then picks up a calculator, and starts to press buttons.)

Gui: Is it this? (She holds up her calculator for him to see the answer.)

Zane: No. Okay. Alright. (Takes out a sheet of paper) You've got 72 kilometers per hour to meters per second. Alright? Okay. No, we'll get it to meters first. So...one thousand meters over one K-M equals 72,000 meters for an hour, okay? And then so I want to find...and so there's...in one hour there's sixty minutes, so I divide by 72000 by sixty alright and then there's 60 seconds in one minute. Twenty meters per second. Do you get how I got that? (Gui nods.) So if it's decelerating at one meter per second squared how many seconds is it going to take it to decelerate?

Gui: Twenty.

Zane: Twenty seconds. That's correct. So it helps to draw a graph. Right here it's cruising at 20 meters per second. Then it slows down (talks as he sketches a velocity time graph for Gui) and then here's that 20 seconds of decelerating right here (draws a diagonal line down to the t-axis) and then it stops for 2 minutes. So how many seconds is two minutes?

Gui: Two minutes?

Zane: It says it stopped for two minutes.

Gui: One hundred twenty?

Zane: One hundred twenty. So what's one hundred twenty plus twenty?

Gui: One forty?

Zane: One hundred forty. There. (He continues to draw the graph.) And then.... (looks back at the book for a moment) and then it accelerates at point five meters per second squared.

Gui: It comes to a stop?

Zane: Yes it does. It's stopping at a train station for two minutes. This is a time graph. Time-velocity. Okay. So it stops here for two minutes and then it can only accelerate at point five meters per second squared. So it's eventually going to make it's way back up to twenty at point five.

Gui: Point five meters per second squared?

Zane: So how long does it take to get to forty? I mean to get to twenty...if it's accelerating at point five... (he points to the graph he has drawn)

Gui: forty-five?

Zane: No, Gui. You need to get up to twenty meters a second and you're going up at one half, right? (Gui picks up her calculator and waits. He thinks for a minute and finally writes 20 divided by one half. Gui keys this into her calculator.)

Gui: Forty?

Zane: Forty. Okay. So it takes forty seconds to do that. So what's 140 second plus 40 seconds?

Gui: One eighty?

Zane: That's just how long it took him to...

Gui: One eighty?

Zane: Yes. That's how long the stop was. Okay? And how much distance did he cover in this time? It's just the area here plus the area here. He points to the two triangular areas under the graph of the velocity time graph he has drawn for her.

Gui: But it doesn't ask for the distance. It doesn't matter.

Zane: It doesn't matter, but that's how you need to figure it out. No we're not done yet. (He reads from the book)...how much time is lost in stopping at the station...so we need to figure out the distance that this took, okay? So what's your...

Gui: This plus this?

Zane: Mm hmm. One-half base plus height. (He goes and borrows a calculator from someone at another table and then brings it back and computes the areas of the two triangles; 200 meters and 400 meters respectively.) So how much is that...200 plus 400?

Gui: six hundred?

Zane: Right. So that's how far the train went while it was stopping. The other train was going 20 meters per second. So how many seconds it going to take for it to go 600 meters?

Gui: (divides on her calculator) Thirty seconds?

Zane: Thirty seconds. So it took [the other train] thirty seconds to cross this whereas it took this one 180 seconds to stop. So how much time is lost?

Gui: Is it this minus this?

Zane: ...and that gives you one hundred fifty seconds...

(Gui nods slowly)

(DHS 10-3-05)

In this episode, Zane is clearly reprising for Gui the way he reasoned through this problem with the aid of a graph. When she asks him for a formula at the outset, he tells her that there is no formula—she just needs to figure it out by thinking about it. His tool for thinking in this instance was a spatial representation. Although it is not clear at the end of this episode that Gui has learned to do the inscription-assisted reasoning in which Zane is engaging, she appears to be able to follow his lead as he takes her down the path he followed through this problem space. It is clear that Zane is able to read many more things from his graph than just velocity and time. He maps the train's journey first as a function of time, interpreting acceleration as a change in the train's velocity, recognizing that the area under his graph represents displacement, and comparing the time intervals it took each train to experience the same displacement. Initially he uses his graph to communicate about the train's journey through time. Once he has accomplished this he uses the same graph to show its journey through space.

An inscription has the additional benefit of being shareable and transportable. Certain inscriptional practices, such as graphing, motion mapping and force diagramming, are well enough developed that the inscriptions they generate are adaptable and reusable to represent a variety of spatial relationships.

Creating and interpreting these graphical and diagrammatic images is a skill that is learned, and in the learning process, students must develop an awareness of how the choices they make in constructing a representation can be interpreted. There is a risk that the task of producing pre-determined inscriptions can become so proceduralized that the inscription becomes an end in itself and it is not employed as a reasoning tool.

Hannah: Okay, so for the first page we did everything pretty much together except for the motion map. What did you guys do for the motion map?

Gui: I got this. Oh, I forgot my dots.

Heather: Like how many, see we all have different things. How many tick marks did you put?

Jimmy: I messed up. (Pause) Oh the first page?

Zane: How many dots?

Hannah: I have, well I just put 10.

Jimmy: Well I didn't use tick marks because you don't really know the velocity.

Hannah: but does it matter if you use them?

Jimmy: (makes a questioning gesture with hand)

Hannah: So did you show that the lines got smaller?

Jimmy: Because you don't really have quantitative evidence...

Hannah: So you just make sure that the lines got smaller?

Jimmy: yeah because it was a negative acceleration. After (counting) seven...

Gui: You didn't have to do that.

Hannah: I just assumed that you could put a bunch of them. Well does it matter how many you have I mean do I have to do it by...

Gui: It doesn't matter.

Hannah: I think you're supposed to put ten.

(DHS 10-5-05)

In the previous episode, Hannah's goal was clearly to make the motion map correctly. At no point in the conversation did anyone express any intention of using their inscription to help them figure something out about the motion of the object they were trying to represent.

As students construct their inscriptions, they map the conceptual structure that the language brings onto a conceptual structure or spatial representation of their own choosing. The information content of a representation can grow and change over the course of a conversation, as seen in this next excerpt, and students may impute much more meaning to it afterward than they did when it was first constructed.

In this episode, middle school mathematics students have been given a multipart problem to work on together in their small groups. We see their inscription and its meaning evolve as they think through the series of questions.

The Candy Problem

1. *Let's say we have 4 cases of M&Ms and 4 cases of Snickers bars. Each case is 4 pounds. How many pounds of candy do we have? (First make a diagram, then see if you can translate this diagram into a mathematical expression, then solve.)*
2. *If each case of M&Ms holds 24 individual bags, how many bags do we have altogether? (Can you use your diagram to help you make the mathematical*

expression for this, or do you need a new diagram? If so, re-draw your diagram.)

3. If there are 2 Snickers bars per pound, how many Snickers bars do you have altogether. (Again, use your diagram, redrawing if necessary, to help you write the mathematical expression you need to solve this.)
4. We want to share the candy fairly (without cutting up the Snickers bars) and we don't want anyone to get more than one Snickers. How many people can share of this candy? How do you know? What is each person's fair share? How do you know? (Be prepared to substantiate your thinking about this with both diagrams and mathematical expressions.)

(The camera is observing a table of three boys: Rigo, Javier and Manuel. Each seems reluctant, at first, to be the one who writes on the whiteboard—this is the first time they have used whiteboards in this class. Rigo finally picks up a marker and after writing the group members' names on the board he draws two columns of four rectangles per column, labels the top rectangle with the names Snickers and M&M respectively, and then puts the number, 4, in front of each of the four rows in the column as shown below.)

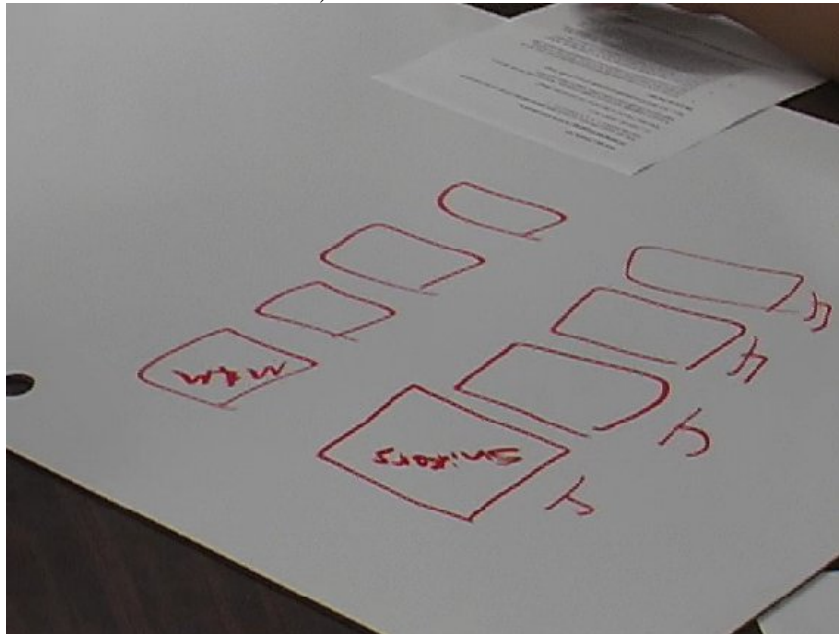


Figure 9. Rigo's initial inscription for part one of the candy problem

Javier: How many pounds of candy do we have?

Rigo: Let's see...

Javier: Four, four (points at each box in turn as he says the number)...four times four...sixteen...and sixteen (points toward second column of boxes)

Rigo: Thirty-two... (Javier taps on the board underneath the diagram and Rigo writes the number 32 there)...do we have to put pounds? (he hesitates and then writes the letter P after the number 32)

(The teaching assistant (TA) comes by their table and questions them on the meaning of the boxes and the number, 4, in front of each row of boxes and the meaning of the P after 32. She directs them to show that P means pounds somewhere on their board and Javier picks up a marker and writes $P = \text{pounds}$ on the bottom left side of the board. She then reminds them that they need to fit all four problems on this board, and when they ask if they should make it smaller, she says they can do this if they want. Rigo erases his diagram and begins again. This time he writes $P = \text{pounds}$ at the top of the board and then draws his columns of boxes underneath it, labeling each box, "4P" and writing underneath "Total 32 pounds". The TA who is operating the video camera asks them if they can write this as a mathematical expression.)

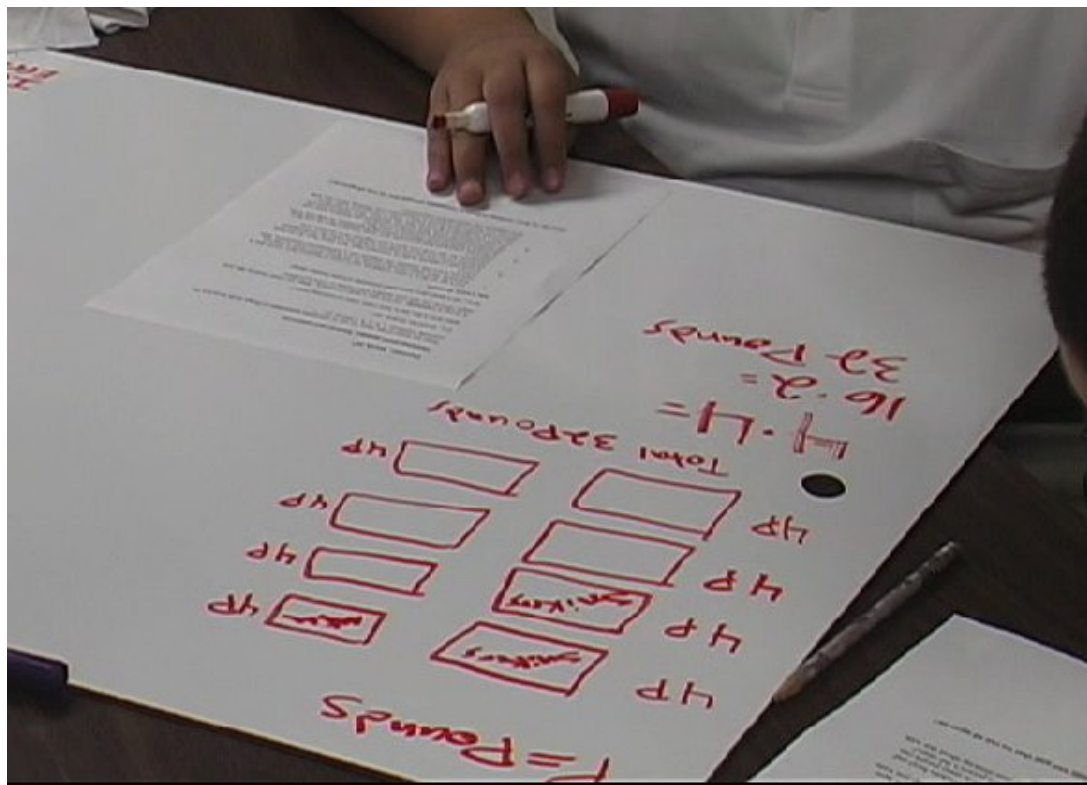


Figure 10. Rigo's redrawn inscription for part one of the candy problem contains more explicit information after he has had to explain it to the TA

They check their representation against the question, seem satisfied and move on to question number 2.

Javier: So each of these has 24 (points to one of the rectangles on the board).

TA: Don't erase your diagram. (Rigo has picked up his eraser cloth and appears to be preparing to erase what he has drawn.)

Rigo: I'm not. I'm just going to erase the names so I can write 24 in the boxes. (He carefully erases the words "Snickers" and "M&Ms" written inside the boxes and writes them above the columns).

Javier: So twenty-four times four? (They lapse into Spanish briefly, apparently clarifying that the problem is just asking about the number of bags of M&Ms.) No. Put it down here. (Javier gestures to the area on the whiteboard underneath the mathematical expression they wrote for problem number one. Rigo draws a line under the work for problem one and writes $24 \cdot 4 =$ and then hesitates...

Manuel: Ninety six...ninety-six bags (Rigo writes this in the next line and then writes the number 24 in each of the rectangles that he has drawn to represent M&M's boxes.)

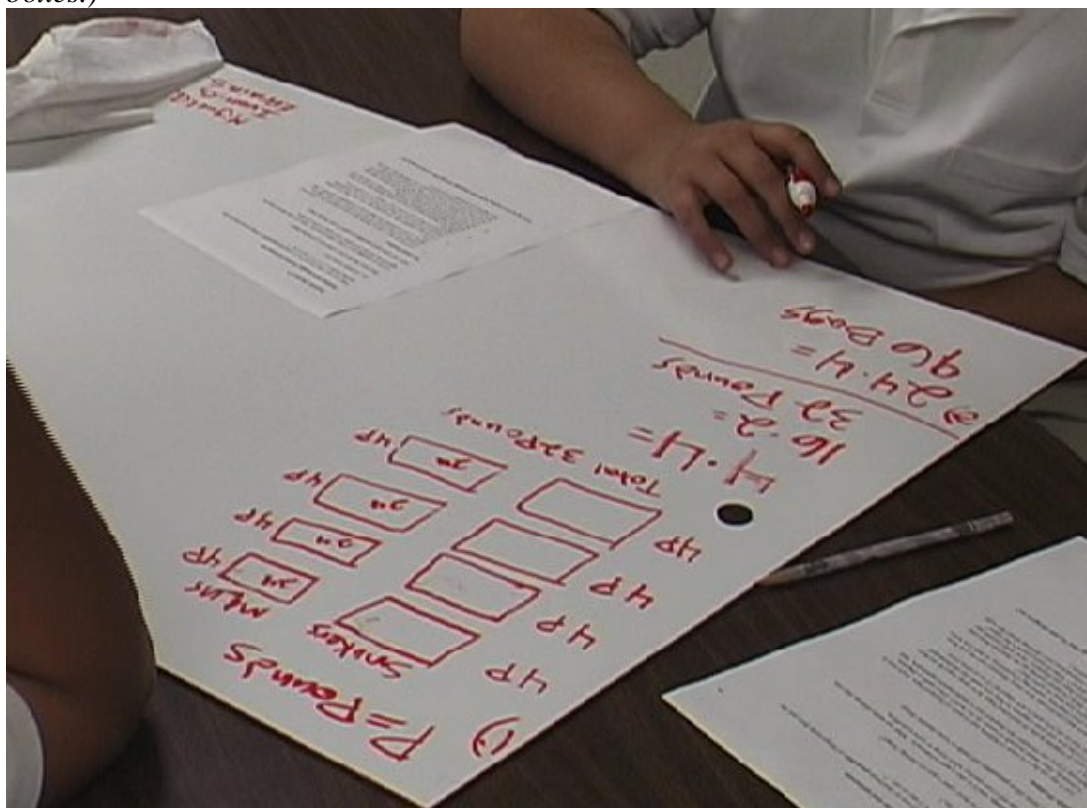


Figure 11. Rigo's inscription evolves as more detail becomes encoded in the representation

They move on to the third part of the problem. After some discussion in Spanish between all three boys, presumably about the fact that two candy bars per pound means that there are eight Snickers in each box, Rigo writes the number, 8, into each of the snickers boxes and then writes $8 \cdot 4 = 32$ Snickers to the right of his diagram.

As he is doing this, Manuel comments that the Snickers bars are much bigger than the bags of M&Ms.

Javier reads the fourth part of the problem aloud and the boys pause for a few seconds as if considering. They are speaking with each other in Spanish. Finally, Rigo reverts to English.

Rigo: Well how many people?

Manuel: That's the question.

Rigo: (after reading the problem aloud to himself)...well there are only 32 people because each one would get one Snickers and then we'd have to...I don't know...

Javier: (rereads the problem aloud and agrees with Rigo...he points to the solution to the third part of the problem)...there's only thirty two Snickers...if each one has to get one that's 32 people...cuz they only want to give one Snicker to each person...

Manuel: (uncaps his marker and prepares to write, then stops and recaps his marker) How do you know?

Javier: We could put 32 Snickers divided by thirty-two people equals one. (He gestures as if writing this with his finger.)

Manuel does not write. They lapse into an extended discussion in Spanish, which at times includes the cameraperson who speaks to them in English although she is fluent in Spanish. They ask her for clarification about problem parameters—whether they need to consider each case of Snickers as a single package or each bar and whether the term “candy” includes M&Ms or just Snickers bars. She goes and asks the teacher and then returns to tell them that candy means all the candy: both Snickers and M&Ms. The three boys discuss again in Spanish. Finally, time is called and the teacher begins to make the rounds of the tables, collecting their whiteboard markers. Manuel hastily writes $32 \cdot 1 = 32$ and tosses his marker into the box she holds out. The teacher has the students stand in a circle with their whiteboards in front of them and after giving them a minute to look at each others' boards, asks for a group to volunteer to go first. Manuel, Javier and Rigo volunteer. Javier holds the board and Rigo begins:

Rigo: Each package is like a snicker and an M&Ms...

TEACHER: Yeah...

Rigo: ...and each package was four pounds...and then we, like... (he gestures to the two columns)...the total was, like, thirty-two pounds...the total of the four...

TEACHER: Okay.

Rigo: ...the boxes...

TEACHER: Okay. And underneath that you have written four times four equals sixteen times two equals thirty-two...

Rigo: Thirty two pounds, yeah.

TEACHER: So, tell us how you got that number sentence from your diagram.

Rigo: The four is like the four boxes of Snickers times the four boxes of M&Ms

Javier: (whispers softly) No!

TEACHER: (repeats Rigo's statement so that everyone can hear) Four boxes of Snickers times four boxes of M&Ms...

Rigo: No. It was the four boxes of Snickers times four pounds...

TEACHER: Four boxes of Snickers times four pounds...per box...

Rigo: ...is 16

TEACHER: ...is sixteen...

Rigo: times two...because there's like...two...

TEACHER: ...times two what?

Rigo: Like the Snickers and the M&Ms.

TEACHER: Oh. There's two kinds of candies...

Rigo: Yeah. And we got 32 pounds in total.

TEACHER: Okay. Did you have a question? (to another student in the class)...no?...um...let's look at other people's boards. You have four times four equals sixteen times two equals thirty-two. There's one thing that bothers me about that.

Rigo: What.

TEACHER: Four times four does not equal sixteen times two... (Rigo and Javier look at each other and laugh)...does it? Four times four equals sixteen. And then we're going to take that sixteen that we got from four times four and we're going to multiply it times two to get thirty two...so...that's a small point. That's more of a shorthand way of writing things and I won't make a big deal about it because I see it on a number of people's boards, but it's something to be careful about.

TEACHER #2: I was going to say, nobody questioned it because everybody understood what they were doing. Which gets easy sometimes, until somebody walks in that doesn't know what we're doing. (She is referring to the fact that the teacher (TEACHER) of this lesson is a guest in this classroom and this is her first day working with these students). Another student raises his hand.

TEACHER: Yes?

Sam: Can you put it in parentheses?

TEACHER: Can you put what in parentheses?

Sam: Like, the, uh...four times four equals sixteen (he brackets this expression on his own whiteboard with his fingers)...and you just put the two out of it...can you do it that way?

TEACHER: (hesitates) Sure, I guess you could put that in parentheses and that would be a good way of setting it off...that just lets us know that the thing in parentheses gets multiplied by two.

Sam: Yeah.

TEACHER: Good point. I'm seeing it different ways on different boards like over here...we have four plus four equals eight times four equals thirty-two. If it were me I would be saying four times four does not equal eight times four, but, you know, we're using our shorthand again here. Just something to be aware to be careful of, and I'm sure your parentheses idea would have applied to that one too.

(WTMS 3-30-06)

In this excerpt, the spatial array of eight rectangles arranged in two columns takes on a succession of meanings as the conversation progresses. Initially they meant Snickers and M&Ms—four boxes of each. With the addition of row labels, they came to mean boxes of candy each weighing four pounds. Then they were taken to mean boxes containing some number of packages of candy, 24 packages per case for M&Ms and 8 bars per case for Snickers. At this point one of the boys, Manuel, recognized that the Snickers were much larger than the bags of M&Ms. This reference to size is most likely

based on a spatial image rather than a numerical comparison, as he did not mention it again when they were trying to do the fair-sharing problem at the end of the problem set. If he had been making a numerical comparison, he would have had no trouble supplying the fact that for every Snickers bar there were three bags of M&Ms.

Metaphors used in the mapping of conceptual structure to a spatial representation can shift as well.

Here Hannah talks about a graph, while she constructs it, as if it is a map from which she is reading information:

Hannah: Okay so, just to give another example, if you had another graph like this and you're A line, like, went up like this, and then your B line still did... (camera zooms in on drawing) I guess it doesn't really matter. Like if your B line was like that then you'd still be looking at your intersection point meaning you'd be taking these distances. So it doesn't matter.

(DHS 10-5-05)

Seconds later Zane is looking at the same graph and talking about it as if it were a journey:

Zane: if we put this on the v vs. t route you'll notice, (pause, drawing) this isn't so accurate, it should be going like this, and then this line, and then so...

(DHS 10-5-05)

Shifts in metaphor-guided mapping onto inscriptions can be reflexive with the evolution of the inscriptions themselves. In the next excerpt, Hannah transforms her understanding of a situation involving energy transfer from a picture to a bar chart and finally to equations. She reasons using container and path following metaphors. She uses the bar charts to construct equations, solves the equations and then uses these to reconstruct better bar charts.

HANNAH: So you have a cart on a hill and it goes down the hill, but it starts out with the initial velocity of zero. And you're assuming that 10% of the energy that happens throughout this process is dissipated (While she is speaking Jimmy is drawing).

ARA: What does dissipated mean?

HANNAH: It means it goes away.

ARA: Away.

HANNAH: So for your graphs... (she is reaching for a marker)

ARA: So it is not moving at first, correct?

HANNAH: Right. (She is drawing)

ARA: *Should we make a little equation for that one? (Hannah is drawing bar charts on the white board—she does not add the equation that Ara suggests.)*

ARA: *So it's zero.*

ARA: *Is there gravitational energy?*

HANNAH: *Well, first you look when you did a graph, and you're going to look and see (drawing energy bar chart) . . . Well, if it's not moving initially, right, you're not going to have any...*

JIMMY: E_k .

HANNAH: E_k .

ARA: *Right.*

HANNAH: *And since it's not, it has no elastic energy because it's not like there is nothing elastic.*

ARA: *Yeah, yeah.*

HANNAH: *So . . .*

ARA: *What did you say was your system?*

HANNAH: *I have the earth, the car and the hill as my system.*

ARA: *So everything? Or do you have more?*

HANNAH: *I put everything in it.*

ARA: *Okay.*

TEACHER: *Okay, three and a half? (teacher is checking how many problems each student did and recording this in on a scoresheet)*

HANNAH: *No, I didn't, I left the hill out because you have friction, you have energy going out of the system.*

JIMMY: *Yeah, I left the hill out too.*

TEACHER: *Okay ½, one, two, two and ½.*

ARA: *Out of how many possible?*

TEACHER: *Out of six.*

JIMMY: *[inaudible] I could [inaudible]*

ARA: *Okay.*

TEACHER: *two and one half.*

JIMMY: *Okay. (Hannah drawing lines on board)*

HANNAH: *Okay. So*

JIMMY: *You explain (to Hannah).*

HANNAH: *Um, this is initial (writing on board)*

ARA: *How did you pick what was your system? Just because that's what you're supposed to do?*

HANNAH: *Well, you look at everything that's in this picture.*

ARA: *Uh-huh.*

HANNAH: *You have your car right?*

ARA: *Uh-huh.*

HANNAH: *And you always have earth because the earth is acting on everything.*

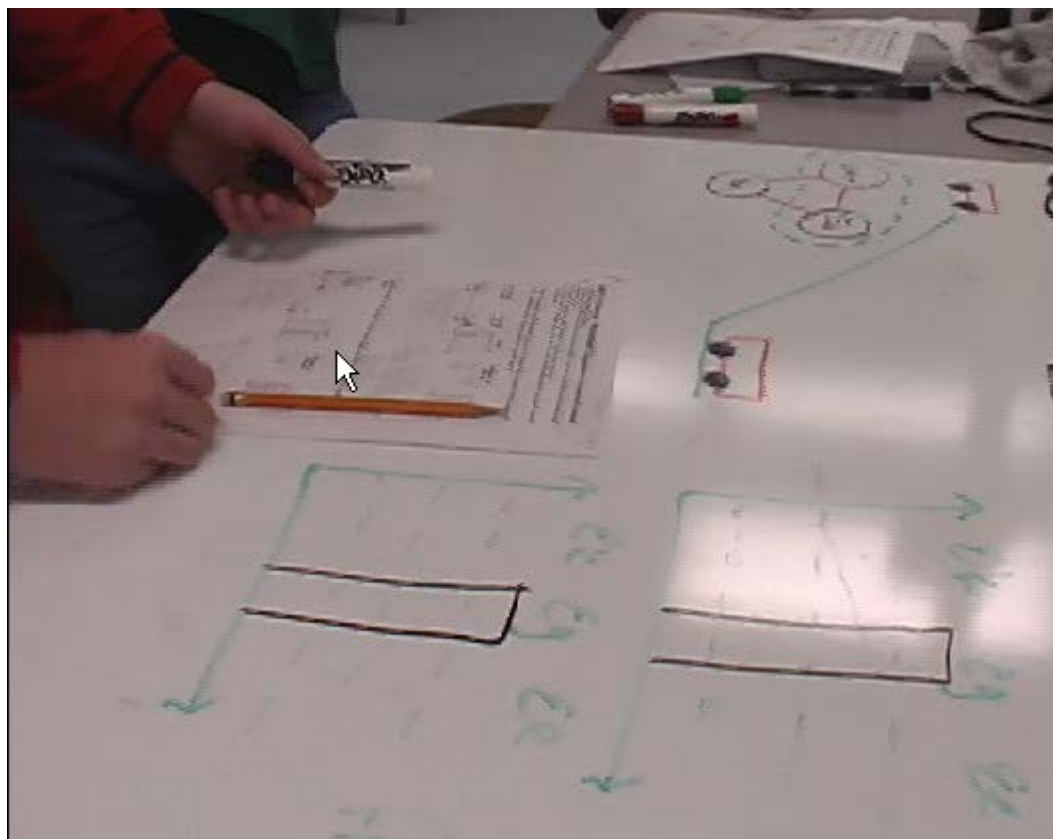


Figure 12. Hannah uses the picture of the cart and the hill to construct energy bar charts

ARA: *Uh-huh.*

HANNAH: *What else is acting on the car?*

ARA: *The ground.*

HANNAH: *The ground. right, so you can call that ground or hill.*

ARA: *I call that it track.*

HANNAH: *Or track. Okay, so that's it. Jimmy do you want to draw that (something) right here.*

JIMMY: *Oh, the thingy.*

HANNAH: *The [] Okay, but when you're looking at the graphs, you're um; actually, do you want to make it a little smaller? Alright we can always make these small. We're supposed to make them small.*

JIMMY: *Here, I'll make a little hill.*

HANNAH: *Okay. When you're looking initially though, it's not moving so you have no E_k .*

ARA: *Uh-huh.*

HANNAH: *It's not; it has not come in contact with anything stretchy so it's not elastic energy. So the only thing acting on it is the earth. Right?*

ARA: *Wait.*

HANNAH: *And you look at that because it's on a hill so there's going to be, um, an elevation change, right?*

ARA: *Wait just a second.*

JIMMY: *[inaudible] If he comes back...*

HANNAH: *He makes me very sad. I tried.*

ARA: *All right. Okay.*

JIMMY: *I did what I could.*

HANNAH: *And I finished some. He said it with such anger.*

ARA: *Okay, so you know there's going to be that much, so then how do you know how much to put.*

HANNAH: *Okay, I put that there was going to be four, because I saw that it was on a hill and that it was going to go down to zero so I wanted to see a big change in gravitational.*

ARA: *Okay.*

HANNAH: *(talking to Jimmy) So...*

ARA: *But I thought we had to like numerically accurately do this.*

JIMMY: *Yeah*

ARA: *So we just can't say, like, four.*

HANNAH: Well, we can always change it if it's not right.

JIMMY: It has to be, yeah.

ARA: Because, like, the way you drew it, only 10% is supposed to be dissipated and not 25%, correct?

JIMMY: Yeah.

HANNAH: Well, then I can add a little bit more to my bar. Maybe a quarter of a bar off. So then it would be like that. (drawing). Anyway, that's just what I did. I put four, and then, so the only initial energy you have is that (pointing at board) because there you know there is going to be an elevation change. Right?

ARA: Uh-huh.

HANNAH: Because the cart is going to roll down the hill. So that's what I put for initial and...

JIMMY: (Pointing at something) But why is that there for that?

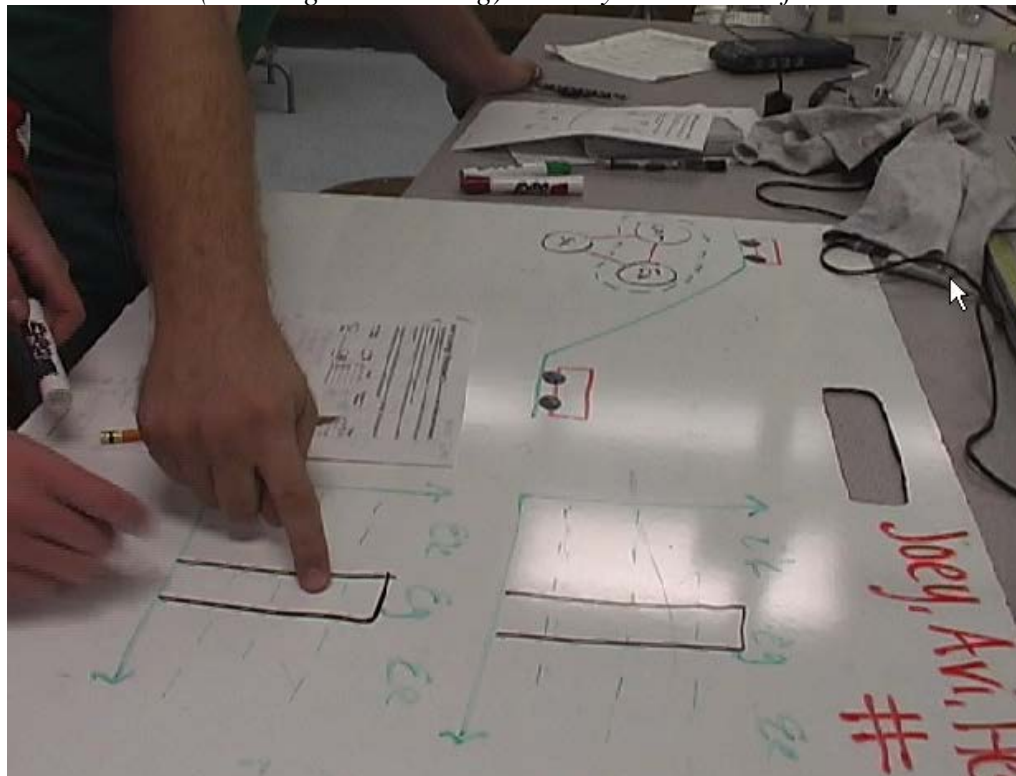


Figure 13. Jimmy points out an inconsistency in Hannah's interpretation of the diagram

HANNAH: *Oh, you're right. Why did I draw that there? (erases the bar under E_g in second bar chart)*

JIMMY: *I don't know.*

HANNAH: *(something about that not being there before either). So, and then your final it loses all of its gravitational, right because it goes down. So you're assuming that it just loses that and it's just going to go somewhere else.*

ARA: *Uh-huh.*

HANNAH: *What happened, as it was moving down the hill? Well, for one I just said, like you would, it moved. So you're going to have what?*

ARA: *I do not understand the question?*

HANNAH: *What, what kind of energy do you always have when something is moving?*

ARA: *Oh, kinetic.*

HANNAH: *Right. So you're going to have kinetic energy. But it says that it's losing 10% of the total energy that it puts out. Because of the friction. So you're not going to have, whatever, four bars.*

ARA: *Yeah.*

HANNAH: *You're going to have less than four. But it'll be in this column.*

ARA: *Right.*

HANNAH: *So I'm going to put my pencil there for now.*

ARA: *But how do you know how much? What equation did you use? How did you?*

HANNAH: *Well, by solving it. That's what you do next. You use, we have the three, I don't know Jimmy if you brought yours. When we have the three equations we had an equation for elastic energy, an equation for kinetic and an equation for gravitational. Since we're finding, like you look at the variables that you have. You have a mass and you have a velocity. So, and you also have an initial gravitational energy. So you use that equation. (Flipping pages.)*

ARA: *The gravitational one?*

HANNAH: Right. It's E_g equals m times g times h .

ARA: Times h ? Cause wait, why? Cause you want to know how much gravitational energy you have in the beginning or what? (girl writing on sheet of paper)

HANNAH: Um, well yeah, you have to know that. (Hannah writing on white board)

JIMMY: Maybe I didn't bring it.

ARA: And then you take 10% of that and you have whatever, so...

HANNAH: Okay, well you have to obviously solve it first. We have to find E_g , gravitational, so you use that equation. And then you're going to plug in values for solving.

ARA: What is g ?

HANNAH: G is 10.

ARA: Okay.

HANNAH: So, we have our mass which is 20

JIMMY: 20 times 10.

HANNAH: 20 kilograms. I didn't know the units for 10.

ARA: Newton's per kilograms I think. I do not know.

HANNAH: And then we know our height is 5 meters.

JIMMY: 5.

ARA: So, 200 times 5?

HANNAH: Is 5,000. (she is writing)

ARA: In meters. So, 1,000 joules

JIMMY: Yeah, 1,000

ARA: So then the kinetic energy is 10% of that, er, minus 10%, 90% of that.

HANNAH: Right, so you take 10% of that minus...



Figure 14. Hannah translates what her bar charts showed her into equations

ARA: So how do you do that in an equation?

HANNAH: Like that. (Writing equation out)

ARA: (paper being handed to her by teacher) What is this out of? 30? Or 25?

HANNAH: Okay. So that 900 joules is your E_g value.

ARA: E_k ?

HANNAH: E_g .

ARA: E_k .

HANNAH: G .

ARA: K.

JIMMY: It's E_k .

HANNAH: Well, yeah, E_g , sorry (erases and puts E_k). Yeah, because you plug that into the division.

ARA: So then 900 joules equals...

HANNAH: I'm going to have to erase, these are too big.

ARA: $\frac{1}{2}$? (Hannah erasing on white board) Meters times velocity squared.

TEACHER: Should we show here (something) multiplication?

HANNAH: So then you take this (draws arrow from bottom to top of board)...

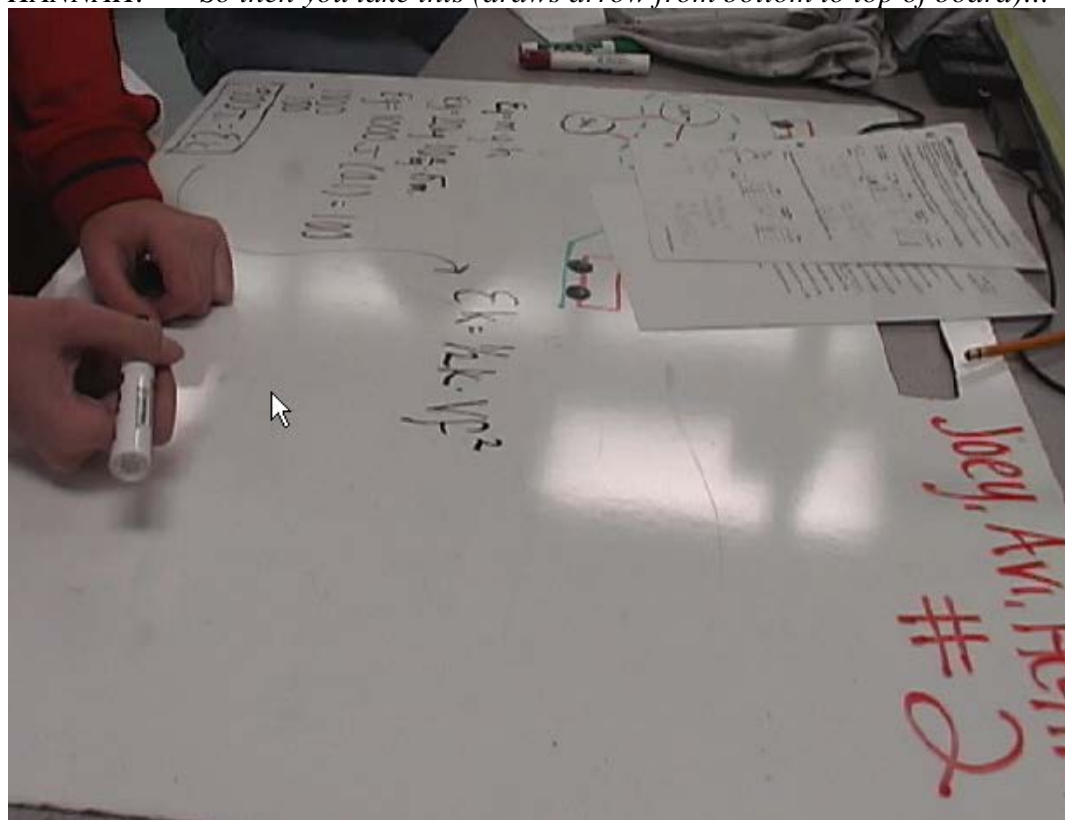


Figure 15. Hannah accounts for energy transfer reasoning from the drawing and from the equations she replaced her spatial representation with

ARA: Wait, wait, wait. But isn't that supposed to . . . okay, never mind. So even when you're at zero you still have gravitational energy?

(showing equation $E_k = \frac{1}{2} k \cdot v^2$)

HANNAH: (mumbling equation) Anyway (laughing)...

ARA: (laughing) Okay. So you're not just going to say the gravitational energy is zero just because you're on the ground, right?

HANNAH: Right. But I don't think, in your final thing, you don't have to acknowledge gravitational energy. (looks at Jimmy)

JIMMY: You mean in your graph.

HANNAH: In your graph. (there is a little more discussion surrounding the units of the final answer and after she finishes solving, she redraws energy bar charts that are quantitative this time at the top of the board.)

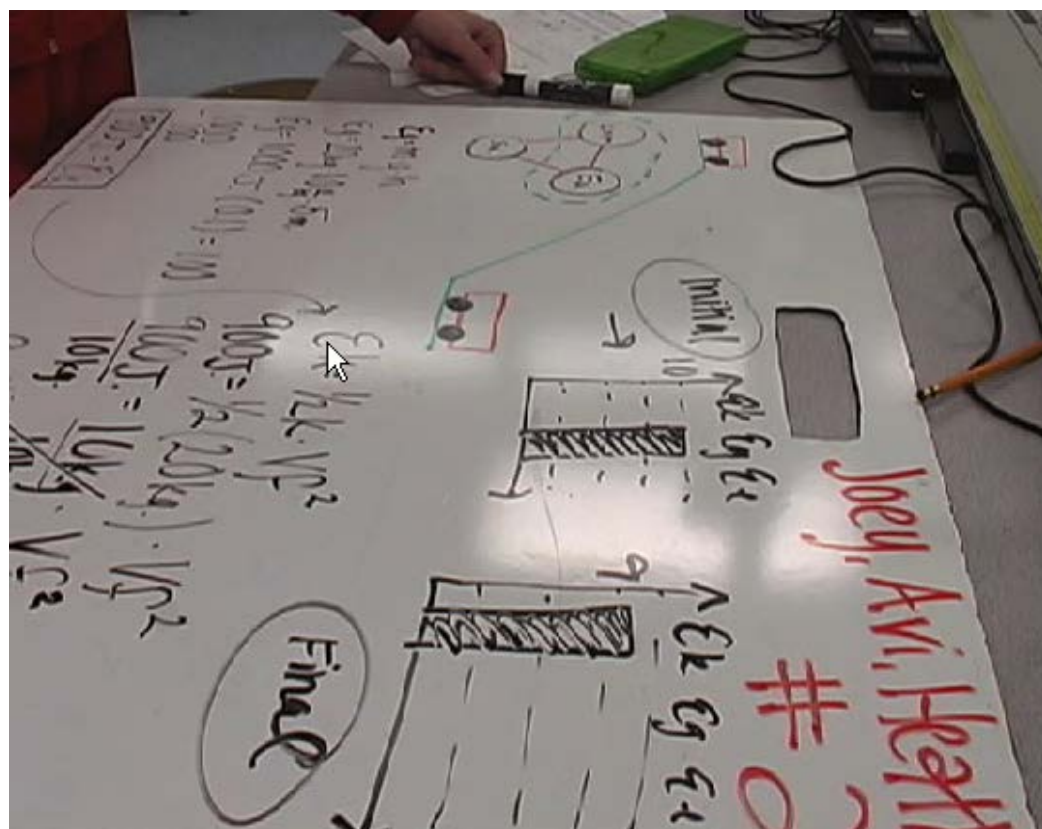


Figure 16. Hannah reconstructs her energy bar chart diagram, this time quantitatively.

(DHS 12-08-05)

At this point in the semester, Hannah has become adept at making and using spatial representations to help her reason about problem situations. The first thing

constructed on this whiteboard is a diagram of the physical situation (the car and the hill) that shows both initial and final conditions. She abstracts information from this representation and re-represents it (incorrectly at first) in bar charts. As she does this, she is clearly thinking of energy as “stuff” that moves around in a container-like system that is made up of sub-containers—for kinetic energy, gravitational energy, elastic energy and dissipated energy. She uses words like *having*, *losing*, *adding*, *putting* and *taking*. When she talks about energy transfers, she talks about energy *going down* or *going somewhere else*. This is more of a journey metaphor, which parallels the “journey” of the cart as it goes down the hill. Only after she has mapped the problem’s conceptual structure to spatial representations does she begin symbolizing the information algebraically. She helps her group-mates through the calculations of energy transfer with questions similar to those often asked by her teacher and when she is finished with the computations, she redraws the energy bar chart representations that were initially qualitative and makes them quantitative.

Clues to students’ mental models

Whiteboards illustrate or at least imply:

1. How students see the problem space—the elements of the conceptual model that they bring to the problem space and how well or poorly they are defined
2. How they fit together (structure, or at least correspondence)
3. How the information in the problem space maps onto these elements of their conceptual model
4. How they navigate their conceptual model to answer the question that the problem poses

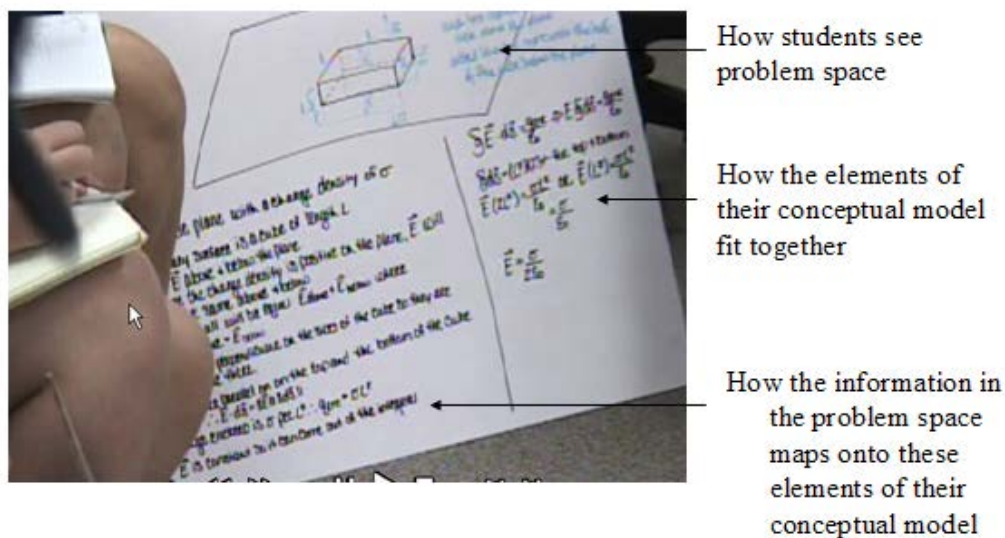


Figure 17. EMCC college physics students conceptualize electric field

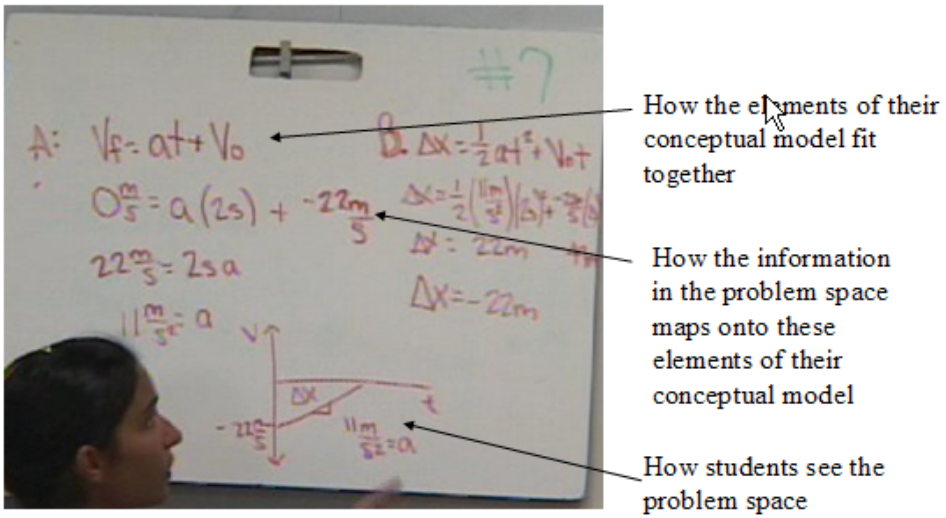


Figure 18. DHS physics students consider the case of an object moving backward and slowing down

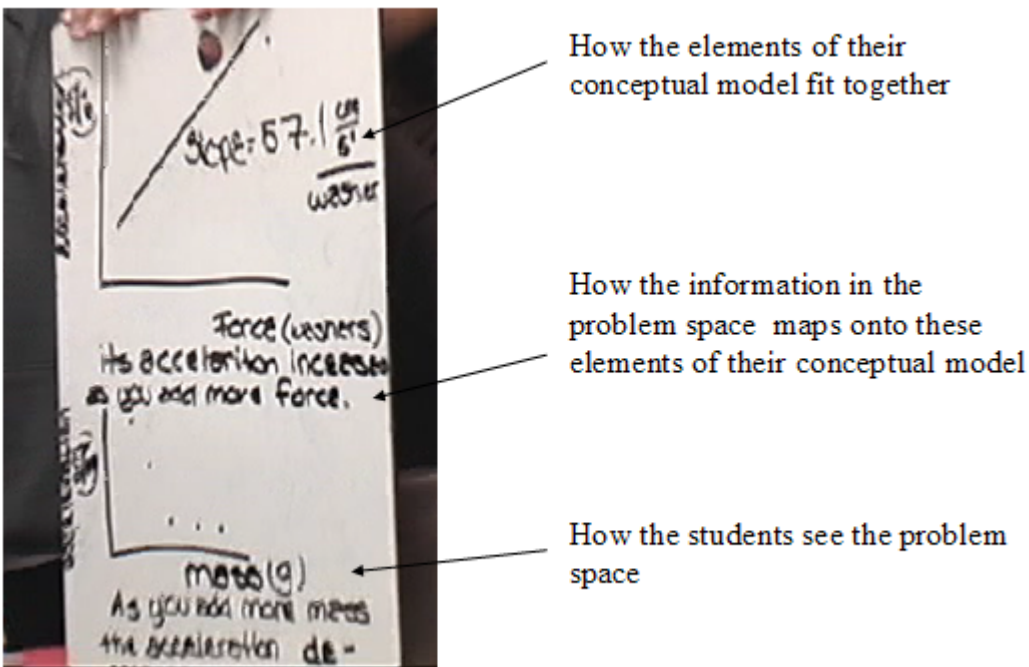


Figure 19: CCHS science students grapple with the relationship between force and acceleration

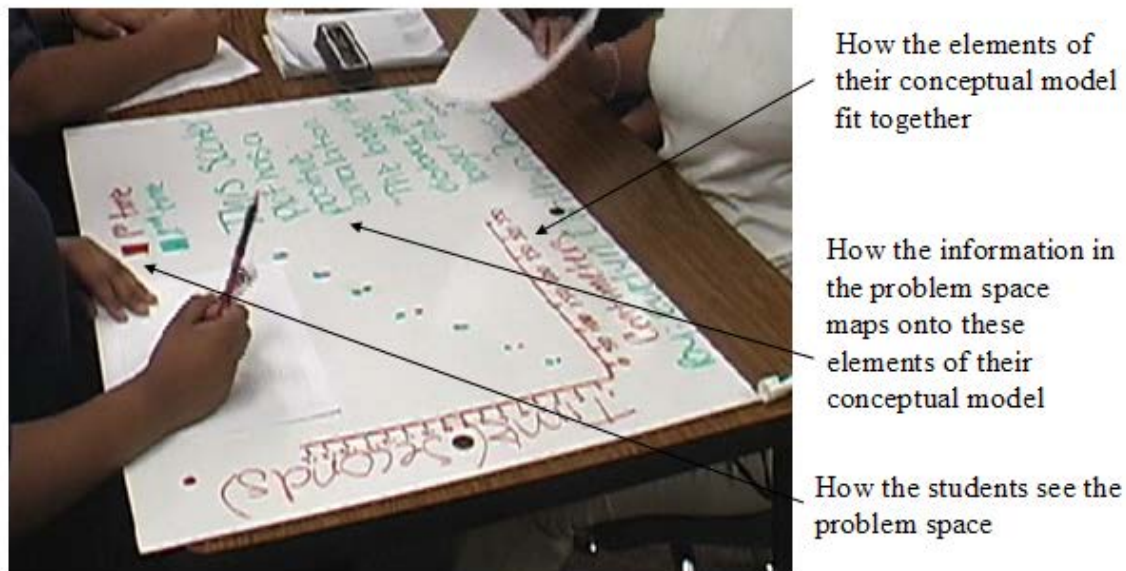


Figure 20. WTMS mathematics students represent the relationship between time and position

The extent to which whiteboards actually show these things is a function of “the rules of whiteboarding” in an individual classroom. Over time, a set of conventions about what should and should not be included on a whiteboard evolves in every classroom. Here are some common ones:

- A whiteboard must contain the names of the members of the group.
- A whiteboard must include graphical, mathematical and diagrammatic representations of the problem.
- A whiteboard must “show work”, i.e., the computations that lead to the answer.
- A whiteboard must show the answer to the problem.

In theory, these appear to be good rules. In practice, they may hide important evidence of how students thought about the problem as they solved it. The most common missing step in the solution process that I observed early in the semester, was a tendency to move straight to the algebraic solution pathway without making or even discussing a graphical or geometric representation, and then once the agreed upon “right answer” was found algebraically, students would go back and construct the required graphical or geometric images that would support the correctness of their solution. In some cases, students omitted this final step entirely recording only an algebraic solution on their board. That they were able to do this repeatedly without comment from the teacher could indicate that the algebraic solution, culminating in “the answer” was “what counted” for these students in this problem solving exercise.

Jen: Okay for number 2 it says a bus traveling at 20 m/s accelerates at a constant speed of -1.5 meters per second squared. What is its final velocity? So we used the equation of the final velocity equals acceleration times time plus the initial velocity and plugged in -1.5 m/s² for acceleration, 7.0s for time and an initial velocity of 20 m/s, and we got 20.5 m/s for the final velocity.

Bonnie: Then for the second one if the car accelerates from rest at a rate of 5.5 m/s^2 how long will it take to reach 28 m/s , and we used the same formula [she appears to be explaining how she plugged values into the formula but she is turned away from the camera and speaking softly... she ends by saying that they got 5.09 s but her voice went up into a question at the end of her statement]

TEACHER: I got a different solution.

Jen: Oh, I was looking at number one. Sorry.

TEACHER: That's everything else stays but you put the number in wrong—7 times 1.5 is 10.5

Jen: I just copied down the wrong number. Same answer. I just put the wrong one... (after a pause, the class claps politely and the students return to their seats.)

Figure 21. No spatial representation is presented here by Jen and Bonnie, and none is asked for.

(DHS 9-30-05)

One way to know where a student's thinking begins or what they think is most important is to look at what is written largest on the board and what is written on the top left side or top center of the board. This is not a foolproof strategy as students may go back and redo their whiteboard after they have agreed upon what it should say and how it should look, but, in general, I found that the whiteboard scribe started in the upper left quadrant of the board, or in the upper center of the board.



Figure 22. In general, the scribe starts in the upper left or upper center quadrant of the whiteboard.

Another clue to what students' believe is important is how they talk to the class about what they have shown on their board. If their board shows a graph, a motion map and an algebraic solution, and all they mention is the algebraic solution, this may mean that this is the only information on the board that they think counts for the listeners.

While the teacher must be aware of *what is missing* in the activity of modeling and model sharing—i.e., what are the elements, relations, operations and rules embodied in the model that the student brings (or not) to the problem space, and which of these are missing from the whiteboard explication—it is not necessarily optimal for the students if the teacher assumes the entire burden of questioning about these aspects. Students must know how to probe one another, and they must feel that asking these questions is expected of them.

They learn how to do this probing by imitation. In virtually all of the classrooms I observed, the students who were active participants in small groups employed the same questioning strategies that the teacher employed.

In the DHS 12-08-05 transcript excerpt, above, of Hannah explaining energy transfer, she questions Ara and Jimmy as she helps them think through the problem:

HANNAH: What happened, as it was moving down the hill? Well, for one I just said, like you would, it moved. So you're going to have what?

ARA: I do not understand the question?

HANNAH: What, what kind of energy do you always have when something is moving?

(DHS 12-08-05)

These questions mirror the sorts of questioning that their teacher routinely does in whole class discussions and whiteboard sessions. Here is the teacher questioning the class on the previous day about a lab activity they had just completed:

Teacher: When we let the cart go, what happened to that energy in the system?

And a few minutes later:

Teacher: What kind of pattern are you seeing?

Students in the community college class frequently questioned each other during whiteboard preparation and board meetings with the types of questions their teacher liked to ask:

Student: So would the concept work both ways?

Teacher: Would it be okay to pretend that it's an infinite plane?

Ninth grade Physical science students question each other as their teacher does also:

Student ...still if you were there, it would be like that, but since you're on the globe would it?

Teacher...I'm holding it here, right from the bottom here, Antarctica means I'm down there, so does it,... when I hold it, is it going to be like that?

One strategy some teachers employ to stimulate student participation is to call on them by name and ask them if they agree with the whiteboard presenter, what they think about some point in the presentation, or what something means. Another strategy involves calling out students who look puzzled or confused and prodding them to share their doubts with the whole group. Since translating their conceptual structures to spatial representations is the step that students most often eschew, this is good place to press for additional input with respect to sense-making. Here are two examples in which teachers use this strategy—one from the community college from the discussion about the infinite plane and the second from the 9th grade physical science class in which students try to come to grips with gravity:

TEACHER: All done? Everybody got it? I see some quizzical looks. What about you Jack?

JACK: (tentatively) Makes sense.

TEACHER: So, does it matter how high above your infinite plane you are? Does that make sense to everybody? Have you ever seen a situation where no matter where you are something is always the same?...

JACK: I'm still a little confused.

(EMCC 10-3-06)

TEACHER: Why do you say yes, James?

JIMMY: I didn't say yes.

TEACHER: You didn't say yes? You threw your voice?

JIMMY: Uh-huh.

TEACHER: *Okay, so you're saying no?*

JIMMY: *I don't know.*

TEACHER: *Does it treat both of these the same?*

JIMMY: *Yes.*

TEACHER: *Why are you saying yes then?(laughter)*

JIMMY: *I don't know. Gravity is pulling...?*

(CCHS 3-01-06)

"Big D" Discourse in physics

Modeling done well involves exteriorization of thought process, comparing one's model to the models of others, subjecting it to reasoned analysis, justifying or discarding it, and identifying its boundary conditions.

MARK: *...but still if this is a cross section then yeah I guess you have to include the wire. But if this isn't really a cross section, this is just an imaginary point, then we're just using this surface.*

RUBEN: *But do you see what it is, the whole point of Gauss's law?*

MARK: *I'm not totally sure if this is actually representative of a cross section of the entire thing.*

KIKI: *But if it's enclosed in a sheath it has to be included in the wire.*

MARK: *True.*

(EMCC 9-30-05, teacher absent)

Teacher: *...it's changing...it isn't just telling me the velocity is changing but how it's changing...what? Suppose that we have another car that's changing from 30 to 15 but that one took 5 seconds. Would it have the same change in velocity?*

Student: *No.*

Teacher: *Would it have the same? If another car just like that one...or, it doesn't matter...but it goes from 30 to 15...if it has the same...does it have the same change in velocity? (Pause) this is not a trick question.*

Students: *yes.*

Teacher: Yeah. If it goes from 30 to 15 it's the same change, right? But if the other one does it in 5 seconds does it have the same acceleration?

Students: No.

Teacher: No. Why is that? What's different?

Students: The time...

(CCHS 2-6-06)

In the preceding two examples, students, in one case on their own and in the other with the help of the teacher, explore the implications of the various elements of the model they are using and their relationships to one another.

Whether or not students are engaging in these activities can be seen in the features of the Discourse (remember James Gee's definition of Big D discourse as the language of a particular community, in this case the communities of physicists or mathematicians, that carries with it particular identities, activities and cultural "stuff") that they attach value to, and what function their Discourse serves for them in the activity of modeling. The Discourse is not the model, but it is an important part of playing the modeling game—it is one of the "rules of the game". At times, however, it can be mistaken for the game itself.

In the following excerpt, honors physics students channel a Greek chorus chanting responses they have learned from repeated drilling over the previous eight weeks, as their teacher talks them through how the conclusion for their laboratory report should be written during a board meeting:

*Teacher: ...Can everybody read what they've got written in yellow here?
[murmuring] Yeah, the text that they've got written in yellow at the bottom right here...*

Chorus: For every one kilogram in mass 9.875 Newtons is increase in force.

Teacher: What's that? What have they done already?

Chorus: Physical interpretation.

Teacher: Yeah. They've done the physical interpretation of the graph. So what was the shape of all the graphs?

Chorus: Straight diagonal line passing through the origin.

Teacher: How is it that we know that they're straight diagonal lines?

Chorus: Linear fit with a correlation of one.

DHS 10-20-05

This “Greek chorus” was often a feature of the board meetings I observed in this classroom, and occurred from time to time during whole class discussions as well. This sort of “target practice” style questioning that called for one or two word answers or the completion of sentences was common in all four of the classrooms observed, but in the classroom excerpted above there was a particular emphasis on using the language of physics. Here the teacher is explicit about his expectation for the students to engage in Big D Discourse:

Student: The graph of elastic energy would be directly proportional to the final velocity squared.

Teacher: How would we know it's directly proportional? Again, this is the important subtle part of the language that you need to get down.

Chorus: By the correlation.

Teacher: What kind of...what would the graph look like to tell you that it was a direct proportion?

Chorus: Straight diagonal line.

(DHS 12-7-05)

Discourse can be used to clarify a student's ideas regarding their conceptual model. It can also be used, however, to constrain or limit those ideas. It is possible to “speak physics” without a clear understanding of the meaning of what one is saying—not unlike the way people bandy about phrases in another language without knowing that language (i.e., mea culpa, mazel tov, coup d'état). If students believe that they can earn points for “speaking Physics”, they will attempt to do so whether or not their words have meaning for them. This is neither big D Discourse nor is it little d discourse (language in use). It may simply be an incantation (i.e., Dominus vobiscum. Et cum spiritu tuo) they believe they are expected to utter at a specified time.

Teacher: No. And actually what we were trying to find out is.....the kinetic energy. Our big idea was that we were putting elastic energy into the system in the beginning and we were having that elastic energy be shifted into the kinetic energy account in the moving cart. So is kinetic energy directly proportional to velocity? (no answers) Is the velocity v. elastic a straight diagonal line?

Ara: No.

Teacher: No. Kinetic energy is not directly proportional to velocity. What shape did the elastic energy v. velocity look like?

Ara: Top opening parabola.

Teacher: A top opening parabola. What's the implication of a top opening parabola?

Ara: That elastic energy is proportional to velocity squared.

Teacher: Again a little bit louder.

Ara: That elastic energy is directly proportional to velocity squared.

Teacher: How are you going to check and see if Ara's point about elastic energy being directly proportional to velocity squared is correct?

Chorus: Test plot.

(DHS 12-7-05)

Distributed Dimension of Whiteboard Mediated Cognition and Modeling

The theory of distributed cognition indicates that the social organization of the group forms a cognitive architecture that determines how information flows through the group. The modeling classroom consists of different kinds of groups and different activities in which these groups participate. The next section will provide “snapshots” of the ways in which cognition is distributed across individuals, tools, artifacts and inscriptions in the different classes I observed.

Small group labs

In laboratory activities, groups of two or three students typically work together utilizing some sort of apparatus whose use is demonstrated by the teacher during a pre-lab discussion. There is a common understanding of the task and the goal of the exercise, which is to discover, in the data they collect, some relationship between two physical quantities. Data, regarding motion or force for example, is frequently collected via computer interface and displayed on a spreadsheet that allows for the automation of certain mathematical manipulations, i.e. linearization or curve fitting. Computerized data collection tools are not always in use, however. Some activities rely on such low-tech tools as meter sticks, spring scales and stop watches.

The student groups I observed invariably had one student, whom I will call The Operator, often the same student in lab after lab, who took the lead in setting up and operating the apparatus. The other members of the group might take the initiative for creating a data table or setting up the computer interface or they might wait to be asked or told by their teammates what to do. Another role that I saw repeatedly was The Measurer. This task usually fell to the same student in each lab. They were the person that read the ruler or the spring scale, sometimes in consultation with others, but often alone. In labs where data was collected on the computer, The Measurer was usually the

one to decide the intervals at which data points would be collected and how many points the group would collect. Another common role was The Data Manipulator.

Table 4

Small group laboratory activity (Note: at times the same individual may play more than one role)

<i>Title</i>	<i>Role</i>	<i>Leader or follower?</i>	<i>Proactive, reactive or passive?</i>
Operator	Manipulates apparatus	Leader	Proactive
Measurer	Uses/reads measuring tools	Either	Reactive
Scribe	Records information	Follower	Passive
Data Manipulator	Manipulates/represents data	Leader	Proactive or reactive

Among the senior high school and community college students, this role shifted among the members of the group over the term. At Darnell High School, this appeared to be related to the growing confidence of individuals in their use of the software. At Echo Mountain Community College, the task seemed to fall to whatever member of the group was not otherwise occupied. In the McKinley middle school mathematics class and the Carlos Cadena High School physical science class, The Data Manipulator was often the same person each time. This person was frequently the same person who filled The Measurer role. When it came to preparing the whiteboard of the lab results, it was typically The Data Manipulator who controlled what was written on the board, sometimes by doing the writing themselves and at other times telling The Scribe what to write.

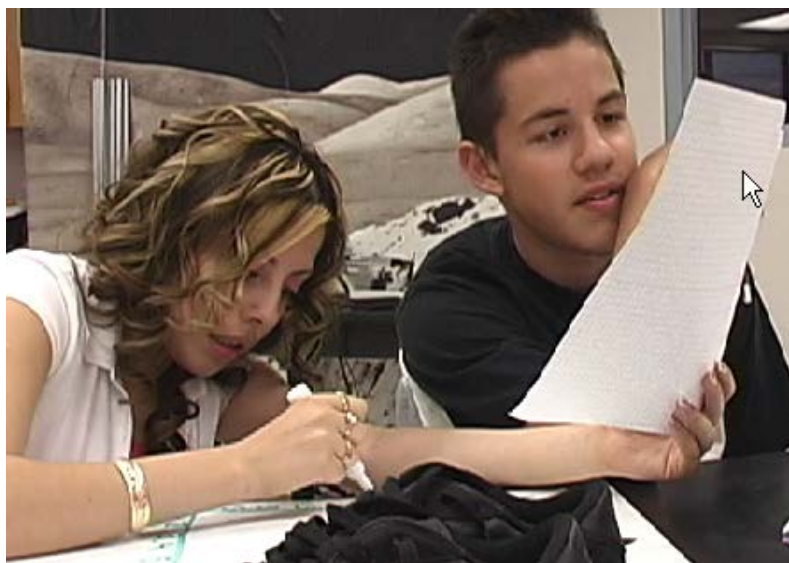


Figure 23. Jose, who functioned as both Measurer and Data Manipulator, tells Sophia, The Operator and Scribe for this lab how to construct a graph of their data



Figure 24. Stephen is both Measurer and Data Manipulator for his lab group

The older the group of students, the more effort went into making sure that all the group members understood and bought into what was written on the whiteboard. With younger students, if there was a group member who did not enter the conversation about what was written on the board, they were usually ignored. Even though these peripheral silent students were frequently questioned directly after their whiteboard was presented, they seemed unconcerned about appearing uninformed before their peers or their teacher, shrugging off questions with an “I don’t get it”.

In general, although students at all grade levels engaged productively in small group collaborative activity, younger students were more apt to be concerned for themselves—not just in terms of how their work would be evaluated by the teacher and their classmates, but whether or not they understood what they were writing—and older students were more apt to be concerned for the group as a whole.

The cognitive architecture of the group was vertical and hierarchical in the middle school and ninth grade classes, with easily identifiable followers and leaders, although the floor changed hands regularly and was not always controlled by the group leader. Information was passed back and forth between group members so that the best possible whiteboard could be constructed. Reasoning about their solution was usually collaborative and usually mediated by their whiteboarded inscriptions, but students did not always hold each other accountable for the conclusions they reached, as long as they were sure that the answers written on their whiteboards were “right”. Senior high school and community college students were more apt to press one another for justification of strategies, tactics and answers, both in small group and in whole group conversations.

Gabe: My only question is would E be constant for a square?

Kiki: Yeah. Because it’s a cube. And that’s the height of your cube...the plane goes right through the middle (gestures with a marker held between her fingers, orthogonal to the plane of her hand...)

Gabe: Well but that’s...

Kiki: What about it? The field lines go straight up from the plane. I mean we've never drawn the field lines diagonally...

Gabe: Well, what I'm saying is... (he gets up and walks over to her board so he can point to particular spots on her drawing)... I guess what I'm trying to say is... it's... from there to there it would be "l" (points from the center of the intersection of the cube with the plane to a point directly above it on the surface of the cube) but from there to there it wouldn't be "l" (points from the same origin to one of the corners of the cube)

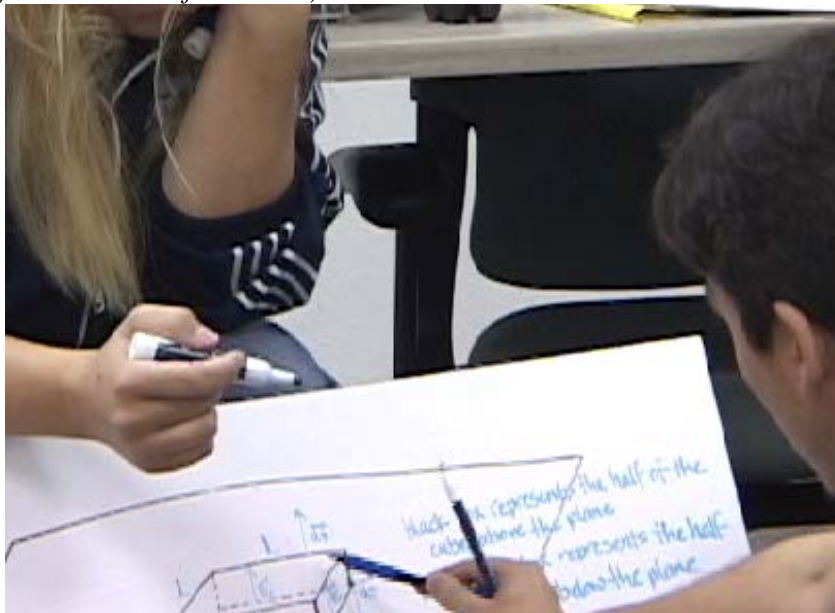


Figure 25. ... but from there to there it wouldn't be "l"...

Mike: ...the electric field wouldn't go like that would it?

Gabe: What I'm saying... (stops for a moment and then walks back to his seat)... never mind. I'm wrong. It's the same.

(EMCC 10-3-05)

In this excerpt, Gabe questions Kiki's choice of a cube to represent a region that enclosed an electric field of uniform density. Kiki justified her choice by invoking the rule about drawing field lines perpendicular to the charged surface.

Small group worksheets

The architecture of small group whiteboarding of worksheet problems was somewhat different. In these exercises, students were usually given a fixed amount of time to represent their solution to some problem from a worksheet they had been asked to do for homework. At other times, they were solving problems that they had seen for the first time only a few minutes earlier and they had had no opportunity to consider them at

any length prior to engaging with the whiteboard group. There were interesting differences in the way the activity unfolded in these two cases.

Going Over Homework – The Power of the Marker

When the activity structure was that of Going Over Homework, the group member who assumed leadership was most often the one who created the important parts of the whiteboard. It was this Decider’s version of the solution that was written down, at times in spite of what other members of the group contributed. At times, some group member would insist that something be added to the solution and The Decider would capitulate and add it to what he or she had written, but, in general, The Decider maintained veto power over the other students’ contributions. I have come to think of this phenomenon as The Power of the Marker. Controlling the marker was, largely, controlling the floor in the whiteboard discussion. The Decider in this setting has the deciding vote in determining what counts out of all the things that students say to each other in constructing a whiteboard representation, and she exercises this control by either writing down or not writing down what other members of the group contribute to the discussion. If there is competition for leadership in the group or if The Decider does not give sufficient attention to what other members of the group contribute, another factor enters the picture: The Power of the Eraser. This happens when some other member of the group than the scribe, picks up the eraser and erases what The Decider has written. Sometimes this will result in a transfer of the marker to the person doing the erasing, but more often it will result in a recreation by The Decider of the representation that is more in line with what The Eraser or other members of the group had in mind. The Eraser exhibits a variety of motives, often fairly benign, including neatness or clarity of expression, but sometimes the motive is clearly control over the content of the finished product. If we look at whiteboard construction in this context through the lens of conversational analysis, we see a lead individual who controls the floor, with challenges that could be considered “turn-sharking”—taking away another student’s turn to speak—or “turn dolphining”—rescuing another student or saving some student from a mistake in her representation. At times The Decider voluntarily relinquished the floor to another member of the group without being challenged. When this happens The Power of the Marker is usually also transferred to this person.

Table 5

Small group whiteboarding - going over homework

<i>Title</i>	<i>Role</i>	<i>Leader or follower?</i>	<i>Proactive, reactive or passive</i>
Decider	Decides what should appear on whiteboard	Leader	Proactive
Scribe	Writes on whiteboard	Follower (when this is not the same person as the Decider)	Reactive
Eraser	Erases what is shown on whiteboard	Leader (turn as leader begins with erasing the whiteboard)	Proactive

Discusser	Discusses what appears or should appear on the whiteboard	Follower	Passive
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Students' goals have a significant impact on the power relations among group members. If group members share the goal of Getting The Right Answer, this has considerable influence on their movement through the problem space as they whiteboard their solution. Often, in this case, The less certain members of the group have very little input in the preparation of the whiteboard. When it comes to the presentation of their board to the class, the Decider is the first, and frequently the only group member to speak, unless the teacher calls on some other member of the group to give the explanation. This, of course, assumes that students' goals can be inferred from what they say aloud or write down.

(A group of three boys, Juan, Jorge and Kevin, nearest the video camera is working on question number four. Jorge gets a meter stick from the back of the classroom and he holds it steady as Juan begins measuring and making a carefully spaced row of dots on their whiteboard. Then after some thought Juan writes on the board $\frac{4 \text{ cm}}{1 \text{ s}}$. He considers what he has written for a few seconds. His partners do not comment. Juan caps his marker.)

Camera operator: "How do you know?"

Juan: I measured the distance from one dot to the next and in each one was four centimeters, and since there is a dot for each second, this means that the object was going four centimeters per second.

...

(A short while later the boys are presenting their whiteboard to the class.)

Teacher: ...Okay you might you might want to pay attention in case you struggled with this on your homework...go ahead guys...

Juan: Okay, from right here, it goes one second per every dot it hits and right here it goes four centimeters from that dot to that dot, so it goes four centimeters per one second...so that's all we got for the centimeters per second.

Maria: What did you find?

Juan: Average acceleration...I mean average velocity.

Teacher: Is it accelerating?

Juan: No.

Teacher: How do you know it's not accelerating?

Juan: Cuz from there to there it goes four centimeters per second and from there to there it goes also four centimeters per second...and from there to there it goes four centimeters (points to each dot in turn) and it keeps going on at a constant speed.

Teacher: Okay. Jorge, If we looked at it ten seconds later, how far would it be from that first dot?

Jorge: I don't know. I don't get this stuff.

(CCHS 2-13-06)

In Going Over Homework, sometimes students were in discourse with each other, and at other times, they appeared to be in dialogue with themselves or with their representation. This latter was the case in the previous excerpt. If the camera operator had not asked Juan how he arrived at his answer from his representation, it is likely that he would have kept his reasoning process private. At times students talked aloud as they were constructing written representations although their words did not appear to be directed to anyone in particular. This was more common in the honors physics and community college classes and may have been related to their greater level of concern that all members of their working group understood the inscription they were constructing.

Practicing With the Model

The activity structure of Going Over Homework differs from that of Practicing With The Model in that The Decider is less apt to be the same person throughout the episode when students are practicing with the model. The marker and the floor change hands regularly during the negotiation of what will ultimately appear on the board, whether or not The Eraser intervenes. There is real crosstalk in this activity structure, with students speaking directly to one another, rather than talking to the teacher about what another student has said or written (something that occurs frequently in whole-group whiteboard presentations, as we will see in the next section). In Practicing With The Model, group members typically contribute to the discourse as co-equals, and in addition to The Decider and The Eraser, described above, other members of the group who do not fill these roles usually participate in the discussion of what should appear on the board. Both turn-sharking (the pre-empting of one another's turns to speak) and turn-dolphining (the protecting or rescuing of one another's turns to speak) may take place in these conversations but they are less about who controls the discussion than about determining what path the reasoning process should be taking.

Table 6
Small group whiteboarding - practicing with the model.

<i>Title</i>	<i>Role</i>	<i>Leader or follower?</i>	<i>Proactive, reactive or passive?</i>
Decider/Scribe	Writes what appears on whiteboard	Leader	Proactive
Eraser	Erases what is shown on the whiteboard	Leader	Proactive
Discussor	Discusses what should be shown on whiteboard	Follower	Reactive

There is more erasing and rewriting in an episode of Practicing With The Model than in Going Over Homework as students try out different ideas that occur to them on the fly by writing them down. Sometimes they are writing these partially formed ideas for the benefit of other members of the group, to help them communicate what they are thinking, and at other times they are writing them down so that they can better visualize how the various elements stand in relation to one another and how they can be manipulated. This may also be an effort to manage cognitive load. If they offload their ideas in the form of written inscriptions, they can free up their working memory for other information. One side effect of this practice is that it serves as a “think-aloud” exercise that allows the other member of the group to know how the person writing is structuring their conceptual system.

When using the whiteboard in this way, as a medium for communicating partially formed knowledge structures, some students appear to prefer to ‘think aloud’ in spatial representations and others prefer to ‘think aloud’ in algebraic representations. Those whose preferred mode of written communication is algebraic are more often talking to themselves than to other members of the group. This is confirmed by the fact that although they may talk as they write, the talk is seldom directed to anyone in particular, and it is seldom answered by the talker’s group-mates.



Figure 26. Christina reasons about the problem space by constructing a diagram

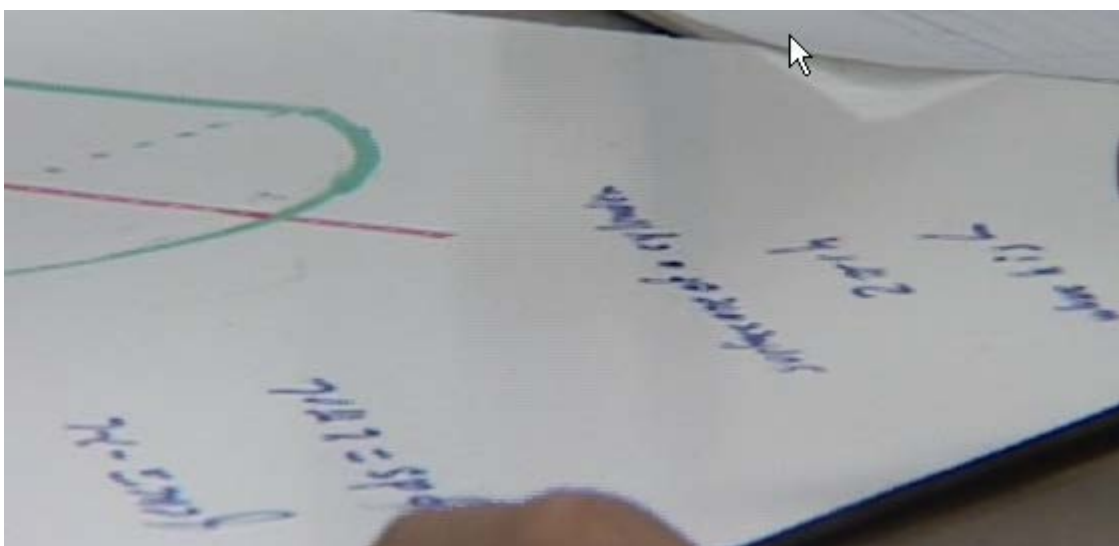


Figure 27. Gabe reasons about the same problem space as Christina above, but he uses equations.

In general, it appears that Distributed Cognition across multiple individuals occurs more often when students are Practicing With The Model than when they are Going Over Homework as the floor and the marker change hands during discourse.

There are two ways that the whiteboarded information described above is typically shared with the whole class: formal presentations, where groups take turns and members of each small group all stand in front of the class and present what they have written on their boards; and board meetings, where the class forms a circle so that everyone can see everyone else's whiteboard at the same time and compare the different boards with one another. These will be described separately in the next two sections.

Whole group whiteboard presentations

The activity structure of whole group whiteboard presentations is fairly rigidly scripted. The teacher controls the floor by nominating some group to present their whiteboard. The group comes to the front of the room and one of its members takes the floor, initiating the Presentation. Often the speaker begins by reading the question that their whiteboard presentation is about. Lemke calls this External Text Dialogue (Lemke, 1990), where the text or worksheet itself is given a voice in the presentation (usually a proxy voice for the teacher). The tone of the presentation changes audibly between External Text Dialogue and Presentation, presumably because the Presentation requires the student to outline their own reasoning about the question while the recitation of the External Text Dialogue is simply a restatement of the information that is given. The presentation typically consists of however many separate parts there are to the problem that is being solved. If the whiteboard presentation is for Going Over Homework, the problems often have multiple parts. Each is described as a separate exercise often preceded by its own bit of External Text Dialogue. Within the Presentation, the speaker usually first identifies the formula that they will use, the pieces of known information that are necessary to solve the problem, and finally, the type of information The Answer is, i.e., a force, or a velocity. Sometimes the speaker will say how they know the various pieces of information that they have identified as necessary, i.e., from the graph, or from the answer to the previous problem. Then they briefly describe how they solved the problem, usually talking about steps in the computation process. They finish with The Answer, which is followed by The Pause, or a Bid for Questions that is followed by The Pause, typically a two to five second interval of time in which Questioning is expected to begin. It is important to note that in whole group whiteboard presentations, the speaker is usually looking at and talking to the teacher. This was true in all three of the classes that employed this format for whole group whiteboard sharing.

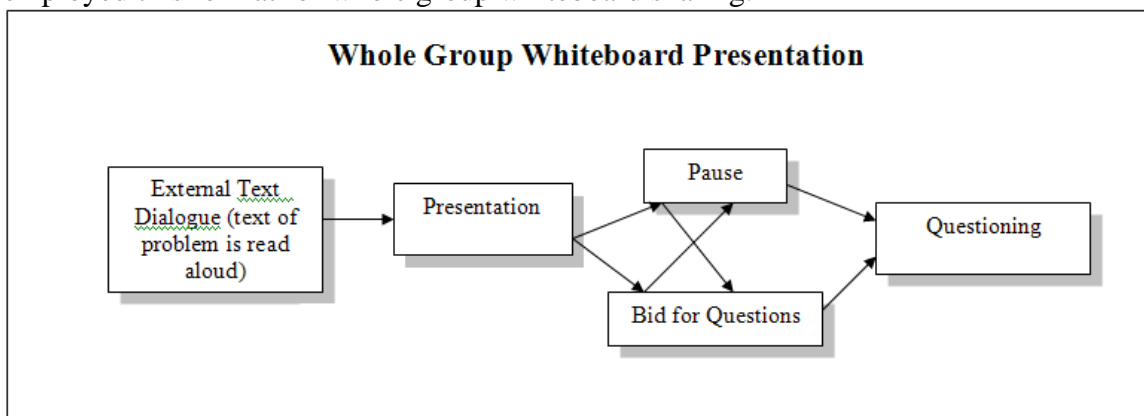


Figure 28. The typical structure of a whole-group formal whiteboard presentation.

Different classes handled The Pause in different ways. In the middle school mathematics class, students often waited for the Speaker or the Teacher to Nominate them before speaking, raising their hand to indicate that they had something to contribute. In the high school classes, students who had questions sometimes just called them out and at other times raised their hands, but most of the time they sat silently, without

making eye contact with the presenters, with their teachers, or even with each other, apparently waiting, either for the teacher to ask the questions or for the group that presented to be allowed to take their seats. The community college class did not employ this formal presentation format so their response patterns will not be considered here.

If there was any teacher questioning, it usually followed The Pause. Often the first question the teacher asked was, “Any questions?” If the audience still had nothing to ask there were several questioning strategies that the teacher employed. He might begin questioning a particular member of the presenting group that he felt had not yet contributed, or just address a question to the presenting group as a whole or he might ask the class what they thought about some detail of the presentation. Sometimes this evolved into a Triadic Dialogue with one or more students, where the teacher asked a question, a student answered and then the teacher offered some sort of evaluation of their reply. This triadic dialogue might take on the form of “target practice” where the teacher has a particular answer in mind and he keeps rephrasing the question until the responding student gives him the answer he is looking for. In most cases, the signal to end The Presentation came from the teacher, either by way of an expression of thanks to the students or directions to the class to “give this group a hand.” Sometimes the students stole this turn from the teacher by starting to clap during The Pause, usually when the presenting group looked like they were making preliminary moves toward their seats. Sometimes the teacher vetoed this by beginning the questioning and other times he would let it go and call on the next group of presenters.

TEACHER: Let’s go on to number 4.

Paul: For number 4 we used final velocity equals acceleration times time value plus initial velocity. Then we plugged in the given numbers and got 9.05 m/s. (Pause) Anybody have any questions?

Their board says:

$$V_f = at + v_0$$

$$3.0 \text{ m/s} = (-2.1 \text{ m/s}^2)(t) + 22 \text{ m/s}$$

$$\underline{-19 \text{ m/s} = (-2.1 \text{ m/s}^2)(t)}$$

$$-2.1 \text{ m/s}^2 \quad -2.1 \text{ m/s}^2$$

$$t = 9.05 \text{ m/s}$$

TEACHER: No questions?

Bob: Nope.

TEACHER: Alright. Let’s give these guys a hand.

...

Olivia: Okay, so the problem is an airplane starts from rest and accelerates at a constant 3 m/s^2 for 30 seconds before leaving the ground. How much runway does the plane cover before it lifts off? So we know the initial velocity is zero meters per second and we know the acceleration is 3 m/s^2 and we know the time is 30 seconds. What we’re trying to find is the change in x so...

The WB says

$$\begin{array}{ll} 5. v_0 = 0 \text{ m/s} & \Delta x = 1/2at^2 + v_0t \\ a = 3 \text{ m/s}^2 & \Delta x = 1/2(3\text{m/s}^2)(30\text{s})^2 \\ t = 30 \text{ s} & \Delta x = 1350 \text{ m} \end{array}$$

Sean: ...it's plug and chug...

Olivia: The change in x equals 1350... One thousand three hundred and fifty meters...

Sean: (Pause) Questions? (after about a second of silence) That's it. (They sit down as their classmates clap. The next group gets up and go to the front.)

(DHS 9-30-05)

In most whole group whiteboard presentations of Going Over Homework, students seemed to be more interested in producing proof (for the teacher) that they solved the problem they had been assigned, than they were in sharing their solutions with their peers. In both the above excerpts, taken from the same class period, the students directed their presentation to the teacher, while the teacher's requests for questions were addressed to the class. Very little time passed (less than 2 seconds) between when the teacher asked if there were questions and when he dismissed the presenting group. Although he went through the motions of asking for questions, is it possible that he did not really expect students to have any?

Whole group board meetings

The activity structure for whole group board meetings is somewhat different from that of formal whiteboard presentations. This activity is usually reserved for occasions when the entire class has worked on the same problem—usually a laboratory exercise or Practicing With The Model problem assigned in class.

The Board Meeting begins when the teacher calls the class to attention and has them get to stand or sit in some sort of circular configuration that allows them all to see each other's boards. Classroom seating arrangements made this somewhat awkward in all but one of the classrooms observed—the community college classroom—which had a large area of open floor space in the center of the room where students could roll their chairs into a circle. They placed their whiteboards on the floor in front of them resting against their knees.

The meeting opened either when the teacher Nominated a particular group of students to begin, when someone volunteered to be first (this latter occurrence was only observed in the community college class) or when someone asked a question of another student group about something shown on their board. The Initial Presentation, if there was one, was fairly similar to the sort given in the whole group whiteboard presentations described in the previous section, but The Pause, was followed by a comparison of the other boards in the circle with the one just presented.

In the middle school and high school classrooms, the conversation that followed The Pause, if any, was usually followed by a teacher-initiated discussion of how other boards were either like or different from the one presented. In the middle school and the two high school classes, the teachers usually retained the floor throughout most of the discussion, periodically calling on different individuals to participate. The middle school students at times managed to sustain discourse without teacher intervention for short periods. When the conversation stalled periodically, the teacher restarted it by asking a question or series of questions. Middle school students did not seem exhibit as much hesitation about taking the floor as the high school students did, as mentioned previously they did not seem to care about whether or not they appeared smart to their classmates.

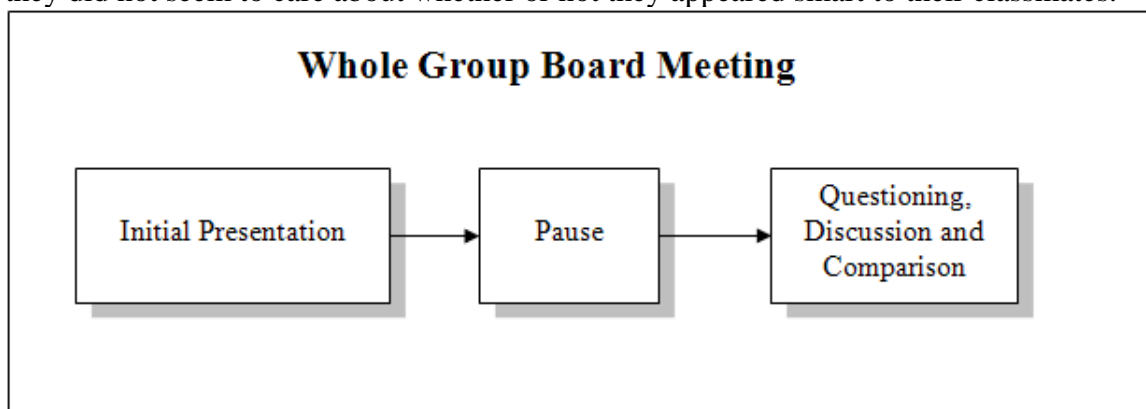


Figure 29. The typical activity structure of a whole-group board meeting.

In the community college class, The Pause was usually brief and questioning by students of one another was spontaneous, often interrupting and asking for clarification while the Initial Presentation was in progress. These differences did not seem to depend on age and maturity so much as on teacher expectations and established classroom norms. The community college students I observed were together as a class for a year before I began collecting data in their classroom. It was a small group and they seemed comfortably familiar with each other. On the few occasions when they had very little to say to each other at board meetings, the teacher prodded them to engage, and chided them if they failed to do so. He watched the non-presenting students closely during The Presentation and if he saw signs of puzzlement on anyone's face, he asked that individual specific questions about the content or the meaning of The Presentation when the follow-up discussion stalled out.

TEACHER: (Kiki is done presenting her whiteboard.) All done? Everybody got it? I see some quizzical looks. What about you John?

John: Makes sense (his tone is tentative).

TEACHER: So does it matter how high above your infinite plane you are?

John: No.

TEACHER: Does that make sense to everybody? A situation where no matter how high you are something's always the same? (Long pause while most of the students sit staring straight ahead) I know it's Monday.

John: When you have a line of infinite charge, length didn't matter.

TEACHER: Length didn't matter...that's good. If I drop my keys right here (about 3 feet off the ground) how would the acceleration of gravity compare if I dropped me keys from up here (holds his keys above his head)? How do we calculate force of gravity? (someone murmurs) A little louder.

Student: Mass times acceleration.

TEACHER: Mass times acceleration of gravity. So it wouldn't matter if it was here or here. So what are we pretending the earth is, when we're doing that?

Student: Infinite.

TEACHER: An infinite flat plane...which we're saying the force is the same. Is the earth really an infinite flat plane?

Student: No.

TEACHER: Not unless you want to go back to the flat earth society. So is it at times okay to pretend something is what it is not? And the answer is...what do you think? (students look at each other but no one speaks for about 10 seconds. Finally someone speaks.)

Student: I guess it's okay.

TEACHER: Okay—three laps around the building till you all wake up...I ask you a simple question like that I don't expect this (blank stare) Is it okay to pretend something is infinite when it's not?

(Murmurs of "yes")

TEACHER: (throws a WB in the middle of the floor...) Let's pretend that it has a charge on it...would it be okay to pretend that this is an infinite plane?

All: yes

TEACHER: Under what conditions?

Student: When it's not a conductor.

TEACHER: Doesn't matter. This is a geometric argument not a physics argument...pause...we're pretending the earth is an infinite plane to accommodate gravity...is it flat? Or even...is the earth flat? ... (students shake heads)...hmmm....so when is it ok to pretend that's an infinite plane?

Ann: When you have to assume it to reach a conclusion?

TEACHER: Well....okay...when you have to...that's a start...that's a mathematical reason...now let's see if we can think of a physical reason.

Ruben: When it's the whole universe?

TEACHER: That's not the whole universe, thank heavens...it'd be awful crowded to fit 4 billion people on there...when can we consider that an infinite plane? What's the definition of an infinite plane?

Mark: When []

TEACHER: Right—does that go to infinity in all directions?

Mark: No

TEACHER: And a plane is how thin?

Mark: Infinitely.

TEACHER: Alright...so that's probably its most plane-like feature it has is its thinness....so when's it ok to consider that an infinite plane?

(EMCC: 10-03-05)

Advantages and disadvantages of the various architectures

Participation, in the form of contribution to the reasoning or problem solving process, was constrained in all cases by the dominance of some person (sometimes a student but often the teacher) who was perceived to have the floor, and by whether or not that person was willing to give it up their control of the floor to someone else. In small group whiteboard sessions, where students were whiteboarding homework problems, there was frequently one student who took the lead, referred to above as The Decider. She was the member of the group who seemed to be most confident in her answer and her ability to demonstrate for others the procedures and reasoning she used to arrive at a solution.

Typically, the Decider held the floor by taking the lead in preparing the whiteboard and answering the questions of other members of the group. When there was

more than one member of a group who exhibited confidence in their understanding of the problem, two people might write on the whiteboard at the same time, one making the graph or diagram while the other wrote the equation and computed the answer. However, when two people were working to prepare the board simultaneously, The Decider was the usually the equation writer and solver, and most often The Answerer of questions posed by other group members. In general, except in the community college class, it appeared that students valued the algebraic solution over the spatial representation—more time was spent talking about it, and there were more conversational references to it than there were to diagrams in both small group and whole group discussions. On the other hand, when conversations turned to questions such as “how do you know...?” or “why did you do that?” The Answerer would often refer to how the diagram was constructed as justification for the algebraic choices made in solving the problem.

More time and attention were given to procedural concerns and algebraic reasoning in cases where students valued getting numeric answers over representing the problem space. This may have been a function of the types of problems they were assigned, i.e., if the problem asked for the acceleration of a 500 gram block sliding down the frictionless surface of a 30 degree incline, students knew that their goal was a numeric value. Problems of this type were the usual fare in the middle school and high school classrooms where worksheets that had been assigned for homework were used as classroom whiteboard exercises. The paradigm labs were exceptions to this, but even these usually resulted in some numeric answer, i.e., “the slope of my graph, which demonstrates that the relationship between change in position and change in time in this situation is directly proportional, was 22 centimeters per second represents the velocity of my battery powered car.” With the exception of the community college students, whose whiteboard exercises most often resulted in diagrams and derivations of formulas rather than “answers”, the students observed showed a strong preference for answers. In the words of Jimmy:

Jimmy: I'm just happy that my answers are right. That's all that matters.

It was the first thing they compared with each other before they began whiteboarding a worksheet problem in small group and “what did you get for number ___?” was the most often asked question when the sat together in their small groups and worked side by side on worksheets during class.

Structuring Dimension of Whiteboard Mediated Cognition and Modeling

If models are systems of elements, operations, relations and rules (Hestenes 2006), then the nature and extent to which students are able to structure their models is fundamental to the usefulness of these models. In this section, I will discuss and illustrate the structuring role of whiteboards and the discourse that surrounds them.

Frames, schemas and metaphors

Semantic frames provide a conceptual structure that defines the relationships among collections of related concepts and the words that describe them. The “driving the car” frame brings along with it not only intentional and representational, but also propositional structure (Lakoff, 1999). It allows us to make sense of cars, drivers, roads, accelerators, brakes, steering wheels and seatbelts. Propositional information such as a driver sitting in his car, wearing a seatbelt, turning the steering wheel, pressing on the accelerator and driving on a road is intentional: it is about cars, roads, driving, etc. The frame represents the structure of the experience of driving. Semantic frames used in physics and mathematics mimic the structure of the model under investigation and facilitate inference-making.

Image and event schemas impose a kind of structure on elements in a system as well. For example, when a student says that energy leaves a system, they have characterized their system as a container from which something is removed. When they say they are using a particular formula for “plug and chug”, they have chosen an imaginative machine (like function machines in algebra) that has slots for inputs (elements) upon which it operates according to pre-defined rules. These operations produce a new element that stands in particular relation to the original elements.

Conceptual metaphors are perhaps the most subtle of the structuring tools of the cognitive unconscious. They allow us to conceptualize one thing in terms of another. For example, if I say that Mother’s Day *comes a week ahead* of my birthday, I am, in effect putting a “front side” on my birthday so that Mother’s day can be in front of it and it can be behind Mother’s Day. In essence, I am characterizing temporal interval in terms of spatial separation.

What are the frames, schemas and metaphors that students use?

Two factors make these ideas extremely difficult to study. The first is that we are so used to searching for, and finding, meaning when we read, listen to discourse, or look at inscriptions that we do not really pay attention to the particular meanings of individual words, pictures or symbols. If I say, “I am *going over* to my friend’s house,” you are unlikely to ask me, “over what, exactly, are you going?” The meaning is all you take from that statement, not the meanings of the verb or of the preposition, which, taken separately, might seem to imply that I will travel a parabolic path that passes over the top of something. The second reason that frames, schemas and metaphors are difficult to study in this particular research project, and in other classroom research as well, I suspect, is that students just do not say very much, even in classrooms where discourse is a large component of classroom activity. Consider the following transcript excerpt from

a board meeting in the physical science class at Carlos Cadena High School (I am the researcher identified in this transcript—I was operating the videocamera while this discussion was taking place):

RESEARCHER: So, so if gravity is a thing that can apply a force, um, yeah, what is, what is gravity? Where, yeah, what is the gravity thing that applies that force? Or is it that gravity is a thing that applies force or gravity's just the name of a kind of force? (Kids all just looking at her). What is that thing?

TEACHER: You guys understand the question?

Giselle: Yes.

Rich: Isn't gravity just something that applies the force?

RESEARCHER: Well, yeah, so what does it look like? I mean, I mean can we say, oh look, there's gravity?

Giselle: What does air look like?

RESEARCHER: Okay, so you think it's maybe coming from the air?

(Laughter, some say no)

Bobby: (Inaudible) Something about the teacher? (More laughter)

RESEARCHER: This is good. This is good because it sounds like a thing to me and yet I've never really, um (kids talking and laughing amongst themselves).

TEACHER: Listen up, this is an excellent question.

RESEARCHER: Yeah, I've never really seen that thing called gravity. I mean we talk about it all the time, and if we talk about it like it's a thing, an object whatever, um, then maybe we have a mental, um, model of it as a thing that occupies a place. I mean we talked about the earth, we talked about the moon and the moon has different gravity and so is gravity just a property of the moon, or a property of the earth? Is it something inside the moon, something inside the earth? Is it something inside the air around the earth, if we talk about gravity as being part of the air? Um, that would kind of leave us up a creek on the moon, though, wouldn't it? Is there any air on the moon?

Some students say yes, some say no.

RESEARCHER: Yeah, yeah.

Students talking but inaudible.

Bobby: Oh mister smart guy now.

RESEARCHER: Yeah, now well, why do they wear those space suits, huh? With those breathing apparatus? It's like almost underwater; they have to have their little scuba suit on to go on the moon. (One boy says something, but can't hear)

RESEARCHER: So where does the moon's gravity come from if gravity is a property of the air?

Kevin: It's invisible.

RESEARCHER: Pardon me?

Kevin: It's invisible.

RESEARCHER: I'm, I'm sorry.

Kevin: It's invisible.

RESEARCHER: It's invisible.

Kevin: Yeah.

RESEARCHER: It's invisible, okay, so is air invisible?

Kevin: No.

RESEARCHER: So does it still, so we're still back to the gravity in the air thing and yet there's no air on the moon.

Maria: Well you can feel it.

Michelle: Maybe it's the property of objects.

RESEARCHER: Maybe it's the property of objects.

(CCHS 3-3-06)

In this excerpt, the average number of words per utterance for the students is just over three, and about half of these utterances were not even complete sentences. It is difficult to infer the mechanisms by which students structure their reasoning when they say so little, and yet there was reasoning happening in this episode—they progressed from thinking of gravity as an object or “thing” to gravity as a property of objects. There

was a sort of ratcheting of meaning as my utterances and theirs were layered atop one another until they added up to a shift in their conceptualization of gravity.

In the sections that follow, I will identify some of the structuring frames, schemas and metaphors that students and teachers employed as they reasoned about problem spaces in mathematics and physics.

Whiteboards as containers

Whiteboards initially served many students as a sort of repository for information. The semantic frame was provided by the problem context. This helped them identify the various elements in their problem space. Laboratory investigations or problems contained a collection of information, and they saw their task as the selection and assembly of this information into some construction called The Answer. In this sense, whiteboards were both containers for information and tools for answer-making.

Whiteboards also provided a system schema of sorts for the problem space that students constructed. If an element was represented on the whiteboard, it was in their problem space. In a sense, the task of creating a whiteboarded representation forced students to identify the elements (and the properties of those elements) they needed from the problem or situation they had been given in order to find some relationship or answer some question.

Whiteboards as road maps

Students identified their system by “depositing” its elements on the whiteboard. They categorized these elements by mapping them onto quantity names. They were generally able to make good use of context clues in assigning them to categories, i.e., slowing down or speeding up meant that the velocity was changing, changes happened with respect to time, and changes in velocity were called accelerations. These quantities were, in turn, mapped onto either spatial representations (i.e., diagrams or graphs) or equations.

Naming at times led to misunderstanding, particularly when elements were named symbolically rather than lexically. For example, there was frequent confusion as to the meaning distinctions between v_o , v_f , and Δv ; t_o , t_f and Δt ; F , ΣF and F_{net} . An utterance (or some bit of text) does not have meaning in and of itself. It has potential for meaning. The meaning it takes on comes from the cognitive framework of the listener (or reader). If the cognitive framework of the listener is limited, the meaning that she can take from an utterance may be incomplete, and this was often the case in the first two months of honors physics at Darnell High School. If the student mapped the elements of her problem space onto a spatial representation, she less often mistook one quantity for another.

Students who mapped information to equations first followed a fairly linear pathway through their problem space. The structural information for their elements and the operations by which they were related were specified explicitly by the mathematical formula. All they had to do was plug and chug. When they chose this path, they came quickly to The Answer. At this point, their exploration of the problem space generally ceased. They might be directed back to the physical interpretation of their answer when they presented their whiteboard to the class if the teacher probed for this, but this

approach left a lot of unexplored territory in their problem space. They made connections at the initial and the end points of their journey but in between they were in the world of mathematics rather than the physical world.

Students who mapped information to a graphical or diagrammatic representation first, navigated more of the problem space in the physical world. They could look at the spatial, temporal and interaction structure of the event they were trying to reason about, move back and forth in space and time, examine instants or intervals, and see points at which things happened or changed. This enabled them to choose equations and map the information onto them with fewer errors or uncertainties.

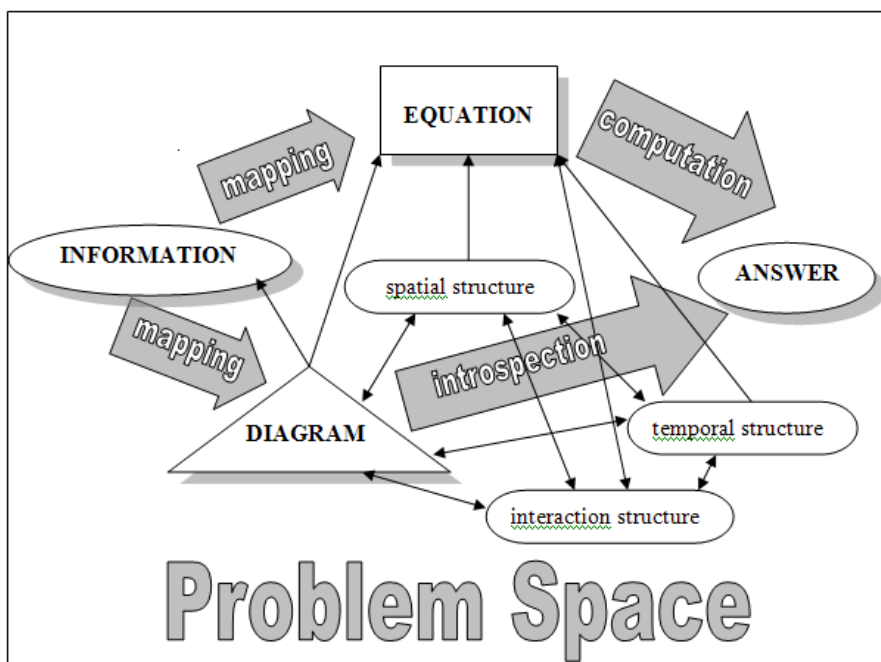


Figure 30. Whiteboards as road maps: problem space navigation pathways.

My observations at Darnell High School took place early in the school year, and for the first four to six weeks, most student whiteboarding was simply about Telling Answers. As time passed, most students progressed to Show and Tell, and the conversations they had with each other as they worked in both small groups and whole groups made meanings with and took meanings from spatial representations before they reduced the information to equations, computations and answers. By the end of the term, the group of students that the videocamera had followed throughout the semester was using whiteboarding to Show Each Other, and to Show Themselves how the elements in their problem space fit together. At this point, their whiteboards had become tools for thinking about real physical situations.

Students at William T. McKinley Middle School, Carlos Cadena High School and Echo Mountain Community College were observed during the second semester of the school year. The high school and college students had all been using whiteboards in their classes for some time, and by the time I arrived, they were, for the most part, past the stage of answer-making. The Echo Mountain students were quite accomplished at Showing Themselves and Others the problem space. The CCHS students were able to

use their boards to Show Themselves the problem space, but they were not always motivated to Show Each Other (or to be shown by each other). In general, whiteboard presentations in this class were aimed at the teacher and only about half of the student observers paid close enough attention to the presenters to ask meaningful questions. The teacher made an effort to involve other members of the presenting group and the class but some students were highly resistant to these efforts. Still for the students who cared enough to attend to the small group discourse that surrounded whiteboard preparation, the whiteboarded representations routinely served as a focal point for their meaning making activities.

The middle school mathematics students I observed had little difficulty learning to use their whiteboards to Show Themselves and Show Each Other the problem space. They readily adopted the practice of including spatial representations (although in some cases these representations were merely object collections) and students were more apt to rely on these as aids to reasoning than on mathematical expressions.

Another form of spatial representation that was used to particularly good effect at the community college was role-play, which put the students right into the system that they were studying as elements in relation to each other, that could be operated upon according to the rules that governed the system. In the next excerpt, students had just begun a board meeting in which they were debating the answers to a series of questions about the microscopic model of what happens in an electrical circuit.

TEACHER: ...Okay. So, is the circuit charged before you close the switch? Is the circuit charged after we close the switch?

Several: No.

TEACHER: No. Hmmmm. So what happens? I'm a charge (stands) I'm going along happily, and the first one that gets to the capacitor, doink, what happens?

Kiki: You got stuck.

Jack: You're stuck.

TEACHER: I'm stuck okay, so game over right? What happens?

Jack: Another charge shows up behind.

TEACHER: Another charge shows up behind me? So, come on other charge. So what happens?



Figure 31. Another charge shows up behind me and what happens?

Mark: You start piling up.

TEACHER: We start piling up?

MARK: You create an electric field.

TEACHER: Well, how many does it take to create that electric field?

Several say one.

TEACHER: One. So take a step back second charge. (Girl steps back). Is there electric field now?

Several say yes.

TEACHER: So why do I, so there's electric field at?

KIKI: In the insulator.

TEACHER: In the insulator. So what happens on the other side?

KIKI: They, it charges the other field and they're repelled.

TEACHER: Okay. The charge, say what you just said again.

KIKI: The charge's field charges the other field and they're repelled.

TEACHER: Okay. Now the real question, and I don't think anyone has asked this question is, how many charges repelled when I build up here, over there.

GABE: Just one?

Several say, all of them.

TEACHER: How many are repelled and start moving down that wire?

Several say, all of them.

TEACHER: All of them, so, in one instance, I put one extra charge on this plate, if all the charges that are the same kind as me on that plate disappear. That's what you just said. (Pointing to boy)

GABE: They move they don't disappear.

TEACHER: Oh, well they moved down the wire, so this; Let's say I'm a negative charge, what would that mean this plate in middle would have how many negative charges, based on what you just said?

KIKI: None?

TEACHER: None. (Looks around at classroom).

KIKI: But that can't be true.

TEACHER: Did everyone get that, can't be true. Okay, so the question you have to, so what would you think if we put a current meter here, and a current meter here? Based on what you know about series circuits, what should they read? Say it a little louder with confidence.

(EMCC 10-5-05)

In this excerpt, the teacher helps students zoom in on the fine structure of their model of charge by “shrinking” them to the size of an electron so they could “walk a mile in the electron’s shoes.” The image schema instantiated in this exercise will likely stay with students when class is over and be available to them when they are reasoning about other situations involving electrodynamics.

How does language evoke spatial and temporal images?

Language can obviously paint a vivid, easily imagined picture of a physical situation. In the following excerpt, Brenda and Ara went straight for the algebraic solution path. They had no trouble mapping the language of their problem statement to the equations necessary to solve the problem. However, their classmate had a bit more difficulty making imaginative sense of the solution, a negative displacement. Brenda substantiated the solution graphically.

TEACHER: Good job on thinking about that. Are you ready for number 7?

The WB shows the following:

$$A: v_f = at + v_0$$

$$0 \text{ m/s} = a(2\text{s}) + -22\text{m/s}$$

$$22 \text{ m/s} = 2sa$$

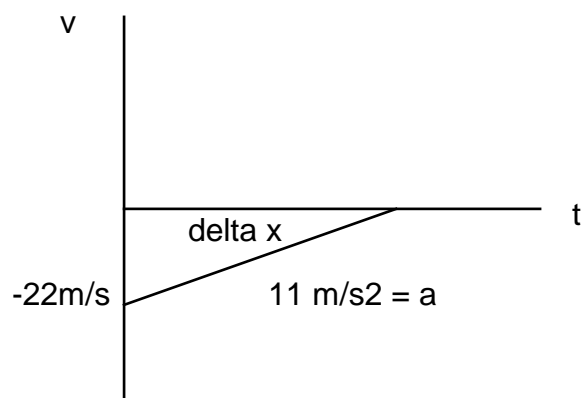
$$11 \text{ m/s} = a$$

$$B. \Delta x = 1/2at^2 + v_0t$$

$$\Delta x = 1/2(11\text{m/s}^2)(2\text{s})^2 + -22\text{m/s}(\Delta)$$

$$\Delta x = 22\text{m} - 44\text{m}$$

$$\Delta x = -22\text{m}$$



Brenda: Okay. Number 7 says a driver brings a car traveling at negative 22 m/s to a full stop in 2 seconds. And the first one is what is the car's acceleration. You use final velocity equals acceleration times time plus initial velocity and you end up getting an acceleration of 11 m/s².

Ara: Okay, so for b you're supposed to find the displacement. So we used the change in x equals 1/2 times acceleration times time squared plus the initial velocity times time, and so we knew the acceleration and the time and the initial velocity so we [plugged them in?] and we got the displacement is negative 22 meters.

Brenda: questions? (Long pause)

TEACHER: Really....

Boy in front: Is it possible for displacement to be negative?

Brenda: Yes.

TEACHER: Oh, that's right—it was a negative initial velocity.

Brenda: Yeah. That's why I drew this. (Points to graph) Because remember how we would find the area? Of the velocity vs. the time? But it was below the t axis so it's negative.

(DHS 10-03-05)

Note that the t axis of Brenda and Ara's graph is positive ("negative time" is not relevant in this problem) while the v axis goes from positive to negative (in spite of the fact that positive velocity is not relevant either). In their descriptions of the solutions to the two parts of this problem, Brenda and Ara's reasoning seem to follow a fairly linear path to the answer. Their graph was at the bottom of their board leading me to suspect that it was drawn after the algebraic solution was obtained, possibly as a justification for the negative sign on the displacement.

How is it different for a diagram?

In the next excerpt, which takes place about six weeks later, students have transitioned to more spatially assisted reasoning. The first inscription on their whiteboard was a free body diagram, from which the equations were then generated. When questioned about their solution by a classmate, Hannah's reasoning was more networked than linear and involved multiple references to different features of her diagram.

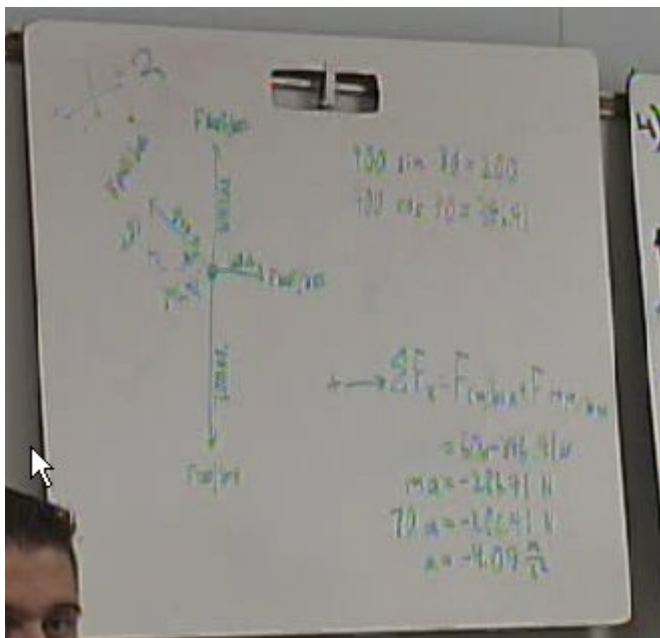


Figure 32 Hannah's leads with a discussion of their free-body diagram

TEACHER: Sometimes instead of using plusses or minuses it's easier to say up down left right, down the ramp up the ramp...so it's not quite so

confusing...who's got the first question for them on number 3...a lot of people had pretty much blank paper for this yesterday (No one looks or talks. Gui finally speaks but we cannot hear her question...something about should all forces add up?)

Jimmy: Yeah well cuz it's not accelerating upward...so we're assuming if the x and the y have enough force...

Hannah: What kind of makes it easier to understand—in an ideal situation you only have 2 forces...the gravity force and the normal (gestures), but in this situation this box is being pulled up at an angle(points at diagram)...so you have this additional force...so you have to kind of...you have these two forces instead of just one...normally this would be the same as this, right? It would be 700...but it's not...because it's moving in another direction at an angle...so you have to compensate, making this one (points at diagram) smaller than this...so ...[] but, like, your frictional force isn't really a factor when it comes to adding them up to equal the 700...because it's not...its not pulling it up an angle...it's not interfering with the direction it's heading.

As noted previously, the propositional structure of algebra is more like language than it is like a diagram. It can give unambiguous structural information, but it gives it very abstractly. It can be quite difficult for physics students to zoom back out to the physical world they are trying to construct a model of once they have entered the world of algebraic manipulation. In the following episode, Gui (who is an English Language Learner) is struggling with the level of abstraction. The teacher's conversation with the class is almost entirely algebraic and most of the students answers to his questions are in unit-less numbers.

TEACHER: So what you're saying if I understand you right, is, you found this total area and that's 72 meters. (murmurs of "yes")...But that's not the bottom line because it only wants the third second—not the first 3 seconds. So you went back and...what'd you get for the orange area?

ARA: 52

TEACHER: 52 meters? So 72 meters minus 52 meters leaves you with...

Voices: 20

TEACHER: 20 meters during that third second. Now is there another way to do this problem that's actually a little bit easier?

JIMMY: that you can just put it altogether, or put the whole thing and then subtract?

TEACHER: Well...at 2 seconds how fast is it going?

JIMMY: It's at 22.

BRENDA: How do you know?

TEACHER: How do you know that?

JIMMY: Because in part a we have to figure out the acceleration which is negative 4...so the first second it's down to 26 and then it's down to 22...

TEACHER: So at 3 seconds...

JIMMY: 18 to 4 so that triangle has...the second leg of the triangle is 4

TEACHER: How many people are seeing this? How many people are totally lost by what Joey is saying? Gui raises her hand).

BRENDA: I see.

TEACHER: You're totally lost by what he's saying? What's its starting velocity?

MARK: 30

TEACHER: What's its acceleration rate?

MARK: Negative 4 meter's per second squared.

TEACHER: Which means what?

GUI: It's getting lower.

TEACHER: It's getting lower and lower...by how much each second?

GUI: four

TEACHER: 4 meters per second during each second. So if it starts at 30, after one second how fast is it going to be going?

GUI: 22?

TEACHER: After ONE second...

GUI: Oh, um, 26?

TEACHER: After 2 seconds

GUI: 22

TEACHER: After 3 seconds

GUI: 18

TEACHER: After 4 seconds

GUI: 14

TEACHER: After 5 seconds

GUI: 10

TEACHER: 10. If it starts out at a velocity of 18 and finishes out at a velocity of...or starts out at 22 and finishes at a velocity of 18 what would its average velocity be for that one second?

Voice: (very tentative after a pause) 20

TEACHER: 20 m/s...and how far does it go in that one second?

Voice: 20

TEACHER: 20 meters. That's another way that you can figure this out. And that's the cool part about this, folks, that I think you guys are missing...is that I can do it like this with areas under the graph or I could use the equations or I could use this idea of an average velocity and what's true about the answers all 3 ways?

Voices: they're the same.

TEACHER: They're the same. You know you always here your teachers telling you to go back and check your work, check your work...you know, particularly on things like AIMS and the SATs. The great part about this is you have alternative ways to check your work. Instead of just looking at your math again to see yeah I put the plus sign in the right place you can approach the problem from a totally different direction and see if you get the same answer. Gui, you're still looking...

GUI: How do you find a distance if you know the average velocity?

TEACHER: The average velocity times time.

GUI: Oh.

TEACHER: That's what we said back in unit 1...in unit 2. That average velocity was change in position over change in time. And yet there's also a short cut I could have done going with this one (points to $\Delta x = 1/2at^2 + v_0t$ and begins plugging in new numbers underneath the equation). Instead of using 3 seconds I could have used one second...but if I did it that way what would I have to say my starting velocity was?

Voice: 30

MARK: 22

(DHS 9-29-05)

This excerpt illustrates the difficulty that students can have in keeping spatial and temporal structure in view when their attention is fixed on algebraic manipulation.

In the next episode, Zane gets stuck on algebraic reasoning and Jimmy bails him out with a graphical solution:

ZANE: So I went through...I used the change in x equation, and I ended up getting...like for acceleration I kept getting a negative one...so I had to do a magical sign change to get it to positive 1...but I mean, I just if you could help me see what I did wrong there...

TEACHER: How were you trying to find the acceleration?

ZANE: I used the $\Delta x = 1/2at^2 + v_0t$.

TEACHER: Where did you get the delta x from?

ZANE: The, uh...I don't know...never mind.

TEACHER: I mean, you've got a speed and a time, and a second speed and a time...

ZANE: I used 12.5 but I don't know where I got that looking at it right now.

TEACHER: Yeah—I can't see where you'd come up with 12.5 either...

ZANE: Yeah.

BRENDA: 12.5 is [] plus time squared...it's down at the bottom [] x...

TEACHER: Ah—I see where you're going...that 12.5 is...what's the graph of v v. t look like for this one? (Long pause)

BRENDA: It's a straight diagonal...or...horizontal line...

TEACHER: Well which one is it? Straight diagonal or straight horizontal?

BRENDA: Straight horizontal at 5 m/s...velocity equals 5 m/s. (The teacher sketches this graph on the board.) And then it's a straight diagonal line in the positive direction...

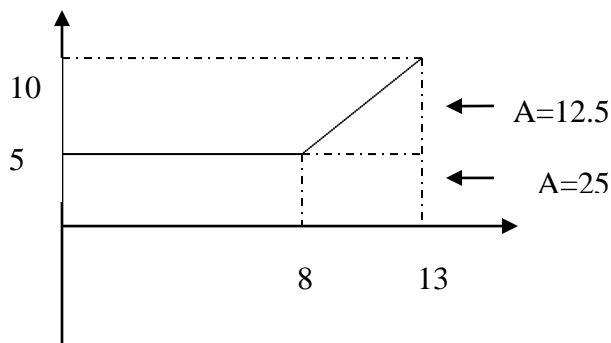
TEACHER: When does it stop being a straight horizontal line?

BRENDA: 8 seconds. Then it goes up. And it stops at 13 seconds and 10 meters per second velocity...

TEACHER: How'd you get 13 seconds—you said it went for 5.

BRENDA: Oh, because eight plus five is 13.

TEACHER: ah. So the delta t for the acceleration is...? 5 seconds.



TEACHER: That area is 12.5. But is that the dog's displacement for those 5 seconds? Why not?

JIMMY: [inaudible] it's all the way down to the x

TEACHER: What's this area?

BRENDA: 25 meters

JIMMY: The distance he would have gone if he kept going.

TEACHER: Again a little bit louder Jimmy so that everyone can hear it.

JIMMY: 12.5 is just how much distance he would have covered if he kept going.

TEACHER: at the constant speed of 5 meters per second...

JIMMY: Like the train problem...

TEACHER: How many people understand what Joey's saying? (Cory raises her hand) He would have only gone 25 meters if his speed hadn't changed. But since he accelerated he ended up covering more than just 5 meters per second each second. At 9 seconds his speed is now 6 meters per second at 10 seconds his speed is now 11 meters per second. He keeps adding one meter per second each second...so he covers extra distance during each second. 12.5 extra meters total.

(DHS 9-29-05)

Students learn to coordinate space and time readily with the use of graphs. These scalable spatial-temporal representations not only encode positions, speeds or accelerations at particular times, their slopes represent rates of changes in these quantities and the areas under the graphs can represent accumulations such as displacement. (See the train problem that Zane helped Gui with for a good example of this.) All of the students I observed were accomplished at not only making and interpreting drawings but also graphs—at least position-time graphs. This supports the notion that by the time they reach the second half of 7th grade, graphing is a skill most students have mastered. The same cannot be said for motion maps and free body diagrams, but this is not surprising as they are proprietary to physics.

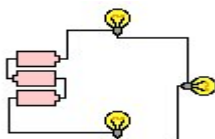
Encoding Spatial Representations (SR) as Conceptual Structures (CS)

Jackendoff's Theory of Representational Modularity posits the existence of an interface between CS and SR. This interface module is bidirectional—that is, it allows information to pass from CSs such as languages, heard or spoken, to SRs, which are essentially geometric. Interfaces do not translate information per se; rather they allow it to be mapped from one domain to another. For example, the word *train* is mapped to the sound (trān), which, in turn, is mapped to its meaning—railroad cars pulled by a



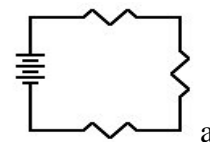
locomotive, and also to its image.

Since our CS-SR interface is bidirectional, we need not start with a word, however. Consider the following image:



. You have doubtless had no trouble mapping its various elements to the words *light bulbs*, *wires*, and *batteries*, and made the attendant phonological and

semantic mappings as well. You may also have mapped the entire picture to a category called *circuit drawing*, or something similar. It is probably not much of a stretch, then, to



map the information in this picture to the following representation: circuit diagram. Our CS-SR interface may have already accomplished some preliminary mapping of these SRs to their corresponding lexical, semantic and phonological structures in the CS (resistors, voltage source). All that remains is to translate them into their symbols, R_1 , R_2 , R_3 and V to make computations according to the operations, relations and rules governing electrical circuits convenient. But wait! What about the wire? You would not have much of a circuit without it, and yet it is not one of the elements that we routinely represent symbolically when construct equations relating the elements of electrical circuits. We have abstracted categorical information from the drawing to make the diagram, and abstracted only certain categories of information from the diagram to symbolize for use in equations.

Why do we preserve and map only some information to symbols such as R and V ? We do it because we encode only what is relevant in the symbols we used for such formulas as $V = IR$. No wire is needed for us to be able to perform the operations specified in this relation. The information that there is a wire is not lost—it is still encoded in the SR. However, the equation CS does not explicitly reveal the presence of the wire. Neither does the following statement: What is the current flowing through a circuit of three light bulbs in series with 40 ohms resistance apiece that is energized by a nine-volt battery?

We were able to think with SRs long before we had CSs such as language or symbol systems. Infants encode three dimensional shapes of objects before they are a year old (Marr, 1982; Rose, 1977). We learn language by mapping spatial representations onto lexical ones. Going from CS to SR comes later.

When students map a word problem onto a symbol set, they are confining their inscription to a representation of the problem's CS. (They may in fact be mapping the problem's words and meanings to internal SRs that are hidden from view unless they happen to mention them.) They abstract information that they can symbolize from the problem but in symbolizing it, they assign it the status of an object. A velocity, v , becomes a thing—a value to substitute into an equation. They lose spatial and temporal information such as trajectory and position, which are geometric and therefore encodable in SR. Cognitive Load theorists tell us that working memory has a limit—seven plus or minus two chunks of information (Gerjets, 2003; Paas, 2003). One of the great advantages of inscriptions is that they allow us to offload information while still retaining its usefulness. In the construction of coherent knowledge structures—models—it makes sense to have all the pieces on the table, particularly when you are doing this constructing jointly with others. SRs that are not represented in inscriptions that students use are not necessarily commonly held and are less likely to contribute to understanding or finding a solution to a problem.

The potential of the SR-CS interface is at the foundation of modeling in physics, and to the extent that mathematics describes things in the real world, it is fundamental to mathematical modeling as well.

My data show that students who move toward reasoning from SRs as well as CSs are better able to choose appropriate models and apply them to solve problems and to explain and their solutions conceptually (not just procedurally) to their peers. In addition, the SRs that are included in whiteboarded inscriptions are available to support and illustrate students' verbal explanations. (Refer to the transcript excerpt in which Hannah explains to Ara how to determine the energy transfers in the system containing a car rolling down a hill, p. 140)

Summary

In this chapter, I have illustrated some of the features of models and the activity of modeling that were problematic for students as they learned physics or mathematics via modeling instruction. I discussed the how the culture of schooling that students (and sometimes teachers) bring along with them into the modeling classroom can sometimes short-circuit the development of skill in modeling, and how a new modeling classroom culture can be used to restructure the learning environment so that it supports modeling activities.

I pinpointed one easily identifiable obstacle that students struggle with—zooming in and out—that teachers can help students with by reframing their problem solving process as they revoice and probe students' understanding in whiteboard presentations and board meetings.

I identified three parallel dimensions of whiteboard-mediated cognition and modeling that formed the broad categories for my analysis of student activity and reasoning in these four classrooms.

The contextual dimension assists students in deciding what information in a problem statement is relevant and provides a frame that helps the student categorize this information and map it onto the drawings, diagrams, graphs, or symbols they use to represent the problem space.

The distributed dimension defines the architecture of the interactions between and among students, their inscriptions, tools and artifacts. It involves communication and coordination or resources.

The structuring dimension describes the cognitive resources students use to identify, categorize and connect the elements, relations and operations that populate and structure their models and highlights the importance of the CS-SR interface.

In the chapter that follows, I will summarize my key findings and recommendations about the effective use of whiteboard-mediated discourse, and offer a practical checklist for teachers to help them optimize their students' potential for fruitful modeling.

CONCLUSIONS

When I embarked on this research, I wanted to find out how students used metaphors to help them reason about space and time. I thought I would hear and see evidence of metaphor use surrounding the preparation and sharing of whiteboards—one of the ‘universal constants’ found in modeling instruction classrooms.

What I found in the first few videotapes I collected of the high school honors physics class initially frustrated me. The kinematics students I observed did not waste much time thinking about space or time. They identified it, assigned it a symbol, plugged it in to the equation they hoped was the right one, did some algebra with it and produced The Answer. The metaphors they used were the basic grounding metaphors of arithmetic (problem solving as object collection, construction, measurement or path following) and the object of metaphorical reference was the physics problem itself. Their problem spaces were containers from which they took the things they needed to reach their goal, and then they plunged into a mathematical ‘wormhole’, emerging at the far end with an answer. The problem space itself remained largely unexplored. Once they entered the spatial or temporal quantities they had collected into an equation, these quantities lost their connection to reality—they might as well have been ‘slithy toves’ or ‘dilithium’.

The algebraic formulations of the problems they solved were the focal points of their whiteboard presentations. Graphs, diagrams and drawings, the SRs of physics, frequently occupied small, out of the way places on their whiteboards (and sometimes were absent altogether) while the algebra was featured prominently, and every step in the algebraic manipulations that led to a solution was shown and described.

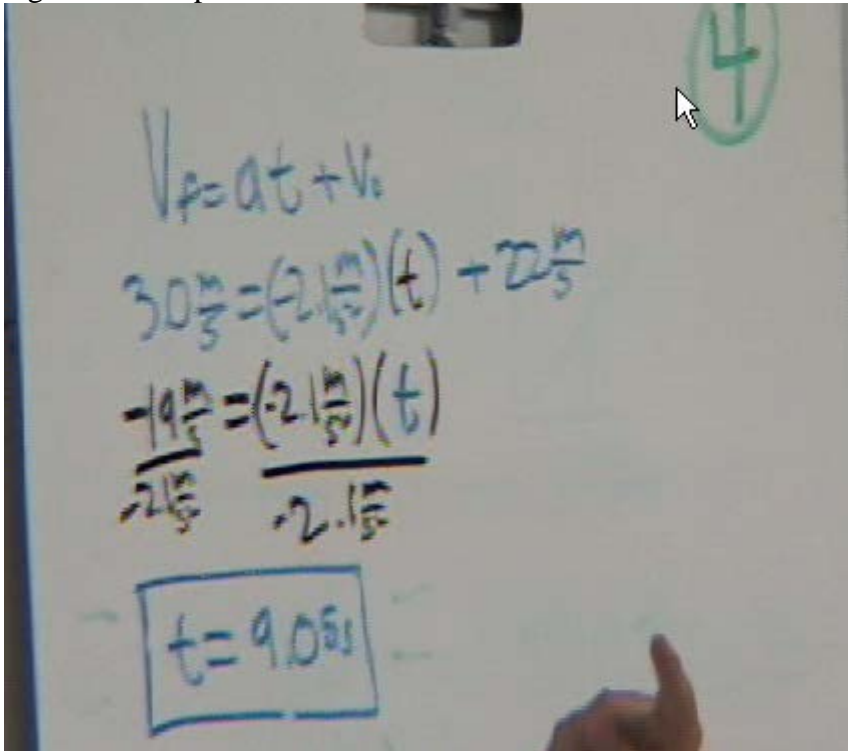

$$v_f = at + v_i$$
$$30 \frac{\text{m}}{\text{s}} = (-2 \frac{\text{m}}{\text{s}^2})(t) + 25 \frac{\text{m}}{\text{s}}$$
$$\frac{-19 \frac{\text{m}}{\text{s}}}{-2 \frac{\text{m}}{\text{s}^2}} = \frac{(-2 \frac{\text{m}}{\text{s}^2})(t)}{-2 \frac{\text{m}}{\text{s}^2}}$$
$$t = 9.0 \text{ s}$$

Figure 33. SRs were sometimes absent altogether while algebra was featured prominently on whiteboards.

The Value of Spatial Representations

As I added more videotapes to my data set, I began to see that students valued SRs in different ways. I saw that their values shifted over time to a greater appreciation of, and reliance upon SRs. Some students used SRs to justify their reasoning while others preferred to use computations. This led me to listen carefully to what students talked about, what counted for them as justification for the choices they made, and to whom they spoke and during whiteboard preparation and sharing, and this gave me some sense of how the SRs they employed functioned to keep them more grounded in the problem space.

At the most elementary level, students drew SRs on their whiteboards because they were required to do so. If they did not include the appropriate diagram or graph on their whiteboard, it would be incomplete and therefore incorrect (i.e., it would not receive full credit). *SRs, then, were about following directions.*

Making SRs might also be about demonstrating a skill. Just as a student might have some computational skill that she demonstrated by setting up and solving equations, she might also possess the skill of drawing an accurate, good-looking graph or diagram. *SRs, in this case, were about showing off.*

Occasionally SRs helped students set up a geometric solution to a problem, i.e., determining the area under a curve. Such solutions were dependent on a student's facility with unitization and thus hinted that some measure of spatial or temporal reasoning was an influence on their choice of solution strategy. Generally, however, they were simply used to justify the use of a certain algebraic formulation of the data. At least this approach included the mapping of graphical representations of space or time to algebraic symbols. *SRs, in this instance, were for justifying equations.*

In all the cases mentioned above, SRs, when they were included, were about getting answers. Moreover, the focus of students' presentations of these whiteboards was The Answer (in particular, the number—units were often overlooked) they had computed. Students who focused on answers were “zoomed in”. Their view of the problem space was very limited, and their answers were not well connected to the problem spaces that gave rise to them. They could zoom back out if prompted but it was effortful, and they were unlikely to do so of their own accord.

As students became more sophisticated modelers, however, SRs were about constructing useful visualizations of physical situations in order to reason about them. When this grew to be the case, the first thing to appear on a whiteboard was the SR, and as it was constructed, its various features and their properties were identified and defined, often in writing. The SR served to keep them zoomed out so that the physical context of the problem remained actively in view in the problem space. Only after this setting of the scene was complete (by the construction of an SR that encoded the elements of the model and their relationships) did students zoom in, abstract out physical quantities and assemble them into a symbolical representation—an equation. Once they had a sensible equation that flowed from the diagram and obeyed the constraints placed on each of its elements by the definitions and properties that were assigned, they were done. Plugging in actual numerical data and solving for some value was, at times, an afterthought. In

some cases, it was not even called for in the tasks students were assigned. *SRs in these instances were for visualizing and making sense of a problem space.*

SRs in this last instance were also an important communication tool in the distributed cognitive sense. There was, at times, a sort of dialogue between the students and their inscription that caused the inscription to evolve as reasoning aloud progressed. In addition to the students working together to create these SRs, the SRs they created functioned as another voice in the exchange of ideas—often a constraining voice that placed limits on the mental models that students were attempting to articulate. One thing that made this dialectic particularly valuable was the opportunity for students to ‘ask’ their SRs questions.

There were many types of questions that students asked in the course of creating and reasoning with SRs. Largely, it appeared that they learned how to ask these questions by imitating the kinds of questioning that their teacher practiced. If the teacher asked fill-in-the-blanks information questions, then that was the kind of questioning they used with each other when creating their whiteboard. If the teacher asked meaning questions or implication questions, then those were the types of questions they were most apt to use with each other when whiteboarding.

Whiteboard sharing with the whole class was substantially different for these two types of collaborations. When SRs appeared first on their whiteboards, students tended to describe how they reasoned through the problem based on their SRs. As a result, they were more apt to be asked questions by other students (or by the teacher) about their diagrammatic choices and interpretations. This resulted in discourse that was more focused on conceptual models and less on procedures. There was rarely a question about a formula or a disagreement about mathematics. When they ran into a physical situation that was counter-intuitive, they would zoom in and look to the mathematics for clues about what mattered, but until they reached some impasse, their reasoning stayed connected with the system schema and its physical interpretation when talking about the models they were constructing. Coordination of representations was possible when their SRs had more than superficial meaning to them.

Implications for Instruction

Modeling instruction can be a powerful instructional practice, but just as in problem solving, we must stay zoomed out. Our primary focal plane or frame must be the model—model construction, model elaboration and model deployment. This is the teacher’s problem space. When we zoom in and fix our focus on details such as computation or the meanings of individual words, we signal to our students that this is where their focus ought to be also—this is what counts. I do not mean to imply that these things are not important, just that they need to be placed in perspective in the bigger picture. We must never forget to zoom back out our primary frame—the model—and help students to do so as well.

Often, teachers describe Modeling Instruction as if it were a technique, a skill or craft. They discuss discourse management techniques, teacher questioning, whiteboarding rules, grading, using technology, pre-lab demonstrations, tricks for explaining concepts (i.e., negative acceleration, Newton’s laws), scheduling and time

management, etc. It is easy to fall into the habit of looking at Modeling Instruction in terms of teacher performance rather than student thinking.

There is a theory about teacher professional development (Fuller & Brown, 1975) that suggests we go through three distinct phases in learning to teach: concern for self, concern for task and concern for students. Even an experienced teacher goes through this developmental sequence when they begin teaching something that is new to them.

And Modeling Instruction definitely qualifies as something new. It is not yet a part of standard teacher preparation programs at the vast majority of colleges and universities in this country. It is not surprising, therefore that we (teachers) focus on our own performance and evaluate it procedurally as if it could be broken down into steps and solved like a mathematics or physics problem. Moreover, once we get comfortable with the procedures that we have identified as modeling, we turn our focus to the task, making our performance of these procedures as technically competent as it can be.

It is not until we take that final step of turning our gaze away from ourselves and our craft, and toward our students, that we are really engaging in Modeling Instruction. Modeling done well is not about the performance that the teacher turns in on a day-to-day basis—it is about the mastery that the students are able to achieve of the set of models they are constructing and using.

What follows is a list of suggestions drawn from this study that may help teachers zoom out and focus on students.

- Design tasks for small group work that focus on probing the problem space rather than tasks that require the use of a certain formula or produce a particular type of answer. Do less of whiteboarding for Going Over Homework and more of whiteboarding for Practicing With The Model. This means that everyone in the class may be doing the same problem. Use board meetings rather than formal whiteboard presentations to share these exercises with the whole group. They are a more effective use of time.
- Expect your students to lead in board meetings. Hand over the floor to them so that they can take turns as leader and interlocutor. Encourage them to talk to their classmates—not to you. Follow their lead. Prompt rather than grill. Probe the CS-SR interface—the boundary between saying and seeing. If students are zoomed in on the computation process, help them zoom out by redirecting their gaze to the problem context and physical situation that structure their problem space. And if an important question is on the table, such as when a model applies, do not let them off the hook by answering it yourself. Make them find the answer themselves, even if you have to come back to it later.
- Encourage the students to examine the conceptual system that they bring to their problem space. If they can see that elements of this system are missing from their whiteboards, *they* should add them.
- In reviewing what a group of students has whiteboarded with the whole class, ask them if it contains all the necessary conceptual elements to build a model of the problem at hand. If not, do the students possess the necessary missing conceptual elements and can they activate them, bring them into their conceptual system and connect them to existing structures in such a way that they are useful for solving

problems? If not, are there students in the class who can help them with this process? Encourage them to identify and use the resources that their classmates possess rather than providing them yourself.

- Attend to students as individuals as well as in groups. Learn to watch and listen to what students who are listening to the presentations of others say and do. Are they engaged? Are they perplexed? What are they taking from what is being said? This is hard to do unless we let someone else have the floor. When we are on deck, we do not have the attention to spare to focus on individual responses to the discourse. We need to practice not taking charge of the conversation.
- Break the habit of soliciting or listening for particular words, phrases or answers—particularly when these answer are just a two or three words long. Sometimes when teachers hear one or more students answer their question with the ‘magic word’ (or words) they are listening for, they take it as a signal that they can move on because the students “get it.” Remember to check on what it is that the student “gets” and consider checking with the rest of the class to see if they are following this student’s line of reasoning.
- Listen for the kinds of things that students think are important enough to question. Where are these things in their concept system? Are they from the CS or the SR? Is the student zoomed in or zoomed out? Change their focal plane and see what happens. Listen for potential gaps in their model that are betrayed by the questions they ask. We must take time to get to know our students—what they value, what they think—what the telltale signs are that reveal when they are bluffing or guessing.
- Ask your students the kinds of questions you would ask yourself if you were trying to set up a problem. Critique them as you would critique yourself. They will learn to critique themselves and others by imitating you.
- Do not let them off the hook by answering important questions for them—and do not move on to a new idea if important questions are not answered to everyone’s satisfaction. Make them arrive at the answer themselves—answers they can justify—not just answers they have guessed right. Make sure they are convinced and can convince each other that they are reasoning correctly about a situation. Then take the vital step of checking with other students to see if they are convinced. And do not take yes for an answer. Make them articulate what they understand in their own words and listen to see if there are any important elements missing from what they say. Make sure they can zoom in and zoom out without their model falling apart.

The whiteboard is not just a tool—it is a medium for communication, for joint attention and cognitive (as well as social) interaction. Whiteboarding is not simply telling what you know, nor is it merely “show and tell.” It is collaborators interacting at their CS-SR interface to show themselves and one another what they think. Until they can write it down and make a convincing case of their reasoning to their peers, their private mental models remain poorly structured. Whiteboards can afford the necessary platform for representation of the structure of a problem space and help students see the relationships between elements within the conceptual system that they bring to this

problem space. Whiteboarding can also facilitate sharing of information and conversion of that information to knowledge that is held in common by members of the group.

APPENDIX A

Final Codes

- (C) – Contextual
- (D) – Distributed
- (S) - Structuring

Codes used to characterize Modeling

zoomed in (C) – tightly focused on bits of information

zoomed out (C) – widely focused on problem space – large scale structure in view

prompting zooming in (C) – a remark that causes student(s) to shift frame from large scale structure to fine structure

prompting zooming out (C) – a remark that causes student(s) to shift frame from fine structure to large scale structure

tool use (D) – data gathering, measurement and data analysis tool use

artifact use (D) – use of raw data

inscription use (D) – use of shared representation

coordination (D) – sharing of info for the purpose of coming to joint understanding

running the mathematical blend-mathematical thinking (S) – mathematically manipulating the data or equations to find something new

running the real blend-physical thinking (S) – manipulating the physical situation to find something new

guessing (S)

target practice - trying to read the mind of the teacher

conjecturing – guessing how a system will behave based on information or intuition

hypothesizing – guessing how the system will behave based on data

Codes used to characterize Contextual Dimension of whiteboard-aided cognition

Invoking the context - reference to physical situation embodied in problem

Representing the context – representing contextual information

CS – algebraically or propositionally

SR – spatio-temporally

Reasoning from the context – invoking contextual information in justifying reasoning

Framing mathematical structure – identifying relevant mathematical model

Framing physical structure - identifying relevant physical relationship

Mapping – representing one kind of information with another, i.e., symbolizing physical parameters graphically or diagrammatically

Codes used to characterize Distributed Dimension of whiteboard-aided cognition

Going Over Homework – type of activity in which students whiteboard homework problems

Practicing With The Model – type of homework activity in which students identify, elaborate or apply some physical relationship

Roles

The Decider – the student who decides what will be written or how it will be described or defined

The Measurer – the student who operates and reads the measuring tool(s)

The Scribe – the student who records what the measurer or decider deems appropriate

The Operator – the person who operates the apparatus

Coordination of representations – reference to multiple representations to reason or justify reasoning

Coordination of individual contributions – reference to multiple individual contributions to reason or justify reasoning

Who's got the floor?

The Presentation – one or more students describes what they have shown on their whiteboard

The Pause – interval of time after the presentation before the teacher or the audience engages with the presenters

Questions

Conceptual – questions about the fundamental physical relationships that underlie what has been presented or how they have been mapped

Procedural – questions about the procedures used to analyze the problem space or solve the problem

Answers

Conceptual – responses that reference the physical context

Procedural – responses that reference the mathematical formalism

Reasoning from CS – reasoning from the propositional or algebraic structures

Reasoning from SR – reasoning from spatial/temporal representations

The Power of the Marker – the influence of the person who has the marker to determine what gets recorded and what gets ignored

The Power of the Eraser – the veto power of the individual(s) who do not control the marker

Codes used to characterize Structuring Dimension of whiteboard-aided cognition

Frames – prototypical preconceptual structures that allow for contextualization

Schemas – knowledge structures

Metaphors – describing one thing in terms of another

CS – conceptual structure

SR – spatial representation, geometric, temporal

CS-SR interface – instances where individuals are observed to transition from CS to SR or from SR to CS

Focus on answer – when the student's goal appears to be a specific numeric value, with or without units

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BIOGRAPHICAL SKETCH

I am a physics teacher. I have been teaching for 20 years, and I have taught many other subjects, but my identity as a physics teacher is as firmly grounded in the texture of my being as my identity as mother, “leftie”, and redhead.

I entered teaching through “the back door”, when my first choice career, medicine, was no longer an option. In elementary school, I wanted to be president of the United States, in high school I was hooked on marine biology, but when I entered college in 1969, I was committed to pre-medicine. Four years later, in classic overachiever fashion, I not only had a bachelor’s degree, but a husband and two children as well. By age 23, I had given birth to my third child and come to the realization that medical school was out of the question. I considered nursing, but considered it likely that I would harbor secret resentment toward the doctors with whom I worked. Teaching seemed the only other practical alternative for someone with a biology major and chemistry minor, so back to school I went. A year later, when I completed student teaching I knew I had, quite by accident, found my calling.

Eighteen years ago I began teaching physics—not a trivial task for someone who had completed college physics 18 years earlier. But how hard could it be, to stay ahead of 12 seventeen year old girls in one section of high school physics? I unearthed my old physics books, bought some new ones, and discovered what I had missed completely the first time around in 1971. Physics is the foundation. It is why chemistry and biology exist. It is both a context and a rationale for mathematics.

It was a life changing realization--like God’s voice from the burning bush. From then on, I wanted to become the best physics teacher I could be. I took classes and workshops. I gave classes and workshops. I discovered that to be a good physics teacher I needed to be a good mathematics teacher, a good cognitive scientist, a good linguist and discourse analyst, even a good philosopher. Although I loved Loretto High School for girls, where I taught in Sacramento, I realized I needed someone to guide my intellectual pursuits, and since I knew David Hestenes slightly, I contacted him and asked if I could be his graduate student. He agreed.

Six years ago I moved to Phoenix, took the job of designing the mathematics and science program for the new Jewish high school, and began my graduate studies.

My research interest is in the integration of physics and mathematics, the physics first sequence of instruction, and the ways in which student discourse shapes thinking and reasoning in physics. Pedagogically, I approach teaching via modeling theory, and cognitively I am particularly interested in the phenomenon of distributed cognition and how a situated group learning experience ultimately distills into individual student understanding. I am also convinced that mathematics and science instruction can and should be integrated. I undertook this with good results for three years at the Jewish high school, and I will build this curriculum design into future research. I hope that my work may eventually help pave the way for large-scale curriculum integration in mathematics and science. At the very least, I hope that by having a foot in both the mathematics education and physics education camps, I can foster a dialogue and an ongoing relationship between the two communities that ultimately enriches both.