THE USE OF MULTIPLE REPRESENTATIONS AND VISUALIZATIONS IN STUDENT LEARNING OF INTRODUCTORY PHYSICS: AN EXAMPLE FROM WORK AND ENERGY

DISSERTATION

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ABSTRACT

In the past three decades, physics education research has primarily focused on student conceptual understanding; little work has been conducted to investigate student difficulties in problem solving. In cognitive science and psychology, however, extensive studies have explored the differences in problem solving between experts and naive students. A major finding indicates that experts often apply qualitative representations in problem solving, but that novices use an equation-centered method. This dissertation describes investigations into the use of multiple representations and visualizations in student understanding and problem solving with the concepts of work and energy.

The concepts of work and energy constitute a fundamental part of the college introductory mechanics course. A multiple-representation strategy was developed to help students acquire expert-like problem solving skills. In this approach, a typical work-energy problem is considered as a physical process. The process is first described in words—the verbal representation of the process. Next, a sketch or a picture, called a pictorial representation, is used to represent the process. This is followed by work-energy bar charts—a physical representation of the same processes. Finally, this process is represented mathematically by using a generalized work-energy equation. In terms of the multiple representations, the goal of solving a work-energy problem is to represent the

physical process in different ways—words, sketches, bar charts, and equations. The abstract verbal description is linked to the abstract mathematical representation by the more intuitive pictorial and diagrammatic physical representations. Ongoing assessment of student learning indicates that this multiple-representation technique is more effective than standard instruction methods in student problem solving.

Internal energy is an abstract concept in work and energy. To help students visualize this difficult-to-understand concept, a guided-inquiry learning activity using a pair of model carts and an experiment problem using a sandbag were developed. Assessment results have shown that these research-based materials are effective in helping students visualize this concept and give a pictorial idea of "where the kinetic energy goes" during inelastic collisions.

The research and curriculum development was conducted in the context of the introductory calculus-based physics course. Investigations were carried out using common physics education research tools, including open-ended surveys, written test questions, and individual student interviews.

Dedicated to my parents

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FIELDS OF STUDY

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CHAPTER 1

INTRODUCTION

About three decades ago, some physicists to their surprise found that there existed a great difference between what was taught and what was learned [1]. Since then, systematic investigations done by physicists about how students learn physics have been growing in number and sophistication. Forming a new community and building a new area in physics—physics education research—researchers have gained deep insight both into students' difficulties understanding physics and into how to help students learn more effectively [2]. At the same time, the goal of learning physics has also been examined and challenged by the needs in the 21st century workplace. Studies by the U.S. Department of Labor [3], the Accreditation Board of Engineering and Technology (ABET) (e.g., ABET Criteria 2000 [4]), the National Science Foundation [5], and the American Institute of Physics [6, 7] unanimously suggest some common skills that the real world workplace wants employees in science and engineering to have: for example, knowing how to learn, scientific investigation skills, problem solving ability, communication and teamwork skills, and others. Therefore, the goals of physics education research are various and different. Some focus more on understanding of physics knowledge, some emphasize developing procedural skills to meet the needs of the workplace, and some do both. For example, *Tutorials in Introductory Physics,* developed by Lillian McDermott and the

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physics education group at the University of Washington, aims primarily to overcome students' persistent conceptual difficulties and to develop scientific reasoning skills [8], while some projects done by the physics education research group of The Ohio State University intend to help students build a solid conceptual foundation and develop procedural skills, such as techniques to solve complex, multipart problems and teamwork skills [9, 10, 11, 12].

Although the goals of physics education research can differ, the procedures for investigations contain some common characteristics [13, 14, 15]:

1) Conduct systematic investigations of student understanding, especially their difficulties in learning physics concepts;

2) Apply the results of this research to develop and implement curricula and instructional techniques; and

3) Assess the effectiveness of the curricula and instructional strategies on student learning.

These steps are not mutually isolated, but form a coherent, continuous, interactive, and iterative process. The Physics Education Group (PEG) at the University of Washington (UW) categorized this process into three components, namely, *research*, *curriculum development*, and *instruction* [14, 15]. These three parts constitute the triangular model (see Figure 1.1) used by the PEG of the UW to conduct their research. Building upon this UW model, the Physics Education Research Group (PERG) at the University of Maryland (UMd) integrated a model of learning into the triangle and developed their wheel model [13], as shown in Figure 1.2. The PERG at the UMd believe that the
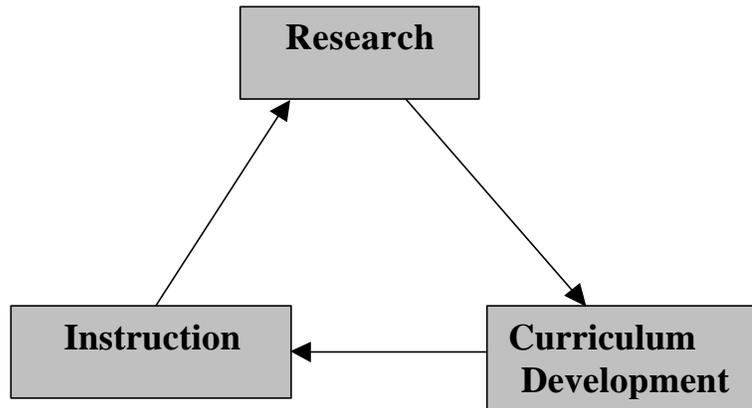
Figure 1.1: The model used by the Physics Education Group at the University of Washington for their physics education research.
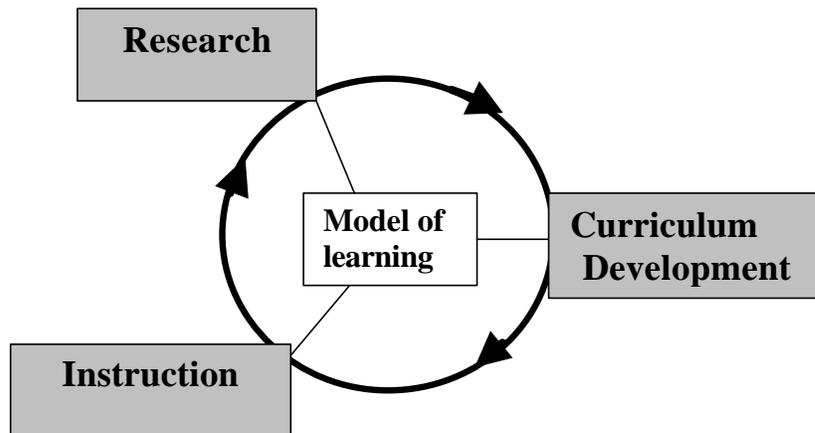
Figure 1.2: The model used by the Physics Education Research Group at the University of Maryland for their physics education research.

learning model is a central component of the research process and can inform the

research cycle at all of its stages.

A model [16] used in the studies described in this dissertation also contains these

three common components of physics education research. As shown in Figure 1.3, this

model includes three parts: the initial state (Ψ_0) of student knowing, the desired state

(Ψ) of student knowing, and a learning system transformer ($\hat{e}$). The initial state of the

student knowing includes student life experience, prior knowledge, skills, as well as

attitudes and beliefs; The desired state of student knowing includes the target knowledge,

skills, attitudes and beliefs, and so forth; and the learning system transformer includes

learning materials, instructional strategies, instructors, classroom implementations, and so

forth. The learning system transformer attempts to help students transform from the

initial state to the desired state.
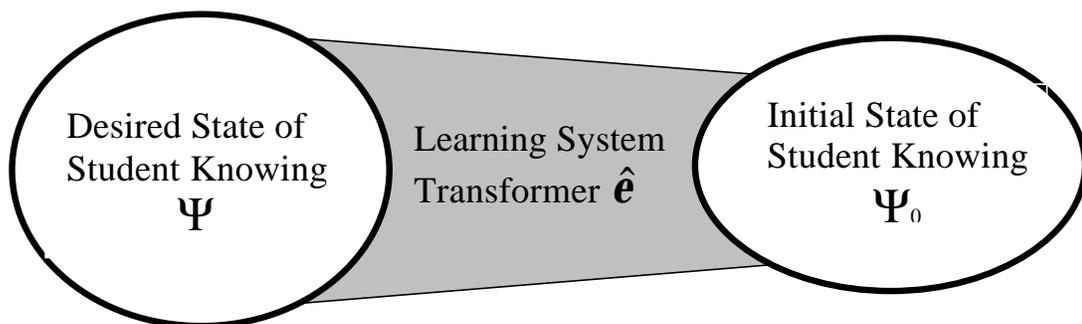
Figure 1.3: The model used in the studies described in this dissertation.

To achieve insight into the student initial state, systematic research is needed. The research results provide a fundamental basis to develop a systematic learning system transformer. To be effective, the learning system transformer must "match" the student initial state and be able to help transform it to the desired state, which is carefully constructed. For example, the learning system transformer must correspond with the model of student learning and must explicitly address student difficulties. After students have been exposed to the learning system transformer, their understanding, as a final state, must be investigated and assessed. The student final state could be the desired state, but might not be. On the basis of the assessment results, combined with results of the research into the student initial state, the learning system is modified, revised, or re-designed. Then the new learning system transformer can interact with students and a new cycle of this model begins. This dissertation illustrates studies, following this model, in the context of work and energy. A primary goal of the research is to help students achieve a desired state in which they grasp expert-like problem solving skills, in particular, the capacity to use both qualitative and quantitative representations in solving work-energy problems.

It is well known that students attempt to solve problems by matching quantities listed in the problem statement to special equations that have been used to solve similar problems. Students move between words and equations, which are very abstract representations of the world, with no attempt to connect either representation to more qualitative representations that improve understanding and intuition. Research [17, 18, 19] into how experts solve problems, however, has discovered that, in a typical procedure of solving dynamics problems, physicists often draw a sketch, develop qualitative

physical representations, such as free-body force diagrams, and *then* apply mathematical equations. Constructing the force diagrams can help experts conceptually understand the problem and set up correctly mathematical equations of Newton's laws. Therefore, to help students develop expertise in problem solving, it is critical to help them learn how to use qualitative representations, in particular, physical representations, in their problem solving. This is not easy. First, it is very difficult to help students overcome the persistent habit of using an equation-centered approach to solve problems, a result of traditional instruction. Second, in many cases, such as in the context of work and energy, there is lack of a good, pedagogical physical representation, which could be used by students to understand work-energy problems conceptually and to set up the work-energy equation correctly. Such physical representations may need to be invented first. Third, even in cases such as dynamics, where good physical representations (e.g., free-body force diagrams) already exist, without systematically learning why and how to use the free-body force diagrams, students are unable to use these physical representations in solving dynamics problems. It is essential to develop a reliable systematic learning system, which explicitly addresses students' difficulties and helps them actively acquire expert-like problem-solving skills.

This dissertation reports a series of studies to develop and assess such a learning system that attempts to help students develop expertise in solving work-energy problems. Alan Van Heuvelen [20] developed a multiple-representation strategy for solving work-energy problems. In this approach, he also developed qualitative work-energy bar charts as a physical representation for work-energy processes. However, as a newly-developed representation, is the work-energy bar chart effective from the point of view of

representation matter and pedagogy? What role does it play in solving work-energy problems? What do students think about this new representation and the multiple-representation technique? Is this approach helpful in getting students to understand the concept of work-energy and to solve related problems? If so, how does learning this strategy help students abandon a naïve, formula-centered method and develop expertise in problem solving? To use the work-energy bar chart and the multiple-representation technique to develop an effective, systematic learning system in student learning of the context of work-energy, it is necessary and critical to investigate these questions.

In addition, as a part of the research effort to develop and assess this learning system, the use of visualization was introduced to enhance student understanding of the concept of internal energy. The multiple-representation technique recommends that two touching surfaces involved in friction be included in the system. In this way, a gain in internal energy of the system is counted, instead of a difficult calculation for work done by the friction. It was found that students had difficulties understanding this abstract concept of internal energy. They lacked a meaningful mental picture of it. But few studies have been conducted to investigate and address the difficulties that introductory physics students have with this concept. In this dissertation, a pair of model carts and an experiment problem using a sandbag, with other learning materials, were developed and used to help students visualize and understand this difficult concept of internal energy.

Overall, the research work conducted in this dissertation attempted to develop and assess a systematic learning system that could help students develop expertise in solving work-energy problems, to demonstrate a process for development and evaluation of new

pedagogical, physical representations in physics problem solving, and to present a method to use visual aids in enhancing student learning.

1.1 Context of the Research

The concepts of work and energy constitute a fundamental part of mechanics, which is a branch of the common course for university introductory physics. There is considerable diversity in the way that physics faculty teach the work-energy portion. There is also no qualitative physical representation (see more detailed discussions in Section 2.1) to represent the conservation of energy involved in work-energy processes. It is well known that most physics professors and teachers solving dynamics problems rely on diagrammatic force representations—a free-body diagram or a force diagram. There is, however, no similar representation for solving work-energy problems. As a physical representation for work-energy processes, Alan Van Heuvelen [20] developed qualitative work-energy bar charts, which serves the same role for analyzing work-energy processes as motion diagrams and force diagrams serve when analyzing kinematics and dynamics problems. Building on his work, a systematic investigation was conducted to probe how naïve students solve work-energy problems, and a series of assessments was done to evaluate student problem-solving performance after they were exposed to the new representation technique. The investigations took place at The Ohio State University (OSU) primarily with honors engineering freshmen in the calculus-based mechanics course from 1997 through 1999.

Applying the approach in the context of work and energy, the concept of internal energy is introduced, rather than having students deal with a difficult calculation of "work done by friction". Internal energy is also a key for helping students to understand

collisions, especially inelastic collisions. We found that many naïve students had difficulties understanding this abstract concept. To help students visualize this difficult-to-understand concept and develop a mental image of it, a guided-inquiry learning activity using a pair of model carts and an experiment problem using a sandbag were developed. These two studies also were conducted at OSU, but the former with the honors engineering freshmen and the latter with regular calculus-based introductory students.

1.2   Research Methods

To understand how students learn physics, the following methods are widely used in physics education research:

1) **Informal observations**: Daily instructional interactions with students inside and outside the classroom provide direct insight into how students learn and where they have conceptual difficulties. Especially in the naturalistic setting of the classroom, there are opportunities to observe, listen to, and communicate with relatively large numbers of students. These informal observations often help initiate some interesting studies.

2) **Individual student interviews:** Learning and understanding is a very sophisticated process. To understand deeply how students learn a particular topic, in addition to the informal observations discussed above, formal individual student interviews are needed. For example, the Physics Education Group at the UW uses individual demonstration interviews, typically lasting 30 to 60 minutes, as the primary data source in investigations of student understanding [15]. The interview demonstration, usually involving real equipment, provides the context for the discussion between the interviewer and the student. They have also found that they could learn more about the students' ideas if the

student is asked to make *predictions* about a particular situation during the interview. Through transcribing and analyzing interview data, students' reasoning and difficulties are identified.

For the research set forth in this dissertation, not only is the individual student demonstration interview used, but also the think-aloud interview [21], which is widely used in cognitive science and psychology. The think-aloud interview, often used for problem solving tasks, typically intends to investigate what a subject thinks about while he or she answers some questions. These two types of interviews differ primarily in the following ways: 1) During the think-aloud interview, the subject is typically instructed to *verbalize aloud* everything he or she is thinking from the very beginning until the task is completed [21], while the student in the demonstration interview is usually instructed to *explain or describe* their reasoning about a particular situation often after they have given answers (see Table 1.1 for details). 2) After a task has been given to the subject, the interviewer is not a part of the think-aloud interview, and communication between the interviewer and the subject is minimized [21]. More social interactions and communication between the interviewer and the student occur during the demonstration interview, and the researcher participates more in the interview procedure. Some detailed characteristics of both interviews are summarized in Figure 1.3.

3) **Written tests**: Carefully constructed written questions are also used to probe student understanding. Compared to the interview technique, a written test has two distinct advantages: 1) It is easy to administer to a large number of students; and 2) Data analysis costs much less time and labor than that for interview data. However, it is sometimes

Demonstration Interview	Think-Aloud Interview
Suitable for	
Demonstration tasks	Written problem solving tasks
Procedures	
Instruction: "…What I am going to do is to show you a few things and ask you some simple questions…I would like you to answer in the best way that you can, perhaps by drawing on some previous experiences you may have had, or on what you may have learned in any previous or current science classes…Don't be concerned if you are unsure about your answers. Actually, I'm more interested in how you are thinking about the questions and the reasons you give for your answers than I am about whether your answers are correct or incorrect. Therefore, I will be encouraging you to explain, as best as you can, how you arrived at your answer…" [22]	Instruction: "In this experiment we are interested in what you think about when you find answers to some questions…In order to do this I am going to ask you to THINK ALOUD as you work on the problem given. What I mean by think aloud is that I want you to tell me EVERYTHING you are thinking from the time you first see the question until you give an answer. I would like you to talk aloud CONSTANTLY from the time I present each problem until you have given your final answer to the question...Just act as if you are alone in the room speaking to yourself…" [21]
	Warm-up questions given
The subject is asked to predict what would happen in a particular situation and to explain his or her reasoning	The subject is reminded to "Keep talking"
More communications between the investigator and the subject	Social interactions between the investigator and the subject are minimized
The investigator is a part of the interview, in addition to a tape recorder or video camera	The investigator is *not* a part of the interview; but just a tape recorder or video camera

Table 1.1: Detailed characteristics of the demonstration interview and the think-aloud interview.

difficult to get deep, accurate understanding of student reasoning, thinking, or problem solving skills from written test results.

To gain deep, comprehensive insight into student learning, all three techniques above were used for the studies in this dissertation.

1.3 Overview of Dissertation

The dissertation consists of two parts, the main body and the appendices. The main body describes the process of research and curriculum development, while the appendices contain student interview data transcribed and original materials developed during the investigation process.

Chapter 2 briefly reviews previous studies related to research about the concepts of work and energy and functions of multiple representations and visualizations in problem solving. Chapters 3-5 report the major studies set forth in this dissertation. Each chapter basically consists of three parts: 1) an investigation that was conducted to identify students' initial difficulties (i.e., Ψ_0), 2) development and implementation of learning materials (i.e., $\hat{e}$, a learning system transformer) that attempt to match the student initial state, to address these difficulties, and to help students achieve the desired state of understanding, and 3) assessment that was done to evaluate the effectiveness of these learning materials by determining how the final state compared to the desired one (i.e., Ψ). In particular, Chapter 3 discusses the use of multiple representations in student problem solving in the context of work-energy, while Chapters 4 and 5 report the use of visualizations to enhance student understanding of the concepts of internal thermal energy and internal potential energy, respectively. The main body of the dissertation ends with Chapter 6, which includes summary and conclusions.

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CHAPTER 2

REVIEW OF LITERATURE

Systematic investigations into student understanding of physics have blossomed since the 1970's. A large number of empirical studies done in the past three decades may be categorized into three areas in terms of their primary emphasis: research about student conceptual understanding and reasoning; research about student problem solving; and research about student attitudes and beliefs [1]. Investigations of student difficulties in conceptual understanding have been more comprehensively documented than those about student problem solving or about beliefs and attitudes. Much of the research into student conceptual understanding has been in kinematics, Newtonian dynamics, electric circuits, and geometrical optics.

Some investigations into student problem solving in physics have been conducted in psychology and cognitive science. A considerable number of studies report differences between experts and novices in problem solving, in varied areas including physics topics and games [e.g., 2, 3, 4, 5, 6]. Some of these research findings and the related theoretical frameworks provide physics education researchers with a useful guide for investigating how students learn physics and what works in physics instruction [7].

It is convenient to divide this chapter into three sections. The first section reviews studies from the literature concerning student conceptual understanding of work-energy

concepts. The second section reviews published documents concerning the role of external representations in problem solving. The last one is a brief summary.

2.1 Research on Student Understanding of Work and Energy

In this section, we first review some studies concerning student conceptual understanding of the work-energy theorem and the conservation of energy principle. Then we review research on student understanding of the internal energy produced during inelastic collisions.

2.1.1 Research on Student Understanding of Conservation of Energy

The concepts of work and energy constitute a fundamental basis for physics at all levels. In introductory physics, of course, work and energy forms a very important topic of the course. However, compared to extensive studies about student understanding of kinematics and Newton's laws, few published documents address students' difficulties in understanding the concepts of work and energy.

R. A. Lawson and L. C. McDermott [8] from the Physics Education Group at the UW investigated student understanding of the work-energy and momentum-impulse theorems. The investigation probed the ability of introductory students to apply these theorems in the analysis of a real motion. The researchers administered the demonstration interviews, including an experimental set-up shown in Figure 2.1, to 28 introductory students—16 from an algebra-based course and 12 from a calculus-based honors section. Using the apparatus shown in Figure 2.1, the students were asked to compare the final momenta and kinetic energies of two dry-ice pucks, one more massive than the other, that move on a glass table. Equal forces are applied to the two pucks by a reversed vacuum
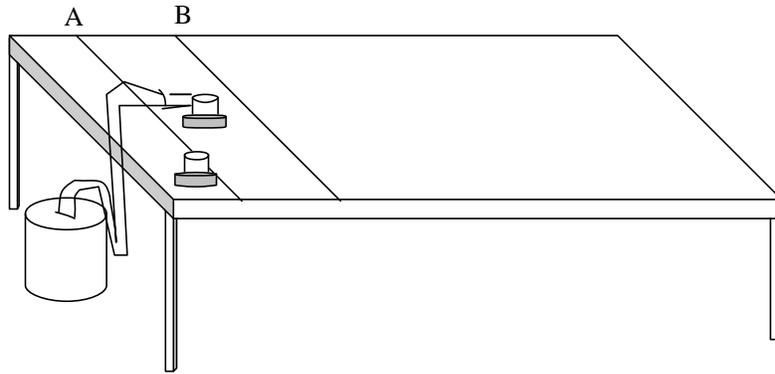
Figure 2.1: The set-up used by the researchers at the UW during individual demonstration interviews to investigate student understanding of work-energy and impulse-momentum theorems.

cleaner as they move straight forward from line A to line B. Although those students had completed the study of energy and momentum, only 50% of the honors students and none of the algebra-based students were able to make a correct kinetic energy comparison before the interviewer intervened.

The authors argued that a meaningful conceptual understanding was unlikely to be achieved after standard instruction in which students learned passively. Many students would miss some fundamentally important features of concepts that were not easily visualized if they were presented only verbally, whether by textbook or in lecture. The researchers claimed that it was necessary to engage students actively in making explicit the connections between the algebraic formalism and real world applications.

A similar study was done by European researchers [9] with secondary school students. The investigation indicates that at least 50% of the sample students (75 18-19 year old physics students from six London secondary schools) apparently lacked

comprehension of the law of energy conservation—they could not apply this principle to simple activities in real life, although they were able to make correct verbal statements. For instance, one sample problem used in the research was: "The spring in a toy car was wound up and the car let go on the floor. Why did the car stop after a while?" About 87% of the students could not correctly explain this phenomenon.

2.1.2 Research on Student Understanding of the Concept of Internal Energy in Mechanics

The concept of internal energy is often introduced to students when they learn about collisions and work done by friction. This is an important concept for students to understand the principle of conservation of energy completely, but it is a very abstract concept. Some studies have been done to investigate student understanding of this abstract concept.

Uri Ganiel [10] developed a pair of model carts: one could be used to demonstrate partially inelastic collisions during which "internal energy" is produced; the other one could be used to represent elastic collisions during which "energy transfer into internal energy" does not occur. Ganiel claimed that his model carts provided a nice visualization for students to understand "where the kinetic energy goes," but he did not show detailed assessment data from students. The effectiveness and limitation of his model carts on student understanding of the concept of internal energy are discussed in detail in Chapter 5.

A study by N. Grimellini-Tomasini and other researchers from Italy and Brazil [11] was particularly interesting, since it focused on probing student's understanding of the role and meaning of both conservation of energy and conservation of momentum laws

in describing the same collision process. One student difficulty, which they documented on the basis of data from a large and diverse population of students (in particular, from fourteen- to sixteen-year-old students), was that students lacked a meaningful understanding about what is transferred during a collision. Many students believed that in a collision something was transmitted from the projectile to the target, but they did not understand what the "something" was. Different students identified the "something" differently, such as motion, energy, force, power, inertia, thrust, velocity, acceleration, etc. For example, the researchers found that it was not difficult for students to realize that a loss of the transmitted quantity may occur during the collision, but difficult for them to give a meaning to the lost quantity. The "loss of energy" during the collision is often justified by students in terms of the colliding objects' mass, velocity, or a combination of the two, rather than energy.

To help students conceptualize energy as the quantity transmitted during the collision, the researchers introduced a visual aid (a spring bumper on a colliding body) and helped students apply familiar concepts such as kinetic and potential energy to look at what is transmitted during the collision. They found that this approach was accepted by students without difficulty. Here is an example of student explanation:

> "…kinetic energy is there before the collision; during the collision no, it's less, after the collision it is there again. But it may always be there, except that it is absorbed by the spring (the spring bumper on one the colliding bodies) during the collision, as energy that the spring has." [12]

Additionally, it was discovered that students were unable to distinguish conceptually between the concepts of energy and momentum. For instance, some students believed that energy and momentum both were conserved in an elastic collision, and that they both decreased in an inelastic collision.

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Furthermore, the study indicated that in situations in which both energy conservation and momentum conservation were applicable, students sometimes applied first one law and then the other, rather than attempting a simultaneous solution that satisfied both conditions. Even when students had mastered both conservation laws separately, they exhibited great difficulty in attempting to solve real-life problems in which they had to choose the approach to use and then make necessary approximations.

2.2 External Representations and Their Roles in Problem Solving

In this section, we first briefly review the nature of representation. Then the role of external representations in human problem solving is discussed. This is followed by a discussion of multiple representations in physics and their roles in problem solving.

2.2.1 Representation and Its Components

A representation is something that stands for something else [13]. This definition of representation seems too simple, but its related issues are extremely sophisticated and important. Many disciplines in science and social science, such as cognitive science, psychology, philosophy, linguistics, education, neuroscience, artificial intelligence, mathematics, chemistry, physics, and others, involve representation issues.

We live in a world full of representations. In real life, for example, it is common to use the symbol "🚹" to represent "man," and "🚺" for "woman;" a green traffic light represents "go," and a red traffic light represents "stop." The symbol or the light here is a kind of representation. Basically, any representation includes the following ingredients [14, 15, 16]:

1) the represented world: that which is to be represented;

2) the representing world: a set of symbols, each standing for something in the represented world;

3) what aspects of the represented world are being represented;

4) what aspects of the representing world are doing the representing; and

5) the correspondence between the two worlds.

These components of a representation are illustrated in an example (see Figure 2.2) shown in Donald Norman's book—*Things That Make Us Smart* [16]. In Figure 3.1, the represented world is the real world in which there exist a tree, mountains, people, and a ball, while the representing world is the drawing on the piece of paper at the bottom. In the representing world, the two drawn symbols represent the tree and the mountains, and the other objects, such as the ball and the people, are ignored. The spatial relation (e.g., the tree in front of the mountains) between the tree and the mountains is correspondingly represented in the relation between the tree symbol and the mountain symbol in the representing world (e.g. the tree symbol in front of the mountain symbol, but, for example, not above the mountain symbol).

2.2.2 External Representations and Their Roles in Problem Solving

Why is representation important? Donald Norman [17] says: "without external aids, memory, thought, and reasoning are all constrained." An external representation is a type of external aid to a human being and can assist people in problem solving.

External representations typically refer to 1) physical symbols, objects, or dimensions (e.g., written symbols, beads of abacuses, shapes or sizes of graphs, etc.) and 2) external rules, constraints, or relations embedded in physical configurations (e.g.,

21

The Represented World

The Representing World
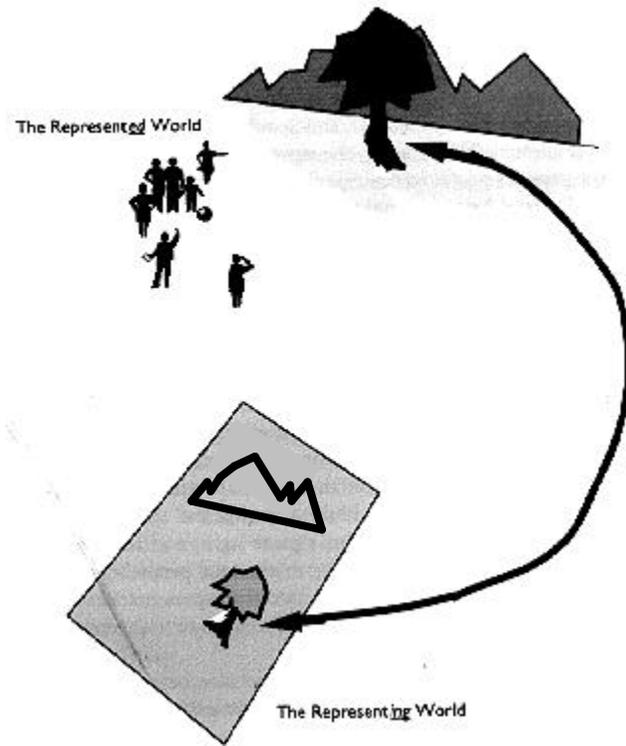
Figure 2.2: The represented and representing worlds. Symbols on the piece of paper at the bottom form the representing world and represent the real world at the top, the represented world.

spatial relations of written digits, physical constraints in an abacus, etc.) [18]. Without the use of external representations, our modern human life would be impossible.

2.2.2.1 The Roles of External Representations in Problem Solving

Below, we briefly review some important roles of external representations in a human's reasoning and problem solving.

First, external representations aid human memory [18]. This might be the most obvious function that external representations play. For example, symbols or words written in one's calendar book are external representations, and they aid one in remembering daily events.

Second, diagrammatic representations sometimes help people recognize problem features more easily and make inferences directly in problem solving. This role of diagrams was shown in a study by Jill Larkin and Herbert Simon [19]. Some details of this research are discussed in section 2.2.3.2.

Third, external representations can change the nature of tasks. Some studies [16,18, 20] in cognitive science show that different forms of graphic displays or different physical constraints for the same task can change the ease or difficulty of the task. One simple example illustrated by Donald Norman [21] is shown in Figure 2.3: three different types of drawings represent the same information—the populations of Tokyo and Beijing. By how much is the population of Tokyo larger than that of Beijing? To answer this question correctly (an estimated answer is that Tokyo is five times as large as Beijing), the bar graph representation shown in Figure 2.3 makes it easier to answer than the other two representations. Norman [21] explains this as a result of human beings being more accurate in estimating line lengths than many other shapes.
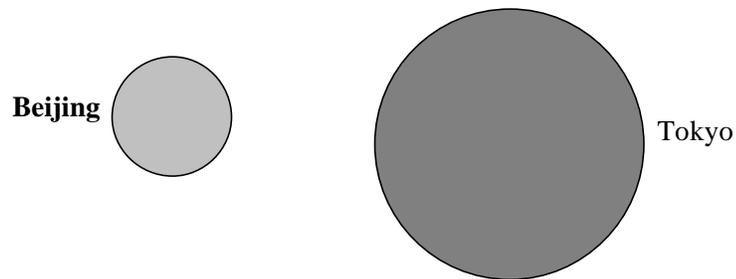
(1) Circle graph
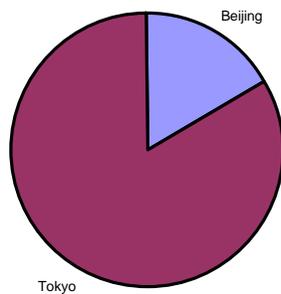
(2) Pie graph

(3) Bar graph

Figure 2.3: The three different graphs represent the same information—population sizes in Beijing and Tokyo. To answer the question "By how much is the population of Tokyo larger than that of Beijing?" the bar graph is the easiest representation to answer this question.

2.2.2.2 Basic Criteria for an Effective Instructional Representation

As discussed above, some representations can make the task easier for a solver, but others can make it more difficult. This raises an interesting and important question related to the research interest of this dissertation: What are effective representations that can help students learn formal knowledge? Swiss psychologist Kurt Reusser [22] indicates some basic criteria for evaluating the effectiveness of an instructional representation.

1) A pedagogically effective representation should help students capture the critical structural features of a problem. Feynman diagrams, widely used in quantum physics, serve as a good example for this case. For instance, electron-proton scattering is a very complex process, but a Feynman diagram vividly describes crucial features of the scattering process.

2) An ideal representation should have an efficient form that reflects the constraints of a problem, directs students to make inference, and allows them to evaluate solutions of the problem.

3) An instructionally valuable representation should mediate between "ordinary language, or the physical view of concrete objects," and "canonical scientific languages and conceptualizations" [22]. In other words, an ideal instructional representation should help our minds make a bridge between concrete daily-life and the abstract scientific domain; it should help students move more easily from the former to the latter.

The commonly used free-body force diagram in physics is an example of an ideal pedagogical representation. The free-body diagrams for the problem (two boxes A and B are pulled by a force *F* on a frictional surface) shown in Figure 2.4 not only show some

Pictorial description

A   B F

Free-body force diagrams

N_A N_B

f_B T_{AB} T_{BA} f_B F

$m_A g$ $m_B g$

Mathematical equations

$$|T_{AB}| = |T_{BA}|$$

$$T_{AB} - f_A = m_A a \qquad\qquad F - T_{BA} - f_B = m_B a$$

$$N_A - m_A g = 0 \qquad\qquad N_B - m_B g = 0$$
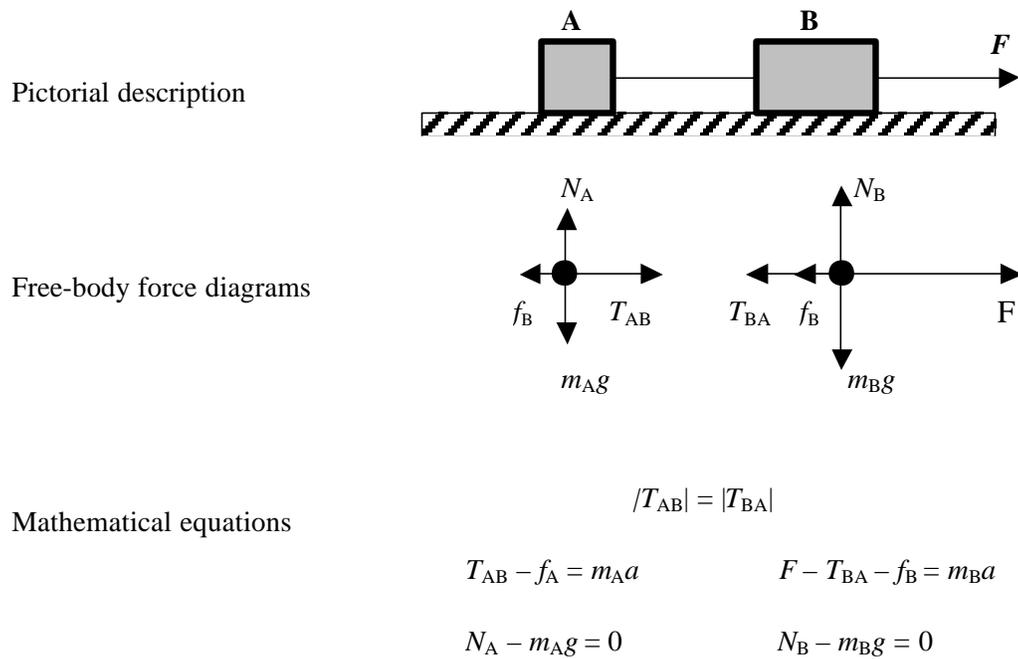
Figure 2.4: The free-body force diagrams illustrate good instructional representations to solve the dynamics problem. They not only show some constraints (e.g., $T_{AB} = T_{BA}$ in magnitude) explicitly, but also serve as an intermediate to lead students from the pictorial description with real objects to abstract formalisms and to help them set up equations correctly.
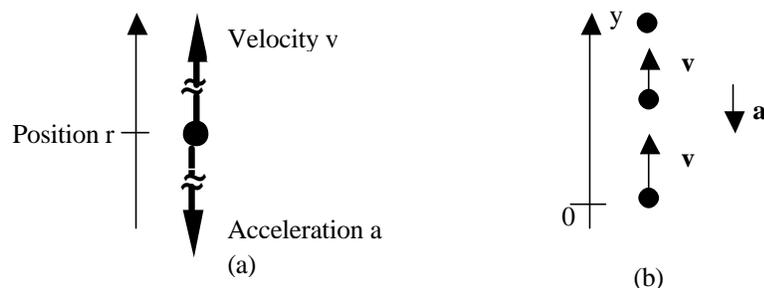
Figure 2.5: Two different motion diagrams representing a coin moving vertically upward. Motion diagram (b) represents better than motion diagram (a) the changing velocity and the relationship between velocity and acceleration.

constraints (e.g., $T_{AB} = T_{BA}$ in magnitude) explicitly, but also serve as an intermediate to

lead students from the pictorial description with real objects to abstract formalisms and to

help them set up equations correctly.

4) A proper representation should also supply students with simple and perceptual

information and reduce the demand for their mental reasoning. For instance, Figures 2.5

(a) [23] and (b) [24] show two different formats of a motion diagram representing a

simple physical process: a coin moving vertically upward. Motion diagram (b) represents

better than motion diagram (a) the changing velocity and the relationship between

velocity and acceleration.

2.2.3 Multiple Representations in Physics and Their Roles in Problem Solving

External representations aid in human problem solving and reasoning in daily life.

They also represent formal knowledge and help transfer and disseminate such knowledge.

In physics, we use a variety of diagrams, graphs, sketches, and symbolic expressions such

as mathematical notations and equations, to help us understand the real world, to build the body of knowledge, and to communicate with each other. All these diagrammatic and symbolic expressions are external representations.

In her studies [6, 25, 26, 27] Jill Larkin categorizes physics problem representations into four types: a verbal statement, a naïve representation, a physical representation, and a mathematical representation. The verbal statement is simply a problem statement in a natural language. The naïve representation is defined by Larkin as a sketch that is composed of real-world objects (such as blocks, springs, pulleys) and describes the problem situation in a real time order sequence [28]. In contrast, the physical representation, very often used by physicists, contains more abstract physics concepts (force, energy, momentum, etc.) and corresponds to physics laws (such as Newton's laws, conservation of energy, or conservation of momentum). An example is a free-body diagram for solving dynamics problems [26]. These diagrams (the physical representation) can aid in qualitative reasoning. For solving a work-energy problem (e.g., a block-sliding problem), an expert could have a qualitative understanding of the problem as "The energy at C1 consists of kinetic energy…and potential energy determined by the known height h_1… " [25], after he or she used a naïve representation but before applying equations. In both naïve and physical representations, the inference rules are qualitative. This is in contrast to quantitative equations, which are called mathematical representations.

On the basis of Larkin's work, Van Heuvelen [24, 29, 30] developed a multiple representation technique to represent physics problems (see Figure 2.6). In his approach, a typical physics problem is considered as a physical process, and this process

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Multiple Representations in Kinematics

Verbal Representation

A car at a stop sign initially at rest starts to move forward with an acceleration of 2 m/s^2. After the car reaches a speed of 10 m/s, it continues to move with constant velocity.

Pictorial Representation

$t_0 = 0$ $a_{01} = + 2$ m/s^2 $t_1 = ?$ $a_{12} = 0$ $t_2 = ?$
$x_0 = 0$ $x_1 = ?$ $x_2 = ?$
$v_0 = 0$ $v_1 = + 10$ m/s $v_2 = v_1 = + 10$ m/s

Physical Representation (Motion Diagram)

a_{01} $a_{12} = 0$

v   v v_1 $v_1 = v_2$

Physical Representation (Kinematics Graphs)
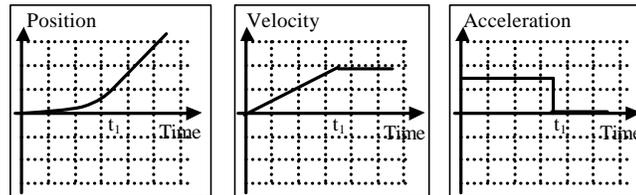
Mathematical Representation

For $0 < x < x_1$ and $0 < t < t_1$	For $x_1 < x$ and $t_1 < t$
$x = 0 + 0 \cdot t + (1/2)(2$ m/s$^2) t^2$	$x = x_1 + (10$ m/s$) t$
$v = 0 + (2$ m/s$^2) t$	$v = + 10$ m/s

Figure 2.6: The kinematics process can be represented in verbal, pictorial, physical, and mathematical representations.

is represented in verbal, pictorial, physical, and mathematical representations, which are similar to Larkin's definitions except for the physical representation. Van Heuvelen explicitly defines the physical representation as physical diagrams and graphs such as motion diagrams, free-body diagrams, kinematics graphs, and others.

2.2.3.1 Differences in Problem Solving between Experts and Novices

In the past thirty years, substantial progress in cognitive science, psychology, and physics education has been made in exploring and explaining differences in problem solving between experts and novices. Researchers found that physicists and introductory students use problem representations differently. Based on studies [6, 26, 27, 31] by Jill Larkin, Herbert Simon, and other researchers, differences between expert and novice performance in solving physics problems can be briefly summarized as:

1) Knowledge difference: experts know many things and can rapidly evoke the particular items relevant to the problem at hand, but novices cannot.

2) Physicists categorize problems as they correspond to physics concepts and principles, but novice students categorize problems as they correspond to surface features (inclined planes, ropes, springs, etc.)

3) Skilled solvers usually solve problems by working forward—from given information to the desired quantities. In contrast, inexperienced students solve problems backwards—from the unknown variables to the given quantities.

4) A typical problem solving procedure by experts often includes three stages: drawing a sketch, developing physical representations such as free-body diagrams, and applying mathematical equations. On the other hand, beginning students usually jump directly

from either the problem statement or the naïve sketch to mathematical formulas, without the use of physical representations.

5) Skilled solvers often carry through as one single step from identifying the right equation, substituting the values of the quantities in it, and solving for the unknown variable. However, inexperienced solvers usually treat substituting values for variables and solving the equation as distinct, separate steps. And also they often ask themselves "What do I do next?" during problem solving and usually make a decision partly by looking at the equations and partly by thinking about the problem goal or sub-goals.

The central difference in problem solving between experts and novices, according to Jill Larkin [27], is that novices have much less ability than experts to construct or use physical representations. Several empirical studies support her point. First, an investigation done by Chi, et al. [31] shows that when beginning students were asked to sort physics problems into categories, they use problem surface features such as inclined planes, springs, or blocks, called naïve representations by Larkin, to group the problems. In contrast, experts sorted the problems using deep structures of physics knowledge such as energy problems or force problems, called physical representations by Larkin. In addition, studies by Larkin et al. [6, 26, 27] have found that, by comparing problem written solutions and think-aloud protocols from experts and novices when solving some typical textbook kinematics and dynamics problems (see two examples in Figure 2.7), skilled physicists generally first make sketch containing real objects in the problem statement, then develop a physical representation such as a free-body force diagram, and apply equations last. However, inexperienced students often jump directly either from the

Kinematics Problem   **Dynamics Problem**

A bullet leaves the muzzle of a gun
at a speed of 400 m/sec. The length
of the gun barrel is 0.5 m. Assuming
that the bullet is uniformly
accelerated, what is the average
speed within the barrel.
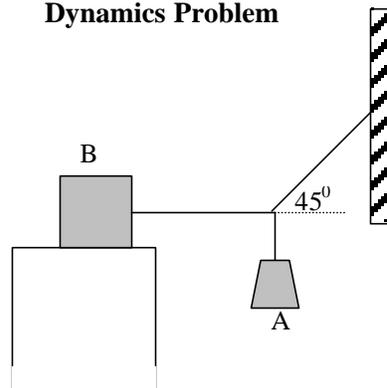
Block B weighs 160 pounds. The coefficient of
static friction between the block and the table
is 0.25. Find the maximum weight of block A
for which the system will be in equilibrium.

Figure 2.7: Example interview problems used by Jill Larkin et al. to investigate
differences in problem solving between experts and novices.

problem statement or from a real-life-type drawing to mathematical formulas, without the intermediate step—the use of physical representations.

A second result supporting experts' use of physical representations is also from the studies by Larkin et al., as mentioned above. They discovered that student subjects solved most of the problems by working backward—starting with unknown quantities to select equations (mathematical representations) containing unknown variables. On the other hand, experts worked out the problems forward—from the given information to physical representations and then to mathematical representations.

Third, two detailed studies by Larkin, one including six physics graduate students and professors to solve a single difficult problem in mechanics [27] and the other including one physics professor to solve five moderately difficult mechanics problems [27, 32], revealed that these physicists spent considerable time developing and testing possible physical representations. Once a physical representation was constructed, the mathematical calculations proceeded rapidly.

2.2.3.2 The Roles of Multiple Representations in Physics

What function does each representation play in problem solving? What special role does a physical representation play? In other words, why do experts use physical representations in their solutions?

Below, a typical physics problem [33] illustrates the important issues identified above. In Figure 2.8, a physical process first is represented in natural language—a verbal representation. Following the verbal representation, we sketch the situation—a pictorial representation. The verbal and pictorial representations are informationally equivalent [19, 34] since each of them can be constructed from the information in the other.

Verbal Representation

A miner pulls a wagon of supplies to the top of a hill where he lives by hanging from a rope attached to the wagon. As the cart moves at increasing speed up the hill, the miner moves downward with increasing speed down the side of a cliff.

Pictorial Representation
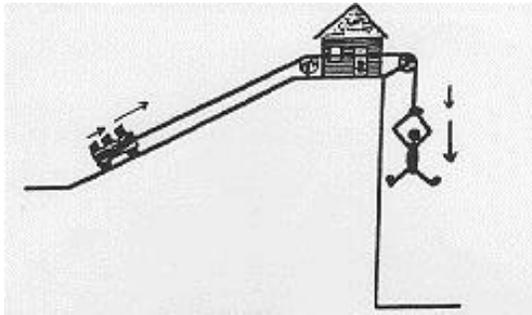
Physical Representations
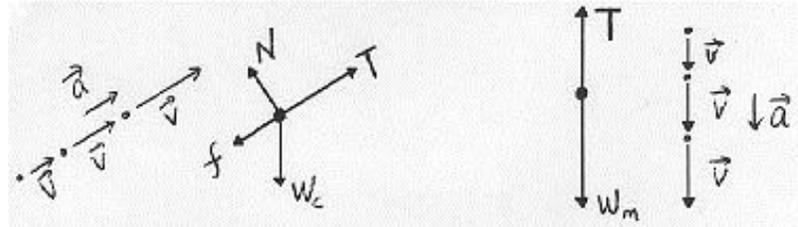
Mathematical Representations

$$T - f - W_c \sin q = m_c a \qquad W_m - T = m_m a$$
$$N - W_c \cos q = 0$$
$$v_f^2 - v_0^2 = 2ad \qquad\qquad v_f = v_0 + at$$

Figure 2.8: The physics process is described in the multiple representations. Each representation has its own functions in problem solving.

Therefore, translating the verbal description into the sketch does not provide new information. However, a study conducted by Larkin and Simon [19] reveals that pictorial representations are superior to verbal representations for solving the problem in the following two ways:

- Diagrams automatically aid in perceptual inferences, which are very easy for humans.

- Diagrams group together all interrelated information using location, thus avoiding a large amount of searching for what is needed in problem solving.

These advantages of pictorial representations usefully and efficiently support one's reasoning in problem solving.

The pictorial representation includes all real objects such as the hill, the rope, the wagon, the miner, and so on; but it does not have physics meaning. It describes the physical process following a time flow. There is a considerable gap to move directly from the pictorial representation to the mathematical representation(s) that corresponds to physical laws and principles. This gap is filled by physical representations; and the pictorial sketch is helpful for constructing the physical representations.

The physical representations in Figure 2.8 are consistent with the pictorial representation and consist of both motion diagrams and free-body force diagrams for the wagon and the miner. From the physical representations, one can see that the motion diagrams are similar to the drawings (such as arrows) used to describe the motions of the wagon and the miner in the pictorial representation, but the motion diagrams are body–free—independent of the surface features of the real objects. This illustrates that the pictorial representation may not be useful for directly constructing the mathematical representation, but it is helpful in developing the physical representations. Similar to the

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pictorial representation, the physical representations also have qualitative features and diagrammatic formats. But they have some unique advantages in general [25], as listed below:

- Physical representations have meanings in physics (e.g. force, velocity, acceleration, etc.).

- They are closely associated with fundamental principles of physics (e.g. Newton's laws).

- They are often time-independent (e.g. constant acceleration, the directions of the forces do not change, etc.)

The physical representations are extremely helpful in setting up the abstract mathematical representations, which can provide quantitative results for the problem.

2.3 Discussion and Summary

In summary, on the basis of the discussions in section 2.1, few studies have been done to probe systematically student understanding and problem solving in the context of work and energy. The published documents show that many students lack a deep understanding of the concepts of work and energy and are unable to apply either the work-energy theorem or the conservation of energy principle to real-life phenomena. Standard instruction in which students passively observe demonstrations and problem solving performed by instructors, is not an effective teaching model enabling most students to visualize the abstract concepts of work and energy and to develop connections between the algebraic formalism and real world applications.

To help students address these difficulties, innovative learning materials and interactive instructional strategies are essential. *Tutorials in Introductory Physics* [35]

illustrates a successful model that uses the research-based curriculum and the interactive engagement teaching technique to help students gain a meaningful conceptual understanding and to develop scientific reasoning skills. For instance, after the tutorial sections of energy and momentum [35], introductory physics students at the UW gained a better conceptual understanding of the work-energy and impulse-momentum theorems than they had after standard instruction, in terms of the student proportion of correctly answering and reasoning about some conceptual questions [36].

The tutorials, a set of supplements to the lectures and textbook of a standard introductory physics course, were developed with more emphasis on the development of conceptual understanding and scientific reasoning skills, but did not emphasize the development of student problem solving skills. A research-based, systematic learning system in the context of work and energy is needed to help students build a solid conceptual foundation and acquire expertise in problem solving.

As the studies reviewed in section 2.2 show, there is ample evidence that external representations play very crucial roles in human problem solving. Herbert Simon [37] says "Solving a problem simply means representing it so as to make the solution transparent." Problem solving in physics is such a case. The multiple representation strategy developed by Van Heuvelen considers a typical problem as a physical process; solving the problem means representing it in verbal, pictorial, physical, and mathematical representations, rather than just figuring out some unknown numbers.

Many studies have been done to explore how and why experts and novices perform differently in problem solving in many different disciplines. On the basis of empirical research findings, some researchers (e.g., Larkin) argue that a central difference

between experts and novices in physics problem solving is the extent to which they can use physical representations. Therefore, to help students develop expertise in problem solving, it becomes significantly critical to help them learn how to use physical representations in their problem solving. The next chapter illustrates a study to help students learn to use a physical representation (i.e., the work-energy bar chart) and a multiple-representation strategy to solve work-energy problems and to help them acquire expert-like problem solving skills.

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CHAPTER 3

MULTIPLE REPRESENTATIONS OF
WORK-ENERGY PROCESSES

Based on the previous research discussed in Chapter 2, it is clear that a major

difference in problem solving between experts and novices is that experts integrate

qualitative representations into their reasoning and understanding processes when they

solve problems, but novices do not (see Section 2.2.3.1). Thus, to help naive students

acquire expertise in problem solving, we need explicitly to help them learn a systematic

problem-solving strategy using multiple representations, especially qualitative

representations, in solving physics problems.

This chapter is composed of four parts. The first part discusses investigations into

student difficulties using qualitative and quantitative representations in solving work-

energy problems. Written test questions and individual student interviews are illustrated

in detail. The second part describes how to develop a multiple-representation approach to

represent work-energy processes and how to help students learn this strategy to solve

work-energy problems. This is followed by extensive assessment results describing the

effectiveness of this multiple-representation method in student understanding and

problem solving in work-energy. This chapter ends with discussion and summary.

3.1 Investigation of Students' Difficulties Using Qualitative and Quantitative Representations in Work-Energy Problem Solving

It is well known that students attempt to solve problems by matching quantities listed in the problem statement to special equations that have been used to solve similar problems [1]. Students move between words and equations, which are very abstract representations of the world, with no attempt to connect either representation to more qualitative representations that improve understanding and intuition. However, few studies have been done to investigate how students solve work-energy problems from a perspective of problem representation. To explore this research question, we carefully analyzed students' solutions on three written-test work-energy problems and conducted in-depth individual student interviews.

3.1.1 Written Test Questions (Autumn 1997 and Winter 1999)

To investigate how students solve work-energy problems using different representations, we consider a physics problem as a physical process. A physical process can be represented in verbal, pictorial, physical, and mathematical representations, which were discussed in detail in Chapter 2. Following this multiple-representation problem-solving strategy, we carefully analyzed some students' final exam solutions of work-energy problems from an OSU calculus-based introductory physics course (Physics 131) in the autumn quarter of 1997 and in the winter quarter of 1999.

3.1.1.1 Student Use of Physical and Mathematical Representations in Solving a Work-Energy Problem (Roller-Coaster Problem, Autumn 1997)

In autumn 1997, we collected some students' final examination solutions from the Physics 131 class. This ten-week class covered the regular concepts of mechanics:

kinematics, Newton's dynamics, work-energy, momentum, and some rotational dynamics. The work-energy concepts were taught in a standard way: The concept and mathematical expression of each type of energy was introduced first. Then, the conservation of energy principle was explored. After this, students practiced solving standard work-energy problems quantitatively. They worked with the work and energy concepts for about five lecture periods, four recitation periods, two laboratory periods, and in their homework assignments. We collected solutions for the final examination from twenty-seven students in one typical recitation section of Physics 131 and carefully analyzed their performances on a work-energy problem (see Figure 3.1. The problem was written by the course instructor).

Question (a) of the problem shown in Figure 3.1 asked for a simple calculation of the potential energy of car at point A, B, and C, based on the definition of gravitational potential energy. For research interest, we carefully analyzed students' responses to questions (b) and (c) from a perspective of multiple representations. Since the problem was given in words with a detailed sketch, we looked carefully at how students used physical and mathematical representations when answering questions (b) and (c). Here the physical representation could be a free-body force diagram or any other type of physics-like graphical representation. About the mathematical representation, we distinguished it into two types, algebraic and numerical representations. An algebraic representation was a symbolic equation or expression, while a numerical representation was a mathematical equation or description using the Arabic numerals. For example, applying the conservation of energy principle for solving question (b) shown in Figure 3.1, one could use two different types of mathematical equations that were consistent

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An 800 kg car rides along the path shown below. It starts from rest at point A and rolls along until it comes to rest at point C. There is a constant force of friction acting on the car. Unfortunately, we don't know what the coefficient of friction is. The distance along the path from point A to point B is 20.0 m, while the distance along the path from point B to point C is 10.0m.
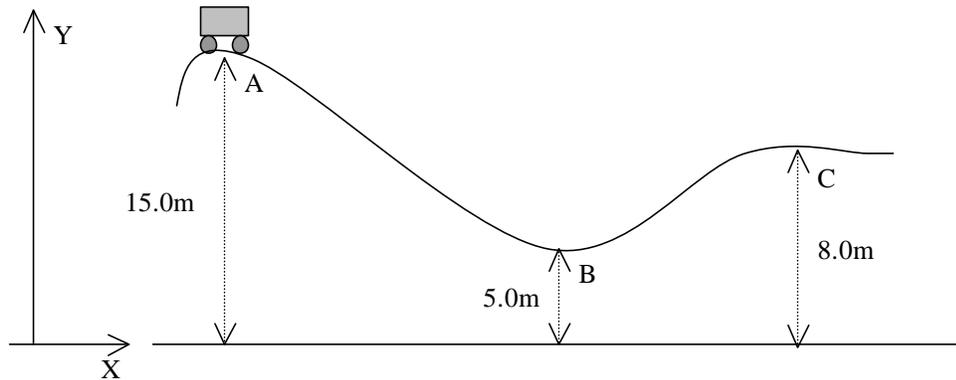
(a) What is the potential energy of the car at points A, B, and C, assuming that the level surface is at y = 0, and U = 0 there?
(b) What is the magnitude of force of friction the road exerts on the car?
(c) What is the speed of the car at point B?

Figure 3.1: The work-energy problem (roller-coaster problem) given on the Physics 131 final examination in the autumn quarter of 1997.

with the conservation of energy principle:

(1) $mgh_A = mgh_C + F_f d,$

(2) $(800 \text{ kg})(9.8 \text{ m/s}^2)(15.0) = (800 \text{ kg})(9.8 \text{ m/s}^2)(8.0) + F_f(20 \text{ m} + 10 \text{ m})$, or

(3) $117600 = 62720 + F_f d.$

Equation (1) above was an example of the algebraic representation, while equation (2)

showed an instance of the numerical representation. When analyzing the students'

solutions, we found that some students wrote the energy conservation equation in the

format of equation (3). We identified it as a type of the numerical representation as well.

(The differences between the algebraic and numerical representations and their roles in

problem solving are discussed in detail in Section 3.1.2) Summaries of the students'

solutions to questions (b) and (c) are reported in Tables 3.1 and 3.2.

3.1.1.2 Discussion of Student Use of Physical and Mathematical Representations in Solving the Roller-Coaster Problem

The problem shown in Figure 3.1 was a typical moderately difficult problem. We

considered a student's solution of question (b) or (c) as a correct one when it had a

correct set-up (i.e., the correct conservation of energy equation) and each quantity in the

energy equation had its correct value. We counted a solution correct if it had the correct

physics, but contained simple calculation errors.

From Table 3.1 we saw that out of twenty-seven students, sixty-two percent

answered the question (b) correctly, nineteen percent answered it incorrectly, and

nineteen percent either had no answers or had unclear ones. Overall, however, only about

eight percent (two students) drew a free-body diagram on their solutions. (The two force

diagrams they drew were not correct in terms of the number of forces exerting on the car

or the force directions.) No other types of physical representations were used in the

	One Recitation Section Physics 131, Autumn 1997 N = 27		
	Correct Solutions	Incorrect Solutions	Unclear or Blank
Total %	**62%**	**19%**	**19%**
Physical Representation	0%	4%	4%
Mathematical Representation — Algebraic Representation	25%	8%	N/A
Mathematical Representation — Numerical Representation	37%	11%	

Table 3.1: Student use of physical and mathematical representations in solving question (b) of the work-energy problem shown in Figure 3.1.

	One Recitation Section Physics 131, Autumn 1997 N = 27		
	Correct Solutions	Incorrect Solutions	Unclear or Blank
Total %	**33%**	**59%**	**7%**
Physical Representation	0%	4%	0%
Mathematical Representation — Algebraic Representation	26%	40%	N/A
Mathematical Representation — Numerical Representation	7%	19%	

Table 3.2: Student use of physical and mathematical representations in solving question (c) of the work-energy problem shown in Figure 3.1.

students' solutions. There could be several reasons for no use of physical representations in the students' solutions. First, there was not a good physical representation for representing work-energy processes. Solving dynamics problems relied on a free-body diagram or a force diagram. There was, however, no similar physical representation for solving work-energy problems. The students did not learn physics-like diagrams to help them in work-energy problem solving. Second, although the students did learn the free-body diagram in dynamics, the role it played in problem solving usually was not emphasized enough. The students could consider that drawing a force diagram was a redundant step. Without the mastery of a systematic, coherent multiple-representation strategy to solve physics problems, the students could not realize that a physical representation such as a force diagram acted as a bridge between the verbal and the mathematical representations and could help them move in smaller and easier steps from words to equations. Third, with the time pressure needed to complete an examination, the students skipped steps, which they felt were not needed to complete the solution.

The results in Table 3.1 also showed that among sixty-two percent of the students who solved question (b) correctly, twenty-five percent started to solve the question (b) with the generalized work-energy equation (called an algebraic representation) such as equation (1) shown in Section 3.1.1.1; but thirty-seven percent started with the work-energy equation including numbers (called a numerical representation) like equation (2) or (3) shown in Section 3.1.1.1. Among nineteen percent of them who could not solve question (b) correctly, eight percent set up the generalized work-energy equation and eleven percent started with the work-energy equation including numbers. Overall, about half of the students just played number games in their solutions.

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In Table 3.2, we saw that only thirty-three percent of the twenty-seven students correctly solved question (c) shown in Figure 3.1. Students' primary difficulties were identified as: 1) not including the work done by friction; 2) confusion about the sign of the work done by friction; and 3) not counting in the gravitational potential energy of the car-earth system at point B.

Again, few students used a physical representation in their solutions. But more students started with the generalized work-energy equation to answer question (c) than to answer question B. Among the thirty-three percent of the students who correctly solved question (c), twenty-six percent started with the algebraic representation. Among the fifty-nine percent of them who incorrectly solved the question, forty percent started with the algebraic representation.

In summary, not surprisingly, we found that students used an equations-centered approach to solve the work-energy problem. There was a lack of a systematic and coherent multiple- representation strategy that could help them develop expertise in work-energy problem solving. In addition, it appeared that many students just played with numbers in their solutions. It was interesting to study what functions an algebraic representation and a numerical representation played in problem solving. This issue is addressed in Section 3.1.2.5.

3.1.1.3 Student Use of Pictorial Representations in Solving Two Work-Energy Problems (Block-Spring Problem and Incline-Loop Problem, Winter 1999)

If a work-energy process was only described in words without a sketch, did students draw a picture or a sketch to help them understand and solve the problem? To answer this question, we analyzed students' solutions of two written questions on the final examination for Physics 131 in the winter quarter of 1999 at OSU. This was a

standard calculus-based introductory mechanics class. The two questions (developed by the class instructor) were shown in Figure 3.2 and 3.3, respectively. Although the questions were given in a multiple-choice format, the student could not answer them just based on conceptual reasoning. Instead, a quantitative calculation applying the conservation of energy principle was needed for one to be able to choose the correct answer. The question shown in Figure 3.2 was a typical block-spring work-energy problem; the question shown in Figure 3.3 was an incline-loop work-energy problem. We thought that drawing a sketch consistent with the verbal description was helpful in visualizing and understanding the physical processes. If a student drew such a picture that was reasonably clear and in which the quantities were labeled, we considered it as a pictorial representation. Solutions of these two questions from twenty-nine students in a typical recitation section of Physics 131 were carefully analyzed to see how they used a pictorial representation in their problem solving. We summarized the students' solutions for the block-spring and the incline-loop questions in Tables 3.3 and 3.4, respectively.

3.1.1.4 Discussion of Student Use of Pictorial Representations in Solving the Two Work-Energy Problems

Choice a) was the correct answer for the block-spring problem shown in Figure 3.2. Out of twenty-nine students, forty-eight percent chose the correct answer (see Table 3.3). But overall, only about ten percent used a pictorial representation in their solutions.

Choice b) was the correct answer to the incline-loop problem shown in Figure 3.3. Out of twenty-nine students, sixty-two percent chose the correct answer (see Table 3.4). Overall, about fifty-five percent drew a reasonably clear sketch in their solutions.

A 2.0 kg block slides across a frictionless surface and collides with a massless spring with a spring constant of 30 N/m. The block compresses the spring by 12 cm from its rest position. What was the kinetic energy in joules of the block just before it impinged on the spring.

  a) 0.22
  b) 0.44
  c) 3.6
  d) 0.47
  e) none of the above

Figure 3.2: The work-energy problem (block-spring problem) appeared on the Physics 131 final examination in the winter quarter of 1999.

	One recitation Section Physics 131, Winter 1999 N = 29		
	Correct choice	Incorrect choice	Total %
Total %	48%	52%	100%
Pictorial representation	10%	0%	10%
Some drawing but not good enough or no sketch	38%	52%	90%

Table 3.3: Student use of a pictorial representation when answering the problem shown in Figure 3.2.

A cart of mass 400 g is set on a frictionless incline at a height of 60 cm above the bottom of a loop-the-loop that has a radius of 5.0 cm. What is the velocity of the cart in meters per second when it passes over the top of the loop?

 a) 3.5
 b) 3.2
 c) 2.8
 d) 2.0
 e) none of the above

Figure 3.3: The work-energy problem (incline-loop problem) appeared on the Physics 131 final examination in the winter quarter of 1999.

	One recitation Section Physics 131, Winter 1999 N = 29		
	Correct choice	Incorrect choice	Total %
Total %	62%	38%	100%
Pictorial representation	41%	14%	55%
Some drawing but not good enough or no sketch	21%	24%	45%

Table 3.4: Student use of a pictorial representation when answering the problem shown in Figure 3.3.

Although these two problems had different surface features, they were both simple work-energy problems and could be answered by directly applying the conservation of energy principle. But it was interesting to find that about fourteen percent more students answered the incline-loop problem correctly than answered the block-spring problem correctly. Also, about thirty-one percent more students drew a sketch in their correct solutions for the incline-loop question than in the correct solutions for the block-spring problem. It seemed that it was more difficult for the student to create a mental picture about the physical process described in the incline-loop problem than that in the block-spring problem. So the student needed to draw an external pictorial representation to help him or her visualize and understand the problem. Drawing a picture appeared to be helpful to the student in answering the problem correctly. But we also found that many students were not accustomed to drawing an effective sketch in their problem solving. A major student difficulty we discovered was that some students did not label the values of relevant quantities in their sketches. As a result, such a sketch could be less helpful to the student in understanding the problem than a sketch with valuables labeled.

3.1.1.5 Discussion of Written Test Results

On the basis of student performance on the three test problems discussed above, it was seemly fair to summarize that:

1) Students seldom used any qualitative, physical representation in solving the work-energy problem. There was a lack of an effective physical representation for work-energy processes and a lack of a systematic, coherent multiple-representation strategy to help students develop expertise in problem solving.

2) Students used pictorial representations in their problem solving. It appeared more natural for students to draw a picture than to use a physical representation. Since a physical representation had special physics meanings, students needed to learn how to construct and use it. A pictorial representation, however, included real-life objects and situations. It was not difficult for most students to draw such a sketch based on their real-life experience. But we found that some students did not indicate any physics quantities and/or their values in their sketches. A drawing without any labels that had physics meanings would not be as effective as one with physics meanings in helping reason about the problem.

3) Students primarily relied on mathematical representations to solve quantitative problems. It was interesting to find that some students set up the generalized work-energy equation (i.e., algebraic representation) first and then plugged in numbers, but a considerable number of students started at the very beginning with plugging in numbers to every known quantity, separately calculated out each related concept such as kinetic energy, gravitational potential energy and so forth, and assembled these numerical results together to have an energy-conservation equation (i.e. numerical representation). It was interesting to probe what functions algebraic and numerical representations play in students' problem solving, which was an undocumented issue.

3.1.2 Think-Aloud Individual Student Interviews (Autumn 1999)

In the autumn quarter of 1999 at OSU, think-aloud individual student interviews were conducted to investigate how students solved work-energy problems. For research interest, the interview was designed to investigate 1) how students solved work-energy problems from a perspective of problem representations before formally learning the

work-energy concepts; 2) how they used a physical representation without completely understanding its roles in problem solving; and 3) what differences existed if a student started with the generalized work-energy equation or with a numerical work-energy equation to answer certain questions.

Fourteen honors engineering freshmen students from an introductory calculus-based mechanics class (Physics 131E) participated in the interviews on a voluntary basis. By the time of the interview, the student had been exposed to the concepts of kinematics and dynamics but not the concepts of work and energy (but all fourteen students were exposed to the context of work and energy in their high school physics classes). So this interview was called the pre-interview in this section.

The student was told that the interview aimed to improve the teaching and learning of physics in the introductory classes; their performance on interview questions would not affect their class grades. Each participant was paid a small amount of money for his/her help and time.

3.1.2.1 Interview Setup and Procedure

During the interview, the student first was instructed how to keep talking about what they were thinking while solving the interview problems. To let the subject become comfortable with this thinking-aloud approach, the student was given three warm-up exercise questions (see Appendix A).

After getting comfortable with how to talk aloud while solving the problem, the student was given an equation sheet with the definitions of different kinds of energy (see Appendix A). The mathematical expression for each type of the energy was explained to the subject by means of a demonstration. For example, to help the subject understand

kinetic energy, we showed the student a moving toy car, which had the kinetic energy. For understanding elastic potential energy, we showed the student a compressed and a stretched spring, which had elastic potential energy.

After helping the student review the basic concepts of work and energy, we introduced the work-energy bar chart and gave the student a short tutorial on it. The work-energy bar chart (see Figure 3.10) is a bar graph describing the conservation of energy for work-energy processes. It has the fundamental features of a physical representation (see more details in Section 3.2.1.) We instructed the student on the meaning of each term in the bar chart and showed him or her the structure of the bar chart, which represented the conservation of energy for a work-energy process. We also showed how to draw a bar under a particular type of energy, which qualitatively represented a certain amount of the energy. The student was informed that the sum of the height of bars on the left should equal the sum of the height of bars on the right, which qualitatively represented the conservation of energy. But we did not explicitly teach the student how to use the work-energy bar chart in solving a work-energy problem. After this introductory part of the interview, the student started to solve three interview problems (see Appendix B).

Notice that, of fourteen students, twelve were given the problems with the work-energy bar chart, as shown in Appendix B. Two students, randomly selected on arriving at the site, were given three similar problems but without the work-energy bar chart (see Appendix C). These two students did not receive the tutorial on the work-energy bar chart during the introduction.

3.1.2.2 Interview Problems

Below are the three interview problems shown in Appendix B:

Problem 1: It consisted of two parts. The first part was a verbal problem statement and a sketch consistent with the statement. The second part included four questions. The problem statement and the four questions were written on five separate cards. They were placed face down and piled in the order of the problem statement, Question 1.0, Question 1, Question 2, and Question 3. The student was instructed to flip over each card and to answer each question in order. When the subject felt satisfied about his or her answer, he or she could go to the next question on the card.

Problem 2: It consisted of two parts as well. The first part was a problem statement including a verbal description of a physical process, a sketch consistent with the description, and a work-energy bar chart. The second part consisted of three questions. Similarly, the problem statement and the three questions were written on four separate cards. They were placed face down and piled in the order of the problem statement, Question 1, Question 2, and Question 3. The student was instructed to flip over each card and to answer each question in order. When the subject felt satisfied about his or her answer, he or she could go to the next question on the card.

Problem 3: It also consisted of two parts. The first part was a problem statement including a verbal description of a physical process, as well as a sketch and a completed work-energy bar chart, which were consistent with the physical process described in the verbal statement. The second part consisted of three questions. Similarly, the problem statement and the three questions were written on four separate cards. They were placed face down and piled up in the order of the problem statement, Question 1, Question 2,

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and Question 3. The student was instructed to flip over each card and to answer each question in order. When the subject felt satisfied about his or her answer, he or she could go to the next question on the card.

Below were the three interview problems without the work-energy bar chart, which are showed in Appendix C:

Problem 1: The problem statement and questions 1, 2, and 3 were exactly the same as those of problem 1 with the work-energy bar-chart, except for question 1.0, which asked the student to complete the work-energy bar chart.

Problem 2: Questions 1, 2, and 3 were exactly the same as those of problem 2 with the work-energy bar chart, except for the problem statement. The problem statement did not include the work-energy bar chart, but the verbal statement and the sketch were the same.

Problem 3: Questions 1, 2, and 3 were exactly the same as those of problem 3 with the work-energy bar chart, except for the problem statement. The problem statement did not include the completed work-energy bar chart, but the verbal statement and the sketch was the same.

Papers and pencils were provided to the students, and they were allowed to use a calculator if needed. Students' paper-pencil solutions were collected and their verbal explanations were recorded.

3.1.2.3 Analysis of the Interview Problems from a Perspective of Multiple Representations in Physics

Since the interviews were conducted before the students learned the concepts of work-energy, the interview problems were designed to be relatively simple. But to investigate the research questions described in Section 3.1.2.1, we carefully constructed

each question of the three interview problems. In this section, we discuss detailed

structures of each interview problem from a perspective of multiple representations in

physics (see more details in Section 2.2.3.).

Problem 1 (see Appendix B): Figure 3.4 showed a problem representation chart

for problem 1 from a perspective of the multiple representations. The figure consisted of

two parts. The right part was composed of the problem statement and the four questions.

The left part indicated how the physical process described in the problem statement could

be represented in the multiple representations—verbal, pictorial, physical and

mathematical representations.

In the problem statement, the verbal and pictorial representations were given. But

in question 1.0 the student was asked to complete the work-energy bar chart, which was

intended to play the role of a physical representation. To answer this question, on the

basis of the verbal and pictorial representations, the student needed to draw a bar with

some height on the slot for the initial gravitational potential energy term and a bar with

some height on the slot for the final kinetic energy term on the chart. The height of each

bar had to be equal, which represented that the energy was conserved during a change

from the initial state to the final state. This was a qualitative question. To complete the

bar chart, one needed to understand the physical process conceptually.

To answer Question 1, one needed to do a quantitative calculation applying the

generalized work-energy equation, which could be either in an algebraic representation or

a numerical representation, as shown in Figure 3.4.

Question 2 was a qualitative reasoning question. It could be readily answered if

one reasoned based on the algebraic representation (the generalized work-energy

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Problem 1
(With Work-Energy Bar-Chart)

Problem statement

Verbal Representation

Pictorial Representation

Physical Representation
Work-Energy Bar Chart

Initial Energy +Work = Final Energy

$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{\text{int}}(\text{friction})$

0

0

Question 1.0: Qualitative Question
(Concepts-Based)

Mathematical Representation

Algebraic Representation

$$mg\, h_A = \frac{1}{2}m\,v^2$$

Numerical Representation

$(60 \text{ kg})(9.8 \text{ m/s}^2)(40 \text{ m})$
$= 23520 \text{ J}$
$23520 \text{ J} = \frac{1}{2}(60 \text{ kg})(v^2)$

Question 1: Quantitative Question

Question 2: Qualitative Question
(Equations-Based)

Question 3: Qualitative Question
(Concepts-Based)

Figure 3.4: A problem representation chart for problem 1 used during the pre-interview. The representation chart is composed of two parts: the left part consisting of the multiple representations of the physical process and the right part consisting of the problem statement and the questions in problem 1. The arrows connecting them indicate which representation could be used to answer which question readily.

question). From the algebraic representation shown in Figure 3.4, it was apparent that the mass m in the equation could be cancelled out, so the speed of the skier at point B was independent of the skier's mass. Thus, the woman and the male skiers should have the same speed when they reached point B. To answer this question, one would need more efforts or mental reasoning if he or she used either the bar chart or the numerical representation. For example, if the work-energy bar chart was used to answer question 2, one had to reason that since the male skier was more massive than the woman skier, the initial gravitational potential energy bar U_{go} would get higher. Based on the conservation of energy principle, the final kinetic energy bar K would get higher as well so that bars U_{go} and K would have the same height. Compared with bar K representing the woman's kinetic energy at point B, bar K for the man's kinetic energy at point B was higher. But just based on a comparison of the height of two kinetic energy bars, one still could not decide about the man's speed at point B, since the kinetic energy was associated with mass as well. Also, this would easily mislead a naïve student to come up an incorrect answer, that the man's speed was greater than the woman's speed since bar K for the man's kinetic energy at point B was higher than the bar K representing the woman's kinetic energy at point B. If using the numerical representation to answer question 2, a student had to replace the woman's mass with the man's mass and then to calculate his speed at point B. Doing so, it would not only take a student more time, but also would become difficult for the student to understand conceptually that the skier's speed at B was independent of the skier's mass.

Question 3 was also a qualitative reasoning question. However, it could be easily answered using the work-energy bar chart. Since the initial potential energy bar kept

constant, adding a gravitational potential energy bar representing the final gravitational

potential energy of the woman-earth system at point C would make the woman's final

kinetic energy bar *K* lower. So it was easy to decide that her speed at C would be less

than her speed at point B.

To distinguish these two different types of qualitative reasoning questions, we

defined questions like question 2 as the equations-based question, while defining

questions like question 3 as the concepts-based question. But this distinction was not very

strict; some qualitative questions could be readily answered in either way. In Figure 3.4,

the arrows linking the right and left parts represent which representation could be used to

answer which question readily.

Problem 2 (see Appendix B): Similarly, a problem representation chart for

problem 2 was analyzed and shown in Figure 3.5. In the problem statement, a work-

energy bar chart was given, but it was not completed. We intended to investigate whether

or not the student constructed it himself and used it in their problem solving.

To answer question 1, one needed to do a quantitative calculation applying the

generalized work-energy equation, which could be either in an algebraic representation or

a numerical representation. Question 2 was a qualitative reasoning question. It could be

answered readily either using the algebraic representation or the work-energy bar chart.

Similar to question 2 in problem 1, question 3 in this problem could be readily answered

using the algebraic representation.

Problem 3 (see Appendix B): A problem representation chart for problem 3 was

shown in Figure 3.6. In the problem statement, a completed work-energy bar chart was

given, which represented the physical process described in the verbal and pictorial

Problem 2
(With Work-Energy Bar-Chart)
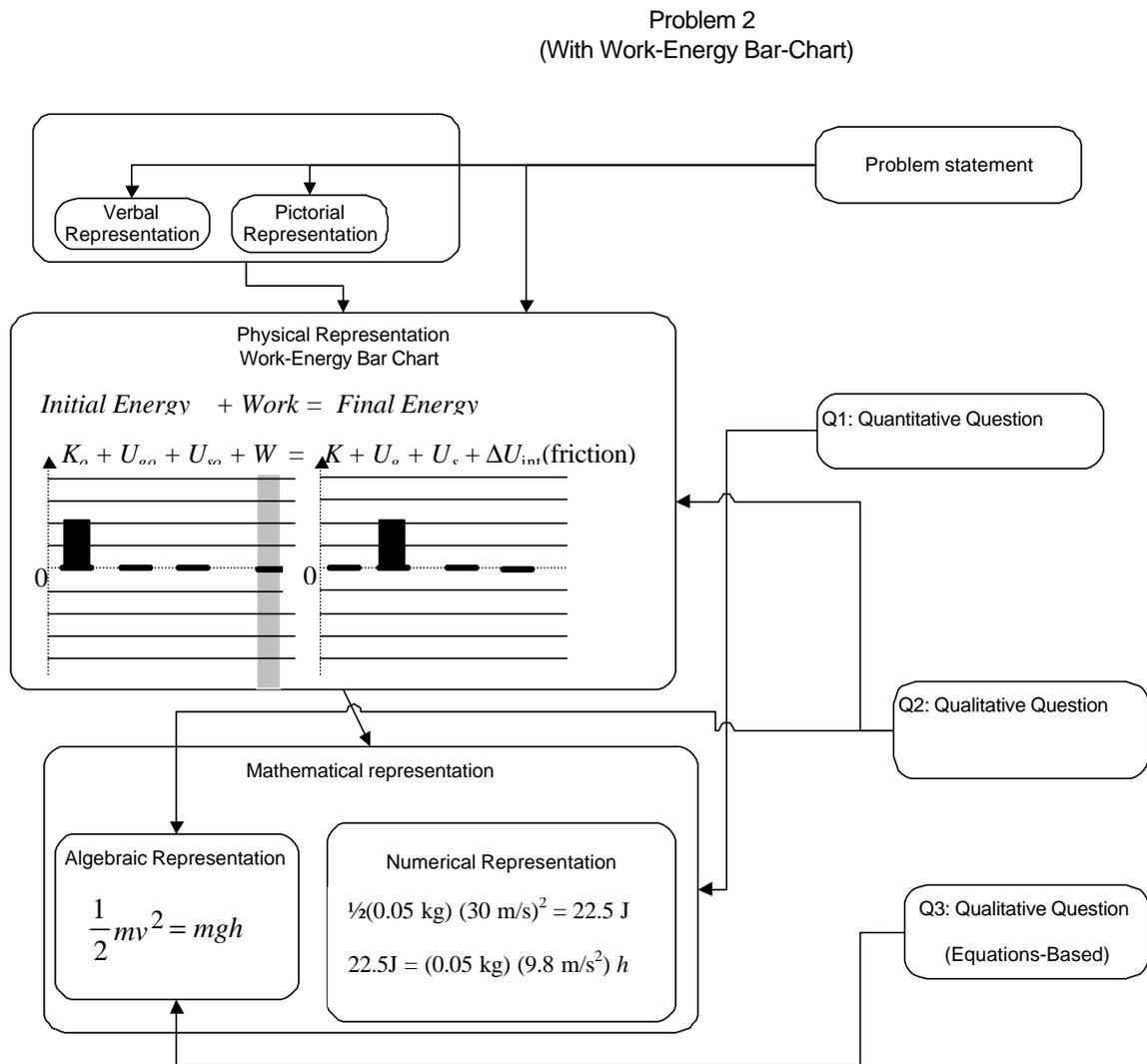
Figure 3.5: A problem representation chart for problem 2 used during the pre-interview. The representation chart is composed of two parts: the left part consisting of the multiple representations of the physical process and the right part consisting of the problem statement and the questions in problem 2. The arrows connecting them indicate which representation could be used to answer which question readily.

Problem 3
(With Work-energy Bar-Chart)

Problem statement

Verbal Representation

Pictorial Representation

Physical Representation
Work-Energy Bar Chart

Initial Energy + Work = Final Energy

$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int}(\text{friction})$

Q1: Quantitative Question

Q2: Qualitative Question

Mathematical representation

Numerical Representation

Algebraic Representation

$\frac{1}{2}K(\Delta x)^2 = \frac{1}{2}mv^2 - mgh$

½(100 N/m)(0.2 m)^2 = 2 J

(0.05 kg)(9.8 m/s^2)(1.0 m) = 0.49 J

2 = ½(0.05 kg)v^2 − 0.49

Q3: Qualitative Question

Figure 3.6: A problem representation chart of pre-interview problem 3. The representation chart is composed of two parts: the left part consisting of the multiple representations of the physical process and the right part consisting of the problem statement and the questions in problem 3. The arrows connecting them indicate which representation could be used to answer which question readily.

representations. Compared with problems 1 and 2, this problem was more complicated. So with the completed work-energy bar chart given, we intended to see whether or not the student could recognize that the bar chart could be useful for him or her to answer the questions, in particular, question 1.

Question 1 was a quantitative question. One needed to do a quantitative calculation applying the generalized work-energy equation, which could be either in an algebraic representation or a numerical representation. Questions 2and 3 both were qualitative reasoning questions. They could be answered readily either using the algebraic representation or the work-energy bar chart.

3.1.2.4 Student Performance on Interview Problems

Of the fourteen students interviewed, twelve were given the problems with the work-energy bar chart while the other two were given the same problems but without the work-energy bar chart.

We first analyzed the twelve students' solutions for question 1.0 in problem 1, which asked the student to construct the work-energy bar chart consistent with the physical process described in the problem statement. On the basis of data analysis, we found that except for one student trying to calculate numbers for each related energy term and to put actual values on the chart rather than drawing bars, the other eleven students did qualitatively draw bars on the chart. Eight of them correctly constructed the bar chart that was consistent with the initial and final states that they chose. One example of the completed bar charts from these eight students was shown in Figure 3.7. The bar charts made by the other three students were shown in Figure 3.8. It was apparent that they had conceptual difficulty dealing with gravity; they did not understand if they should

Initial Energy + Work = Final Energy

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int}(\text{friction})$$
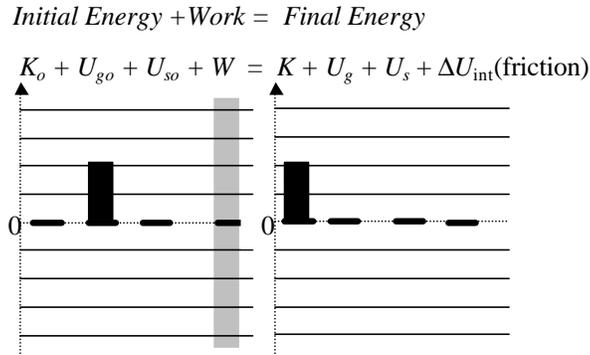
Figure 3.7: One example of the work-energy bar charts that students correctly completed for the interview problem 1.

include work done by the gravitational force or count it as gravitational potential energy, or both. Two of them had no difficulty understanding the structure of the bar chart, but one did.

Among these twelve students who solved the interview problems with the work-energy bar chart, two could not finish all the problems within the interview time, which lasted about one hour. One student talked too softly during the interview and his voice could hardly be transcribed from the tape. So except for these three students, we analyzed the data from the other nine students. Of these nine students, only one used the work-energy bar chart to help him solve problem 3, the rest of the students did not use it in their problem solving. We also analyzed the data from the two students who solved the interview problems without the work-energy bar charts. Overall, from a perspective of the multiple representations, students' solutions for the problems with the work-energy

Initial Energy +*Work =* *Final Energy*

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{\text{int}}(\text{friction})$$

Initial Energy +*Work =* *Final Energy*

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{\text{int}}(\text{friction})$$

Initial Energy +*Work =* *Final Energy*

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{\text{int}}(\text{friction})$$
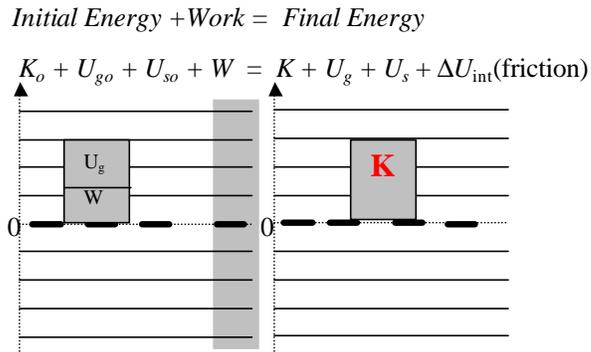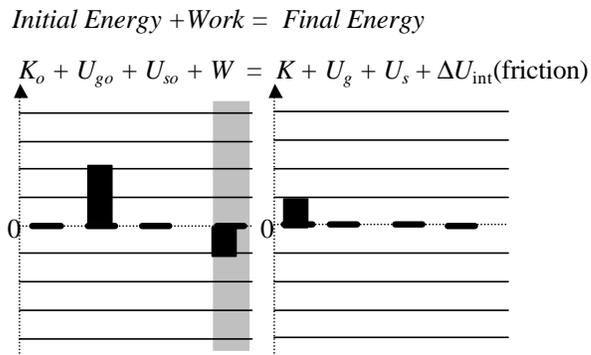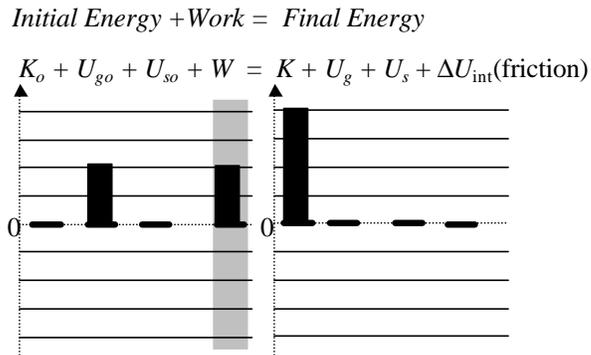
Figure 3.8: Three work-energy bar charts made by three students when answering the question asking for completing the work-energy bar chart for interview problem 1.

bar charts were similar to these for the problems without the work-energy bar chart. Two students' detailed solutions for the interview problems with the work-energy bar charts are illustrated in Appendix D, while one student's detailed solutions for the interview problems without the work-energy bar chart are shown in Appendix E.

Student performance on problem 1: As shown in Appendixes D and E, all these three students used the numerical representations to answer question 1, which was a quantitative question. None of them started with the algebraic representation, as shown in Figure 3.4. For question 2 (qualitative question), only one student used the algebraic representation to reason about the question. The other two students both still used their numerical representations from question 1 to answer this question. As a result, one could not correctly response the question; the other one got the correct answer but it took him a long time and more efforts to arrive at the correct answer. To answer question 3, all three students primarily relied on the pictorial representation. Two of them used the concepts of work and energy to explain their reasoning, but one used projectile motion ideas to answer the question, shifting his knowledge domain from work-energy to projectile motion.

Student performance on problem 2: To answer question 1, all three students used the numerical representations. For qualitative question 2, it was interesting to see that they all used the numerical representations from question 1, plugged in the new value for the initial speed, and calculated the new height. This question could be easily answered either using the algebraic representation or the physical representation. Student performance on question 3 was even more interesting. One student used the algebraic representation and quickly reached the correct answer. Another student first assumed a

numerical value for mass of the lighter ball and then used the numerical representations to calculate the maximum vertical height. For the last student, apparently, he first attempted to reason the question qualitatively. He imagined a real-life situation he experienced (throwing up a golf ball and a baseball) and tried to link it with the question. His mental picture was a type of internal pictorial representation, because 1) unlike an external representation, it was a mental picture; and 2) his mental image consisted of real-life objects rather than abstract physics concepts. Then he attempted to think mentally about some formula (seemly to be equations for projectile motion, but too hard for him to do it mentally). Then he said: "It's the best and easiest way to calculate it mathematically." He assigned a numerical value for mass of the lighter ball and used the numerical representations to calculate the maximum vertical height. After he found that his final results had the same value for both the more massive and less massive balls, he realized that the mass of the ball had to not matter. Then he set up the generalized work-energy equation and immediately saw that mass m could be cancelled out.

Student performance on problem 3: This problem was more complex than the first two problems. One student used the given work-energy bar chart when he answered all three questions. First, when struggling with question 1, he realized that the work-energy bar chart was helpful for him to understand the problem, in particular, to understand about the gravitational potential energy. Then he used the bar chart to help himself set up the generalized work-energy equation. So he quickly solved the question correctly. When he answered questions 2 and 3, apparently, he used the work-energy bar chart to help reason about and evaluate his answers.

For the other two students, one did not use the work-energy bar chart to answer the questions, although it was given; one was not given the work-energy bar chart. They basically used the numerical and pictorial representations to answer all the questions.

3.1.2.5 Discussion of Interview Results

The student performance on the interview problems, as discussed above, provided some empirical evidence about how students solved work-energy problems from a perspective of the multiple representations and how they used the work-energy bar chart in their problem solving before formally learning about it.

First, it was apparent that most students did not have difficulty understanding the format and the structure of the work-energy bar chart, even though they just had a short tutorial on it. This provided evidence that the work-energy bar chart as an external representation was easy for the student to understand. Research in psychology found that a bar graph was easy for human beings to reason about some tasks (see more details in Section 2.2.2.1).

Second, the data indicated that students relied more on the pictorial representation than on the physical representation to reason in problem solving. Students could correctly construct the work-energy bar chart for problem 1, but did not use it to answer the following questions, some of which could be answered easily using the work-energy bar chart. It seemed fair to conclude that, without understanding why and how to use the work-energy bar chart in problem solving, students did not use it. Although the work-energy bar chart had an easy-to-understand format, it had physics meanings. We needed to treat the physical representation itself as physics knowledge and to explicitly teach the student how to use it.

In contrast, the pictorial representation consisted of a daily situation and concrete objects; the student was familiar with them based on his or her real-life experience. In addition to its diagrammatic format (see more details in Section 2.2.3), the pictorial representation was helpful for the student to visualize what was going on in the problem and to reason in problem solving. However, including real objects in the pictorial representation could become disadvantageous. It appeared that it was difficult for the student to answer certain qualitative questions using the pictorial representation. It provided surface features that could trigger inappropriate reasoning.

Third, to understand student problem solving from a perspective of the multiple representations, it was necessary to distinguish two types of mathematical representation: the algebraic and the numerical representations. There was empirical evidence that they played different functions in student problem solving. Apparently, the naïve student basically used the numerical representation to solve quantitative questions and to answer some qualitative questions as well. A strategy that the student used could be summarized as: represent related concept in numbers first, assemble these numbers together based on a law or a principle, and calculate out a number for the unknown variable. By doing this, it appeared that the meaning of each concept became invisible or lost; the student met difficulty conceptually understanding the physical process.

Apparently, it was difficult for the student to use the numerical representation to answer some qualitative questions. It would either take the student a longer time and/or more effort to reach the correct answer or be misled to make an incorrect answer. Compared with the numerical representation, the algebra representation had some advantages. It was helpful for the student to answer some qualitative questions quickly.

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Additionally, it could aid in student conceptual understanding of the physical process. But the algebraic representation was an abstract, symbolic representation. Maybe it was too abstract for the beginning student, so he or she wanted to avoid using it or use it as little as possible.

Fourth, we also found that the student went back and forth between pictorial representations (sometimes verbal representations) and numerical representations in problem solving. It seemed to the student that these were concrete, familiar, and manageable representations. But they contained few physics concepts or meanings; they were less helpful to reason in the physics domain. Apparently, an intermediary representation was needed 1) to provide physics concepts or meanings, 2) to have a diagrammatic format so as to link closely with the pictorial representation, and 3) to be closely associated with fundamental laws or principles in physics so as to aid in understanding and setting up the algebra representation. A physical representation could help close this gap. Previous studies (See Section 2.2.3.1) had found that the central difference in problem solving between experts and naïve students was that experts integrated physical representations into their understanding and reasoning while solving problems, but novices did not. To help students develop expertise in solving work-energy problems, it was essential to introduce a physical representation in the context of work and energy and to help students apply the physical representation in their own problem solving.

Could the work-energy bar chart be used as an effective physical representation in solving work-energy problems? How could we help students use it in their problem solving? What functions would the work-energy bar chart play in student problem

solving? Would students perform more expert-like in problem solving after they are exposed to this new technique? In the rest of this chapter, we attempt to address and investigate these questions.

3.2 Development and Implementation of a Systematic Multiple-Representation Strategy

To address the students' difficulties and to help students acquire expert-like problem solving skills, we developed an approach to representing work-energy processes in multiple ways. In this method, a typical physics problem is considered as a physical process. The process is first described in words—the verbal representation of the process. Next, a sketch or a picture, called a pictorial representation, is used to represent the process. This is followed by a physical representation that involves more physics-like quantities and descriptions such as free-body diagrams and graphs. Finally, the process is represented mathematically by using basic physics principles to describe the process. Since a physics principle usually can be either represented by a more generalized algebraic formula or a numerical equation, for a mathematical representation we can distinguish two formats: the former as an algebraic representation, and the latter as a numerical representation. The pictorial and physical representations are often called qualitative representations, in contrast to the quantitative mathematical representation. In this chapter, the use of verbal, pictorial, physical, and mathematical representations is called multiple-representation problem solving. An example of multiple representations for a kinematics process is shown in Figure 3.9. In terms of multiple representations, the goal of solving physics problems is to represent physical processes in different ways— words, sketches, diagrams, graphs, and equations, rather than solving for some unknown

Multiple Representations in Dynamics

Verbal Representation

A cat fell off a roof, and landed in snow. It stopped after sinking 0.2 m into the snow. Just before hitting the snow, the cat was falling at a speed of 10 m/s. Determine the average force of the snow on the 5 kg cat while it was sinking into the snow.

Pictorial Representation

$y_0 = 0, v_0$

System

0

$y, v = 0$

t_0

t
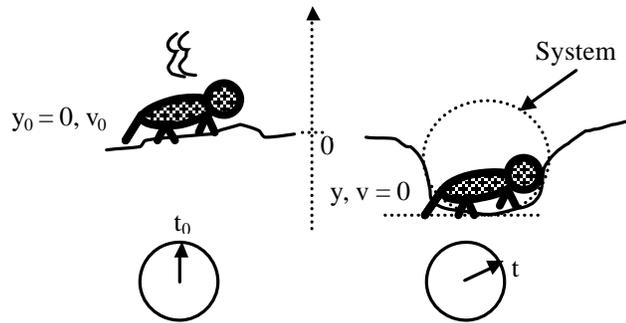
Physical Representations
(Motion Diagram and Force Diagram)

v_0

v

$\bar{a}$

y

N (snow on cat)

W (Earth's mass on cat)
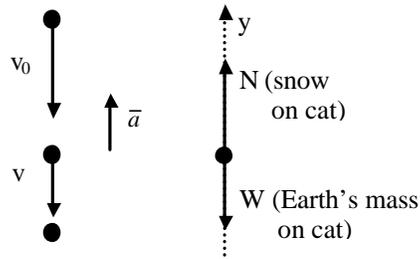
Mathematical Representation

$$2\,\bar{a}\,(y - y_0) = v^2 - v_0^2$$

$$N - W = m\bar{a}$$

Figure 3.9: Multiple representations of a dynamics process.

quantity. This multiple representation process helps students develop meaning for the abstract algebraic symbols by linking them to the more concrete pictorial and physical representations.

3.2.1 Development of Work-Energy Bar Chart as a Physical Representation of Work-Energy Processes

As we know, most physics professors and teachers solve dynamics problems by relying on diagrammatic force representations—a free-body diagram or a force diagram. There is, however, no similar representation for solving work-energy problems. As was discussed in Section 3.1.1 and 3.1.2, when solving work-energy problems, most introductory physics students just play with equations and numbers, and seldom use any qualitative physical representations to help them understand the problem.

Van Heuvelen [2] developed work-energy bar charts (see Figure 3.10) that serve the same role for analyzing work-energy processes as motion diagrams and force diagrams serve when analyzing kinematics and dynamics problems. The basic structure of the bar charts as shown in Figure 3.10 relies on the principle of conservation of energy, which says the initial energy of a system plus work done on the system equals its final energy. Hence, the bar chart is composed of three parts. The first part includes the initial system energy, which can have different types, such as kinetic energy (K_o), gravitational potential energy (U_{go}), and elastic potential energy (U_{so}). Work (W) is the second part in the bar chart. To conceptually distinguish between work and energy (because work is a process quantity, but energy is a state quantity), we indicate the work part in the bar chart in shadow. The last part is the final system energy that includes kinetic energy (K_o),

$$Initial\ Energy + Work = \quad Final\ Energy$$

$$K_o + U_{go} + U_{so} + W \quad = \quad K + U_g + U_s + \Delta U_{int(friction)}$$
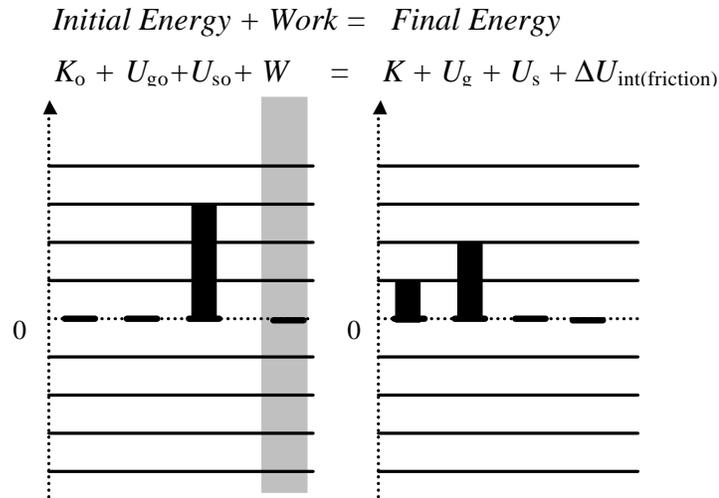
Figure 3.10: Work-energy bar charts that serve the same role for analyzing work-energy processes as motion diagrams and force diagrams serve when analyzing kinematics and dynamics problems.

gravitational potential energy (U_{go}), elastic potential energy (U_{so}), and internal energy [$\Delta U_{int(friction)}$], which is the last term in the bar chart (See Section 3.2.3 for detailed discussions about the internal energy).

The relative magnitudes of the different types of energy in the bar chart are initially unknown, just as the magnitudes of forces in a free-body diagram are often initially unknown. The chart does, however, allow us to conserve energy qualitatively—the sum of the height of bars on the left equals the sum of the height of bars on the right. For example, Figure 3.10 shows that a system initially has some elastic potential energy, say, three units (no need to know the exact magnitude). During a process that the system changes from its initial state to its final state, no external work is done on the system.

Therefore, the sum of the final energies, that is, kinetic energy and gravitational potential energy, should equal three units. The kinetic energy could be one unit, and the gravitational potential energy could be two units, as shown in Figure 3.10. However, the kinetic energy could be two units, while the gravitational potential energy is one unit, so that the sum of these two bars equals three units.

The work-energy bar chart has all the important features of a physical representation (see Section 2.2.3.2). First, the bar chart has meaning in physics—it represents the conservation of energy. Second, as discussed above, it *qualitatively* represents the conservation of energy. Third, it has a diagrammatic format. A bar chart usually is an effective external representation to help people reason about information that it represents [3]. Thus, theoretically, the work-energy bar chart can be used as a physical representation for work-energy processes. How to use it by students in work-energy problem solving is discussed below.

3.2.2 Multiple Representations of Work-Energy Processes

To help students forgo a formula-centered problem solving method and acquire a coherent expert-like problem solving strategy, we developed a multiple-representation approach for solving work-energy problems. That is, a work-energy process can be represented in verbal, pictorial, the bar chart (as a physical representation), and mathematical representations, as shown in Figure 3.11. Notice that the pictorial representation used in this strategy refers to a detailed sketch that identifies the system and its initial and final states, includes a coordinate system, and indicates the values of relevant quantities in the system's initial and final states. The mathematical

Verbal Representation

A large rock initially at rest slides down a hill
and is moving at the bottom of the hill.

Pictorial Representation

System

y t_0, y_0,
          v = 0

t, y = 0
v > 0

d

0

Earth

Physical Representation
(Work-Energy Bar Charts)

Initial Energy+Work=Final Energy

$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int(friction)}$

0   0
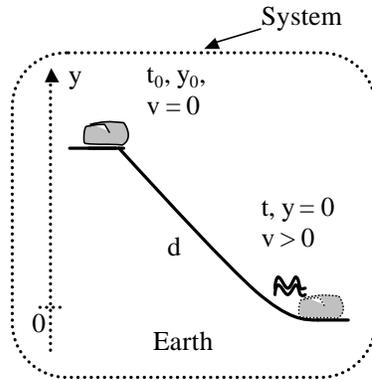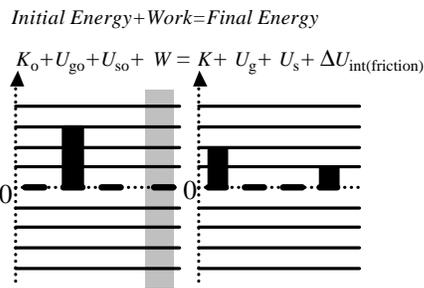
Mathematical Representation

$$m g\, y_0 = \frac{1}{2} m v^2 + f\, d$$

Figure 3.11: Multiple representations of a work-energy process.

representation is defined as the generalized work-energy equation (i.e., algebraic representation of the conservation of energy principle), as shown in the bottom of Figure 3.11. To use this approach with the concepts of work and energy, students must learn certain conceptual ideas and skills, including:

• Choosing a system—the object or objects of interest for the process being considered;

• Characterizing the initial state and the final state of the process;

• Identifying the types of energy that change as the system moves from its initial state to its final state and the signs of the initial and final energies of each type;

• Deciding if work is done on the system by one or more objects outside the system as the system changes states;

• Developing the idea that the initial energy of the system plus the work done on the system leads to the final energy of the system—the energy of the universe remains constant;

• Constructing an energy bar chart—a qualitative representation of the work-energy process; and

• Converting the bar chart to a mathematical representation that leads to a problem solution.

3.2.3 Systems and Work

To use this multiple-representation approach with the concepts of work and energy, we need to pay a lot of attention to the system, to its changing characteristics, and to its interaction with its environment. The book by Bridgman [4] provides a nice introduction to the effect of system choice on the description of the work-energy process.

Burkhardt [5], Sherwood [6], Sherwood and Bernard [7], Arons [8], and Chabay and Sherwood [9] also discuss the importance of system choice. A system includes an object or objects of interest in a region defined by an imaginary surface that separates it from its surroundings. Choosing a system is the key for deciding what energy changes occur and what work is done. Work is done only if an object outside the system exerts a force on an object inside the system and consequently does work on the system as the internal object moves.

This idea is illustrated in Figure 3.12 where the same process (neglecting friction) is analyzed by using three different systems (an idea provided by Bob Sledz at Garfield High School in Cleveland). Notice that in Figure 3.12 (a) the cart and the spring are in the system, as is Earth. The process is not affected by objects outside the system. Thus, no external work is done on the system. Potential energy change occurs when objects in a system change their shape or their position relative to other objects in the system. For example, the change in separation of a mass relative to Earth's mass in the system causes a gravitational potential energy change. The change in separation of two electric charges in the system causes a change in electrical potential energy. The change in the shape of a spring when it compresses or stretches causes a change in its elastic potential energy. In Figure 3.12 (a), the system's initial energy, the elastic potential energy of the compressed spring, is converted to the system's final energy, the kinetic energy of the cart and the gravitational potential energy due to the separation of the cart and Earth.

In Figure 3.12 (b), the system has been chosen with the earth outside the system. Thus, the earth's mass exerts an external gravitational force on the cart and consequently does work on the cart as it moves to higher elevation. In this case, we do not count the

(a)

System

y

x_0

0

t, y, v

$t_0, y_0 = 0$

$v_0 = 0$

Earth

Initial Energy+Work=Final Energy

$K_\mathrm{o}+U_\mathrm{go}+U_\mathrm{so}+W = K+ U_\mathrm{g}+U_\mathrm{s}+\Delta U_\mathrm{int(friction)}$

Apply the work-energy equation to the process represented above.

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + mgy$$

(b)

System

y

x_0

0

t, y, v

$t_0, y_0 = 0$

$v_0 = 0$

Earth

Initial Energy+Work=Final Energy

$K_\mathrm{o}+U_\mathrm{go}+U_\mathrm{so}+W = K+ U_\mathrm{g}+U_\mathrm{s}+\Delta U_\mathrm{int(friction)}$

Apply the work-energy equation to the process represented above.

$$\frac{1}{2}k\,x_0^2 - mg\,y = \frac{1}{2}mv^2$$

(c)

System

y

x_0

0

t, y, v

$t_0, y_0 = 0$

$v_0 = 0$

Earth

Initial Energy+Work=Final Energy

$K_\mathrm{o}+U_\mathrm{go}+U_\mathrm{so}+W = K+ U_\mathrm{g}+U_\mathrm{s}+\Delta U_\mathrm{int(friction)}$

Apply the work-energy equation to the process represented above.

$$W_{spring} - mg\,y = \frac{1}{2}mv^2$$

Figure 3.12: The different systems are chosen for the same physical process. In (a), the cart, the spring, and Earth are in the system. In (b), however, the cart and the spring are in the system, but not Earth. The system in (c) includes only the cart. For each chosen system there is one work-energy bar chart and the corresponding generalized work-energy equation. In practice, it would be easy for students to use a system that includes Earth and the spring, although the way to choose the system does not affect the physical results.

system's gravitational potential energy as changing because the earth is not in the system. As we know, the negative work done by the Earth's gravitational force on the left side of the chart in Figure 3.12 (b) has the same effect as positive final gravitational potential energy on the right side of the chart in Figure 3.12 (a).

For the system in Figure 3.12 (c), the cart alone is in the system. Thus, we now include the effect of the spring by analyzing the work done by the external force of the spring on the cart and the work done by the external earth's gravitational force. The sum of these two work terms causes the system's kinetic energy to increase. There is no elastic potential energy change in the system since the spring is now outside the system.

In the above example, the system chosen in Figure 3.12 (a) is probably easiest to use in physics instruction. Gravitational potential energy is usually emphasized in high school and college introductory physics courses and is an easier concept for students to understand. It may, in practice, be easier for students to solve problems if they choose systems that include the earth. Similarly, it is more difficult for students to calculate the work done by a spring than to calculate its elastic potential energy $\frac{1}{2}kx^2$ at the beginning and end of a process.

What about friction? Bridgman [10], Sherwood [6], Sherwood and Bernard [7] Arons [11], and Chabay and Sherwood [9] have discussed examples such as these shown in Figure 3.13—one object sliding along a surface. Let us discuss a non-real but simple situation first. In Figure 3.13 (a), we imagine the block is a point particle, and this point-particle block is moving until it stops because of the floor's frictional force. The system includes only the point-particle block. So the floor exerts on the point-particle block an

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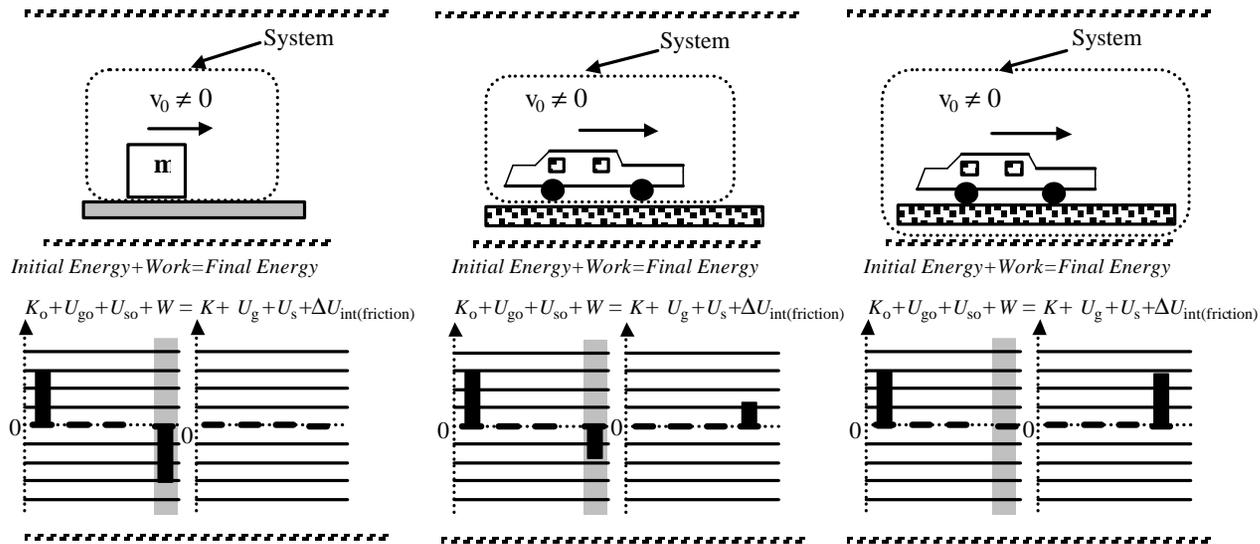
Figure 3.13: The physical processes involve friction. In (a), a point-particle block slides to a stop on a floor with friction. The system includes only the point-particle block. So the floor exerts on the point-particle block an external frictional force, and this frictional force does a negative amount of work, which has the same magnitude as the block's initial kinetic energy. In (b), a real car skids to stop on a rough road. The car is the only object in the system. Thus, the road that touches the car causes an external frictional force and a difficult work calculation. Chabay and Sherwood argue that for such a real system that includes the car, it has less negative work done by the friction of the road on the car than the car's initial kinetic energy. The remaining amount of the car's kinetic energy is transferred into the internal energy in the car (see Ref [9], pp. 236-241 for detailed discussions about this advanced topic). In (c), we see a recommended system choice that includes the objects and the frictional interfaces between the objects in the system. In this way we can readily include the system's internal energy change due to friction rather than dealing with a complex work calculation.

external frictional force which points opposite to the direction of motion. This frictional

force does negative work. This negative work in stopping the point-particle block has the

same magnitude as the block's initial kinetic energy. But in a real and complex situation,

such as the car skidding to a stop on a rough road as shown in Figure 3.13 (b), the car is a

real object. If the car is the only object in the system, the road that touches the car causes

an external frictional force. The work done by this frictional force is very difficult to

calculate, if we look carefully at what really happens at the boundary between the car

tires and the road. As Arons says [11]:

> What happens at the interface is a very complicated mess: We have abrasion, bending of
> "aspirates," welding and unwelding of regions of "contact," as well as shear stresses and
> strains in both the block and the floor.

In this situation, it is very difficult to deal with the formal definition of work done by the

frictional force. Ruth Chabay and Bruce Sherwood in their new textbook give a nice and

detailed explanation about how to calculate the work done by the frictional force in such

a situation—see *Matter and Interactions* [9]. According to Chabay and Sherwood's

arguments, the work done by this frictional force is still negative, but the magnitude of

this work is less than the initial kinetic energy (and not equal to $-F_{friction}d$ either); the

remaining amount of the initial kinetic energy is transferred into internal energy in the car

(see the following for detailed discussions about the internal energy).

Rather than dealing with this difficult work calculation, Bridgman [4] and Arons

[8] recommend that the two touching surfaces, such as the car tires and the road, be

included in the system as in Figure 3.13 (c). Since friction is no longer an external force,

we look for energy changes in the system. Friction causes objects rubbing against each

other to become warmer—the random kinetic energy of the atoms and molecules in the touching surfaces increases. Friction can also cause atomic and molecular bonds to break. This causes the potential energy holding atoms and molecules together to change. This happens, for example, when snow melts as a person's skis rub against the snow or when a meteorite burns due to air friction. A large number of bonds between rubber molecules are broken when a car skids to a stop. Thus, if surfaces with friction are in the system, the system's internal energy increases due to friction. The internal energy of the system is expressed by the last term, $\Delta U_{\text{int(friction)}}$, in the qualitative work-energy bar chart.

3.2.4 Instruction

How do we use the bar charts and the multiple-representation strategy with students? To help students overcome their dependence on a formula-centered method to solve work-energy problems, to help them develop a qualitative understanding of the work-energy concepts, and to acquire a systematic, coherent expert-like problem solving strategy, we explicitly expose students to such learning experience as the following:

- Learn to draw a pictorial representation from a verbal statement.

- Construct a qualitative work-energy bar chart for the process represented by the pictorial and verbal representation.

- Set up the generalized work-energy equation based on the bar chart. That is, there is one term in the equation for each term in the bar chart.

- Learn to move among representations in any direction. For example, construct a detailed sketch and a verbal description that is consistent with a complete bar chart or with a mathematical equation that is the application of the conservation of energy principle for some physical process.

- Solve quantitative problems using the multiple-representation strategy.

3.2.4.1 Qualitative Representations of Work-Energy Processes

In this section, some examples (Figures 3.14 – 3.17) are given in detail to illustrate how students learn to use the work-energy bar chart and the multiple representation strategy to qualitatively analyze work-energy processes [12]. Usually, the processes shown in the worksheets are demonstrated with real objects in the lecture or laboratory. In Figure 3.14 (a), a work-energy process is described in words and in a sketch. Students are asked to construct a detailed sketch that identifies the system and its initial and final states, includes a coordinate system, and indicates the values of relevant quantities in the system's initial and final states. Construction of a sketch such as that in Figure 3.14 (b), a pictorial representation of the process, is perhaps the most difficult task for students.

Having completed a pictorial representation of the process, students next construct a qualitative work-energy bar chart for the process (as in Figure 3.15). The student looks at the initial situation and decides whether the system has kinetic energy, K_o. If so, the student places a short bar above the initial kinetic energy slot. If there is no initial kinetic energy, no bar is drawn. The bar for initial gravitational potential energy, U_{go}, depends on the initial location of an object in the system relative to the origin of the vertical coordinate system. The student draws a positive bar if the object is at a higher position than the origin of the y-axis, no bar if at the same elevation as the origin, or a negative bar if lower than the origin. Similarly, the bar for the initial elastic potential energy, U_{so}, depends on whether compressed or stretched elastic objects are initially in the system. The work W bar depends on the presence of objects outside the system that

(a)

(b)

System

t_0, y_0
$v_0 > 0$

y

0

t,
y = 0,
v = 0

Earth

The cable lowers the elevator,
initially moving down, so that it
comes to a stop at a lower elevation.
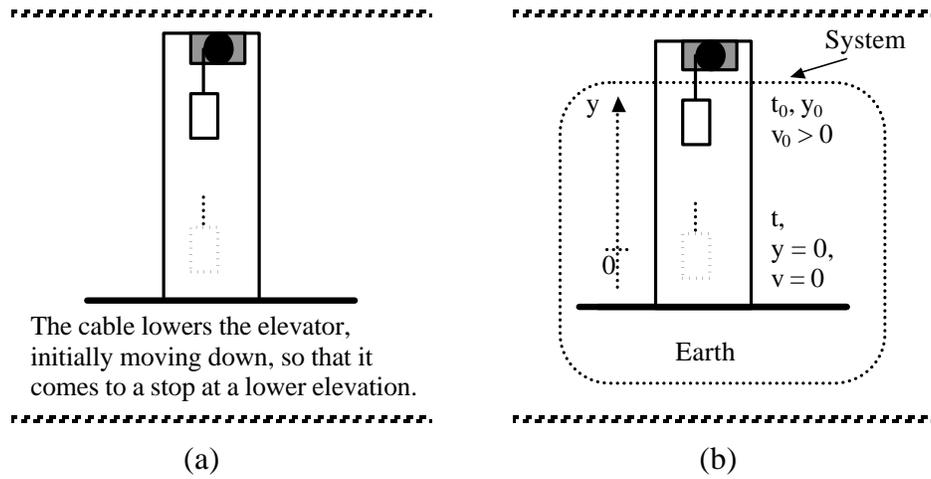
Figure 3.14: The work-energy process is described in words and in a sketch as shown in (a). Students are asked to construct the pictorial representation, including a system choice and a coordinate system, and indicating the values of the quantities in the system's initial and final states as shown in (b).

The toy car initially at rest slides
down a track and is moving at the
bottom of the track.
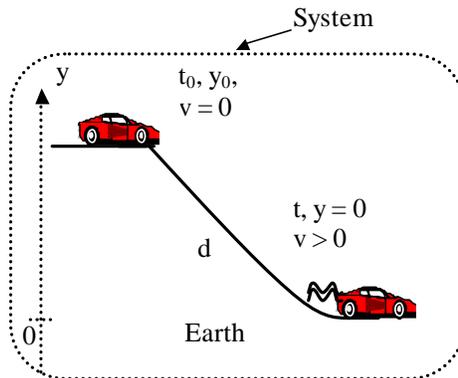
System

$t_0, y_0,$
$v = 0$

y

$t, y = 0$
$v > 0$

d

0

Earth

Initial Energy+Work=Final Energy

$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int(friction)}$

0   0
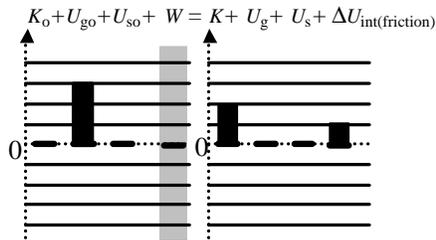
Apply the work-energy equation
to the process represented above.

$$mg\,y_0 = \frac{1}{2}mv^2 + fd$$

Figure 3.15: For the given work-energy process, students are asked to construct the
detailed sketch, and then convert it to an energy bar chart. Finally, they use the bar chart
to apply the generalized work-energy equation.

exert forces on objects inside the system as the system moves from its initial state to its final state. The sign of the work depends on the direction of the external force relative to the direction of the displacement of the object in the system. The final energy of the system is analyzed in the same way as the initial energy.

Having identified non-zero energy terms by placing short bars in the chart, we can now emphasize the conservation of energy principle by making the sum of the lengths of bars on the left equal to the sum of the bars on the right. The relative magnitudes of the bars on one side are usually unknown, just as the relative magnitudes of force arrows in a force diagram are often unknown.

Having constructed the bar chart, it is now a simple task to construct the generalized work-energy equation to describe this process. There is one term in the equation for each term in the bar chart. When detailed expressions for the types of energy are developed, students can include these expressions in the equations, as in the bottom mathematical representation in Figure 3.15. For the problem in Figure 3.15, students could be asked to think about how the chart and the actual process would change if the coefficient of kinetic friction was doubled.

In Figure 3.16, students start with a complete bar chart and are asked to invent a verbal and pictorial description of a physical process that would lead to that bar chart, and to construct the mathematical representation of the physical process. (There are many processes that could lead to a particular chart.) In Figure 3.17, students start with the mathematical representation of a process and are asked to construct a bar chart that is consistent with the equation, and to invent a process that would produce the equation and the chart—a so-called Jeopardy problem [13]. Students should now have a good

Verbal Representation

Pictorial Representation

Initial Energy+Work=Final Energy

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int(friction)}$$

0   0

Mathematical Representation
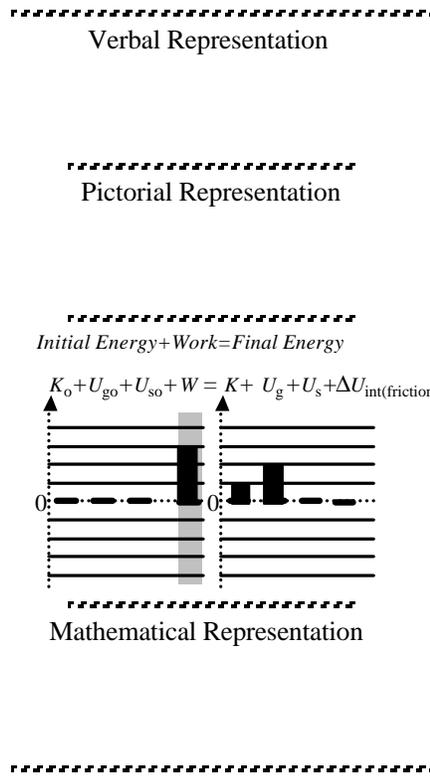
Figure 3.16. A work-energy process is described by a work-energy bar chart. Students start with the bar chart and invent a sketch, a real-world situation in words and a generalized work-energy equation that is consistent with the bar chart.

Verbal Representation

Pictorial Representation

Initial Energy+Work=Final Energy

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int(friction)}$$

0   0
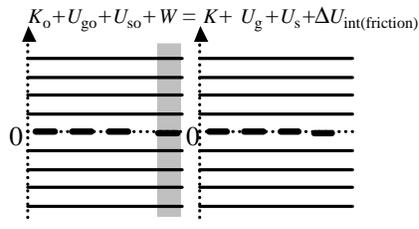
Mathematical Representation

$$(10\text{kg})(10\text{m/s}^2)(20\text{m} \sin 35^0)$$
$$= 0.5\, k\, (20\text{m})^2 + (30\,\text{N})(20\text{m})$$

Figure 3.17. The so-called Jeopardy problem starts with the mathematical equation for a work-energy process. Students are asked to construct a bar chart that is consistent with the equation, to draw a sketch, and to invent a process that would produce the equation and the chart.

qualitative understanding of work-energy processes—at least much better than when the processes are introduced first using a formal mathematical approach.

3.2.4.2 Quantitative Work-Energy Problem Solving

Students next solve quantitative problems using this multiple-representation strategy. They go from words, to sketches and symbols, qualitative bar graphs, a generalized work-energy equation, a solution, and finally an evaluation to see if their solution is reasonable. An example is shown in Figure 3.18. A set of Active Learning Problem Sheets (the ALPS Kits) [12] has a Work-Energy Kit that includes 38 qualitative questions [such as illustrated in Figure 3.14 (a)] and 16 quantitative problems (such as illustrated in Figure 3.18). Students buy the ALPS Kits at the beginning of the quarter. They solve some of these problems during lectures, during recitations, and for homework. Problems from the text are assigned with the proviso that the same format should be used on these problems. In addition, a set of 14 qualitative and quantitative work-energy problems (included as part of the ActivPhysics1 CD and workbook[14]) is used as active learning activities during lectures and laboratories. The problems are computer simulations, which include the dynamic work-energy bar charts to help students visualize energy transformations and conservation during physical processes.

3.3 Assessment of the Effectiveness of the Multiple-Representation Strategy

In the autumn quarters of 1997 and 1999, honors engineering freshmen in a calculus-based introductory mechanics class (Physics 131E) at OSU learned the multiple-representation strategy with the concepts of work and energy. This ten-week class covered the regular concepts of mechanics: kinematics, Newton's dynamics, momentum, work and energy, and some rotational dynamics. The students worked with the work and

A 500-kg cart, including the passengers, is initially at rest. When the spring is released, the cart is launched for a trip around the loop-the-loop whose radius is 10 m. Determine the distance the spring of force constant 68,000 N/m must be compressed in order that the cart's speed at the top of the loop is 12 m/s. Ignore friction. Assume that $g = 10$ m/s^2.

System

A

Earth

B
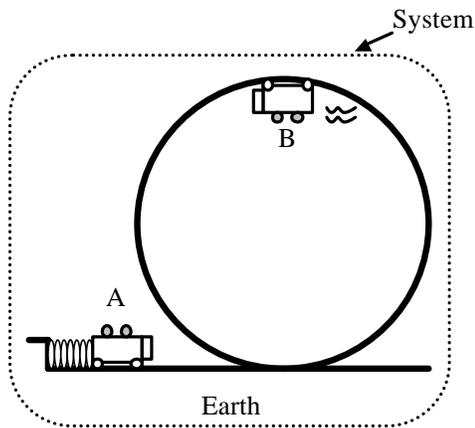
(a) Construct a qualitative work-energy bar chart for the process at the left.

Initial Energy + Work = Final Energy

$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int}\text{(friction)}$

0   0
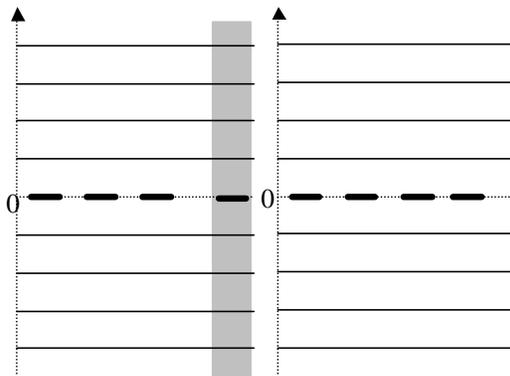
(b) Use the work-energy bar chart to help construct the work-energy equation for this process.

(c) Rearrange the above to determine the unknown distance that the spring must be compressed.

(d) Evaluation

- Does the answer have the correct units?

- Does the answer seem reasonable?

- How would the answer differ if the loop has a smaller radius? Does this agree with the equation in part (c)?

Figure 3.18: One of the quantitative problems included in the Active Learning Problem Sheets. Students solve those problems using the multiple-representation strategy after having developed skills in constructing qualitative representations. These multiple-representation problems help students develop qualitative understanding about the physical processes and develop problem-solving expertise, instead of using only an equation-centered method.

energy concepts for about six lecture periods, three recitation periods, one lab period, and in their homework assignments. Below, we report results of assessing the effectiveness of the multiple-representation problem solving strategy on student understanding and problem solving in work-energy.

3.3.1 Evaluation Results (Autumn 1997 and Winter 1998)

In the autumn quarter of 1997 and in the winter quarter of 1998, we conducted a series of assessment activities for the honors engineering freshmen in Physics 131E who learned this multiple-representation strategy with the concepts of work-energy. We administered a survey to evaluate student attitudes towards the work-energy bar chart and the multiple-representation approach, gave a tutorial-type question to assess students' conceptual understanding, and analyzed students' solutions of a work-energy problem on the final exam.

3.3.1.1 Student attitudes towards the energy bar charts and the multiple representation approach of solving work-energy problems

Most of the honors engineering freshmen in Physics 131E took high school physics that is taught in the standard way. So it becomes important to know what they think about the multiple-representation problem-solving strategy, which is innovative and counter to the traditional formula-centered instruction. A survey was administered to evaluate what students thought about the roles of the work-energy bar charts and the multiple-representation strategy. The survey included three free-response questions. The survey questions and students' answers on each of the three survey questions are summarized in Tables 3.5, 3.6, and 3.7. Below, we discuss in detail students' responses to survey question 1 and 2, which are summarized in Table 3.5 and 3.6. The students' responses to survey question 3 are discussed in Section 3.3.1.3.

Table 3.5 reports a summary of the students' responses for question 1 on the survey: "Did using the energy bar charts help you learn energy concepts and solve work-energy problems? Explain why they were useful or not useful." We found that out of 67 students, overall, ninety-two percent thought that the energy bar charts were useful. Sixty-four percent of them thought that the energy bar charts helped them visualize what is happening to different types of energy in energy problems, and helped them set up the correct equations to solve the problems easily. Fifteen percent of them thought that the energy bar charts helped them better understand the abstract energy concepts and energy conservation. Ten percent of the students thought that the energy bar charts were helpful in learning the energy concepts at the beginning, and after a while they could create the bar charts mentally rather than write them out. But eight percent of the students preferred using equations directly rather than the energy bar charts.

Based on the students' comments, apparently students think that the work-energy bar charts as a physical representation help them visualize the process of energy transformation and "see" the conservation of energy. Also, students think that the work-energy bar charts play a crucial role in helping them set up equations correctly and readily.

Table 3.6 reports a summary of the students' responses for question 2 on the survey: "Did representing the work-energy processes in multiple ways—words, sketches, bar charts and equations—help you learn energy concepts and solve energy problems? Explain why they were useful or not useful." Out of 67 students, eighty-four percent thought that this multiple-representation strategy was helpful: helped them understand the concepts better; set up the problems correctly; and was a good teaching method for a

	92% Useful (N = 67)
64%	These students thought that the energy bar charts helped them visualize what is happening to different types of energy in energy problems, and set up the right equations to solve the problems easily. *Two examples of the students' responses:* • Greatly helped, because they provided visual representation of what's going on and made figuring out the equations easier. • The energy bar charts were extremely useful because they provided direction for each problem. You were able to <u>see</u> what types of energy were being used at different stages of the problems and what individual energy equations to apply to the problems.
15%	These students thought that the energy bar charts helped them understand the abstract energy concepts and the energy conservation better. *Two examples of the students' responses:* • Yes, they were very helpful. They allowed me to see how and in what forms energy was conserved. • These were <u>extremely</u> helpful. Doing this made me change my way of think *[sic]* from thinking in terms of equations to concepts. Once I had the concepts down, then I can choose the right equations.
10%	These students thought that the energy bar charts were helpful in learning the energy concepts at the beginning, and after a while they could create the bar charts mentally rather write them out. *One example of the students' responses:* • They were useful since they gave a visual way to go about the problems. After a while I got to the point that I automatically did them in my head and did not need to really write them out.
3%	These students thought that the energy bar charts were helpful, but they did not explain their reasons.
	8% Not Useful (N = 67)
8%	These students preferred using equations directly rather the energy bar charts. *One example of the students' responses:* • No. I had trouble finding relationships between the energies involved using the energy bar charts. I was better off using the equations. The energy bar charts only showed before, after, which increased, or decreased, and which energy was more. Personally, I needed exact values, which working equations provided. I did not gain a better understanding of energy using the charts.

Table 3.5. Students' responses for question 1 on the survey: Did using the energy bar charts help you learn energy concepts and solve work-energy problems? Explain why they were useful or not useful.

84% Useful (N = 67)	
84%	These students thought that the multiple-representation strategy was helpful to understand the concepts better, and to set up the problems correctly, and was a good teaching method to help a variety of students learn physics. *Two examples of the students' responses:* • Doing these problems in many different ways with different descriptions helped to visualize the deeper basic concepts underlying them. Seeing a problem only one way, or learning a concept only one way, causes your knowledge of that concept to be very one-dimensional. Representing multiple ways helps me to see all sides of a concept. • They did help me by letting me see the problem different ways. Also I would like to note that this is a good teaching method to help a variety of students learn physics. Often times a student doesn't understand one method but sees another.
9%Useful and Not Useful (N = 67)	
9%	These students thought that the multiple representations were helpful to understand the energy concepts, but were sometimes a waste of time for quantitative problem-solving. *One example of the students' responses:* • Sometimes they were useful but doing each type of description for each problem was overkill and wasted a lot of time.
7% Not Useful (N= 67)	
7%	These students thought that the multiple representations were not helpful except equations. *One example of the students' responses:* • No. Only the equations were of any use to me. The rest just seemed to get in the way.

Table 3.6. Students' responses for question 2 on the survey: Did representing the work-energy processes in multiple ways—words, sketches, bar charts and equations—help you learn energy concepts and solve energy problems? Explain why they were useful or not useful.

95% Use in Problem Solving (N = 67)	
46%	These students used the energy bar charts and the multiple representations to solve problems at the beginning, but less and less as they became more familiar with the problems. *Two examples of the students' responses:* • I used them less as I became more of an expert. I began to go straight to writing out the formulas (for each type of energy) and the equations. But, I kept the bar chart ideas in the back of my mind. • I did use them. I first used the bar chart just to see what parts I wanted in the equations, such as work from friction or potential spring energy. Then I would write the equation down and solve. Now I've become more able to solve problems. I still use the bars whenever I get confused. They really help to limit mistakes in writing the initial equation.
33%	These students used the energy bar charts and the multiple representations most of time when solving energy problems. *Two examples of the students' responses:* • I used the charts and representations throughout the process to be sure I was setting up the problem right without overlooking important details. Converting one representation to another was just one more way I double checked my work. • I used the charts every time, although I didn't always write it down on paper. It's a safe guard *[sic]* that you are not neglecting any form of energy.
16%	These students used the energy bar charts and the multiple representations in other ways. *Two examples of the students' responses:* • I personally only used the bar charts if I felt the situation was difficult enough to require a further analysis. However, at the beginning I used the bar charts for every problem and they really helped me grasp the concepts. • I used them very little in the very beginning, but then I realized their uses. Once I became familiar with them, I was able to be more comfortable solving problems. Now I have worked with many different problems, I use them to check my work.
5% Not Use in Problem Solving (N = 67 in total)	
5%	These students did not use this method unless it was required. They just preferred to use equations or got help from the textbook.

Table 3.7: Students' responses for question 3 on the survey: Did you (or how did you) use the energy bar charts and the multiple representations of work-energy processes to solve energy problems while doing homework, group problems in recitation, problems in lab, and/or on exam problems? Did you use them when first becoming familiar with the concepts and then less as you become more expert at solving work-energy problems?

variety of students. Nine percent of the students thought that the multiple representations were helpful in understanding the energy concepts, but were sometimes a waste of time for quantitative problem solving. Seven percent of the students still just liked to use equations directly in their problem solving.

From the students' responses, we find that this multiple-representation technique is helpful to most students in learning the concepts of work and energy and in solving related problems. It helps students develop a deeper conceptual understanding while solving problems.

3.3.1.2 Student Performance on a Tutorial-Type Question

Although the work-energy bar charts and the multiple-representation approach are developed as a problem-solving strategy to help students acquire problem-solving expertise, they emphasize the development of qualitative understanding and reasoning about work-energy concepts and processes. Does performing this type of qualitative analysis improve student scores on conceptual work-energy questions? To address this question, we examined our students using a tutorial-type work-energy problem (shown in Figure 3.19) developed by the Physics Education Group at The University of Washington (UW) [15, 16].

The problem in Figure 3.19 asks students to compare the final kinetic energy of two pucks having different masses pushed by the same force across the same distance. The OSU honors engineering freshmen were given this problem on a survey test at the end of winter quarter of 1998—one quarter after they had studied the introduction of the work-energy method in the autumn quarter of 1997. Out of the 56 students, about *60%* of them gave correct answers with correct reasoning in words or by using equations. The same problem was given on the final exam to a regular OSU engineering calculus-based

The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces. Which puck has the greater kinetic energy upon reaching the finish line? Explain your reasoning.

Finish

Start

m   4m

F   F
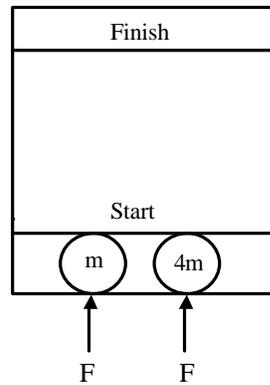
Figure 3.19: The problem originally developed by the Physics Education Group at UW was administered to the OSU honors engineering freshmen one quarter after they had learned the work-energy method. The students were told that their scores on the problem did not affect their class grades. This problem was also given to 147 OSU regular calculus-based introductory physics students after standard instruction in which the bar charts and the multiple-representation strategy had not been used.

physics class after standard instruction in which the bar charts and the multiple representation strategy had not been used. Only about *20%* of 147 students in this class provided correct explanations for the problem.

The paper by T. O'Brien Pride and *et al.* [16] reports that after standard lecturing for 985 regular and honors calculus-based physics students from UW, just *15%* got correct answers with correct reasoning on this question. In addition, for 74 UW tutorial instructors (most of them graduate teaching associates) before teaching the Washington work-energy tutorial sections, *65%* answered correctly with correct explanations. Furthermore, it is interesting to see from this paper that before the tutorials, *65%* of 137 physics faculty from the national tutorial workshops successfully produced correct answers to the question shown in Figure 3.19. The proportion of these different groups successfully answering this question is summarized in the bar graph in Figure 3.20. The OSU calculus-based honors engineering freshmen learning the multiple-representation strategy and using the energy bar charts performed on this problem much better than regular and honors calculus-based physics students both from OSU and from UW, and they did almost as well as physics faculty and physics graduate students.

3.3.1.3 Student use of the multiple-representation strategy in problem solving

How did students use the multiple-representation method to solve energy problems? In the survey with the three free-response questions administered to the 67 honors engineering freshmen, the last question asked how the students used the work-energy bar charts and the multiple representations for solving problems. A summary of the student responses are reported in Table 3.7. We find that most of the students used this method to solve problems. Although about half of the students used it less and less
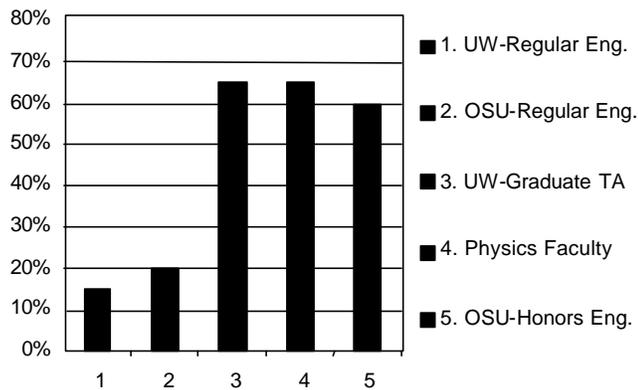
Figure 3.20: The graph shows the proportion of each group that correctly answered the question shown in Figure 3.19. Bars 1 and 2 indicate that no more than 20% of over one thousand calculus-based physics students from UW and OSU answered and explained the problem correctly after standard instruction. Bars 3 and 4 indicate that 65% of about two hundred physics graduate students and faculty correctly provided the answers and reasoning for the problem on pretests before the UW tutorials. Bar 5 indicates that 60% of more than fifty OSU honors engineering freshmen successfully answered the question with correct reasoning.

when becoming more familiar with energy problems, many of them still kept the energy bar charts in their mind as a useful problem-solving tool in evaluating their solutions.

But did the students actually apply the multiple-representation method when rushing through their exams? On the final exam problem (see Figure 3.21) given in this calculus-based honors engineering freshmen class, *66%* of the students constructed a pictorial representation of the process. Fifty-seven percen*t* of the students solved the problem using the work-energy method, but *43%* of them applied Newton's second law. Among the *57%* of students who used the work-energy approach, about *16%* of them drew a qualitative work-energy bar chart, and *79%* of them correctly constructed the generalized work-energy equation.

From the above results, we saw that many students made a pictorial sketch to help them understand the problem, but not many actually constructed a work-energy bar chart on the final exam problem. There could be several reasons for this. With the time

A Soap Box Derby car starts at rest and coasts 50 m down a steep 20^0 incline. The effective coefficient of friction due to all sources is 0.10. How fast is the car moving after traveling 50 m? Assume g = 10 m/s^2. You must show all of the work supporting your answer or no credit will be given.

Figure 3.21: A problem that appeared on the final examination in a calculus-based class for honors engineering freshmen in the autumn quarter of 1997.

pressure needed to complete an exam, students skip steps that they feel are not absolutely necessary to complete the solution. Students' responses on the survey question in Table 3.7 indicate that many students had become more familiar with work-energy problems and had learned to draw the bar charts mentally—they no longer needed to draw them explicitly. A similar effect can be found for students using the motion diagrams to solve kinematics problems. The students explicitly drew the motion diagrams less and less frequently as their expertise developed and they could construct the motion diagrams mentally. Alternatively, this final exam problem may not have been difficult enough for most of the students to need to use the work-energy bar charts to complete the solution. Finally, it might be that some students felt the bar charts were not useful in problem solving, and so there was no reason to use them.

3.3.2 Discussion of the Assessment Results

From the measurements of student responses on the written survey questions, on the conceptual reasoning question, and on the final exam problem, we find that more than 80% of the students thought the work-energy bar chart and the multiple-representation strategy was useful for them in understanding the work-energy concepts and in solving related problems. Students' performance on the tutorial-type question indicates that representing a work-energy process in multiple ways, especially using qualitative representations, helps the students develop meaningful understanding about the physical processes. However, we also find that not many students actually constructed the work-energy bar chart on their solutions to the final exam problem. There are still some critical research questions that need to be investigated further. For instance, do students have better performance on complex energy problems using this method instead of using other

approaches? How do students solve work-energy problems after learning this multiple-representation strategy? That is, do they still use a formula-centered approach or do they use this multiple-representation method? To address these research questions, we conducted a series of assessment activities in the autumn 1999 at OSU. In Section 3.3.3 below, we report these evaluation results.

3.3.3 Evaluation Results (Autumn 1999)

In the autumn quarter of 1999, we conducted a series of assessments in a calculus-based introductory mechanics class at OSU for honors engineering freshmen (Physics 131E) that learned this multiple-representation strategy. In addition to the survey questions given in the autumn quarter of 1997, the evaluation also included in-depth individual student interviews and a comparison of student performance for this class and another honors class on a complex work-energy problem. This ten-week class covered the regular concepts of mechanics: kinematics, Newton's dynamics, momentum, work and energy, and some rotational dynamics. The students studied work and energy concepts for about six lecture periods, three recitation periods, one lab period, and in their homework assignments.

3.3.3.1 Student Attitudes towards the Energy Bar Charts and the Multiple Representation Approach of Solving Work-Energy Problems

The survey questions of the autumn 1999 version are slightly different from the ones given in the autumn quarter of 1997: the first two questions are the same, but we slightly modified the last question. The last question from autumn 1997 asks: "Did you (or how did you) use the energy bar charts and the multiple representations of work-energy processes to solve energy problems while doing homework, group problems in recitation, problems in lab, and/or on exam problems? Did you use them when first

becoming familiar with the concepts and then less as you become more expert at solving work-energy problems?" We think this question, especially the second part, is a leading question. Hence, we revised the last question as: "Did you (or how did you) use the energy bar charts and the multiple representations of work-energy processes to solve energy problems while doing homework and/or exam problems?"

This revised survey was administered to the honors engineering freshmen in 131E to evaluate what they thought about the role of the work-energy bar charts and the multiple-representation strategy. Students' answers on each of the three free-response survey questions are summarized in Tables 3.8, 3.9, and 3.10. Below, we discuss in detail the students' responses for Questions 1 and 2, which are listed in Table 3.8 and 3.9. The students' responses for Question 3 are discussed in Section 3.3.3.5.

Table 3.8 reports a summary of the students' responses for question 1 on the survey: "Did using the energy bar charts help you learn energy concepts and solve work-energy problems? Explain why they were useful or not useful." This question is the same as the one given in the autumn 1997. We found that of 76 students, eighty-six percent thought that the energy bar charts were useful. Fifty-seven percent thought that the energy bar charts helped them visualize what is happening to different types of energy in energy problems, and set up the right equations to solve the problems easily. Seventeen percent thought that the energy bar charts helped them understand the abstract energy concepts and the energy conservation better. Seven percent thought that the energy bar charts were helpful in learning the energy concepts at the beginning, and that after a while they could create the bar charts mentally rather than write them out. Fourteen percent of the students preferred using equations directly rather the energy bar charts.

86% Useful (N = 76)	
57%	These students thought that the energy bar charts helped them visualize what is happening to different types of energy in energy problems, and set up the right equations to solve the problems easily. *Two examples of the students' responses:* • The bar charts were helpful. It's a technique I've never really seen before. They were helpful because they give an easy-to-understand graphical representation of what's going on. From the graphs, it's then pretty easy to write out the work-energy equations. • Yes. It helps me to visualize the conservation of energy throughout the processes. Once you have the bar chart done all you have to do is plug in equations and values.
17%	These students thought that the energy bar charts helped them understand the abstract energy concepts and the energy conservation better. *Two examples of the students' responses:* • I think they were useful in visualizing the concept of energy conservation. • I think that they are very useful to the students when first learning about conservation of energy. It teaches the students to make initial and final energy balance out and not to forget any of the energy.
7%	These students thought that the energy bar charts were helpful in learning the energy concepts at the beginning, and after a while they could create the bar charts mentally rather write them out. *One example of the students' responses:* • These charts were helpful at first because they helped us see which terms to use from the "universal" energy conservation equation. But eventually I could pick up on which terms to use without the bar charts.
5%	These students thought that the energy bar charts were helpful, but they did not explain their reasons.
14% Not Useful (N = 76)	
14%	These students preferred using equations directly rather the energy bar charts. *One example of the students' responses:* • No, they were annoying. They were actually more confusing because I had to think how much energy was in what form. That's what you can see after you do the equation.

Table 3.8: Students' responses for question 1 on the survey: Did using the energy bar charts help you learn energy concepts and solve work-energy problems? Explain why they were useful or not useful.

82% Useful (N = 76)	
82%	These students thought that the multiple-representation strategy was helpful to understand the concepts better, and to set up the problems correctly, and was a good teaching method to help a variety of students learn physics. *Two examples of the students' responses:* • Having the concepts in several forms helps me to see how the energy is conserved and helps to explain what is going on. Each representation shows in a different way what is happening in the system. This makes the parts of the problem easier to understand and pick out. • I think that multiple descriptions definitely help with the learning process in physics. By giving the students multiple descriptions, it gives the students a complete understanding of how things work. .
10%Useful and Not Useful (N = 76)	
10%	These students thought that the multiple representations were helpful to understand the energy concepts, but were sometimes a waste of time for quantitative problem-solving. *One example of the students' responses:* • It helped in some cases, but when it was required and not really needed, it was annoying. If we can figure out a problem without going through everything, we should be able to once in a while.
8% Not Useful (N= 76)	
8%	These students thought that the multiple representations were not helpful except equations. *One example of the students' responses:* • I prefer numbers over all that stuff. It's a pain to require us to do a bunch of busy work.

Table 3.9: Students' responses for question 2 on the survey: Did representing the work-energy processes in multiple ways—words, sketches, bar charts and equations—help you learn energy concepts and solve energy problems? Explain why they were useful or not useful.

	88% Use in Problem Solving (N = 76)
19%	These students used the energy bar charts and the multiple representations to solve problems at the beginning, but less and less as they became more familiar with the problems. *Two examples of the students' responses:* • When first doing these problems I used the bar chats to assist in visualizing the problem. And this made it easier to set up the equations… After I got the hand of it though, the bar charts were no longer necessary to actually draw. • I used them at first, but after I became accustomed to the concept, I used the bar charts less frequently.
31%	These students used the multiple representations, especially the energy bar charts, most of time when solving energy problems. *Two examples of the students' responses:* • Whenever possible I always tried to use energy bar charts including on homework, midterm, and even a few practice problems. I think using energy bar charts greatly increases my overall understanding of the problem. • I always made a bar chart before working with equations. This way I knew visually what equations to use and I was more sure on what to use in my equations.
30%	These students explained that they used the bar charts to help set up equations correctly when solving the problem. *Two examples of the students' responses:* • I used bar charts to outline the setup for a problem, and then used sketches and arrows to reinforce my bar charts and check vector directions and overall problem setup. After this, setting up equations was easy, just correspond pictures and energy bars to formulas. • I created the bars for the applicable forms of energy, then derived my equation from this bar chart. When having the choice between using work-energy problem solving method or another method, the work-energy is almost always simple and more straightforward. If often makes the problem easier to understand, the work involved easier to follow, and thus overall, less mistakes are made.
8%	These students used the energy bar charts and the multiple representations in other ways. *One example of the students' responses:* • Honestly, when working by myself I rarely used them, again except when the problems were multi-part and complex.
	12% Not Use in Problem Solving (N = 76 in total)
12%	These students did not use this method unless it was required to do so. They just preferred to use equations.

Table 3.10. Students' responses for question 3 on the survey: Did you (or how did you) use the energy bar charts and multiple representations of work-energy processes to solve energy problems while doing homework or exam problems?

Question 2 in the survey is same as the one given in the autumn 1997 as well. Table 3.9 reports a summary of the students' responses for Question 2: "Did representing the work-energy processes in multiple ways—words, sketches, bar charts and equations—help you learn energy concepts and solve energy problems? Explain why they were useful or not useful." Out of 76 students, eighty-two percent thought that this multiple representation strategy was helpful: helped them understand the concepts better; set up the problems correctly; and be a good teaching method for a variety of students learn physics. Ten percent thought that the multiple representations were helpful in understanding the energy concepts, but were sometimes a waste of time for quantitative problem solving. Eight percent still just liked to use equations directly in their problem solving.

From the students' responses for the Questions 1 and 2, we see that these results are very similar to those from autumn 1997. The consistency of the students' responses from these two classes indicates that most students appreciated the use of this multiple-representation strategy in helping learn the concepts of work and energy. Many students realized that the work-energy bar chart helps them visualize the energy transformation among different types of energies and "see" the conservation of energy during a physical process. Also, a large number of students were conscious of the crucial role that the work-energy bar chart plays in helping them set up equations correctly and readily. Furthermore, we find that the multiple-representation technique is helpful for most students to learn the concepts of work and energy and to solve related problems. It helps students develop deeper conceptual understanding while solving problems.

3.3.3.2 Student Performance on a Written Test Question (Bullet-Block-Spring Problem)

Did students who experienced the multiple-representation technique have better performance on work-energy problem solving than others who learned some other approach with the work-energy concepts? To address this research question, we administered a written test problem (see Figure 3.22) to the Physics 131E class on the final examination in autumn 1999. Also, a similar test question (see Figure 3.23) was given to the Physics 131H class, an honors class for physics majors, who were exposed to the concepts of work-energy in other approaches. In this section, we report how the 131E students performed on the problem shown in Figure 3.22. In the next section, we discuss how the 131H students performed on the problem shown in Figure 3.23.

The question shown in Figure 3.22 was administered to 77 students who were exposed to the concepts of work-energy using the multiple representation strategy. This question can be divided into two small sub-problems. The first one includes the interaction between the bullet and the block, a momentum conservation problem. The second sub-problem includes the interaction between the block-bullet system and the spring, an energy conservation problem. To determine the bullet's speed before it hits the block, the student needs to solve both sub-problems correctly. Overall, out of 77 students, *72%* solved the problem correctly. For research interest, below we report in detail about students' performance on the second part of the problem—the interaction between the block-bullet system and the spring, an energy conservation problem. This part is referred to as the energy problem in the following.

A 0.10-kg bullet is fired into at 1.90-kg block. The block is attached to a spring of force constant 1000 N/m. The block slides for 0.40 m while compressing the spring after the bullet runs into the block. Determine the bullet's speed before it hit the block. Assume that the gravitational constant is 10 m/s^2. You must show all of the work supporting your answer or no credit will be given.
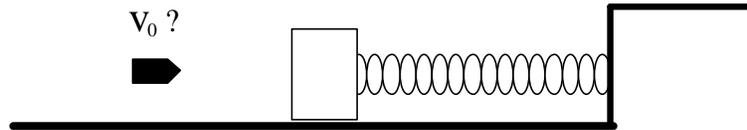
Figure 3.22: The problem was given on the final exam for the honors engineering freshmen who had learned the work-energy bar charts and the multiple-representation strategy with the concepts of work and energy.

			131E Autumn 1999 N = 77		
	% Total	**Physical representation (Work-Energy Bar Chart)**	**Mathematical representation**		
			Algebraic representation	**Numerical representation**	
Correct Answer	77%	38%	73%	4%
Incorrect Answer	22%	6%	18%	4%
Not include mass of bullet	8%	3%	7%	1%
Incorrectly apply the conservation of energy	9%	3%	9%	0%
Apply Newton's laws	5%	0%	2%	3%
Unclear	1%	N/A		

Table 3.11: Students' performance on the energy problem that is a sub-part of the problem shown in Figure 3.22. This energy problem includes the interaction between the bullet-block system and the spring.

The performance of the 77 Physics 131E students on this energy problem is summarized in Table 3.11. Overall, 77% solved the energy problem correctly, 44% drew the work-energy bar chart in their solutions, 91% set up the generalized work-energy equation, and only 8% played with numbers in their solutions. It is also interesting to see that among the 77% students who correctly answered the problem, about 50% drew work-energy bar charts in their solutions. But among the 22% students who incorrectly answered the problem, only about 27% drew work-energy bar charts.

3.3.3.3 Student Performance on a Similar Bullet-Block-Spring Problem after Standard Instruction

In autumn quarter 1999 at OSU, we also collected data from a calculus-based introductory mechanics class for honors physics majors (Physics 131H). This was a ten-week standard lecture-based class. It covered the regular concepts of mechanics: kinematics, Newton's dynamics, momentum, work and energy, and some rotational dynamics. The students worked with the work and energy concepts for about twelve lecture periods, one lab period, and in their homework assignments. The class was taught by an experienced professor and placed emphasis more on theoretical inferences and on mathematical derivations.

The problem shown in Figure 3.23 was administered on the final examination in the Physics 131H class in the autumn quarter of 1999. This question is similar to the one (see Figure 3.22) given to the Physics 131E class who learned the work-energy bar chart and multiple-representation strategy with the concepts of work-energy. Similarly, it could

A bullet (1.0 g) is fired into at a stationary190 g block that is attached to a spring (k = 100 N/m). The collision between the bullet and the block occurred so quickly that the block did not move significantly. After the bullet embedded itself in the block, the block slid 0.40 m before it stopped.

a) Immediately after the collision between the bullet and the block, how fast was the block moving?

b) Suppose that friction between the block and the underlying surface was negligible. From the given maximum compression of the spring, how fast was the bullet moving before it hit the block?
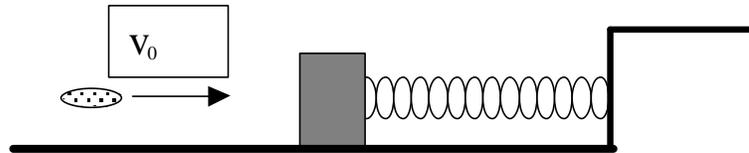
Figure 3.23: The problem was given on the final exam for the honors physics majors who had learned the work-energy concepts in standard instruction.

	131H Autumn 1999 N = 52			
	% Total	Physical representation (Work-Energy Bar Chart)	Mathematical representation	
			Algebraic representation	Numerical representation
Correct Answer	**48%**	**0%**	**36%**	**12%**
Incorrect Answer	**52%**	**0%**	**28%**	**24%**
Not include mass of bullet	6%	0%	4%	2%
Incorrectly apply the conservation of energy	6%	0%	2%	4%
Apply Newton's laws	11%	0%	3%	8%
Not solve the problem	21%	0%	19%	2%
Other errors	8%	0%	0%	8%

Table 3.12: Students' performance on the energy problem that is a sub-part of the problem shown in Figure 3.23. This energy problem includes the interaction between the bullet-block system and the spring.

be divided into two small sub-problems. The first one includes the interaction between the bullet and the block, a momentum conservation problem. The second sub-problem includes the interaction between the block-bullet system and the spring, an energy conservation problem. A major difference between the problems shown in Figures 3.22 and 3.23 is that the two separate questions in Figure 3.23 had explicitly broken the problem into the two small sub-problems. This might have made the problem in Figure 3.23 easier, since less effort is needed by a student to identify and break the problem as a whole into the smaller ones.

Overall, out of 52 physics majors, *25%* correctly answered both question a) and b) of the problem shown in Figure 3.23. Student performance on question a), which is the energy problem, is summarized in Table 3.12. Overall, 48% of the students correctly solved this energy problem; 64% started the problem with the generalized work-energy equation; but 36% still played with numbers in their solutions. Not surprisingly, none of the students used the work-energy bar chart s or any other physical representations in their solutions, since they had learned only to use mathematical expressions to represent work-energy processes.

3.3.3.4 Comparison of Student Performance on the Two Bullet-Block-Spring Problems

To make this comparison meaningful, we carefully checked some factors that could have significantly affected the comparison results (See Table 3.13). First, the two professors taught these two classes were both very experienced instructors; both had taught these classes before. Second, honors engineering freshmen in Physics 131E class were exposed to the concepts of work and energy for about two weeks, while honors

		131E	131H
Student academic background		Honors engineering freshmen	Honors physics or mathematics majors
Pretest scores of FCI	Autumn 1999	63.4%	57.1%
	Autumn 1998	59.8%	64.5%
	Autumn 1997	unknown	65.3%

Table 3.13: Pretest scores of the FCI from honors engineering students and honors physics or mathematical majors. The test was given at the very beginning of each quarter.

physics and mathematics majors were exposed to the concepts of work and energy for about three weeks. Third, we kept track of students' pretest scores on the Force Concept Inventory (FCI) [17] from the autumn quarter of 1997 through the autumn quarter of 1999. On the basis of the test scores, we saw that the two classes had very similar performance on the pre-FCI tests. Therefore, it seemed reasonable to compare the student performance on the two bullet-block-spring problems.

Comparing data in Tables 3.11 and 3.12, we found that overall the honors engineering freshmen who learned the multiple representation strategy performed better than the honors physics or mathematics majors on the energy problems of the two similar bullet-block-spring problems. Furthermore, the former performed more expert-like than the latter. Of 77 honors engineering students, in addition to about the half using the work-energy bar chart, about ninety percent set up the generalized work-energy equation and only eight percent just played with numbers in their solutions. In contrast, of 52 honors physics or mathematics majors, overall, sixty-four percent set up the generalized work-energy equation and thirty-six percent played with numbers in their solutions. To further investigate how the students from these two classes solved work-energy problems, we

solicited some volunteers from these two classes and conducted think-aloud individual

student interviews. Interview results are discussed in Section 3.3.3.6.

3.3.3.5 Student Use of the Multiple-Representation Strategy in Problem Solving

In autumn 1999, we also investigated how students use the multiple-

representation method to solve energy problem. It was very important to understand

whether this strategy was helpful in student solving work-energy problems and, if so,

how it was helpful. In addition to a survey question that asked how students used this

strategy, we analyzed students' solutions of one examination problem. Below, we discuss

students' responses to the survey question and their solutions for a work-energy problem

that appeared on their examination.

In the survey with the three free-response questions administered to the 76 honors

engineering freshmen, the last question asked the students: "Did you (or how did you)

use the energy bar charts and multiple representations of work-energy processes to solve

energy problems while doing homework or exam problems?" Students' responses to this

question are summarized in Table 3.10. We found that, overall, eighty-eight percent of

the students commented that they used this method to solve problems, while only about

twelve percent responded that they just liked to use equations to solve work-energy

problems unless they were required to use qualitative representations such as the work-

energy bar charts. Furthermore, it appeared that about thirty-one percent of the students

used this strategy, especially the work-energy bar charts, most of time when solving

energy problems. Again, we found that about nineteen percent of the students used it less

and less as they became more familiar with energy problems. But many of them still kept

the energy bar charts in their minds as a useful problem-solving tool in evaluating their

solutions. It was also interesting to see that about thirty percent of the students explained that they used the work-energy bar charts to help set up equations correctly when solving the problems.

But did the students actually apply the multiple-representation method when rushing through their examinations? We collected students' solutions for a midterm examination problem from two of the three recitation sections of this calculus-based honors engineering freshmen class. By this midterm examination time, the students had learned the concepts of work and energy. The problem shown in Figure 3.24 was one of the problems given on their one-hour examination. This problem could be readily solved applying the conservation of energy principle. Students' solutions were analyzed from a perspective of the multiple representations. The results are summarized in Table 3.14.

Overall, out of 55 students, seventy-eight percent answered the problem correctly. Seventy-three percent drew the work-energy bar chart in their solutions, and eighty-eight percent constructed the algebraic representation of the generalized work-energy equation. We also found that all of these seventy-three percent of students set up the work-energy equations that were consistent with the bars in their work-energy bar charts.

On the basis of the results in Table 3.14, it appeared that most students constructed the physical representation to help them understand the problem. Comparing the assessment result we got in the autumn quarter of 1997 (see Section 3.3.1.3), we found that more students constructed the work-energy bar chart in their solutions for the problem shown in Figure 3.24. There could be several reasons for this. The problem shown in Figure 3.24 was difficult to be answered applying Newton's laws, but the

A 3200 N/m spring initially compressed
2.0 m launches an 80 kg cart up a 370
incline. A 100 N friction force opposes
the motion. How far up the incline will
the cart travel before coming to a stop?
Assume that the gravitational constant is
10 N/kg. Be sure to show your work.
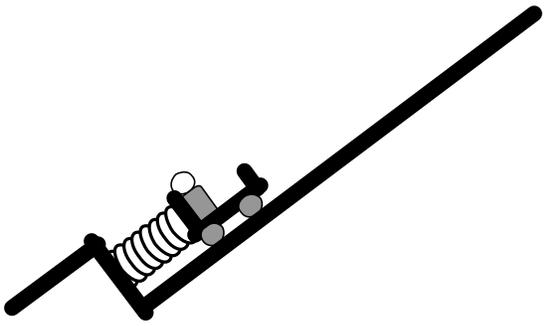
Figure 3.24: The work-energy problem given on the midterm exam of 131E class. The exam was given when the students just had learned the multiple-representation strategy with the concepts of work and energy.

	131E Autumn 1999 $N = 55$ **(From two of the three recitation sections)**			
	% Total	Physical representation (Work-Energy Bar Chart)	Mathematical representation	
			Algebraic representation	Numerical representation
Correct Answer	78%	64%	73%	5%
Incorrect Answer	20%	9%	15%	5%
Unclear	2%	N/A		

Table 3.14: Students' performance on the problem shown in Figure 3.24.

problem shown in Figure 3.21 could be answered easily either applying Newton's laws or

the conservation of energy principle. Thus, more students applied the conservation of

energy principle to solve the problem shown in Figure 3.24; there were more possibilities

for students to use the work-energy bar chart in problem solving. The problem shown in

Figure 3.24 was harder than the one shown in Figure 3.21. Students needed to further

reason about the former. So the work-energy bar chart could help them understand the

problem and set up the equation. The problem in Figure 3.24 was given in the midterm

examination in autumn 1999 when students just learned the concepts of work and energy.

But the problem in Figure 3.21 appeared on the final examination in the autumn quarter

of 1997.Students might keep the multiple-representation technique fresher right after they

learned it than after a while.

Overall, apparently, many students used the physical representation in their

problem solving. For research interest, we conducted in-depth think-aloud individual

student interviews to probe how students used the work-energy bar chart in their problem

solving.

3.3.3.6 Think-Aloud Individual Student Interviews

After honors engineering freshmen (the Physics 131E students) learned the

concepts of work and energy using the multiple-representation technique in the autumn

quarter of 1999, we conducted follow-up interviews. Six students were interviewed from

this class on a voluntary basis. None of these six students were interviewed during the

pre-interview that was conducted before students learned the concepts of work and

energy. For research interest, we also interviewed, on a voluntary basis, four honors

students from the Physics 131H class in which honors physics or mathematics majors learned the concepts of work and energy in other approaches.

Interview Procedure and Problems: During the interview, the students first were instructed how to keep talking about what they were thinking while solving the interview problems. To let them become comfortable with this think-aloud approach, the students were given three warm-up exercise questions (see Appendix A).

After getting comfortable with "thinking aloud" while solving the problems, the students were given three interview problems. We provided them an equation sheet, which includes all basic equations from kinematics, Newtonian dynamics, work and energy, momentum, and rotational dynamics. They were also allowed to use a calculator.

The student completed the interview problems (see Appendix F) in the following consequence: Problem 1, Problem 2, and Problem 3. Each problem consisted of two parts. The first part was a verbal problem statement. The second part included three questions. The problem statement and the three questions were written on four separate cards. They were placed face down and piled up in the order of the problem statement, Question 1, Question 2, and Question 3. The students were instructed to flip over each card and to answer each question in order. When they felt satisfied about their answers, they could go to the next question on the card. When finishing all three questions of Problem1, the student could get into Problem 2 and so on.

Problem 1 (see Appendix F) is a conceptual reasoning question. The correct answer for Question 1 is that the two sleds have the same kinetic energy when they reach the finish line. This question can be readily answered if one applies the work-energy theorem. But it also can be answered applying Newton's second law plus kinematics

equations. Based on the answer to question 1, it is easy to answer Question 2: Sled B has the greater speed since it is less massive. For Question 3, again, based on the work-energy theorem, we know that sled A has the greater kinetic energy. Notice that there are no numbers given in this problem. We attempted to probe whether or not the students could use algebraic representations to answer the questions.

Problem 2 (see Appendix F) involves a complex physical process. To answer Question 1, one needs to do quantitative calculation either applying the conservation of energy principle or by Newton's laws plus kinematics equations. But it is much easier to use the concepts of work-energy than those of dynamics and kinematics. Questions 2 and 3 are both qualitative questions. The correct answer for Question 2 is that the speed of the toy car will be the same right before it lands on the floor. For question 3, the correct answer is that the speed of the toy car will become less right before it lands on the floor, if the height of the table is lowered.

Problem 3 is also a complex work-energy problem. It is easier to be answered applying the conservation of energy principle. Questions 1 and 2 are quantitative questions and Question 3 is a qualitative one.

Notice that the three problem statements do not include any sketches. We intended to investigate how students used pictorial, physical, and mathematical representations to solve the problems. Working papers and pencils were provided to the student and a calculator was allowed to use if needed. Student's paper–pencil solutions were collected and his or her verbal explanations were recorded.

3.3.3.7 Student Performance on the Interview Problems

We carefully analyzed two sets of problem solutions produced by two students, one from the Physics 131E class and one from the Physics 131H class. The former learned the multiple-representation strategy with the concepts of work and energy, while the latter learned those concepts in other approaches. Their detailed solutions are illustrated in Appendices G and H, respectively.

Solving for Problem 1, the 131E student used the concepts of Newton's second law and kinematics. After reading the problem statement, he drew a sketch and labeled related known information. To answer Question 1 in the problem, he started with the concept of kinetic energy (the question goal). To compare the kinetic energy of sled A with the kinetic energy of sled B at the finish line, he realized that he needed to figure out their final speeds. Then he used Newton's second law and a kinematics equation (not including time) to derive speed squares (V^2) for both sleds A and B. After this, he plugged each V^2, respectively, into the sled A's kinetic energy equation and the sled B's kinetic energy equation. By comparing the two kinetic energies, he correctly answered Question 1. To answered Questions 2, he referred to the speed square (V^2) equations that he got from Question 1. To answer Question 3, he used the final kinetic energy equations in his solution for Question 1 to reason. He correctly answered Questions 2 and 3.

The 131H student did not make any drawings but wrote down given information after reading the problem statement in Problem 1. To answer Question 1, she started with a work-energy theorem idea. She explicitly realized that the work done on sleds A and B was the same since the forces exerted on both and the distances they traveled were the same. She also recalled: "the work done equals the change of kinetic energy." But she

gave up with this idea after she realized that the work done equaled the change of kinetic energy, but not kinetic energy itself, which was asked to solve for. She shifted to use Newton's second law. After she found that the acceleration of sled A would be different from that of sled B since they had the different masses, she realized that the more massive sled could move slow. Then she shifted to look for the kinetic energy of each sled at the finish line. After she plugged the mass into the kinetic energy equations of sleds A and B, she said: "if speeds for sleds A and B are the same, then they have the same kinetic energy at the finish line. But I don't know if their speeds are same or not." Then she looked back for Newton's second law and used it with the velocity equation (including time) to derive the speeds of sleds A and B. Finally, she plugged the speed expressions back to the kinetic energy equations and derive the final kinetic energies of sleds A and B at the finish line. Since she ignored the time differences in motions of sleds and B, she incorrectly answered Question 1.

To answer Question 2, this 131H student used her speed equations from Question 1. But she still could not correctly answer Question 2. For Question 3, she started with a work idea, and then tried to use the work-energy theorem to solve for the kinetic energy.

To answer Problem 1, both the students used algebraic representations; none of them assigned numbers for physics quantities used in the problem. But compared their solutions, apparently, the 131H students shifted more often between different concepts such as the work-energy theorem, Newton's second law and kinematics, and the definition of kinetic energy. She made a decision to shift her idea each time partly by looking at the equation sheet and partly by thinking about the problem goal or sub-goals.

This is one of the characteristics of naïve problem solving identified from a study done by Jill Larkin et al. (see Section 2.2.3.1 for more details).

For solving Problem 2, the 131E student first drew a pictorial representation—a clear sketch with given information labeled. Then he constructed a simplified work-energy bar chart (see Figure 3.25) to help him understand the problem: he wrote down each possible energy term, represented by each letter, in the chart, and used it as a guide to check whether each type of energy was zero or not in the initial and final states. He drew a bar for each non-zero type of energy, and also made the final kinetic energy bar as high as the sum of the initial gravitational potential energy bar and the initial elastic potential energy bar (i.e., spring energy bar). Then he used the complete work-energy bar chart to set up the generalized work-energy equation: one expression of energy term in the equation for each bar in the bar chart (see Figure 3.25). After he constructed the generalized the work-energy equation, he substituted numbers into the equation and solved for the unknown variable.

To answer Question 2, which is a qualitative question, he used the generalized work-energy equation to reason. For Question 3, which is also a qualitative question, he used the work-energy bar chart to reason about it.

Clearly, this 131E student solved Question 1 by working forward—from the given information to the desired quantity. His problem solving procedure explicitly consisted of three stages: drawing a sketch, developing a physical representation—the work-energy bar chart, and applying a mathematical equation—the generalized work-energy equation. Also, after he set up the generalized work-energy equation, he treated as one single step by substituting the values of the quantities in it and solving for the

$$mgh \;+\; \frac{1}{2}kx^2 \;=\; \frac{1}{2}mv^2$$

.05 kg (10 m/s^2) (1 m) + ½ (100 N/m)(.2 m)^2 = ½(.05 kg)v^2

.5 + 2 = .025v^2

2.5/.025 = v^2

v = 10 m/s

Figure 3.25: A part of the solution produced by the 131E student for solving Question 1 in the interview Problem 1. He constructed the work-energy bar chart, used it to set up the generalized work-energy equation, and then plugged numbers into the equation to solve for the unknown quantity.

unknown variable. All these features were identified, from studies by Jill Larkin et al., as problem solving strategies often used by experts.

The 131H student also started by drawing a sketch and labeling known information to help her understand Problem 2. By looking their paper solutions, we found that the drawings that both of the 131H and the 131E students made were almost identical and equally good. After this step, their solutions become very different. To answer Question 1, the 131H student started with the goal—find the speed of the toy car just before hitting the floor. Then she looked at the equation sheet and decided to solve for the kinetic energy of the toy car just before the hitting. So she started to look at the top of the ramp and to use the concepts of work and energy to solve the question. Her detailed solution is shown in Figure 3.26. She did not set up the generalized work-energy equation. Instead, she separately calculated the initial energy (i.e., $P = mgy$) and the final energy (i.e., $1/2mv^2 = 0.9$ J). Apparently, she relied on numerical representations to answer the question. She used her sketch (i.e., a pictorial representation) to reason about Questions 2 and 3 and incorrectly answered them.

Apparently, the solution of Problem 2 produced by this 131H student reflected the following features: 1) jumping from the sketch to mathematical equations; 2) using a numerical-representation-centered approach, instead of constructing the generalized work-energy equation—an algebraic representation; and 3) relying on the pictorial representation to reason about the conceptual questions. These are identified as naïve problem solving characteristics by other researchers (see Section 2.2.3.1 for more details) and studies done in this dissertation.

$$P = m \cdot g \cdot y$$
$$P = 0.05\,kg \cdot 10\,m/s^2 \cdot 1.8\,m$$
$$P = 0.9\,J$$

$$K = 0.9\,J$$
$$\frac{1}{2}mv^2 = 0.9\,J$$
$$mv^2 = 1.8\,J$$
$$v^2 = 36\,m^2/s^2$$
$$v = \pm 6\,m/s$$
$$= -6\,m/s$$

Figure 3.26: A part of the solution produced by the 131H student for solving Question 1 in the interview Problem 2. She did not set up the generalized work-energy equation. Instead, she separately calculated the initial energy (i.e., $P = mgy$) and the final energy (i.e., $1/2mv^2 = 0.9$ J).

For Problem 3, both Questions 1 and 2 are quantitative ones, but Question 3 is a qualitative question. As shown in Appendix G, the solutions produced by the 131E student consisted of three steps: a sketch with known information, the work-energy bar charts, and the generalized work-energy equations. From a perspective of the multiple representations in problem solving, his solutions consisted of three representations: the pictorial, physical, and mathematical representations. He used the generalized work-energy equation from Question 2 to reason about Question 3. Apparently, his solutions reflected critical features of expertise in problem solving.

To answer Problem 3, the 131H student also constructed a sketch with related known information labeled. To solve for Question 1, she started with a work-energy idea. She tried to identify what types of energy that the model cart had at the point leaving the plateau, at the top of the first hill, and at the top of the second hill. She got a hard time to think about this mentally and to memorize the different types of energy. Then she labeled to the above three points as 1), 2), and 3), and wrote down

at 1) $P + K$,

at 2) $P/2 + K$,

at 3) $P + 0$, and

where P represented gravitational potential energy and K represented kinetic energy. Then she assumed that the above equations had to be equal because of the conservation of energy. She assigned U for a constant and revised the above equations as

at 1) $P + K = U$,

at 2) $P/2 + K = U$, and

at 3) $P + 0 = U$.

Then she tried to see how the potential energy and kinetic energy change at these three points. She said: "I'm trying to visualize what's happening to the energy at these different points." She used her sketch and started to think about the potential and kinetic energies at point 1) (at the top point of the plateau). After she realized that she only knew the potential energy but not the kinetic energy at this point, she wrote down $P_1 = mgh$, plugged into numbers, and calculated $P_1 = 1\ J$. Then she tried to reason about how the potential energy and kinetic energy changed at point 2) (at the top of the first hill). She said: "the potential energy would be 0.5 J since its height is 0.5 m. About the kinetic energy, it should be as twice as that at the point 1). But I don't know how large it (the kinetic energy) is at point 1)." Then she looked at point 3) (at the top of the second hill) and realized that the kinetic energy should be zero, but the potential energy was unknown since she did not know the height of the hill, which was the desired variable. Then she was stuck there. She tried to link the potential energy at point 3) with the energy at the beginning point 1), but she realized that she did not know the kinetic energy at point 1). Then she looked at her sketch and went back and forth between these three points for several times, and had a difficulty imaging how the energy changed and the relationship of the energies at different position. Finally, she looked at the equation sheet and said: "I'm looking at equations, and hope these magic formula can tell me how to do it." From the equation sheet, she found the potential energy expression for a spring. She seemingly got a cue from this equation and realized that the energy stored in the spring initially should equal the kinetic energy that she was looking for. Then she used the given numbers to calculate the spring potential energy. Finally, she added together two

numbers, the value for both the potential energy and the kinetic energy at the point of the plateau, and solved for the desired variable.

To answer Question 2, she started by drawing a sketch. Then she used the numerical result U = 2.6 J from her solution for Question 1 to calculate the potential energy and kinetic energy at the second hill with the new height. After she got the value for the kinetic energy of the cart at the beginning of the track, she tried to solve for the speed of the cart at that point. Then she realized that this speed was not the desired variable. She went back to look at the question and then used the work-energy theorem to calculate the frictional force. As shown in Appendix H, she used numerical representations to answer this question. To answer Question 3, she used her numerical representations from Question 2 to reason.

This 131H student started with the concepts of work and energy and worked forward to solve Questions 1 and 2. But, apparently, she struggled with imagining the energy changes at the different points during the physical process described in the problem. She wrote down symbolic equations to help her reason and memorize different types of energy at the different points. But those symbols such as P or K could not let her see visually how large they were in magnitude; she had to infer and memorize mentally. It appeared that she needed an external, qualitative visual aid, between the pictorial and symbolic representations, to help her "see" easily how the different types of energy changed at the different points and understand the problem conceptually. It seemed that she could not access such an external visual representation, and finally, she looked at the equation sheet and hoped that some "magic formula" could help her. She used a bunch of numerical representations to finish her solutions.

3.3.3.8 Discussion of Student Performance on the Interview Problems

Based on the student performance on the interview problems, we find that the student who learned the multiple-representation strategy used this technique to solve the work-energy problems. In particular, this student's solutions for Problem 2 and 3 reflect expertise in problem solving. We also find that he used the work-energy bar chart to help him understand the problems conceptually first, to set up the generalized work-energy equations, and to reason about some questions qualitatively.

The solutions produced by the 131H student who learned the concepts of work-energy using other approaches primarily reflect naïve problem-solving features, but contain some expertise, such as working forward and recognizing the problems using the concepts of work and energy. But it appears that without being able to access a physical representation, it is difficult for the student to understand and infer the problem conceptually before she used equations. In other words, it is unlikely that the student will uses expert-like problem solving strategies without learning how to use a physical representation to aid in the problem solving.

3.4 Discussion and Summary

It is well known that students attempt to solve problems by matching quantities listed in the problem statement to special equations that have been used to solve similar problems. Research into differences in problem solving between experts and non-experts has found that experts often use physical representations to help them qualitatively understand the problem. This is the central, fundamental difference, argued by some researchers (see Chapter 2).

But few studies have documented how naïve students use an equation-centered approach to solve problems from a perspective of multiple representations in physics. Research reported in this chapter reveals that, to understand how naïve students solve problems, it is necessary to separate two types of a mathematical representation: an algebraic representation and a numerical representation. The former is defined as symbolic physics equations; the latter is defined as numerical physics equations. Beginning students use a numerical-representation-centered approach in their problem solving: representing related concepts in numbers first, assembling these numbers together based on a law or a principle, and calculating a number for the unknown variable. Using this naïve method, we have found that it is difficult for students to gain a conceptual understanding while solving problems. Playing with numbers, students loose the connection between physics concepts and the mathematical symbolic expressions and the connection between mathematical expressions and real physical processes.

It is also discovered that naïve students often go back and forth between pictorial representations and numerical representations in problem solving. Apparently, they seldom use any physical representations and rarely use algebraic representations in solving work-energy problems.

To help students address those difficulties (i.e., the initial state) and to help them develop expertise in problem solving (i.e., the desired final state), Alan Van Heuvelen developed a problem solving strategy of representing work-energy processes in multiple ways: verbal, pictorial, work-energy bar chart, and mathematical representations. A systematic series of assessment activities were conducted to evaluate the effectiveness of this new learning technique on student conceptual understanding and problem solving in

the context of work and energy after it was implemented in introductory physics classes for honors engineering freshmen.

As a newly developed representation, the work-energy bar chart meets the basic criteria for an effective, pedagogical representation and possesses basic features of a physical representation:

1) It has physics meaning—each slot represents a type of energy or work.

2) It has a bar graph format—easy for students to use and to make inferences.

3) It diagrammatically represents the critical structure of a work-energy problem—the initial energy of a system plus the work done on the system leads to the final energy of the system. That is, it qualitatively represents the conservation of energy principle.

4) It is independent of time and the surface features of real objects.

The studies reported in this chapter provide empirical evidence that the work-energy bar chart plays a very important role in student problem solving.

1) The work-energy bar chart helps students reason about work-energy problems qualitatively and conceptually first. Being independent of surface features of real objects such as ropes, springs, or inclines, it aids students in using the concepts of work and energy to understand the problem before they apply mathematical equations. In a bar graph format, it also helps students qualitatively "see" the conservation of energy.

2) The work-energy bar chart helps students set up the generalized work-energy equation correctly and easily. A complete work-energy bar chart provides visual aid for students to construct the mathematical equation—there is one term in the equation for each bar in the bar chart.

3) The work-energy bar chart directs students to make inferences and allows them to evaluate their problem solutions. Students use it to answer conceptual reasoning questions and to check their results.

Overall, from the point of view of the multiple representation of work-energy processes, the work-energy bar chart, as a physical representation, plays a central role in students' solving of work-energy problems. Its function is like a bridge, helping students move easily with smaller steps from the abstract realm of words, or from real-life sketches with surface features, to the abstract realm of scientific and mathematical notations. By linking pictorial and mathematical representation together, the work-energy bar chart can also help students produce mental images for the different energy quantities and to make mathematical symbols meaningful.

Pictorial and mathematical representations also have their own functions in student problem solving. We have found that it is relatively natural for students to draw a sketch to help them understand the problem. In a diagrammatic format, it helps students visualize, infer, and reason about the physical process. A sketch with its system choice also aids students in constructing the work-energy bar chart. On the basis of the empirical data presented in this chapter, it has been discovered that students use a sketch combined with the qualitative bar chart to reason about physical processes without using mathematics and to answer conceptual questions such as being asked to predict qualitatively how changes in various factors will affect the process. But a pictorial representation—a drawing including real objects such as springs, pulleys, and others—can "trigger" students' responses based on surface features.

A mathematical representation can include algebraic and numerical representations. The generalized work-energy equation, identified as an algebraic representation, quantitatively represents the conservation of energy. We have found that it is useful for students in answering some qualitative questions. But it is abstract. A numerical representation can help students solve for unknown information or a value of the desired quantity in the problem. To some students, the numerical representation is less abstract and more easily manageable than the algebraic representation. But the former has the least physics meanings.

This strategy constitutes one part of the learning system (the transformer), which attempted to match the students' initial state, to address their difficulty, and to help students develop expert-like problem-solving skills (the desired state). The actual classroom strategies are also very important. Students learn to learn better if they understand the reasons for various pedagogical strategies. This research has indicated that it is relatively natural for students to use pictorial representations but not physical representations. A physical representation is physics knowledge after all. In addition, many students have experienced only formula-centered didactic instruction. Some students like only equations and think that it wastes time or is a redundant task to represent a problem in different ways. Thus, students must understand why they need to use the qualitative, physical representation in their problem solving, must explicitly learn how to use it, and must have opportunities to apply it in different situations.

To help students address the difficulty in using a numerical-representation-centered approach in their problem solving, it is necessary for students to set up explicitly, on the basis of a complete work-energy bar chart, the generalized work-energy

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equation without any numbers. Students seem to accept qualitative representations more easily, understand them better, and use them more effectively for qualitative reasoning and problem solving if the qualitative representations are introduced BEFORE the corresponding mathematical equations are introduced.

Finally, when students have learned all of the representation types, their understanding improves if they learn to move among representations in any direction. For example, they might be given a numerical representation that is the application of the conservation of energy principle for some process. Their task is to construct other representations of that process—playing Equation Jeopardy. They learn to "read" the mathematics language of physics with understanding, to make numbers and symbols meaningful in physics, and to make a link with real physical processes.

In summary, empirical evidence has shown that in the learning system including the technique representing a work-energy process in verbal, pictorial, physical and mathematical representations, students do apply this strategy in their own problem solving. In particular, students use the work-energy bar chart to help them conceptually understand the problem, qualitatively infer about the physical process, and set up the generalized work-energy equation. In conclusion, this technique helps students abandon their naïve problem solving method and develop expertise in problem solving.

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CHAPTER 4

VISUALIZATION OF INTERNAL THERMAL ENERGY PRODUCED DURING INELASTIC COLLISIONS

As discussed in Chapter 3, ample assessment results show that the multiple

representation strategy, especially the use of the work-energy bar chart as a physical

representation of work-energy processes, helps students acquire a meaningful

understanding of the concepts of work and energy and develop expertise in problem

solving. To use this technique with work-energy processes, we need to pay a lot of

attention to a choice of system. Especially when we deal with physical processes in

which friction cannot be negligible, the choice of the system is crucial. This multiple-

representation approach recommends that two touching surfaces involved in the friction

be included in the system. In this way, we count a gain in internal energy of the system,

rather than deal with a difficult calculation for the work done by the friction. In light of

our classroom observations and the assessment results, we found that few students had

difficulty accepting inclusion of the frictional boundaries within the system and in

calculating a change of the system's internal energy. But we observed that many students

did not understand the concept of internal energy itself. This concept seemed too abstract

for them; they lacked a meaningful mental image or model of it.

Moreover, an understanding of the concept of internal energy is a key for students in applying the conservation of energy principle in collisions, in particular, in inelastic collisions. Few introductory physics textbooks [1], however, introduce this concept in an easy-to-understand way or provide any visual representations to aid in understanding. Without full understanding of this concept, many students recognize elastic collision and inelastic collisions by observation of surface features (see Section 5.1 for details). In addition, some physics instructors often either leave out introduction to this concept or use the term "heat" to describe "where the initial kinetic energy goes" during inelastic collisions. As a result, to a large number of students, "internal energy" and "heat" mean the same thing, but they do not know what that thing is (see Section 4.3.3.1for details).

In this chapter, we discuss studies that investigate and address student difficulties understanding the concept of *internal thermal energy*. Next chapter, we report research that investigates and addresses student difficulties understanding the concept of *internal potential energy.*

This chapter is divided into four parts. The first part reports studies concerning student difficulties with the internal thermal energy produced during inelastic collisions. We discuss in detail individual student demonstration interviews and student responses to a written question. The second part describes how to develop a guided-inquiry learning activity to help students visualize the abstract concept of internal thermal energy and to develop a meaningful mental image of it. In addition to student worksheets, the activity includes classroom demonstrations using a pair of rubber balls and a pair of model carts. The third part discusses extensive assessment results of the effectiveness of the learning activity. The chapter ends with discussion and summary.

4.1 Investigation of Students' Difficulties Understanding the Concept of Internal Thermal Energy

Internal thermal energy is a very abstract concept in physics. It is defined as average random kinetic energy that molecules have at the microscopic level. It is associated with the temperature of a physical object. The greater an average random kinetic energy that the object's molecules have, the higher the object's temperature is. To help students visualize this not-easy-to-see concept, Uri Ganiel [2] developed a pair of model carts. The model cart shown in Figure 4.1 was used for demonstrating inelastic collisions, while the other one in Figure 4.2 was for showing elastic collisions (see Section 4.1.1.1 for details). Uri Ganiel used the model carts to help in-service teachers visualize "where the kinetic energy goes" during the inelastic collision. But he did not evaluate further how the students understood the physics that the model carts showed. Based on his work, we built a pair of Ganiel's model carts. Using the model carts and a pair of rubber balls (happy and unhappy balls), we designed an individual student interview to elicit students' ideas about internal thermal energy and to assess the effectiveness of the model carts on students' understanding of this concept.

4.1.1 Individual Student Demonstration Interviews (Spring 1998)

The individual student demonstration interview was conducted in spring quarter 1998 at OSU. Five students participated in the interview on a voluntary basis. Three were honors engineering freshmen, who learned the multiple-representation strategy with the concepts of work and energy in the Physics 131E class in autumn quarter 1997. The other two came from a regular calculus-based introductory physics class (Physics 132). Thus,
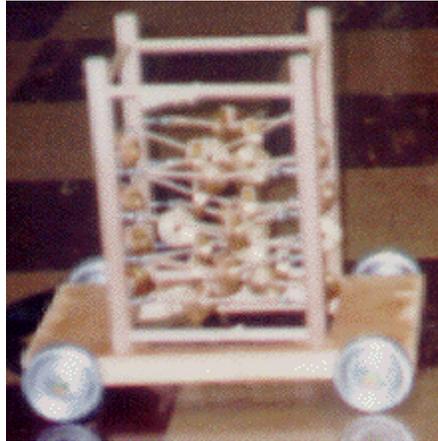
Figure 4.1: Developed by Uri Ganiel, this model cart is used for demonstrating *inelastic* collisions.
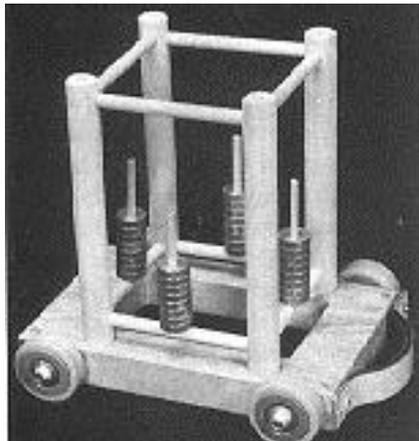
Figure 4.2: Developed by Uri Ganiel, this model cart is used for demonstrating *elastic* collisions.

by the interview time, they all had learned the concepts of mechanics from their respective calculus-based introductory physics classes.

The students were told that the interviews aimed to improve the teaching and learning of physics in the introductory classes; their performance on interview questions would not affect their class grades. Each participant was paid a small amount of money for his or her help and time.

4.1.1.1 Interview Demonstrations

The interview attempted to investigate: 1) after instruction, what did students understand about the energy transformation during collisions? In particular, what did they understand about the internal thermal energy produced during inelastic collisions? 2) How could the model carts help students understand and visualize the concept of internal thermal energy and energy conservation during inelastic collisions? 3) How could we use the model carts effectively with students in a large classroom setting? Therefore, three demonstrations were performed during the interview: 1) a happy ball and an unhappy ball were dropped onto a table from the same height; 2) the unhappy ball and a clay ball were dropped onto a table from the same height; and 3) the two model carts collided with a wall. Below, we discuss in detail the demonstrations using the happy and unhappy balls and the model carts, while leaving the demonstration using the unhappy ball and clay ball to be discussed in Section 5.1.

Happy and unhappy ball demonstration: Happy and unhappy balls are a pair of identical-looking black balls. The pair used for the interview, sold by Arbor Scientific [3], have identical diameter of 2.5 cm. They have slightly different masses: the happy ball has a mass of 8.60 grams, while the unhappy ball has a mass of 9.85 grams. These

identical-looking balls' behavior would surprise you when they are dropped onto a table from the same height. After hitting the table, the happy ball bounces back close to the release height, but the unhappy ball barely bounces at all. Due to this remarkable property, the happy and unhappy balls are widely used by physics teachers to show students a more elastic collision and a more inelastic collision, respectively.

Why do the happy and unhappy balls behave so differently? To understand this mystery, we need to look into their molecular structures. The happy ball is made of neoprene rubber, whose macromolecular structure [4] is shown in Figure 4.3. In these chain macromolecules, although there exist large chlorine groups that restrict rotation on every fourth carbon, the carbon atoms (held together by single bonds) are still able to rotate around their axes. Depending on the temperature, therefore, it could be expected that these chains would be in a constant twisting motion and that they would thus be badly entangled with each other. Any deformation of the neoprene rubber will tend to "straighten out," or uncoil, the entangled mass of contorted chains, but subsequently they will tend to coil up and return to their original positions [4]. By virtue of these violent contortions and restorations of long chains of the rubber, the happy ball can bounce back after it hits the table [5].

From the point of view of energy, the happy ball makes a more elastic collision. Before being dropped, it has gravitational potential energy due to the separation between the ball and Earth. Just before it hits the table, most of the happy ball's potential energy has been converted to kinetic energy (choosing the table as a reference level). As the ball touches the table, it starts to deform. But the entangled macromolecular chains inside the

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Figure 4.3: The macromolecular structure of neoprene rubber, of which the happy ball is made.

Figure 4.4: The macromolecular structure of polynorbornene rubber, of which the unhappy ball is made.

ball lack degrees of freedom to vibrate, so they tend to resist deformation. Thus, the kinetic energy is not readily converted into vibrational energy of the macromolecules, a type of internal thermal energy. (We ignore that the small amount of the kinetic energy that could be converted into the internal energy of the table.) As a result, the happy ball bounces back high.

How about the unhappy ball? It is made of polynorbornene rubber. Its macromolecular structure is shown in Figure 4.4 [6]. Due to the carbon ring structure, the macromolecular chains of polynorbornene are not able to twist and entangle. With more degrees of molecular freedom, at room temperature this polymer cannot quickly return to its original shape during the collision with the table. Thus, the kinetic energy of the unhappy ball can be converted easily into vibrational energy of the macromolecules. That is, the unhappy ball gains internal thermal energy during the collision, defined as an inelastic collision.

Since the students could not see visually what happened to the energy transformation at a molecular level during the collisions that the happy and unhappy balls made, we used this demonstration to probe students' qualitative understanding of the concept of internal thermal energy and of the conservation of energy principle during elastic and inelastic collisions. Instead of asking the students to answer, by memorization, such questions as "what do the terms 'internal thermal energy' or 'heat' mean to you", we asked the students to observe this demonstration and explain their reasoning as they observed the results.

Model cart demonstration: We make the model carts shown in Figures 4.1 and 4.2 collide with a rigid wall on a smooth hallway floor. A strong spring bumper in the

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front of each cart hits the wall. (See Section 4.2.1 for detailed information about structures of the model carts and how to build them.) After hitting the wall, the cart with the rigid rods (shown in Figure 4.2) bounces back significantly farther than the one with the rubber bands (shown in Figure 4.1). Moreover, small disks tied to the rubber bands randomly vibrate much more strongly just after the collision. Obviously, some of this cart's kinetic energy before the collision is transferred to kinetic energy of the small disks, which represents a gain of "internal thermal energy" in the cart. Thus, this rubber-band cart demonstrates a more inelastic collision.

The rigid-rod cart demonstrates a more elastic collision. Since the small disks on this cart are fixed and have no degrees of freedom to vibrate, the kinetic energy of the cart cannot be converted into vibrational energy of the small disks. Therefore, this cart bounces back significantly farther after the collision.

During the interview, the students first were informed that the two carts had identical mass, that the spring bumper in the front of each cart were identical, and that each one had four identical wheels. Then the students were shown the two carts, starting at the same starting line, pushed simultaneously with the same initial speed toward the wall. We asked the students to observe what happened to the carts before and after the collisions and to explain their reasoning as they observed the results.

The use of the model carts as a simple analogy of the balls: Up to this point, we have seen why the happy ball experiences a more elastic collision while the unhappy ball experiences a more inelastic collision, and why the rigid-rod cart demonstrates a more elastic collision while the rubber-band cart demonstrates a more inelastic collision. Is it reasonable to use the two model carts as a simple analogy to the happy and unhappy

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balls? Actually, Uri Ganiel's [2] major purpose for inventing the carts originally was his intention to use the carts to model microscopic structures of solids and to demonstrate the energy transformation, on a molecular scale, experienced by the solids during elastic and inelastic collisions. With associations such as the small disks representing the "molecules" of a solid and the rubber bands representing the "bonds" between molecules, the rubber-band cart can be considered as a simple model of the unhappy ball and the rigid-rod cart as a simple model of the happy ball. For the rubber-band cart, during the collision a gain of the random vibrational energy of the small disks can demonstrate a gain in internal thermal energy of the unhappy ball. For the rigid-rod cart, during the collision, without access to "vibrational degrees of freedom," no energy transfer into the motion of the small disks can occur. This can represent the lack of degrees of freedom to vibrate of the entangled macromolecular chains of the happy ball, so its kinetic energy cannot easily be converted into internal thermal energy of the ball. Thus, the carts provide simple models for elastic and inelastic collisions. The rubber-band cart helps students especially visualize "where the kinetic energy goes," "see" the internal thermal energy produced, and have an idea of how energy conservation works during inelastic collisions.

What do students think of this analogy? How can we use the model carts to help them visualize the abstract concept of internal thermal energy and develop a meaningful mental image of it? Given our interest in these questions, we conducted individual student demonstration interviews to try to address these questions.

4.1.1.2 Student Responses and Reasoning

Upon arrival, the students were first instructed to verbalize their thinking and were encouraged to explain their reasoning as best as they could. Then the three

demonstrations mentioned above were shown in sequence. Before each demonstration, the students were asked to make a prediction first and to give their reasoning. After each demonstration, the students were asked to explain their reasoning again on the basis of their observations. A protocol of the interviews is shown in Appendix I. Not every single question on the protocol had been asked to each student. But the listed questions served as a guide to help the interviewer ask each student the same questions in the same way.

Student responses and reasoning related to the happy and unhappy ball demonstration: All five students predicted that the two balls would bounce back after they hit the table. Three honors engineering students explicitly mentioned that if the balls were made of the same material and had the same size, they should bounce back to the same height at the first bounce. Two regular engineering students explained that the two balls would bounce back because the table would push them to bounce back.

After the students saw that one ball bounced back high but the other almost did not bounce at all, they gave a variety of explanations, as shown in Table 4.1. It was apparent that the two regular students had difficulty understanding the energy transformation during the unhappy ball's collision with the table. They thought that the initial kinetic energy of the unhappy ball was absorbed by the table, since the unhappy ball stopped after the collision. As being asked "in what form the absorbed energy existed in the table," one answered in the form of kinetic energy. He mentioned that the table would move a tiny, tiny bit, but one could hardly see it since the table was too heavy. The other one considered the absorbed energy as heat and sound, but she had no idea about what she meant by "heat."

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	Students' reasoning about why the unhappy ball did not bounce back but the happy ball did. **(N = 5, individual student demonstration interview, spring 1998)**
Honors engineering students	**Student I:** The balls were made of different materials. One (happy ball) was a rubber ball, and the other one (unhappy ball) was a metal ball. (The interviewer asked him to explain what happened to the energy of the unhappy ball just after the collision.) The kinetic energy of the unhappy ball was absorbed by the ball as internal energy, like heat and sound.
	Student II: The bouncing ball (happy ball) made an elastic collision. The not-bouncing one (unhappy ball) made an inelastic collision; some energy was converted into internal energy of the ball.
	Student III: Molecules inside the one did not bounce back could be more like beans in a bag. After the hit, they moved around and were compressed. So the ball could not bounce back. Molecules inside the bouncing ball (happy ball) were more like springs. During the hit, the molecules were compressed a little bit but pushed back. So it bounced back.
Regular engineering students	Both students reasoned that the two balls had different density. The bouncing one (happy ball) was denser. (The interviewer asked them to explain what happened to the energy of the unhappy ball just after the collision.) The kinetic energy of the ball was absorbed by the table.

Table 4.1: Students' reasoning about why the unhappy ball did not bounce back but the happy ball did after the students observed the demonstration.

All three honors students thought that the initial kinetic energy of the unhappy ball was converted to its "internal energy", but they had different understandings of the term "internal energy". Student I thought that the internal energy was heat and sound, and "heat" meant something from the ball into the air and dissipated in the air. Student II thought that "internal energy" was a type of energy that atoms and molecules had internally. From the explanation given by student III, it appeared to him that "internal energy" was the energy to move and compress molecules.

Student responses and reasoning related to the model cart demonstration: The students were first asked to predict what would happen if the two carts were pushed at the same speed toward a wall in a hallway. At that moment, the two model carts were covered and the student could not see the inside structures of the carts. All five students predicted that the two carts would bounce back the same distance since they had identical mass, bumpers, and wheels.

After the demonstration was actually shown, the student observed that one bounced back a significantly shorter distance than the other one. To explain this phenomenon, five students provided different reasoning. All three honors engineering students thought that this would result from different mass distributions inside the two carts. For example, one thought that the not-bouncing cart would have something like a pendulum hung inside. After the collision, the pendulum swung back and forth. As a result the cart was slowed down. One regular student thought that some of the initial kinetic energy of the not-bouncing cart was absorbed by the wall. The other regular student thought that the two carts had different structures inside: one (bouncing cart) had a solid structure and the other one (not-bouncing cart) had a loose structure.

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After the cover was taken off from each model cart, we showed the students the same demonstration again and asked them to explain why they thought one cart bounced back a significantly shorter distance than the other one. This time, the students could see that the rubber-band cart bounced back a significantly shorter distance than the rigid-rod cart. But we did not explicitly ask the students to observe closely about what would happen to the small disks on each cart before and after the collisions.

One honors student noticed that the small weights on rubber bands vibrated much more than before and he provided his reason that since the small weights absorbed some of the initial kinetic energy, the rubber-band cart could not bounce back farther. The other two honors students still thought that the difference in mass distribution between the two carts resulted in the difference in two carts' behaviors. The two regular students still gave the same reasoning for this as before.

Finally, as being asked to match the model carts with the balls, one regular student matched the happy ball with the rubber-band cart (since the cart had rubber bands on it and the happy ball was made of rubber or spring-like material) and matched the unhappy ball with the rigid-rod cart (since the cart was rigid and the unhappy ball was made of dense material). The four other students matched the happy ball with the rigid-rod cart (since they both bounced back high or far) and matched the unhappy ball with the rubber-band cart (since they both could not bounced back that much).

4.1.1.3 Discussion of Interview Results

The interview data showed that the honors and regular students all had difficulty understanding "where the kinetic energy goes" during the inelastic collision between the unhappy ball and the table. Apparently, they lacked a mental picture of the concept of

internal thermal energy. They used the terms "internal energy" or "heat" without a meaningful understanding of them.

It appeared that the students were distracted by the different-looking structures of the two model carts. They thought that the two carts' different behaviors (one bouncing back a significantly shorter distance than the other) resulted from their different mass distributions, even after they actually observed the cart demonstration. Clearly, the students used surface features to reason about the physical phenomena. To avoid this surface distraction and to help students visualize and understand easily "the physics that the carts were intended to show", we modified the rigid-rod cart and developed a new pair of model carts. This is discussed in detail in Section 4.2.1.

4.1.2 Written Question (Autumn 1998)

In the autumn quarter of 1998, to elicit a large number of students' ideas about the concept of internal thermal energy and their understanding of the energy transformation during inelastic collisions, we showed the demonstration using the happy and unhappy balls to a regular calculus-based introductory physics class (Physics 131). This ten-week class covered the regular concepts of mechanics: kinematics, Newtonian dynamics, momentum, work-energy, and some rotational dynamics. The happy and unhappy ball demonstration was shown after the students had studied the concepts of work and energy, momentum, as well as collisions. After observing the demonstration, the student was asked to answer the question: Two "identical" black balls are dropped onto a table surface from the same height. Provide one or more reasons why you think one of the balls did NOT bounce back, but the other did?

Thirty-seven students handed in their responses to the question. Thirty-two percent thought that the happy and unhappy balls were made of different materials. For example, the unhappy ball was a metal one and the happy ball was a rubber one; or the unhappy ball was made of denser material. Fourteen percent thought the two balls had different elasticity. The happy ball was more elastic than the unhappy ball. Eleven percent thought that the internal structures of the balls were different. For example, the happy ball was a solid one, but the unhappy ball was a hollow one. Five percent did not answer the question.

We also found that thirty-eight percent in total used "an energy idea" to explain why the unhappy ball did not bounce back. Thirty percent used the term "internal energy" or "molecules' energy" to describe "to what the kinetic energy was lost" during the unhappy ball's collision with the table, while eight percent thought that the kinetic energy of the unhappy ball was converted into "heat", The proportion of the students who used the term "internal energy" is larger than what we expected it. We noticed that the class professor gave his explanation about the demonstration using the concept of internal (thermal) energy. It appeared that some students might just copy down the professor's explanation, rather than trying to answer the question first themselves (although the students were asked to do so). We did not think that the students acquired a meaningful understanding of the concept, although they could use the term.

In summary, it appeared that the students used surface features, such as the material, density, or structure differences, to reason about the elastic and inelastic collisions. They did not use such a deep structure of knowledge as energy or momentum ideas to explain what happened during the collisions. Even some students used the term

"internal energy" to describe "to what the kinetic energy was lost", but they lacked a meaningful mental picture of this abstract concept.

4.1.3 Discussion of Student Difficulties Understanding the Concept of Internal Thermal Energy

Overall, it is apparent that internal thermal energy is a very difficult concept for many students. Lacking a meaningful understanding of it, students use surface features to understand elastic and inelastic collisions. Also, some students use the term "heat" to describe "to what the kinetic energy is lost". In physics, heat is defined as transfer of internal thermal energy [7], but not internal thermal energy itself. Heat is a process quantity, but internal thermal energy is a state quantity. Clearly, introductory physics students do not understand these distinctions between these two concepts. But it appears that many students have no ideas about either of them. They just use the terms without knowing them.

4.2 Development and Implementation of a Guided-Inquiry Student Learning Activity

During the process of curriculum development to help students gain a meaningful understanding of the concept of internal thermal energy, we need to consider the following factors. First, in addition to being a not-easy-to-understand concept itself, internal thermal energy is also associated with the definitions of heat and temperature, two other abstract concepts. Teaching by telling these concepts would be useless to help students gain a conceptual change. Second, we do not think it is vital to let students know, by memorization, the differences and relationships among those complex concepts in an introductory physics class. Third, in many daily-life situations and even in some classrooms, "heat" or "heat energy" is often used as a substitute for the concept of

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internal thermal energy. This misuse of the language would make it more difficult for students to distinguish clearly between the concepts of internal thermal energy and heat. However, it is *important* to help students visualize "where the kinetic energy goes" during inelastic collisions. The visualization could help students develop a mental image that makes the abstract concept of internal thermal energy meaningful—much as the picture of an apple gives meaning to the word "apple." Lack of a meaningful understanding of "to what the kinetic energy is lost" is identified as a major students' difficulty understanding inelastic collisions. Linking the abstract concept with a concrete mental picture could help students address this difficulty. This could also help students understand the conservation of energy during an inelastic collision and recognize an inelastic collision from the point of view of the "internal energy" produced. Helping students develop a mental image of the concept of internal thermal energy on a molecular scale would be useful for them in learning thermodynamics. Therefore, we developed a guided-inquiry learning activity, using the modified model carts, intended to help students visualize the internal thermal energy produced during inelastic collisions and to develop a mental model of this concept.

Below, we first describe in detail how to construct a pair of the modified model carts. This is followed by an introduction to the development of student worksheets. Last, we discuss how to use this guided-inquiry learning activity with students in a classroom.

4.2.1 Construction and Modification of Model Carts (Spring and Autumn 1998)

Because of the different-looking structures of Ganeil's model carts, we found that it was not easy for students to visualize and understand the physics that the model carts

attempted to demonstrate. As shown in Figure 4.5, a new model cart was developed, replacing the one shown in Figure 4.2. This modified rigid-rod cart has a structure and mass distribution similar to those of the rubber-band cart in Figure 4.1. The detailed information about how to make this new pair of model carts is shown in Figures 4.5 and 4.6.

4.2.2 Development of Student Worksheets

To have students be exposed to an inquiry-like learning activity, it is essential to develop a written curriculum to guide them in developing their own understanding about related concepts. In addition to two demonstrations using happy and unhappy balls and using the model carts, we developed student worksheets. Questions on the worksheets could be divided into three groups: 1) Questions about the demonstration using the happy and unhappy balls; 2) questions about the demonstration using the model carts; and 3) questions about the analogy between the model carts and the rubber balls.

4.2.2.1 Student Worksheets (Autumn 1998)

The worksheets developed in the autumn quarter of 1998 consisted of 12 carefully-constructed questions (see Appendix J) that could guide students through a reasoning process to visualize the concept of internal energy and to develop a mental model of it.

The first four questions asked students to observe the demonstration of the happy and unhappy balls and to provide their explanations about how they made sense of what they observed. The questions intended to *elicit* students' common sense ideas about internal energy. Questions 5 to 9, asking about the demonstration of the model carts, were collisions and *confront* students' common sense ideas about the concept of designed to
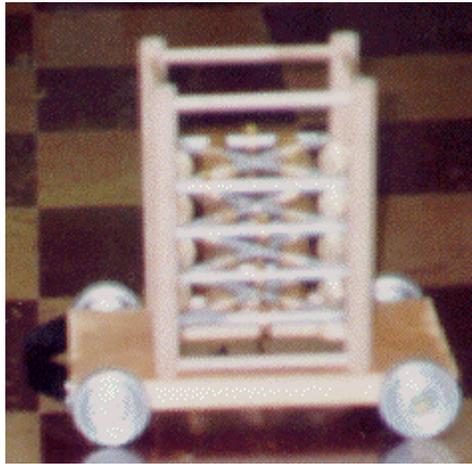
The information about how to construct the model cart shown as above

Base:
Frame: Rectangular wood frame with 24 by 16 cm (1.8 cm thick) and in the center a rectangular cut off with 13 by 11 cm.
Four wheels: With low friction and ordered from Frey Scientific.
Spring Bumper: A steel strip with 16 by 2.5 cm bent to a bow (the strip is cut off from 0.025 t x 6''w x 50'' steel coil)

Body:
Four vertical rods: 20 cm high
Eight horizontal (thinner) rods: 13 cm long each
16 Aluminum-hollow rods: On sides, 13 cm long each; on diagonals, about 19 cm each.
Brass disks: 5 mm thick and the outer diameter of 24 mm; in the center of each disk, a hole with a diameter of 6 mm; a mass of 18 g each.

Figure 4.5: The information about how to construct the rigid-rod model cart, as shown in the picture (this model cart can be used for demonstrating elastic collisions).

<div style="border: 1px solid black; padding: 10px;">

The information about how to construct the model cart shown as above

Base:
Frame: Rectangular wood frame with 24 by16 cm (1.8 cm thick) and in the center a rectangular cut off with 13 by 11 cm.
Four wheels: With low friction and ordered from Frey Scientific.
Spring Bumper: A steel strip with 16 by 2.5 cm bent to a bow (the strip is cut off from 0.025 t x 6''w x 50'' steel coil)

Body:
Four vertical rods: 20 cm high
Eight horizontal (thinner) rods: 13 cm long each
Hooks: eight on each vertical rod
Brass disks: 6 mm thick and the outer diameter of 24 mm; in the center of each disk, a hole with a diameter of 6 mm; a mass of 20 g each
Rubber bands: some

</div>

Figure 4.6: The information about how to construct the rubber-band model cart, as shown in the picture (this model cart can be used for demonstrating inelastic collisions).

help students visualize the energy transformation during elastic and inelastic internal energy. Question 10 in the worksheets introduced students to the microscopic molecular structure of a solid and asked them to use this model to explain what happened to the initial energy of the rubber-band cart after the collision. This question intended to help students make a link between the microscopic model of a solid and the model carts. By explicitly asking about matching the carts with the balls, Question 11 attempted to help students realize that "what happened to the carts" modeled "what happened to the balls at the 'invisible' microscopic level," and to visualize the energy transformation from the model carts' collisions. The last question introduced the definition of internal (thermal) energy and asked students to use this concept to explain again why the unhappy ball did not bounce back. Noticed that in this question, we did not explicitly introduce the internal energy as internal thermal energy. We attempted to help students visualize molecules' random kinetic energy as a type of internal energy and develop a mental image of it, rather than to memorize the term. Questions 10 to 12, the last three, were designed to help students *resolve* their common-sense ideas and develop a meaningful mental model of the abstract concept of internal (thermal) energy.

4.2.2.2 Student Worksheets (Autumn 1999)

In the autumn quarter of 1999, we modified the worksheets developed in the autumn 1998. In addition to minor wording revisions, comparing the version in the autumn 1998, major modifications for the autumn 1999 version (see Appendix K) are discussed below.

1) From student responses for Question 5 in the autumn 1998 version, we found that a majority of the students predicted that the two carts would bounce back the same

distance. So this question did not serve well to elicit a variety of students' ideas. We revised this question to ask students to observe and to write down what happened to cart A and cart B after they collided with a fixed block.

2) Question 10 in the autumn 1998 version was divided into two questions, questions 10 and 11 in the autumn 1999 version. This attempted to help students think explicitly about why the rubber band model cart bounced back a significantly shorter distance than the rigid-rod cart and to "see" clearly a link between the structures of the model carts and the molecular model of a solid.

3) The concept of internal (thermal) energy was introduced while the molecular structure inside a solid was introduced in Question 11. It seemed better than Question 12 in the 1998 version. When analyzing students' responses to Question 12 in the 1998 version, we found that many students just answered that the kinetic energy of unhappy ball was converted into its internal energy but without further explanations. We thought that it could result from the way that this question worded. The question introduced the concept of internal energy first and then asked the students to use this concept to explain why the unhappy ball did not bounce back, so this could mislead the students just to use the term "internal energy" to answer the question. Thus, we revised Question 12 into Questions 11 and 12 in the 1999 version.

4) We added Question 13, the last question, in the autumn 1999 version of the worksheets. This question explicitly asked students to apply the concept of internal energy for collisions. It attempted to help students understand the energy transformation during elastic and inelastic collisions.

4.2.3 Instruction

How can we implement these learning materials with students in a classroom? We assembled the happy and unhappy ball demonstration, the model cart demonstration, and the written worksheets into a coherent guided-inquiry learning activity, which aimed to help students visualize the concept of internal thermal energy and understand the energy transformation during collisions. After having exposed them to the concepts of energy, momentum, and collisions, students actively learned this activity in a large classroom. During the activity the students were required to write down their explanations on the worksheets, which were collected at the end of the classes.

The only role that the professor played in the classroom was to show the demonstrations, rather than to provide answers for the worksheet questions. Basically, in the class, the happy and unhappy ball was demonstrated first. Then the students were asked to answer, based on their observations and reasoning, the first four questions related to this demonstration. By the time most students had written down their answers, the professor asked for the students' attention and showed the demonstration with the model carts. The model carts with covers were first pushed to collide with fixed lead bricks on the lecture table. While the students answered the related questions on the worksheets, behind the lecture table, the professor took off the covers from the carts. Then the demonstration using the model carts, without the covers, was shown again. After observing this, the students were asked to answer the rest of the questions themselves. The worksheets were collected for an evaluation purpose after the instruction. Notice that, just for evaluating each individual student performance, during the instruction the students were not encouraged to discuss their answers with their

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neighbors or partners. This activity, however, was designed for students to work interactively with their peers.

4.3 Assessment of the Effectiveness of the Guided-Inquiry Learning Activity

In the autumn quarters of 1998 and 1999, honors engineering freshmen in a calculus-based introductory mechanics class (Physics 131E) at OSU learned the guided-inquired learning activity. This ten-week class covered the basic concepts of mechanics: kinematics, Newtonian dynamics, momentum, work and energy, and some rotational dynamics. The concept of internal energy was first introduced when the students learned the multiple-representation strategy in the content of work-energy. They studied the guided-inquiry learning activity after having learned the concepts of work and energy, momentum, and collisions. Below we report the results of assessing the effectiveness of this guided-inquired learning activity.

4.3.1 Evaluation Results (Autumn 1998)

In the autumn quarter of 1998, we collected the student worksheets after the Physics 131E class learned the guided-inquiry activity. Students' responses on the worksheet questions were analyzed (see Section 4.3.1.1. for details). In addition, two multipart problems, including inelastic collisions, appeared on a written test for this 131E class at the end of the quarter. The test [8] was designed to assess whether students were able to recognize and to divide some complex, multipart problems into simpler and smaller sub-problems, which could be solved by applying single physics principles. Student performance on these two written test questions are described in Section 4.3.1.2.

4.3.1.1 Student Responses to the Worksheet Questions

Seventy-five students handed in their worksheets after the guided-inquiry learning activity. We randomly divided the worksheets into two equal sets and carefully analyzed one set.

Of twelve questions on the worksheets (see Appendix J), the first four questions asked about the demonstration using the happy and unhappy balls. In particular, Question 2 attempted to elicit student common-sense ideas about the elastic and inelastic collisions, while Question 4 intended to explore what students thought about the concept of internal energy and the energy transformation during the inelastic collision. Student responses to these two questions are summarized in Tables 4.2 and 4.3.

For Question 2, some students provided more than one reason why they thought one of the balls did not bounce back, but the other did. We counted each different reason given by the same student separately. So in Table 4.2, the total proportion of student responses exceeds one hundred percent

Table 4.2 shows that the students primarily relied on either surface features or common-sense ideas to reason about happy and unhappy balls' collisions. For example, thirty percent thought that the two balls were made of different materials. This is true, but the students did not understand what this difference was and why it could make the balls behave differently. Some students thought that the happy ball was a rubber ball, while the unhappy ball was a metal. Thirty-five percent thought that the difference could be elasticity—the happy ball with high elasticity and the unhappy ball with low elasticity. We did not think the introductory students really understood the concept of elasticity. Twenty-seven percent thought that the unhappy ball was more massive than the happy

	Student responses to Question 1 on the worksheet: Provide one or more reasons why you think one of the balls did NOT bounce back, but the other did. (Calculus-based honors engineering freshmen, N = 37, Autumn 1998)	
		Students' responses
35%	Unhappy ball is less elastic than happy ball or the former with low elasticity and the latter with high elasticity	
30%	Two balls made of different materials	
27%	Unhappy ball is heavier than happy ball	
16%	Unhappy ball is denser than happy ball	
14%	Unhappy ball is harder than happy ball	
16%	Force is absorbed by table; unhappy ball's initial energy or momentum is lost into table	
5%	Unclear	

Table 4.2: Students' responses to the worksheet question: Provide one or more reasons why you think one of the balls did not bounce back, but the other did.

	Student responses to Question 4 on the worksheet: What do you think happened to the energy of the "unhappy" ball just after the collision (the "unhappy" ball is the one that did not bounce)? (Calculus-based honors engineering freshmen, N = 37, Autumn 1998)	
		Students' responses
32%		Into the table
22%		Heat and/or sound
27%	19%	Internal energy and/or sound
	5%	Absorbed by molecules inside the ball and/or table
	3%	The ball and table became warmer
5%		Lost to friction
14%		Unclear, blank, or others

Table 4.3: Students' responses to the worksheet question: What do you think happened to the energy of the unhappy ball just after the collision?

ball, so the unhappy ball could not bounce back. This was a students' common-sense idea; an underlined reasoning could be that the unhappy ball was too heavy to be pushed back by the table.

Question 4 explicitly asked the students to use an energy idea to explain what happened to the initial energy of the unhappy ball just after the collision. Table 4.3 shows that thirty-two percent thought that the initial (kinetic) energy of the unhappy ball went to the table. Twenty-two percent thought it was converted to heat and/or sound. Twenty-seven percent in total gave responses related to an internal energy idea. Nineteen percent thought the unhappy ball's initial energy was converted to internal energy of the ball and/or the table. But they did not provided detailed explanations about what they meant by "internal energy." It appeared that few students could apply this concept to explain "to what the initial kinetic energy was lost" during the unhappy ball's collision. Five percent thought that the initial energy of the unhappy ball was transferred to molecules either inside the ball or the table.

After the students observed the model cart demonstration and answered Questions 5-10 on the worksheets, Questions 11 attempted to probe whether or not it was easy for the students to match the model carts with the balls. All thirty-seven students thought that the rubber-band cart modeled the unhappy ball and the rigid-rod cart modeled the happy ball.

Finally, Question 12 attempted to investigate what the students thought happened to the initial energy of the unhappy ball using the concept of internal (thermal) energy. As shown in Table 4.4, a majority of the students used the concept to answer the question. Forty-nine percent thought that the initial kinetic energy of the unhappy ball was

		Student responses
	Student responses to Question 12 on the worksheet: Kinetic energy that individual molecules have is one example of internal energy. The faster molecules inside a solid move and rotate, the larger their internal energy, and the "hotter" the solid is. So now, using the concept of internal energy, explain why the "unhappy" ball did not bounce back, but the "happy" ball did. (Calculus-based honors engineering freshmen, N = 37, Autumn 1998)	

		Students' responses
81%	49%	Internal energy
	27%	Internal energy to make molecules move or vibrate faster
	5%	Unhappy ball became warmer
5%		Heat
5%		Structure different and dense different
9%		Unclear, blank, or others

Student responses to Question 12 on the worksheet: Kinetic energy that individual molecules have is one example of internal energy. The faster molecules inside a solid move and rotate, the larger their internal energy, and the "hotter" the solid is. So now, using the concept of internal energy, explain why the "unhappy" ball did not bounce back, but the "happy" ball did.

(Calculus-based honors engineering freshmen, N = 37, Autumn 1998)

Table 4.4: Students' responses to the worksheet question: Kinetic energy that individual molecules have is one example of internal energy. The faster molecules inside a solid move and rotate, the larger their internal energy, and the "hotter" the solid is. So now, using the concept of internal energy, explain why the "unhappy" ball did not bounce back, but the "happy" ball did.

converted to its internal energy, twenty-seven percent explicitly explained that the

unhappy ball's initial energy was transferred to the kinetic energy of its molecules and to

make the molecules move or vibrate more. Here is an example of the students' responses:

> "The unhappy ball's kinetic energy was transferred to its molecules which moved. The happy balls' molecules don't move as easily. So less energy was able to be transferred internally."

4.3.1.2 Discussion of the Student Responses to the Worksheet Questions

When the students experienced this guided-inquiry learning activity, they had

already learned the concepts of collisions. The concept of internal energy was introduced

both in collisions and in work and energy using the multiple-representation strategy.

Apparently, before the model cart demonstration was shown, few students could apply

this concept to explain what happened to the initial kinetic energy of the unhappy ball

just after the collision. It appears that many students did not acquire a meaningful

understanding and a mental picture of this abstract concept before any visualization was

applied to help them learn this concept.

Comparing the students' responses to the similar questions (i.e., Questions 4 and

12 on the worksheets), it is apparent that many more students, after experiencing the

model cart demonstration than before, applied the concept of internal energy to explain

why the unhappy ball did not bounce back. We also found that some students explicitly

stated that the initial kinetic energy of the unhappy ball was transferred to its' molecules

thus making them vibrate more. Apparently, visualizing the vibration of the small disks

on the rubber-band cart after the cart's collision helped students imagine what would

happen to the motion of unhappy ball's molecules after the collision. This would help the

students develop a mental picture of the abstract concept of internal (thermal) energy and make it meaningful.

Does the students' mental image of internal energy help them better understand the conservation of energy and be better able to recognize inelastic collisions? Two written test questions, including inelastic collisions, were administered to both students who learned the guided-inquiry learning activity using the modified model carts and to students who did not experience this activity. Below, we report student performance on the written test questions.

4.3.1.3 Student Performance on Written Questions

The Problem Decomposition Diagnostic test [8], developed by David Van Domelen at OSU, was given to the honors engineering freshmen who learned the guided-inquiry activity. The test, with six problems in total, asked the students to divide the problems into small sub-problems and to indicate what each sub-problem is, either by surface features such as "it is a spring problem" or by deep structures of physics such as "it is a work-energy problem." Two test problems, the Pendulum-Box Bash Problem (see Figure 4.7) and the Block Catcher Problem (see Figure 4.8), include an inelastic collision as a small sub-problem.

The test also was administered to a calculus-based introductory class for honors physics and mathematics majors (Physics 131H), who had not experienced the guided-inquiry activity. Taught by an experienced professor, this ten-week class also covered the fundamental concepts of mechanics, but with an emphasis on logical arguments, mathematical derivations, and problem solving.
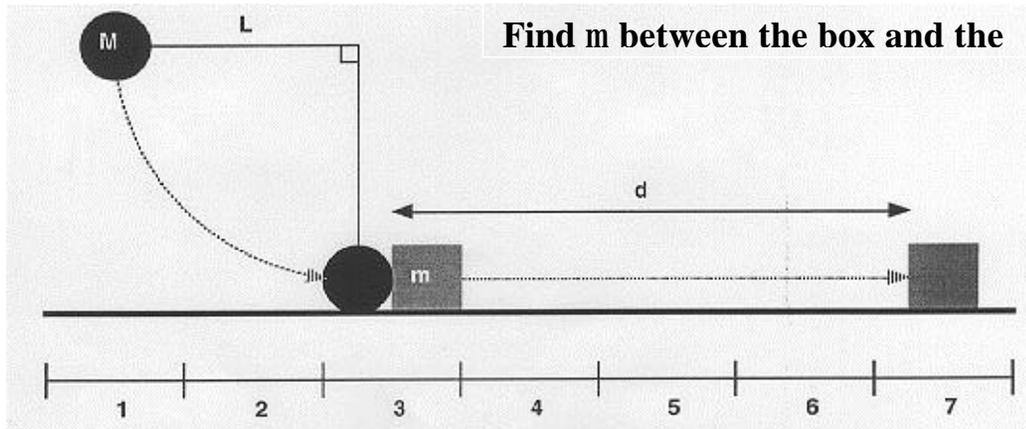
Figure 4.7: The Pendulum-Box Bash Problem in the Problem Decomposition Diagnostic Test.
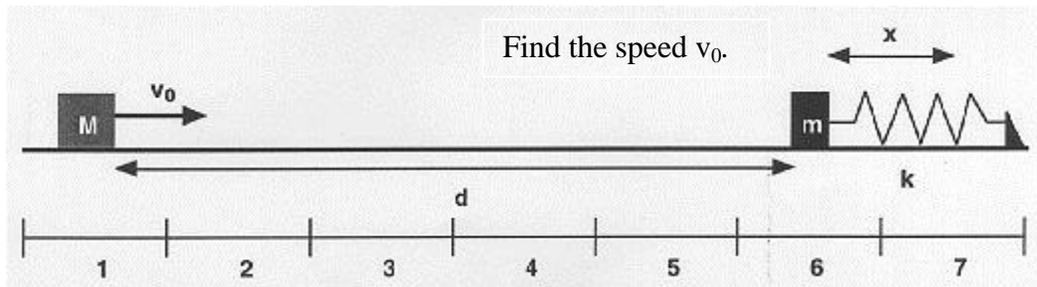
Figure 4.8: The Block Catcher Problem in the Problem Decomposition Diagnostic Test.

Ninety-three students from the honors engineering freshmen class (131E) and forty-one students from the honors physics and mathematics majors class (131H) took the test, and we analyzed their performance on the two problems shown in Figures 4.7 and 4.8. Table 4.5 shows the proportion of students who correctly recognized the collision as a sub-problem. The results indicated that the students who learned the guided-inquiry activity performed much better than the honors physics students after standard instruction.

As discussed in Section 3.3.3.4, honors students from these two classes had similar performance on pre-tests of the Force Concept Inventory [9] in the autumn quarters of 1998 and 1999. On the basis of these test scores, it appeared that the students in both classes had a similar initial understanding of Newtonian dynamics. It was seemly fair to conclude that, in terms of student background and ability levels, the honors students (majors in physics or mathematics) in the Physics 131H class were comparable

Students	Pendulum-Box Bash Problem % correct	Block Catcher % correct
131E **N = 93**	**78%**	**71%**
131H **N = 41**	**35%**	**14%**

Table 4.5: Proportion of students who correctly recognized the collision as a small sub-problem in the two complex problems.

with the honors engineering freshmen in the Physics 131E class. The difference in student performance shown in Table 4.5 could result from several reasons.

First, as discussed in Section 4.3.1.2, apparently, the guided-inquiry learning activity helped the honors engineering freshmen visualize the internal energy produced during the inelastic collisions. This could help them infer about the energy transformations and recognize easily an inelastic collision.

Second, the honors engineering freshmen were taught explicitly about how to solve multipart problems in the class. They could become more skillful in dividing this type of problems into smaller parts than the honors physics majors who learned problem solving with other approaches.

Third, a cooperative learning approach was used both in "lectures" and "recitations" for the Physics 131E class. The honors engineering freshmen interactively participated in learning activities and in problem solving.

Therefore, we need to conduct further investigations on assessment of the effectiveness of the inquiry-learning activity on student understanding.

4.3.2 Evaluation Results (Autumn 1999)

In the autumn quarter of 1999, the honors engineering freshmen class (131E) included two large classroom sections. The guided-inquiry activity using the model cart was introduced in one of the two sections. Students in these two sections were very comparable in terms of their academic background in high school and their pretest scores on the Force Concept Inventory [9]. Two professors taught the two sections, but they covered the same concepts. Three recitation instructors taught the students from both sections, and all the students had the same laboratory activities and homework

assignments. A cooperative learning strategy was used as the basic format of the two sections. After having learned the concepts of work and energy, momentum, and collisions, the students in one section experienced the guided-inquiry learning activity, while during the same day the other students learned to solve some complex multipart problems involving inelastic collisions. The same professor taught these two sections corresponding to these two different classroom activities.

We collected and analyzed students' responses to the questions on the worksheets. Additionally, a written question appeared on the class midterm examination asked about the internal energy produced during an inelastic collision. Moreover, we gave a similar problem including an inelastic collision on the final exams for both of these two sections. Below, we discuss in detail student performance on the worksheets and on the written test questions.

4.3.2.1 Student Responses to the Worksheet Questions

Of thirteen questions on the worksheets (see Appendix K), the first four questions asked about the demonstration using the happy and unhappy balls. In particular, Question 2 attempted to elicit student common-sense ideas about the elastic and inelastic collisions, while Question 4 attempted to explore what students thought about the concept of internal energy and the energy transformation during the inelastic collision. Student responses to these two questions are summarized in Tables 4.6 and 4.7.

For Question 2, some students provided more than one reasons why they thought one of the balls did not bounce back, but the other did. We counted separately each different reason given by the same student. So in Table 4.6 the total proportion of student responses exceeds one hundred percent.

Overall, Table 4.6 shows similar results to that in Table 4.2, which we got from the autumn quarter of 1998. A major difference between these results was that some students (twenty-one percent) from the autumn quarter 1999 stated that because the unhappy ball made an inelastic collision, it did not bounce back, while the happy ball made an elastic collision, so it bounced back. We thought that this could result from the time difference in learning this guided-inquiry learning activity for these two classes. In the autumn quarter of 1998, this activity was introduced just after the students learned the concepts of collisions. Being new with the concepts of collisions, the students might not be comfortable to use these concepts to answer the questions. In the autumn quarter of 1999, the students learned the activity after the collision section was almost completed. The students had practiced solving some complex collision problems, so they might be more comfortable to use the terms "elastic collision" or "inelastic collision" to answer the worksheet question.

We also analyzed the students' responses to Question 4, which asked the students to explain what happened to the unhappy ball's initial energy just after the collision. Results shown in Table 4.7 were similar to those obtained from the autumn quarter of 1998. But we found that twenty-three percent of the students form the 1999 class just gave their explanations, as "the initial energy of the unhappy ball was absorbed by the ball," or "the initial energy of the unhappy ball was absorbed by the ball and the table." Without their detailed reasoning available, we did not know what they thought about "the absorbed energy." But it appeared that these students did not understand what happened to this "absorbed energy." In other words, the students might not know in what form the "absorbed energy" existed inside the ball or table.

	Student responses to Question 1 on the worksheet: Provide one or more reasons why you think one of the balls did NOT bounce back, but the other did. (Calculus-based honors engineering freshmen, N = 39, Autumn 1999)
	Students' responses
28%	Unhappy ball is less elastic than happy ball or the former with low elasticity and the latter with high elasticity
21%	Unhappy ball has an inelastic collision; happy ball has an elastic collision
36%	Two balls made of different materials
18%	Unhappy ball is heavier than happy ball
10%	Unhappy ball is denser than happy ball
5%	Unhappy ball is harder than happy ball
3%	Unhappy ball's energy is lost
3%	Unclear

Table 4.6: Students' responses to the worksheet question: Provide one or more reasons why you think one of the balls did NOT bounce back, but the other did.

		Student responses to Question 4 on the worksheet: What do you think happened to the energy of the "unhappy" ball just after the collision (the "unhappy" ball is the one that did not bounce)? (Calculus-based honors engineering freshmen, N = 39, Autumn 1999)
		Students' responses
18%		Into the table
5%		Heat and/or sound
23%		Absorbed by the ball
30%	20%	Internal energy and/or sound
	10%	Internal energy and heat
10%		Lost to friction
9%		Unclear or others

Table 4.7: Students' responses to the worksheet question: What do you think happened to the energy of the unhappy ball just after the collision?

Student responses to Question 12 on the worksheet: We can consider the two carts as a simple analogy for the two black balls. Now explain why the "unhappy" ball died on the surface after the collision.		
(Calculus-based honors engineering freshmen, N = 39, Autumn 1999)		
		Students' responses
100%	21%	Internal energy
	76%	Internal Energy to make molecules move or vibrate faster
	3%	Unhappy ball hotter

Table 4.8: Student responses to Question 12 on the worksheet: We can consider the two carts as a simple analogy for the two black balls. Now explain why the "unhappy" ball died on the surface after the collision.

Question 12 asked the students to provide their reasoning again why the unhappy ball "died" on the table just after its collision, after they observed the model cart demonstration and answered related worksheet questions. As shown in Table 4.8, seventy-six percent explicitly stated that because the unhappy ball's initial energy was converted into vibrating the ball's molecules thus increasing its internal energy, so the unhappy ball did not bounce back. Twenty one percent used the term "internal energy" to describe "where the kinetic energy goes," but they did not explicitly mention the vibration of the ball's molecules. Here was an example of the students' responses:

> "The unhappy ball's atoms must have been allowed to vibrate must faster thus increasing the internal energy. However, the "happy" ball's atoms must not have been allowed to vibrate as freely, thus the kinetic energy of the falling ball could not be transferred into internal energy during the collision with the table and the kinetic energy simply caused the ball to bounce back."

Comparing to the students' responses (in Table 4.4) to the similar question in the autumn 1998, a larger number of students in the 1999 class explicitly stated that the unhappy ball's initial energy was transferred to internal energy and to make the ball's

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molecules to vibrate more. We thought that this could primarily result from the two questions being worded differently. Question 12 used on the worksheet of the 1998 version directly asked the students to use the concept of internal energy to explain why the unhappy ball did not bounce back. It could mislead some students to just use the term "internal energy" to answer the question. So we revised this question, and it was apparent that the 1999 version question was better able to let the students give their reasoning, rather than to rephrase the term.

The data above show that after the students experienced the model cart demonstration and answered the related questions on the worksheets, they could visualize the vibration of small weights on the rubber band cart, and imagine this to model "what happened to the unhappy ball at a molecular level." Apparently, they gained a mental picture of the internal (thermal) energy—vibrational energy of molecules.

4.3.2.2 Student Performance on a Midterm Examination Question

In this section, we discuss student responses to a test question related to their understanding of the concept of internal energy. Of 78 students in this class, 38 students, called the cart group, studied the guided-inquiry learning activity, while the other 40 students, called the non-cart group, learned to solve some complex, multipart problems, which included an inelastic collision as a sub-problem. Except for this different learning experience, these two groups of students participated in all the same learning activities both in large classroom settings and in small classroom settings.

Of the five questions in the problem shown in Figure 4.9, question b) asked students 1) to calculate how much the initial energy is "lost," and 2) to explain what

A 400 g rubber ball is dropped from a fourth floor window of Smith Lab., 15 meters above a sidewalk. Assume negligible loss due to air drag. a) What is the velocity of the ball just before it strikes the ground? b) If the speed is reduced by 5% upon rebounding, what fraction of the initial energy is lost and what happened to it? c) What will be the maximum height reached after the first bounce? d) What is the impulse acting on the ball during its collision with the sidewalk? e) If the ball is in contact with the ground for 12 milliseconds, what is the average force acting on the ball while in contact with the sidewalk?

Figure 4.9: The problem appeared on the midterm examination for one section of the honors engineering freshmen class in the autumn of 1999. Question b) asked the students 1) to calculate how much the initial energy is lost and 2) to explain what happens to the "lost" energy. In our research, student answers for the second part of question b) are analyzed and summarized in Table 4.9.

happens to the "lost" energy. For our research, student responses for the second part of question b) were analyzed and summarized in Table 4.9.

To make a reasonable comparison of the student responses from the cart group and the non-cart group, we graphed distributions of the student final class letter grades (i.e., A, B, and C & below) for both groups. In light of the bar graphs shown in Figure 4.10, it appeared that the students with different ability levels distributed very similarly in these two groups. Students from these two groups also achieved similar mean scores on the post-test of Force Concept Inventory [9].

Up to this point, we thought that the students in these two groups were "matched," and any differences in their performance on the second part of question b) should have resulted from the different treatments (that is, the cart group learned the guided-inquiry activity, while the non-cart group studied problem solving). Based on the results shown in Table 4.9, we found that for the cart-group, of 38 students, fifty-five percent explained that the "lost" energy was converted into internal energy, while thirteen percent described that the "lost" energy was converted into heat or sound. Eight percent and eleven percent, respectively, mentioned that the energy was lost due to friction or because of non-elastic collision. For the non-cart group, on the other hand, of 40 students, ten percent used the concept of internal energy to describe "where the 'lost' energy goes," while about forty-three percent thought the energy was "lost" into heat or sound. Also, about eighteen percent explained that the energy was lost due to friction, and fifteen percent mentioned it being due to the inelastic collision between the ball and the sidewalk.
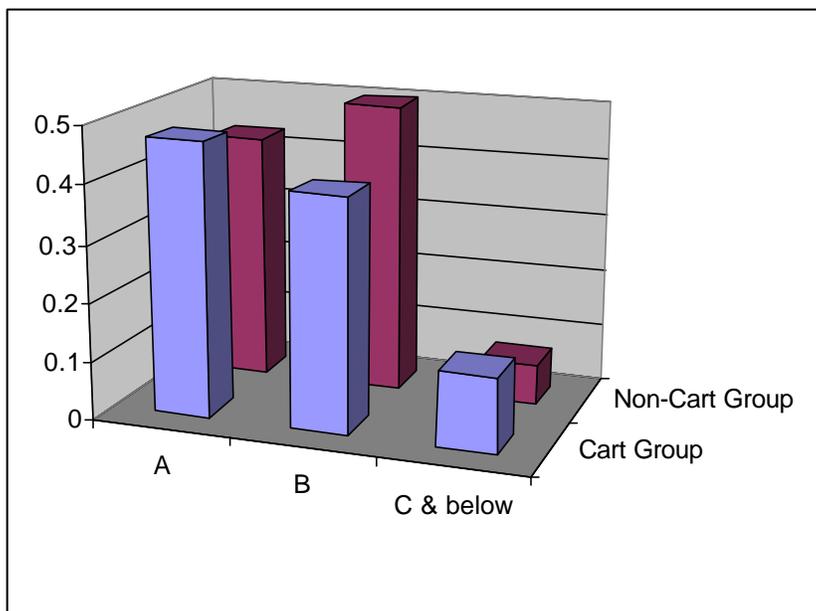
Figure 4.10: Distributions of students' final class letter grades from the cart group and the non-cart group.

Students' responses for the second part of question b) shown in Figure 5.9 N = 78, 131E, Autumn 1999						
Cart Group	**Lost to**		**Due to**			
	Internal energy	Heat, sound, etc	Friction	Inelastic collision	Other	Unclear or blank
N = 38						
	55%	13%	8%	11%	5%	8%
Non-Cart Group N = 40	**Lost to**		**Due to**			
	Internal energy	Heat, sound, etc	Friction	Inelastic collision	Other	Unclear or blank
	10%	42.5%	17.5%	15%	2.5%	12.5%

Table 4.9: Student responses for the second part of question b) of the problem shown in Figure 4.9. The cart group refers to the students who learned the guide-inquiry activity using the model carts, while the non-cart group refers to the students who learned how to solve some complex multipart problems including inelastic collisions.

4.3.2.3 Discussion of Student Performance on the Midterm Examination Question

It is apparent that forty-five percent more in the cart group than in the non-cart group used the term "internal energy" to describe "to what the kinetic energy was lost" during the ball's collision with the sidewalk. On the basis of students' responses to the worksheet questions, we could tell that most students from the cart group gained a mental image of the concept of internal (thermal) energy. It is reasonable to believe that the students from the cart group used the term with a meaningful understanding, although many of them did not give further explanations, maybe as a result of rushing through the examination. In contrast, forty-two percent from the non-cart group used the term "heat" to answer the same question. Apparently, the students misused this concept, but it appeared that they had no ideas about the meaning of it (see Section 4.3.3.1 for details).

4.3.2.4 Student Performance on a Final Examination Question

It is apparent that the guided-inquiry learning activity helped students visualize the concept of internal (thermal) energy and develop a meaningful mental picture of it. Using the same time, the other students explicitly learned how to solve some complex, multipart problems. Did visualization of the concept of internal energy help students better solve an inelastic collision? Or did this conceptual activity hurt student problem solving? To explore these questions, we analyzed student performance on the final examination problem (see Figure 4.11), including an inelastic collision as a sub-problem. Below, we discuss student performance on the inelastic collision part in the problem.

The problem shown in Figure 4.11 appeared on the final examination for the section in which about half of the students learned the guided-inquiry learning activity, while the other half were exposed to problem solving. This problem could be divided into

two small sub-problems. The first one included the interaction between the bullet and the block—a momentum conservation problem. The second sub-problem included the interaction between the bullet-block system with the table—an energy conservation problem or a Newton's law problem. To determine how far the block would travel after the bullet hit, the students needed to solve both sub-problems correctly. Overall, of 80 students, 80% solved the problem correctly. Below we report in detail student performance on the perfectly inelastic collision between the bullet and the block—a momentum conservation problem.

Of 80 students in total, 38 learned the guided-inquiry activity using the model cart, while 42 studied solving some complex, multipart problems including perfectly inelastic collisions. By analyzing student solutions, shown in Table 4.10, we found that these two groups of students performed similarly well on the inelastic collision part. The results made it apparent that helping students understand the concept of internal energy did not hurt their problem solving performance.

A bullet of mass 50 gram is moving horizontally at a speed of 150 m/s when it strikes a block of wood of mass 1.45 kg at rest on a table and becomes imbedded within it. The coefficient of kinetic friction between the block and the table is 0.25. How far will the block travel after being struck by the bullet before it again comes to rest on the table?

Figure 4.11: The problem appeared on the final examination for one section of the Physics 131E class, in which about half of the students studied the guided-inquiry learning activity using the model carts.

	One section of the Physics 131E class N = 80, Autumn 1999	
	Correct answer	**Not recognizing the inelastic collision**
Cart group (N = 38)	**90%**	**5%**
Non-cart group (N =42)	**83%**	**10%**

Table 4.10: Student performance on the inelastic collision part of the final examination for one section of the Physics 131E class. The cart group refers to the students who learned the guided-inquiry activity using the model cart, while the non-cart group refers to the students who did not learned this activity, but studied solving some multipart problems including perfectly inelastic collisions as sub-small parts.

4.3.3 Individual Student Demonstration Interviews (Spring 1999)

In the spring quarter of 1999, we conducted individual student demonstration interviews using the happy and unhappy balls and the modified model cart. We attempted to investigate some research questions that had not been answered from the data discussed above. As reported in Section 4.1.1, the different-looking structures of Ganiel's carts made it difficult for the student to visualize and realize "where the kinetic energy goes." Did students have similar difficulty or any other difficulties with the modified model carts? We were also interested in probing what students understood about "heat." We did not expect that introductory physics students really understand the physics meaning of "heat." But, apparently, this term was often used by students to describe "to what the kinetic energy is lost." Understanding what students mean by "heat" could help us address student difficulty with the concept of internal thermal energy. Note that the

interview did not emphasize helping students visualize the internal thermal energy

produced during the inelastic collisions. Instead, we were more interested in evaluating

the effectiveness of the modified model carts on demonstrating "where the kinetic energy

goes".

Ten students were solicited from a regular calculus-based introductory physics

class (Physics 131) on a voluntary basis. They had been exposed to the concepts of work

and energy, momentum, and collisions, but not to the guided-inquiry learning activity

using the model carts. We carefully transcribed and analyzed verbal data from six

students. Below, we report interview tasks and student reasoning in detail.

4.3.3.1 Interview Demonstrations and Student Responses

Basically, two demonstrations were used during the interview: 1) A happy ball

and an unhappy ball were dropped onto a table from the same height, and 2) the two

model carts collided with a wall. Questions used to ask students during the interviews

were basically the same as the ones on the student worksheets (See Appendix K)

Demonstration of happy and unhappy balls: Most students predicted that the

two black balls would hit the table at the same time and bounce back to the same height,

since they were "identical." After the demonstration was shown, students' explanations

about the phenomenon they observed were interesting (see Table 4.11). A common

response from the six students was that the balls were made of different materials. For

example, one (happy ball) was rubber and the other one (unhappy ball) was plastic. Some

other explanations also appeared. For instance, that the happy ball was solid and the

unhappy ball was hollow; that the happy ball was lighter and the unhappy ball was

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Student responses to questions related to the demonstration using happy and unhappy balls (Calculus-based introductory students, N = 6)	
Student	**Could you provide one or more reasons why you think one of the balls did NOT bounce back, but the other did?**
I	The bouncing ball has an elastic collision, and the other one has an inelastic collision. They are made of different materials. The bouncing one is rubber, and the other is plastic. Or maybe one is solid, and the other is hollow.
II	The bouncy one is elastic, and the other one is inelastic.
III	I don't know why. Maybe they are made of different materials. Maybe the energy (of the unhappy ball) goes to the table and sound.
IV	I don't know why. I remember that my professor demonstrated the balls similar to these. He (professor) explained that energy was transferred to heat, sound or something else, maybe. I cannot remember why. (Then the interviewer asked him to use his own ideas to try to explain.) They (the balls) are maybe made of different materials. Or maybe this is hollow (unhappy ball), and this one is solid (happy ball).
V	The bouncing one is more rubbery, and the not-bouncing one is more plastic.
VI	This one (happy ball) lost a little bit of energy; most were converted back. So it bounced back. But this one (unhappy ball), the potential energy was converted to kinetic energy, just before hitting. After the hit, the energy was lost to sound and heat.

Table 4.11: Student responses to interview questions related the demonstration using the happy and unhappy ball during the individual student interviews.

heavier; and that the happy ball had an elastic collision and the unhappy ball had an inelastic collision.

Basically, the students did not use the concept of energy to explain why the happy ball bounced high but the unhappy ball did not. When students were asked about what kind(s) of energy each ball had right before and right after the collisions and further what they thought happened to the energy of the unhappy ball after collision, their reasoning appeared much more interesting. All six students more or less correctly explained that for the happy ball, it had kinetic energy right before and after the collision (a reference point on the table). But for the unhappy ball situation, they used the term "heat" to explain about "to what the kinetic energy of the unhappy ball was lost" (see Table 4.12). But when being asked to explain more about what they mean by "heat," none could give any scientific explanations. Five basically had no ideas about the meaning of this term. To student V, "heat" meant more like the concept of "internal thermal energy."

For the model cart demonstration, four students were asked to explain why one bounced back a significantly shorter than the other one, after they observed the demonstration. At this point, the two carts were covered, so the students could not see structures inside the two carts. As shown in Table 4.13, three of them thought that the difference in bouncing between the two model carts could result from the difference between their inside structures: 1) the bouncing cart had the center of mass lower; 2) it had more mass on the side toward the cart's bouncing direction; 3) it had something like a pendulum swinging or moving in the same direction as the cart's motion direction; or 4) it had something stationary. In a word, the students thought that either the different mass distribution or something moving inside the carts made them bounce or not bounce much.

Student responses to questions related to the demonstration using happy and unhappy balls (Calculus-based introductory students, N = 6)	
SS	**Student reasoning about what happened to the energy of the unhappy ball right after the collision**
I	After the collision, the unhappy ball had no kinetic energy and no potential energy. The energy was lost to friction, sound, or heat. `(The interviewer asked her to explain more about what she meant by "heat.")` I am not sure. The lab TA told me "heat". It is internal friction, maybe?
II	For this ball (unhappy ball) kinetic energy was lost to sound and heat. `(The interviewer asked him to explain more about what he meant by "heat.")` "Heat" is like "thermal energy." It makes you feel warm, I guess. I am not sure.
III	Before, it (unhappy ball) had kinetic energy. After, no kinetic energy, no potential energy. The energy went to the table, sound, and thermal energy, maybe. `(The interviewer asked her to explain more about what she meant by "thermal energy.")` To be honest, I don't know. In order to be elastic, it (unhappy ball) should bounce back to the same height. But it didn't. Its energy had to be lost to somewhere—I called it "thermal energy". I don't know what I mean.
IV	As I said, I cannot remember what my professor said about this. Maybe the energy is transferred to heat, sound or something else. `(The interviewer asked him to explain more about what he meant by "heat.")` It means friction force. They say friction is always "heat." I really don't know. I just take the word.

V	The energy goes to the table since it (unhappy ball) stays on the table.
	 `(The interviewer asked him to explain in what form he thought the energy existed in the table.)`
	 Internal thermal energy. I mean, like heat, sound, or friction.
	 `(The interviewer asked him to explain more about what he meant by "heat.")`
	 It is something like that, internal particles start moving a little faster. Then the kinetic energy goes into internal thermal energy.

VI	Just before the hit, this ball (unhappy ball) had kinetic energy. After the hit, it's lost to sound and heat. `(The interviewer asked him to explain more about what he meant by "heat.")` Heat would be outside sources to lose energy, like sound, or like friction. I really don't know.

Table 4.12: Student reasoning about what happened to the initial kinetic energy of the unhappy ball right after its collision.

	Student responses to questions related to the demonstration using the two model carts (Calculus-based introductory students, N = 6)	
SS	Student reasoning about why one cart bounced back a significantly shorter distance than the other one (before two covers of the carts were take off)	
I	Cart A* has more friction on its wheels. (The interviewer told her that the two carts had the same type of wheels.) Their masses may be distributed in different locations. For example, Cart B's center of mass may be lower, so it moves faster.	
II	Maybe something inside the carts is like a pendulum. It can shift or move one way or another. It could make Cart A slow down, but make Cart B speed up. Or maybe the carts have different mass distributions. More mass on this side could make Cart B move fast (he pointed to the non-bumper side of Cart B. Notice that this side pointed toward the cart's motion direction when it bounced back). But if more mass is on this side (he pointed to the bumper-side of Cart A. Notice that this side did not pointed toward the cart's motion direction when it bounced back.), Cart A cannot move that fast. So it slows down.	
III	N/A	
IV	I don't why. It's like the balls' case. One bounced; one stopped. I really cannot think of why it's like that.	
V	N/A	
VI	Maybe something inside Cart A moving around. It moves in this way (pointing to the opposite direction of the Cart A's motion), but Cart A bounced back in this way. So it slowed down. Something inside Cart B is stationary. Like jelly, it cannot move around much. So Cart B bounced back a long distance.	

* Note that the rubber-band cart was labeled cart A and the rigid-rod cart was labeled cart B during the interview.

Table 4.13: Student reasoning about why one cart bounced back a significantly shorter distance than the other one (before two covers of the carts were taken off).

After the covers were taken off from the carts, the students were shown again that the two carts collided with a wall. This time, they could see that the rigid-rod cart bounced back farther than the rubber-band cart. Without being asked to pay attention to observing what would happen to small weights on the two carts before and after the collisions, the students were asked to provide their reasoning about why the rubber-band cart bounced back a significantly shorter distance than the rigid-rod cart. Table 4.14 indicates that all six students observed that the small weights on the rubber bands vibrated more after the collision, but the small weights on the rigid rods could not move. They all thought this difference resulted in the rubber-band cart slowing down. But two used an energy idea to explain this, and three used a motion idea to explain this. Student III's response was not understood.

Finally, the students were asked to match the balls with the carts. Students I and III matched them in two ways. First, they matched the happy ball with the rubber-band cart since the cart had rubber bands on it and the happy ball was made of rubber or spring-like material; the unhappy ball was matched with the rigid-rod cart since the cart was rigid and the unhappy ball was made of rigid material like plastic. But they also thought that, on the basis of their behaviors, the happy ball should match with the rigid-rod cart since they both bounced back high; the unhappy ball should match with the rubber-band cart since they both could not bounced back that much. The four other students just matched the balls with the carts in the latter way.

4.3.3.2 Discussion of the Interview Results

Of the six students interviewed, one received A, three received B, and the other two receive C and D as their final class letter grades. On the basis of the data discussed,

	Student responses and explanations on questions related to the demonstration using the two model carts (Calculus-based introductory students, N = 6)
SS	Student reasoning about why the rubber-band cart bounced back a significantly shorter distance than the rigid-rod cart (after two covers of the carts were take off)
I	The small weights on the rubber bands can move independently in all different directions. So they slow down this cart (rubber-band cart). But the weights attached to this cart (rigid-rod cart) only can move in the same direction as the whole cart's.
II	You can see that the small weights vibrate back and forth after the hit. So they make the cart (rubber-ban cart) slow down. But for this one (rigid-rod cart), the small weights cannot move around. So nothing can slow it down.
III	This rigid cart bounced back longer. It had an elastic collision. Its mass is concentrated on the rods. But for this rubber-band one, forces from the hit push these little ones (i.e., small weights) to vibrate. Its mass is concentrated on the rubber bands. This is the difference between the two carts. I can visualize it, but I cannot put it in words.
IV	The energy used up to move these small weights on the rubber bands. So the rubber-band cart slowed down.
V	The energy was transferred to move these weights around. So the energy was lost to that; this cart (rubber-band cart) could not bounce back that much. But this one (rigid-rod cart) could not lose energy to move the weight around. So the energy just went to move the cart back.
VI	The small weights can move around on this cart (rubber-band cart), but cannot move around on this rigid one. So one is slowed down, and the other is not.

Table 4.14: Student reasoning about why the rubber-band cart bounced back a significantly shorter distance than the rigid-rod cart (after two covers of the carts were taken off).

it is apparent that no matter whether they were high-ability or low-ability students, they all had similar difficulties understanding the concept of internal thermal energy. A major difficulty with this concept is that the students misused the term "heat" to describe "to what the kinetic energy lost," and had no idea about what the "heat" is.

It appears that the students had no difficulty with the structures of the modified model carts. With similar structures and similar mass distributions, they were easy for the students to catch the difference in motion of the small weights on them.

4.4   Discussion and Summary

Overall, internal (thermal) energy is an abstract and not-easy-to-understand concept for introductory students. They consider "heat" as "internal thermal energy", but they do not understand either of them. Students lack a meaningful mental picture of the concept of internal thermal energy. "Learning by passively listening" is not effective for most students to understand and visualize this abstract concept. Without a meaningful understanding of this concept, it is difficult for students to understand physical processes of work-energy and collisions. Students either use surface features (e.g., material difference or structure difference) or common-sense ideas (e.g., mass difference) to understand about inelastic or partially inelastic collisions.

The guided-inquiry learning activity was developed to help students visualize this "invisible" concept and to develop a meaningful mental picture of it through interactively participating in the learning activity. This activity was developed on the basis of investigations reported in this chapter and research by others.

A pair of modified model carts were developed based on empirical data that we collected from students. Our investigation indicates that the model carts originally

developed by Uri Ganiel are not easy for students to visualize "where the kinetic energy goes." Some students thought that the difference in mass distributions between the two carts could result in the two carts' different behaviors (i.e., one bounces back a significantly shorter than the other one). A pair of the modified model carts has similar structures and mass distributions. There is empirical evidence that this modified pair is easy for students to visualize "vibration of small weights", which models a gain of internal thermal energy during the cart's collision.

We developed a guided-inquiry learning activity based on the approach—*elicit, confront,* and *resolve*—used by the Physics Education Group at the University of Washington [10]. This activity consisted of two parts: student worksheets and demonstrations using unhappy and unhappy balls and the modified model carts. The ball demonstration and the first several related questions on the worksheets attempted to *elicit* students' ideas about elastic and inelastic collisions, in particular, their reasoning about "where the kinetic energy goes" during the inelastic collision. The model cart demonstration and associated worksheet questions were intended to *confront* the students' common-sense ideas and surface-features-based reasoning that elicited from the first part of the activity. Questions asking to match the model carts with the atomic model of a solid and to match the model carts with the balls aimed to *resolve* the students' difficulty and to visualize "where the kinetic energy goes" during the inelastic collision and to develop a meaningful mental picture of the concept of internal thermal energy.

To evaluate the effectiveness of the guided-inquired learning activity, we conducted a series of assessment activities, including written tests and in-depth individual student demonstration interviews. By analyzing students' responses to the worksheet

questions, we found that after learned the activity, many students could imagine that the initial kinetic energy of the unhappy ball must go to its internal energy—its molecules inside vibrate or move more afterward. For example, seventy-six percent of the students from the autumn 1999 provided such responses to explain why the unhappy ball did not bounce back. Also, most students (seventy-six percent from the autumn 1998 and ninety-seven percent from the autumn 1999) could use the concept of internal (thermal) concept to explain "where the kinetic energy goes" during the unhappy ball's inelastic collision.

Written test data have shown that students who learned the guided-inquiry learning activity gain a better conceptual understanding of the conservation of energy during inelastic collisions. Helping students build such a conceptual understanding does not hurt their problem solving performance.

In summary, the research discussed in this chapter illustrates an example of how to use visualization to help students gain a meaningful mental picture of the abstract concept of internal thermal energy. Seeing "where the kinetic energy goes " during inelastic collisions could help students understand the conservation of energy. The mental image of internal energy could help them in inelastic collision problem solving.

ENDNOTES OF CHAPTER 4

1. For instance, *Fundamentals of Physics* (5th ed., John Wiley & Sons, Inc, New York, 1997) by Halliday, Resnick, and Walker, is a widely-used textbook for introductory physics classes. In this book, the concept of internal energy is first introduced when the work done by a kinetic frictional force is discussed. Here is a quote from the book: "Thermal energy is said to be an internal energy of an object because it involves random motion of the atoms and molecules within the object…"(See Vol. 1, p. 166.) When the concept of an inelastic collision is introduced, the book says: "An inelastic collision is one in which the kinetic energy of the system of colliding bodies is not conserved…The kinetic energy that is lost in any inelastic collision is transferred to some other form of energy, perhaps thermal energy." (See Vol.1 p. 222.) Just by reading these sentences, it is hard for a student to develop a meaningful understanding of the abstract concept of internal (thermal) energy and to visualize "where the kinetic energy goes" during an inelastic collision.

2. Ganiel, U., "Elastic and inelastic collisions: A model", Phys. Teach. **30**, 18-19 (1992).

3. Arbor Scientific (P.O.Box 2750 Ann Arbor, MI 48106-2750. Call toll-free 1-800-367-6695). See more details on the Arbor Scientific web site.

4. Morton, M. (ed.), *Rubber Technology* (Van Nostrand Reinhold Company, New York, 1987), pp. 8-9, 14.

5. This phenomenon is very temperature dependent. A happy ball will not bounce well if it is cooled down.

6. See the written material coming along with an order of the happy/unhappy balls from the Education Innovation, Inc. (151 River Road, Cos Cob, CT 06807, and email: Edinnov@aol.com).

7. Arons, A. B., *Teaching Introductory Physics* (John Wiley & Sons, Inc., New York, 1997), in part I, pp. 139-142, pp. 150-153 and in part III, pp. 67-81.

8. Van Domelen, D., *The Development of the Problem Decomposition Diagnostic,* Ph.D. dissertation, The Ohio State University, 2000.

9. Hestenes, D., Wells, M., and Swackhamer, G., "Force Concept Inventory," Phys. Teach. **30**, 141-151 (1992).

10. Wosilait, K., *Research as a guide for the development of tutorials to improve student understanding of geometrical and physical optics*, Ph.D. dissertation, University of Washington, 1996.

CHAPTER 5

VISUALIZATION OF INTERNAL POTENTIAL ENERGY PRODUCED DURING INELASTIC COLLISIONS

In a solid, in addition to the internal thermal energy, atoms also have internal potential energy, which is due to electrical and quantum mechanical interactions among atoms. The interaction between two neutral atoms is approximately represented by the Lennard-Jones potential energy function [1] (see Figure 6.1),

$$U_{LJ} = 4e\left[\left(\frac{s}{r}\right)^{12} - \left(\frac{s}{r}\right)^{6}\right],$$

where e is the depth of the potential energy at its minimum, and s is the distance at which the potential energy is zero. Since the negative slope of the potential energy curve is the force, the energy graph in Figure 6.1 shows that two neighbor neutral atoms attract each other at some intermediate distances but repel each other strongly within a very short distance. Inside a solid, this interaction makes one atom bond together with several surrounding atoms. These bonded atoms could form a molecule and many of these molecules constitute a macroscopic solid.

When a solid is compressed or stretched by a mechanical force (i.e., a certain amount of work done on the solid), the separations between atoms, especially atoms near
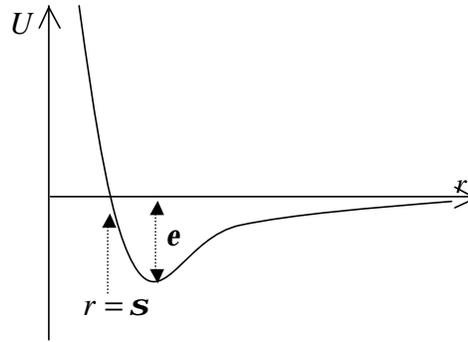
Figure 5.1: Lennard-Jones potential energy function, which approximately describes the potential energy of two neutral atoms.

the surface of the solid, are changed (become shorter or longer). Correspondingly, the

potential energy of two neighboring atoms is changed. Therefore, in either situation, the

solid gains internal potential energy due to the mechanical work done on it.

   This type of internal potential energy, associated with a change of the

configuration of the atoms, and internal thermal energy are commonly produced during

inelastic collisions. For example, in a car accident, two colliding cars very often become

deformed after bumping together. Some atoms inside deformed metal parts of the cars are

either stretched or compressed so that they have a gain of internal potential energy [2]. As

a result, the two-car system gains internal potential energy from the kinetic energy of the

two cars before their collision. Of course, as discussed in Chapter 4, the bent bodies of

the two cars can also become hot since some of the initial kinetic energy of the two-car

system is also converted to internal thermal energy (larger random kinetic energy of the

atoms). In addition to these two types of internal energy produced during the car

collision, a small amount of the initial kinetic energy of the system is also radiated away as sound waves—the bang sound produced during the collision [2].

To help students develop a comprehensive mental picture of the internal energy produced and the energy transformation during an inelastic collision, it is important for them to be able to visualize internal potential energy, in addition to internal thermal energy. The energy associated with the sound wave is not emphasized in this dissertation, as 1) the sound energy, produced during macroscopic collisions usually accounts for only a small fraction of the initial kinetic energy; 2) the sound can be heard so that it is not difficult for a student to realize that it is produced during collisions; and 3) it appears that many students do not have difficulty understanding sound energy (see Section 5.1.2 for details).

This chapter reports research concerning student difficulties in understanding the concept of internal potential energy. The chapter has four sections: 1) Investigating students' initial understanding, in particular, their difficulties with the internal potential energy produced and the energy transformation during inelastic collisions; 2) Developing and implementing an experiment problem as a hands-on laboratory activity to help students visualize the internal potential energy produced during a partially inelastic collision; 3) Assessing the effectiveness of the learning activity on student understanding; and 4) Discussing results of the studies.

5.1 Investigation of students' difficulties understanding the internal potential energy produced during inelastic collisions

We might hypothesize that the concept of internal potential energy could easily be visualized, since it is associated with a change in the physical shape of a solid. But, does a student really understand that a substance gains internal potential energy when it has a change in its shape? To investigate this question, we conducted individual student demonstration interviews with a small number of students and administered a written test question to a large number of students. Below we report the details of these studies.

5.1.1 Individual Student Demonstration Interviews (Spring 1998)

In spring 1998 the individual student demonstration interviews, as discussed in Section 4.1.1, were conducted to investigate what students understand about the concept of internal energy (in particular, the concept of internal thermal energy) and energy transformation during inelastic collisions. Among three major demonstrations used (see Section 4.1.1.1), one involved dropping an unhappy ball and a modeling clay ball onto a table from the same height. The students were informed that these two balls had identical masses. Then they were asked to predict what would happen to the balls after being dropped and to give their reasoning. This was followed by an actual demonstration, and students were asked to explain their reasoning on the basis of their observations. Five students, three honors engineering freshmen and two regular engineering students participated in the interviews.

5.1.1.1 Student Responses and Reasoning on the Task

All five students predicted that the unhappy ball and the modeling clay ball would not bounce back if they were dropped onto the table. For the unhappy ball, since the students had seen from the first demonstration that it did not bounce back, it was not surprising that they "predicted" (actually based on their previous observations) that the unhappy ball would not bounce back (see Section 4.1.1 for more details). However, all five students predicted that the clay ball would not bounce back either and they all mentioned that the clay ball would have a change of its shape afterward. The two regular students reasoned that the clay ball would absorb the force from the table since the clay ball would be compressed or smashed in. One honors student explained that because the clay ball had a lower density, it would get flat at its bottom. So the clay ball would stay on the table. The other two honors students explained that because the clay ball would be compressed, molecules inside the clay could not be pushed back to the original shape, so the clay ball could not bounce back.

After the clay ball was actually dropped, the students could physically see that the part of the clay ball touching the table was flattened or deformed. This was consistent with the students' prediction. Then the interviewer asked the students about what types of energy the clay ball had right before it hit the table. All five students answered that the clay ball had kinetic energy. Their responses to the question about "where the initial kinetic energy of the clay ball goes after the collision" became interesting. Two regular students believed that the kinetic energy was transferred to the *force* to change the clay ball's shape. One honors engineering student explained that some kinetic energy was transferred to internal energy of the clay ball to make molecules inside bent and

compressed; some kinetic energy went to the table as well to compress molecules inside the table. Another honors student proposed that the initial kinetic energy of the clay ball was transferred to energy, at an atomic level, such as to breaking bonds between molecules, to compressing, or to stretching molecules. The last student's response was that the initial kinetic energy of the clay ball went into deforming the clay ball. He said that the table exerted a force on the clay ball and pushed molecules inside over some distance.

5.1.1.2 Discussion of the Interview Results

It was apparent that all five students knew that clay ball would stay on the table after the collision. But it appeared that the honors engineering students had a better understanding in terms of the reason that the clay ball could not bounce back. The three honors students more or less realized that the deformation of the clay ball was related to a type of internal energy associated with separations of molecules or bonds between molecules. But obviously, they did not know what term in physics to use to describe this type of energy.

The two regular students used a force idea to explain why the clay ball stayed on the table. Even after being explicitly asked to use an energy idea to explain "where the kinetic energy goes," they still reasoned that the kinetic energy was transferred to the *force* needed to deform the clay ball. Apparently, they had difficulty understanding the internal potential energy, modeled by the deformation of a clay ball, which was produced during the inelastic collision. They were unable to link a change of the shape of a substance with a gain in its internal potential energy. But it appeared that two students

somewhat realized that the initial kinetic energy was "something" related to the deformation of the clay ball. But obviously they did not know what the "something" was.

5.1.2 Pretest Problem (Spring 1999)

In the spring quarter of 1999, a written test question (see Figure 5.2) was administered to the physics 131 students in their laboratory sections. This 131 class, including two lecture sections, was a regular calculus-based introductory physics class for engineering and science students at OSU. The two lecture sections were taught by two different professors, but covered the same content: kinematics, Newtonian dynamics, momentum, work and energy, and some rotational dynamics. Students from these two lecture sections were mixed together in sixteen laboratory sections, scheduled from Mondays to Wednesdays. The problem shown in Figure 5.2 was administered to the students at the beginning of each laboratory section during the last week of the quarter, when the students had learned the concepts of work and energy, momentum, and collisions. The question was called a pretest problem since it was given to the students after standard instruction but before they were exposed to a research-based laboratory activity.

5.1.2.1 A Multipart Problem Including a Partially Inelastic Collision

The physical process described in the pretest problem in Figure 5.2 included three small sub-parts: 1) The pendulum bob swung down until just before hitting the box—a conservation of energy problem; 2) the bob collided with the box—a conservation of momentum problem; and 3) the box slid on the table until it comes to a stop—a

A pendulum bob is released horizontally from the rest and swings down to hit a block at rest on a table. The mass M of the block is larger than the mass m of the bob. Jut after the collision, the bob stops and the block starts to move.

Just before the collision: The bob has a speed *v,* and the block is at rest.

Just after the collision: The bob stops, and the block moves forward with speed *u.*

(a) Is the kinetic energy of the pendulum bob just before the collision **greater than, equal to or less than** the kinetic energy of the block right after the collision? Please clearly explain your reasoning.
(b) Is the magnitude of the momentum of the pendulum bob just before the collision **greater than, equal to or less than** the magnitude of the momentum of the block right after the collision? Please clearly explain your reasoning.

Figure 5.2: The pretest problem was administered to students in the OSU regular calculus-based physics 131 class in the spring quarter of 1999. When taking this test, the students had learned the concepts of work and energy, momentum, and collisions.

conservation of energy problem or a dynamics problem. The collision between the pendulum bob and the box was partially inelastic since the bob and the box did not stick together but some internal (thermal and potential) energy was produced during the collision. The pendulum bob stopped after the collision. Some of its initial kinetic energy was converted to the internal energy of the bob-block system and the rest was transferred as the kinetic energy to the block so that the block started to slide just after the collision. A problem such as the one discussed above is called a multipart problem since it could be divided into two or more smaller, simpler sub-problems that could be solved by a single, basic physics law [3].

During the collision part of many introductory physics courses, physics instructors emphasize elastic collisions and completely inelastic collisions, which are relatively simple ideal situations. However, in daily life, collisions such as traffic accidents are neither elastic nor completely inelastic collisions (since some internal energy is always produced and the colliding objects do not stick together after the collisions). Such collisions are partially inelastic. To help students develop skills in solving complex, real-world problems (see more details in Section 5.2.3.1), it is important to expose students to real-life situations, such as partially inelastic collisions.

The pretest problem, including two sub-questions (a) and (b), emphasized students' understanding of the partially inelastic collision. Question (a) attempted to probe what students understood about the concept of internal energy and about the energy transformation during the partially inelastic collision. Question (b) investigated what students understood about conservation of momentum. In our teaching, we found that many students could recite the conservation of energy and momentum principles, but

were not able to apply them correctly in solving collision problems. (Similar results were found in some previous studies as well—see Chapter 2 for more details.) Thus, to avoid triggering some students to recite the laws when answering questions (a) and (b), we attempted not to use terms or phrases such as "conservation," "energy is conserved," or "momentum is conserved" in the pretest problem. Furthermore, the problem statement explicitly mentioned that the mass M of the block was larger than the mass m of the bob. We knew that students had learned head-on elastic collisions, especially a simple one, in which the target and the projectile have *equal mass* and they just exchange velocities (i.e., after the collision, the projectile stops but the target starts to move with the initial velocity of the projectile). The partially inelastic collision described in the pretest problem was very similar to a head-on elastic collision in which two colliding objects have identical mass. To avoid confusion, the problem explicitly stated that the block had larger mass than the bob. In this situation, only a partially inelastic or inelastic collision can occur. Otherwise, the bob would bounce back if it were elastic.

5.1.2.2 Student Responses and Reasoning

Student responses and reasoning on questions (a) and (b), which came from 135 students in one lecture section of the physics 131 class, were analyzed and are summarized in Tables 5.1 and 5.2.

For the 135 students answering question (a), forty-seven percent thought the kinetic energy of the pendulum bob right before the collision was *greater than* the kinetic energy of the block right after the collision. Forty percent thought the kinetic energy of the bob was equal to the kinetic energy of the block right after the collision, and eleven

Calculus-based physics 131 in the spring quarter of 1999 N = 135				
Answer			**Explanation**	
Greater than	**47%**	25%	9%	Due to friction
			7%	Energy is lost
			7%	Energy is lost to heat, sound, or other forms of energy
			2%	Energy is lost to internal energy
		7%		The bob has greater velocity before the collision. Although the mass of the block is greater, velocity square counts more than mass for the kinetic energy
		4%		Inelastic collision
		11%		Blank, unclear, or other reasoning
Equal to	**40%**	16%		Elastic collision
		17%		Because energy is conserved or because the bob stops after the collision, all kinetic energy is transferred
		7%		Blank, unclear or other reasoning
Less than	**11%**	Inelastic collision or the block has more mass, and some other reasoning		
Blank/ unclear	**2%**			

Table 5.1: Students' responses and explanations for question (a), shown in Figure 5.2, which asks to compare the kinetic energy of the pendulum bob just before the collision with the kinetic energy of the block just after the collision.

Calculus-based physics 131 students in the spring quarter of 1999 N = 135			
Answer		**Explanation**	
Greater than	**7%**	4%	Due to friction or energy lost
		3%	Other reasoning
Equal to	**85%**	51%	Momentum is always conserved
		6%	Conservation of energy
		5%	Mass and velocity are balanced out
		4%	No outside forces
		3%	Elastic collision
		16%	Blank or other reasoning
Less than	**7%**	2%	Due to friction or energy lost
		5%	Others
Blank/unclear	**1%**		

Table 5.2: Students' responses and explanations for the question (b), shown in Figure 5.2, which asks to compare the magnitude of the momentum of the pendulum bob just before the collision with the magnitude of the momentum of the block just after the collision.

percent thought the kinetic energy of the bob was less than the kinetic energy of the block.

From the data in Table 5.1, about thirty-three percent of the students thought that the kinetic energy of the bob before the collision equaled the kinetic energy of the block after the collision. Since the bob and the block did not stick together after the collision, many students thought that the collision between the bob and the block was not inelastic. Hence, it had to be elastic and kinetic energy was conserved. In addition, some students thought the collision was an elastic one since the bob stopped and the block started to move afterward.

Second, of the forty-seven percent of the students who had the "correct" final answers, most did not provide correct explanations. Nine percent mentioned that, due to friction, the initial kinetic energy of the bob was greater than that of the block after the

collision, but they did not explain further how the friction caused that. Another nine percent explicitly mentioned that during the collision some energy was lost to heat, sound, internal energy, or to other forms of energy, then the initial kinetic energy of the bob was greater than the final kinetic energy of the block. As discussed in Chapter 4, these students could have just used these terms without meaningful understanding. In other words, they lacked mental images for these abstract concepts.

For question (b), Table 5.2 shows that eighty-five percent answered that the magnitude of the momentum of the bob right before the collision equaled that of the block right after the collision. Seven percent thought that the magnitude of the momentum of the bob was greater than that of the block right after the collision, and another seven percent thought that the magnitude of the momentum of the bob was less than that of block right after the collision.

It is interesting to see that fifty percent of the students held the idea that momentum was always conserved for collisions. This was correct for dealing with simple elastic and inelastic collision problems, but they did not develop a fundamental understanding of why the momentum was conserved for these collisions. Only four percent of the students explained that, due to absence of outside forces, the magnitude of the initial momentum of the bob was equal to the magnitude of the momentum of the block after the collision. This was the correct answer.

5.1.2.3 Discussion of Pretest Results

From the data shown in Tables 5.1 and 5.2, it appears that after standard instruction very few students have acquired a deep understanding of the conservation of

energy and momentum principles, fundamental laws to be used to describe collision processes. Most students had a variety of difficulties understanding collisions.

First, most students lacked a meaningful understanding about what "the loss" was during inelastic collision. Some students comparing colliding bodies' masses, velocities or a combination of the two to decide whether or not the kinetic energy was conserved during the collision. This result was similar to a research finding reported by Grimellini-Tomasini et al. [4] (see detailed discussions in Chapter 2).

Second, some students attempted to use the idea of friction to explain "why the kinetic energy was lost." They did not understand how friction could reduce the kinetic energy to be lost.

Third, the data in Table 5.1 indicate that some students seemingly knew that some energy was lost during the collision. But it was apparent that they did not understand "to what the energy was lost."

Fourth, it was difficult for some students to distinguish conceptually energy and momentum. It seemed to them that both the conservation of energy and the conservation of momentum were decided by a single undefined mechanism. The data in Tables 5.1 and 5.2 show that some students thought that if the energy was conserved, momentum was conserved as well; if energy was lost, momentum was lost too. This difficulty for students in understanding of collisions was also identified by Grinellini-Tomasini et al. [4].

Fifth, some students used surface features to identify different types of collisions. For instance, they considered the collision as elastic if colliding objects "not stick together" or "one was moving and one stationary before and after the collision."

Finally, many students just memorized phrases (e.g., momentum is always conserved during collisions) to justify whether the momentum was conserved or not during the collision. They lacked an understanding of the condition that was necessary for the conservation of (linear) momentum held true (i.e., no net external force acting on a closed or isolated system).

In summary, most students did not develop a meaningful understanding of "the lost energy," the internal thermal and potential energy, produced during inelastic collisions. They had difficulty understanding the energy transformation conceptually during inelastic collisions. Additionally, many students did not acquire a conceptual understanding of the fundamental conditions that determine the conservation of momentum and kinetic energy.

5.1.3 Discussion of Student Difficulties Understanding the Internal Potential Energy Produced during Inelastic Collisions

Overall, on the basis of the written test data from a large number of students and the interview data from a small number of students, it is apparent that most introductory physics students after standard instruction lack a conceptual understanding of internal energy, which is a very abstract but critical concept involved in partially or completely inelastic collisions. Student difficulties with the concept of internal potential energy could be identified as: 1) Students lack a conceptual understanding of internal potential energy. That is, students do not know how atoms interact at an atomic level. And they do not know that the interaction is defined in one way by potential energy (e.g., Lennard-Jones potential energy function) and the potential energy is a function of the separations between atoms. 2) Students are unable to link a change of a solid's shape at a

macroscopic level to a gain in its internal potential energy at a microscopic level. 3) Students do not know what concept or term in physics should be used to describe the energy associated with the deformation of a colliding object.

5.2 Development and Implementation of an Experiment Problem as a Laboratory Activity

Student difficulty could result from many reasons. For example, problems in the text describe situations already simplified and often explicitly inform students that a collision is either elastic or completely inelastic [5]. Students do not need to decide if the conservation of kinetic energy law or the conservation of momentum is the most appropriate concept needed to solve the problems. They could simply find associated equations and plug numbers into them to solve the problems. There is ample evidence from physics education research [6] that student deep conceptual understanding does not often come as a result of solving standard, quantitative problems. Developing a mental image of the concept of internal potential energy is very important to help students develop a conceptual understanding of the conservation of energy principle and to understand "to what the kinetic energy is lost" during inelastic collisions. Below we report a curriculum development and implementation to help students visualize internal potential energy.

5.2.1 Development of Pendulum-Box Bash Experiment Problem (Spring 1999)

On the basis of the investigation discussed above, an interactive laboratory activity using a Pendulum-Box Bash (PBB) experiment problem was developed to address student difficulties and to help students visualize internal potential energy

produced during a partially inelastic collision. A detailed setup of the PBB experiment problem and student laboratory worksheets are shown in Appendixes L and M.

5.2.1.1 Problem Setup

The physical process for this PBB laboratory activity was similar to the pretest problem. It can be divided into three small sub-parts: 1) The pendulum bob swings down until just before hitting the stationary box—a conservation of energy problem; 2) the bob collides with the box—a conservation of momentum problem; and 3) the box slides on the table until it stops—a conservation of energy problem or a dynamics problem. The collision between the pendulum bob and the box was partially inelastic (see the section below for details). By adjusting the mass of the box, the bob could come to a stop after the collision.

5.2.1.2 The Use of a Sandbag versus a Super ball as the Pendulum Bob

Two versions of the PBB laboratory activities were designed: one using a sandbag as the pendulum bob, the other one using a super ball as the pendulum bob. The other apparatus and the laboratory setup were identical.

Sandbag version: When a sandbag serving as the pendulum bob was released from rest initially (with the string horizontal), it collided with the stationary box (paperboard box) at the bottom of its swing. After the collision, the sandbag almost stopped and was significantly deformed. A student could see the deformation, which illustrated that some internal potential energy of the sandbag changed during the collision.

Super ball version: Compared with a sandbag, after colliding with the box, a super ball did not have a visible change of its shape. A student could not physically see

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any deformation of the super ball. By adjusting the mass of the box, the super ball could be made to come to a stop after the collision. There was still internal potential energy produced (such as a small deformation of the paperboard box).

5.2.2 Development of Student Laboratory Worksheets

In addition, student laboratory worksheets (see Appendix M) were developed for this PBB laboratory. The worksheets consisted of two parts: the problem statement and follow-up questions. The problem statement, like telling a story, described the students' task. The problem statement characterized a complex, multipart situation and was poorly defined.

The follow-up questions attempted to address the student difficulties discussed in Section 5.1. For example, questions (a) and (b) were intended to help students understand that the initial kinetic energy was partly converted to internal potential energy (the energy to deform the colliding objects, the sandbag–box system or the super ball-box system). Using experimental data, students could calculate the kinetic energy of the pendulum bob just before the collision and the kinetic energy of the box just after the collision and could see if they were the same or different. Students could see that about 20%~30% of the initial kinetic energy of the bob was converted to the internal potential energy of the ball-box system when using a super ball as the pendulum bob and about 35%~45% of the initial kinetic energy when using a sandbag as the pendulum bob [7].

Questions (c) to (e) attempted to help students understand the conservation of momentum. From their calculations, students could see that the initial momentum of the bob equaled that of the box just after the collision.

The last question, (f), aimed to help students recognize the collision as a partially inelastic one (the momentum of the bob-box system was conserved, but not the kinetic energy).

5.2.3 Instruction

Students were exposed to this experiment problem in a laboratory setting (the Physics 131 lab) during the last week of the spring quarter of 1999. Of sixteen lab sections, students in the first eight laboratory sections used a sandbag as the pendulum bob, while students in the other eight laboratory sections used a super ball as the pendulum bob.

Five graduate and undergraduate teaching associates taught these sections, and each of them taught both laboratory versions. All the laboratory instructors attended a training session in which they learned how to teach the laboratory activity from a laboratory coordinator (another graduate student teaching associate).

5.2.3.1 Experiment Problem

The Pendulum-Box Bash laboratory, an experiment problem [8, 9, 10], consists of a set of apparatus that can be used to perform some task that is stated as a problem. To solve the experiment problem, a student usually needs to take the following major three steps:

1) Plan a solution

- Draw a sketch;

- Organize and access conceptual knowledge;

- Define a poorly-defined problem;

- Divide a complex problem in smaller, simpler problems;

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- Represent the parts in multiple representations; and

- Reassemble the parts together to get the solution to the complex problem.

2) Measure and estimate physical quantities

- Measure related quantities and record data;

- Make proper approximations and estimations; and

- Put solutions of the parts together with values and make a prediction for the outcome of the experiment.

3) Evaluate the solution

- Perform the experiment;

- Enumerate and evaluate approximations; and

- Assess and modify the solution.

For solving the PBB experiment problem, students basically were exposed to the problem solving procedure as above.

5.2.3.2 Sandbag versus Super Ball Laboratory

In a laboratory room, a group of three or four students worked together to complete the laboratory task in a period of two hours. During laboratory sections in which students were exposed to the PBB experiment problem using a super ball as the pendulum bob, laboratory instructors found that they had to tell students that the collision was partially inelastic. But for the laboratory activity using a sandbag as the pendulum bob, students could see that the sandbag became significantly deformed during the collision. The student realized that some initial kinetic energy was "lost" to the energy to deform the sandbag. Major differences between these two versions of the laboratory activity were summarized in Table 5.3.

	Sandbag	**Super ball**
Differences	Significant and apparent deformation	No visible change of shape
	Deformation modeling internal potential energy production	Students were told some internal potential energy was produced

Table 5.3: Major differences in using a sandbag as the pendulum bob and using a super ball as the pendulum bob for the Pendulum-Box Bash Experiment Problem Laboratory activity.

5.3 Assessment of the Effectiveness of the Pendulum-Box Bash Experiment Problem as the Laboratory Activity (Spring 1999)

In the spring quarter of 1999, we conducted a series of measurements, including written test problems and individual student demonstration interviews, with regular 131 students who had performed the PBB experiment problem in the laboratory. The assessment attempted to answer the following questions: 1) was it effective to use the deformation of a sandbag in helping students visualize "to what the kinetic energy is lost to" or "where the kinetic energy goes" during partially inelastic collisions? 2) does such a visual aid (the change in shape of the sandbag) help students develop a mental image of "a type of internal energy" produced during inelastic collisions? and 3) does student visualization of "the kinetic energy goes to the internal energy associated with deformation" affect their performance in recognizing and approaching a collision part of complex problems. The next three sections report the assessment results of the written tests and the student interviews, and the last section discusses the evaluation results.

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5.3.1 Student Performance on Post-Test Problem

At the end of the PBB lab, a test problem was given to the 131 students to assess their understanding about the concept of internal potential energy and the energy transformation during a partially inelastic collision. The post-test problem, shown in Figure 5.3, included two sub-questions, one that probed students' understanding of internal energy and the energy transformation during the collision and the other one investigated students' understanding of the conservation of momentum. The post-test problem looked different in its surface features from the pretest problem shown in Figure 5.2 and the PBB experiment problem shown in Appendix M. A student could see a deformation of the boxing bag after the collision. But in the problem statement, it did not explicitly mention it. Furthermore, the post-test problem had an emphasis on the collision, which was partially inelastic as well.

5.3.1.1 Student Responses and Reasoning

Of 135 students from one section of the Physics 131 course, 67 took the PBB laboratory using the sandbag as the pendulum bob, while the other 68 took the PBB laboratory using the super ball as the pendulum bob. Students' responses on the post-test questions (a) and (b) are summarized in Tables 5.4 and 5.5. Table 5.4 shows that, of 67 students in the sandbag laboratory, eighty-two percent answered that the initial kinetic energy of the fist was greater than the kinetic energy of the bag right after the collision. Sixteen percent answered that the initial kinetic energy of the fist was equal to the kinetic energy of the bag right after the collision. Similarly, of 68 students in the super ball laboratory, eighty-five percent answered that the initial kinetic energy of the fist was

A karate expert strikes with his fist of mass m a free-hanging boxing bag of mass M. The mass M of the bag is larger than the mass m of his fist.

Just before the collision: The fist has a speed *v*, and the bag is at rest.

Just after the collision: The fist stops, and the bag swings up with speed *u*.
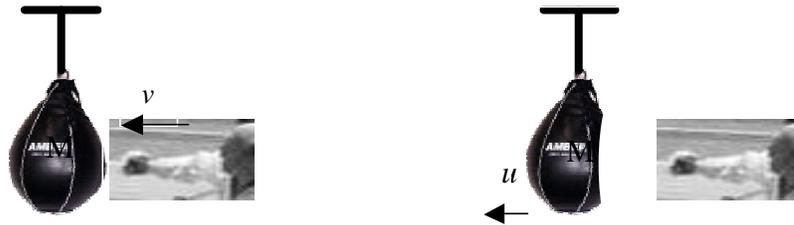
(a) Is the kinetic energy of the karate expert's fist just before the collision *greater than, equal to or less than* the kinetic energy of the boxing bag just after the collision? Please clearly explain your reasoning.

(b) Is the magnitude of the momentum of the karate expert's fist just before the collision **greater than, equal to or less than** the magnitude of the momentum of the boxing bag just after the collision? Please clearly explain your reasoning.

Figure 5.3: The post-test problem was administered to students in one section of the calculus-based physics 131 course in the spring quarter of 1999. The students took this test at the end of the Pendulum-Box Bash Experiment Problem laboratory.

Calculus-based physics 131 in the spring quarter of 1999 N = 135				
Laboratory	**Student responses on the post-test problem**			
Sandbag (N = 67)	**Greater than**	82%	49%	Deformation or a change in the shape of the boxing bag
			27%	Heat, sound, or due to friction
			6%	Unclear or other reasoning
	Equal to	16%	Energy was conserved, no friction, the bob stopped afterward, or no explanations	
	Unclear	2%		
Super ball (N = 68)	**Greater than**	85%	28%	Deformation or a change in the shape of the boxing bag
			50%	Heat, sound or due to friction
			7%	Unclear or other reasoning
	Equal to	12%	Energy was conserved, no friction, the bob stopped afterward, or no explanations	
	Unclear	3%		

Table 5.4: Students responses and explanations, grouped by the sandbag and the super ball laboratory, on question (a) of the post-test problem shown in Figure 5.3, which asked to compare the kinetic energy of the karate expert's fist just before the collision with the kinetic energy of the boxing bag just after the collision.

Calculus-based physics 131 in the spring quarter of 1999 N = 135				
Laboratory	**Student responses on the post-test problem**			
Sandbag (N = 67)	**Equal to**	97%	79%	Conserved or always conserved
			2%	No external forces
			2%	Conservation of energy
			14%	Blank, unclear or other reasoning
	Greater than	3%	Energy is lost or no reasoning	
Super ball (N = 68)	**Equal to**	94%	69%	Conserved or always conserved
			3%	No external forces
			2%	Conservation of energy
			20%	Blank or unclear reasoning
	Greater than	6%	Energy is not conserved or lost to deform the bag	

Table 5.5: Students responses and explanations, grouped by the sandbag and the super ball laboratory, on question (b) of the post-test problem shown in Figure 5.3, which asked to compare the magnitude of the momentum of the karate expert's fist just before the collision with the magnitude of the momentum of the boxing bag just after the collision.

greater than the kinetic energy of the bag right after the collision. Twelve percent answered that the initial kinetic energy of the fist was equal to the kinetic energy of the bag right after the collision.

However, when student explanations were analyzed in detail, some significant differences appeared between these two groups. Among 67 students from the sandbag laboratory, forty-nine percent [11] explained that some initial kinetic energy of the fist was lost to "deformation of the boxing bag" or "a change in the shape of the boxing bag." Twenty-seven percent explained that some initial kinetic energy of the fist was "lost" to "heat," "sound," or "due to friction." But among 68 students from the super ball laboratory, twenty-eight percent [11] explained that some initial kinetic energy of the fist was "lost" to "deformation of the boxing bag" or "a change in the shape of the boxing bag." Half of them still explained that some initial kinetic energy of the fist was lost to "heat," "sound," or "due to friction."

Table 5.5 showed that student responses on question (b) of the post-test problem were similar to each other for students from the sandbag and the super ball laboratory. It appeared that most students from both groups believed that the momentum was conserved during the collision, but few correctly understood why. To compare student performance on the post-test problem more meaningfully, we also broke student responses on the pretest questions into the sandbag and the super ball group. The results for questions (a) and (b) were summarized in Tables 5.6 and 5.7, respectively. Data in Table 6.6 showed that, among 67 students before taking the PBB laboratory using the sandbag as the pendulum bob, forty-two percent answered that the initial kinetic energy

Calculus-based physics 131 in the spring quarter of 1999 N = 135				
Laboratory	**Student responses on the pretest problem**			
Sandbag (N = 67)	**Greater than**	42%	21%	Energy is lost to heat, sound, internal energy or due to friction
			8%	The bob has greater velocity before the collision. Although the mass of block is greater, velocity square counts more than mass for the kinetic energy
			13%	Unclear and other explanations
	Equal to	46%	18%	Elastic collision
			16%	Energy is conserved or all kinetic energy is transferred
			3%	The bob stops after the collision, so all energy is transferred to the block
			9%	Other explanation or blank
	Less than	11%		
	Unclear/ blank	0%		
Super ball (N = 68)	**Greater than**	52%	29%	Energy is lost to heat, sound, internal energy or due to friction
			6%	The bob has greater velocity before the collision. Although the mass of block is greater, velocity square counts more than mass for the kinetic energy
			17%	Unclear and other explanations
	Equal to	34%	15%	Elastic collision
			7%	Energy is conserved or all kinetic energy is transferred
			6%	The bob stops after the collision, so all energy is transferred to the block
			6%	Other explanation or blank
	Less than	12%		
	Unclear/ blank	3%		

Table 5.6: Students responses and explanations, grouped by the sandbag lab and the super ball laboratory, on question (a) of the pretest problem shown in Figure 5.2, which asked to compare the kinetic energy of the pendulum bob just before the collision with the kinetic energy of the block just after the collision.

Calculus-based physics 131 in the spring quarter of 1999 N = 135				
Laboratory	**Student responses on the pretest problem**			
Sandbag (N = 67)	**Equal to**	86%	57%	Conserved or always conserved
			3%	No external forces
			4%	Conservation of energy
			0%	Elastic collision
			4%	Mass and velocity are balanced out
			18%	Blank, unclear or other reasoning
	Greater than	5%		
	Less than	9%		
Super ball (N = 68)	**Equal to**	84%	46%	Conserved or always conserved
			4%	No external forces
			7%	Conservation of energy
			6%	Elastic collision
			6%	Mass and velocity are balanced out
			15%	Blank or unclear reasoning
	Greater than	9%		
	Less than	5%		
	Unclear/ Blank	2%		

Table 5.7: Students responses and explanations, grouped by the sandbag lab and the super ball lab, on question (b) of the pretest problem shown in Figure 5.2, which asked to compare the magnitude of the momentum of the pendulum bob just before the collision with that of the block just after the collision.

of the bob was greater than the kinetic energy of the box right after the collision. Forty-six percent answered that the initial kinetic energy of the bob was equal to the kinetic energy of the box right after the collision. But among 68 students before taking the PBB laboratory using the super ball as the pendulum bob, fifty-two percent answered that the initial kinetic energy of the bob was greater than the kinetic energy of the box right after the collision. Thirty-four percent answered that the initial kinetic energy of the bob was equal to the kinetic energy of the box right after the collision.

Data in Table 5.7 showed that student responses on question (b) of the pre-test problem were similar to each other for students from the sandbag and the super ball laboratory. Eighty-six percent of the students who took the sandbag laboratory answered that the magnitude of the initial momentum of the bob was equal to that of the box after the collision, while eighty-four percent of the students who tool the super ball laboratory answered that the magnitude of the initial momentum of the bob was equal to that of the box after the collision.

Students' final class letter grades also were analyzed. Among 67 students who took the sandbag version laboratory, 27 (40%) received A and B (higher grades), 38 (57%) received C and D (lower grades), and 2 (3%) received E (failure). Similarly, among 68 students who took the super ball version laboratory, 25 (37%) received A and B, 40 (59%) received C and D, and 3 (4%) received E (failure). A distribution of the number of students, from the sandbag and the super ball laboratory, who received final class letter grades as A and B, C and D, as well as E is summarized in Figure 5.4.
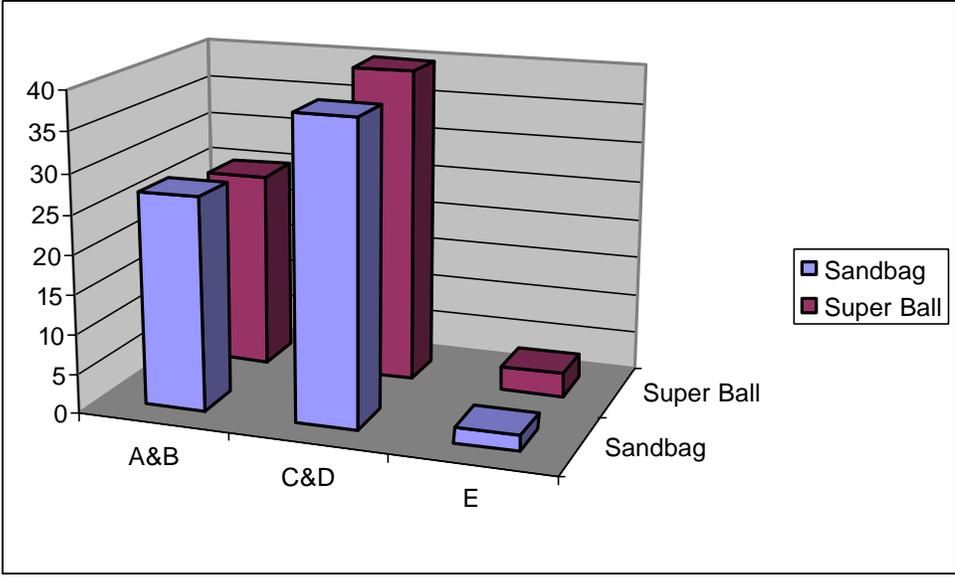
Figure 5.4: Distribution of the number of students, from the sandbag and the super ball laboratory, who received final class letter grades as A and B, C and D, as well as E.

5.3.1.2 Discussion of Post-Test Results

On the basis of data discussed above, it was interesting to see that some differences in student performance on the post-test problem appeared between the sandbag and the super ball group. First, it was apparent that more students from the sandbag laboratory than from the super ball laboratory realized that some of the initial kinetic energy of the fist was "lost" to the energy to make the boxing bag deformed. About half of the students from the sandbag laboratory but less than one third of the students from the super ball laboratory gave such answers. Second, half of the students from the super ball laboratory used scientific words that they did not understand (for example, "heat" or "due to friction") to describe "how or to what the kinetic energy was lost". But less than one third of the students from the sandbag laboratory gave such answers.

The two groups had similar responses to question (a) on the pretest. Thus, we think that the difference in student responses to the post-test question (a) resulted from two different versions of the PBB experiment problem. The deformation of a sandbag helped students visualize "where the kinetic energy goes" during the inelastic collision.

Similar results appeared for these two groups for question (b). On an average, about ninety-five percent students from each group answered that the magnitude of the momentum of the fist before the collision equaled that of the boxing bag after the collision. Seventy to eighty percent of the students said that the momentum was always conserved. The sand bag and the super ball as the pendulum bob versions of the experiment had the same effect on students' understanding of why the conservation of momentum principle was appropriate to apply for solving the partially inelastic collision.

5.3.2 Student Performance on a Final Examination Problem

This section discussed student performance on the final examination problem (see Figure 5.5). The problem included three sub-parts, one of which was a partially inelastic collision. The first part, the pendulum bob swinging down to the bottom, was a conservation of energy problem. The second small part was the collision between pendulums bobs A and B, a partially inelastic collision and conservation of momentum problem. The third part was also a conservation of energy problem, the pendulum B swinging up after the collision. This test problem differed in some surface features from the PBB experiment problem, but was equal in terms of physics concepts.

This test problem primarily attempted to assess the effectiveness of the PBB laboratory activity on student understanding of partially inelastic collisions. In particular, it was intended to evaluate if it was helpful for students to recognize an inelastic collision after they learned the deformation of a colliding body, which modeled internal potential energy change. We hypothesized that the sandbag and the super ball version laboratory activities had the same effect on student recognition of a partially inelastic collision. Therefore, to avoid triggering students who took the sandbag version laboratory to recall the deformation of the sandbag, the test problem did not explicitly indicate that either pendulum A or pendulum B became deformed after the collision, nor did the problem pictorial representation provide any visual aid about the deformation. When analyzing student solutions on this problem, we counted the correctness of the solution. We also considered carefully 1) if the solution included the collision as a separate part or a sub-problem and 2) how the collision part was solved.

A pendulum bob A of mass 2m is released from rest from a height H so that it is initially horizontal. Bob A swings down and at the bottom, it collides with another stationary bob B of mass 3m. After the collision, bob A stops, and bob B swings up. What is the maximum height bob B swings up after the collision? Show all of your work and reasoning for credit.
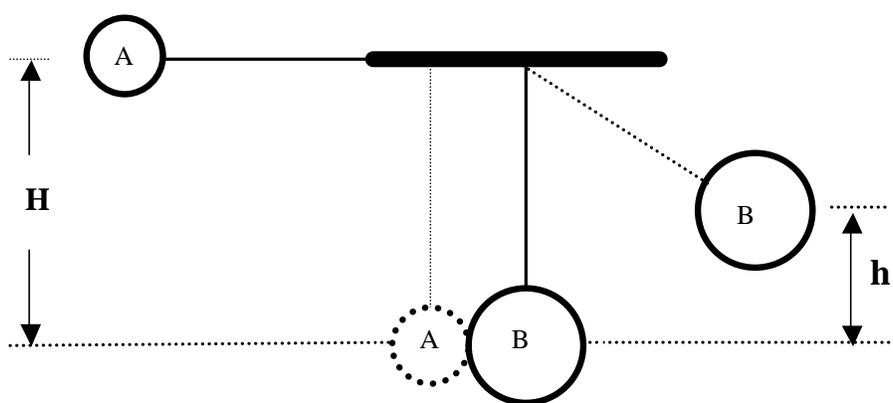
Figure 5.5: The problem appeared on the final examination for one section of the 131 class in the spring quarter of 1999. In this section, half of the students took the Pendulum-Box Bash laboratory using a sandbag as the pendulum bob and the other half took the same laboratory but using a super ball as the pendulum bob.

Half of the students from this Physics 131 section took the PBB lab activity using a sandbag as the pendulum bob, and the other half undertook the same activity but using a super ball as the pendulum bob. This was the only different learning experience of these two groups of students.

5.3.2.1 Student Performance

Student performance on the problem shown in Figure 5.5 is reported in Table 5.8. Of 135 students, seventy percent divided the problem into small parts and included the collision as a sub-problem. Fifty three percent correctly applied the conservation of momentum principle for solving the collision part. Among this half of students, it is difficult to know why they chose to apply the conservation of momentum to solve the collision part. As discussed in Section 5.3.1, it appeared that many students did not develop a deep understanding of why the momentum was conserved during the collision, even after they learned the PBB laboratory activity. To evaluate whether or not the use of the deformation of a sandbag, which modeled the internal potential energy produced, had any impact on student understanding of the energy transformation during the inelastic collision, we carefully analyzed the problem solutions in which students incorrectly applied the conservation of energy for the problem as a whole and incorrectly applied the conservation of kinetic energy for the collision part. In addition, grouped by the sandbag and the super ball laboratory and by student final class letter grades, a comparison was made between those final exam solutions and the matched pretest solutions.

As summarized in Table 5.9, among 27 students who received either A or B for their final class letter grades and also took the sandbag version laboratory, fifteen percent *incorrectly* applied the conservation of energy to the final exam problem as a whole or

Physics 131 class in the spring quarter of 1999 N = 135 calculus-based students			
24%	Applied conservation of energy for the problem as a whole		
6%	Unclear or blank		
70%	**Divided the problem into small parts and included the collision part**	**53%** **(Correct)**	Conservation of momentum
		2% (Incorrect)	Conservation of momentum
		6% (Incorrect)	Conservation of kinetic energy
		7% (incorrect)	"Conservation of velocity"
		2% (Incorrect)	Velocity equations derived from the conservation of momentum and the conservation of kinetic energy for an elastic collision

Table 5.8: Student overall performance on the final examination problem shown in Figure 5.4.

Class grades	Pretest question (a)	Final exam problem
A & B students	% of students who *incorrectly* answered that the initial kinetic energy of the bob was equal to the kinetic energy of the box after the collision	% of students who *incorrectly* applied the conservation of energy to the problem as a whole or *incorrectly* applied the conservation of kinetic energy to the collision part
Sandbag (N = 27)	48%	15%
Super ball (N = 25)	40%	8%

Table 5.9: Proportion of students who incorrectly answered for pretest question (a) that the initial kinetic energy of the bob was equal to the kinetic energy of the box after the collision and percentage of students who incorrectly applied the conservation of energy to the problem as a whole or incorrectly applied the conservation of kinetic energy to the collision part. Those students received either A or B for their final class letter grades and were grouped by the sandbag and the super ball laboratory.

Class grades	Pretest question (a)	Final exam problem
C, D & E students	% of students who *incorrectly* answered that the initial kinetic energy of the bob was equal to the kinetic energy of the box after the collision	% of students who *incorrectly* applied the conservation of energy to the problem as a whole or *incorrectly* applied the conservation of kinetic energy to the collision part
Sandbag (N =40)	45%	30%
Super ball (N = 43)	30%	49%

Table 5.10: Proportion of students who incorrectly answered for pretest question (a) that the initial kinetic energy of the bob was equal to the kinetic energy of the box after the collision and percentage of students who incorrectly applied the conservation of energy to the problem as a whole or incorrectly applied the conservation of kinetic energy to the collision part. Those students received C, D or E for their final class letter grades and were grouped by the sandbag and the super ball laboratory.

incorrectly applied the conservation of kinetic energy to the collision part. But before being exposed to the PBB sandbag laboratory, forty-eight percent incorrectly answered in the pretest question (a) that the initial kinetic energy of the bob equaled the kinetic energy of the box after the collision. Among 25 students who also received either A or B for their final class letter grades but took the super ball version laboratory, eight percent *incorrectly* applied the conservation of energy to the final exam problem as a whole or *incorrectly* applied the conservation of kinetic energy to the collision part. But before being exposed to the PBB super ball laboratory, forty percent *incorrectly* answered, for the pretest question (a), that the initial kinetic energy of the bob was equal to the kinetic energy of the box after the collision.

Table 5.10 showed that among 40 students who received C, D or E for their final class letter grades and took the sandbag version laboratory, thirty percent *incorrectly* applied the conservation of energy to the final exam problem as a whole or *incorrectly* applied the conservation of kinetic energy to the collision part. But before being exposed to the PBB sandbag laboratory, forty-five percent *incorrectly* answered in the pretest question (a) that the initial kinetic energy of the bob equaled the kinetic energy of the box after the collision. Among 43 students who also received C, D or E for their final class letter grades but took the super ball version laboratory, forty-nine percent *incorrectly* applied the conservation of energy to the final exam problem as a whole or *incorrectly* applied the conservation of kinetic energy to the collision part. But before being exposed to the PBB super ball laboratory, thirty percent *incorrectly* answered, for the pretest question (a), that the initial kinetic energy of the bob was equal to the kinetic energy of the box after the collision.

5.3.2.2 Discussion of Final Exam Results

It appeared that the PBB laboratory, when using either a sandbag or a super ball as the pendulum bob, helped many above-average students overcome the difficulty of incorrectly applying the conservation of kinetic energy for the partially inelastic collision. But for average or below-average students, it appeared that the PBB laboratory using a sandbag as the pendulum helped some of them overcome the difficulty. The PBB laboratory using a super ball as the pendulum made some of them confused and even caused some to incorrectly apply the conservation of energy principle to the final examination problem as a whole or incorrectly applied the conservation of kinetic energy to the collision part of the problem.

It was not surprising to find that there existed no significant difference in the use of either the sandbag laboratory or the super ball laboratory to help the above-average students address the difficulty. Top students seem to learn successfully no matter how they were taught. For instance, a study [12] showed that top 25% of the students could became Newtonian thinkers even after traditional instruction. But it was more difficult to help a substantial fraction of the non-top students to master the physics knowledge that was taught. There was a gap between what those students learned and what was taught [6].

But it appeared that the deformation of a sandbag helped the average and under-average students from the sandbag laboratory visualize the internal (potential) energy produced during the partially inelastic collision. This visual aid could be effective for them to recognized that the kinetic energy was not conserved during the collision and to develop a mental image of "to what the kinetic energy was lost" On the other hand, the

average and under-average students from the super ball laboratory could not visually see any deformation of a super ball produced during the collision. In addition, the use of a super ball could provide those students a visual effect or a surface feature that misled them to recognize the partially inelastic collision as elastic one. Some students might think that, based on their daily-life experience, a super ball always bounced high and a high-bouncing ball always made an elastic collision.

5.3.3 Individual Student Demonstration Interviews (Spring 1999)

To help decide if the deformation of the sandbag was an effective visual aid to see the internal potential energy produced and to understand "where the initial kinetic energy goes" during inelastic collisions, individual student demonstration interviews were conducted. Ten students from the Physics 131 class were solicited on a voluntary basis. The interviews were carried out during the last week of the spring quarter of 1999 when the students had just finished the PBB laboratory activity using a super ball as the pendulum bob. These students had all studied the concepts of work and energy, momentum, and collisions.

5.3.3.1 Interview Demonstrations and Questions

Basically, the following three tasks were included in the interview:

1) A sandbag and a super ball, with identical mass, were dropped from the same height simultaneously onto a table. The subject was asked to predict what would happen to the bag and the ball after they hit the table and to explain his or her reasoning. Then the demonstration was shown and the student was asked to explain his or her reasoning about what was observed.

2) After this, the interviewer showed the student a box and a super ball, which were used during the PBB laboratory. The interviewer used the box and the super ball to show the student the PBB experiment problem and to help him or her recall the PBB laboratory. Then the student was asked to answer two questions: 1) "in the Pendulum-Box Bash laboratory, was the kinetic energy of the super ball right before the collision greater than, equal to, or less than the kinetic energy of the box right after the collision? Please explain your reasoning;" 2) "in this Pendulum-Box Bash laboratory, was the magnitude of the momentum of the super ball right before the collision greater than, equal to, or less than the magnitude of momentum of the box right after the collision? Please explain your reasoning."

3) Last, the interviewer showed the subject a sandbag and a box. The student was informed that the problem was repeated only with the sandbag as pendulum bob. The student was asked to answer two questions on the basis of their predictions: 1) "in this situation, would the kinetic energy of the sandbag right before the collision be greater than, equal to, or less than the kinetic energy of the box right after the collision? Explain your reasoning;" 2) "in the same situation, would the magnitude of the momentum of the sandbag right before the collision be greater than, equal to, or less than the magnitude of the momentum of the box right after the collision? Please explain your reasoning."

Note that the students were asked to answer these two questions before the real experiment was shown. But all the students had seen that the sandbag stayed on the table and became deformed after it hit the table from the first demonstration. For research purpose, it was interesting to investigate whether or not the students could imagine that a deformation would happen to the sandbag during its collision with the box and whether

or not they could recognize that the initial kinetic energy would go to the energy to make the sandbag deformed. If so, it would tell us that the students could develop a mental image in which deformation of a sandbag led to internal energy production.

5.3.3.2 Student Responses and Reasoning

Demonstration of dropping sandbag and super ball: For this demonstration, except for one student who was not asked to make a prediction, all nine students predicted that the sandbag would stay on the table and the super ball would bounce back after they were dropped and hit a table. Their explanations why the super ball would bounce back could be summarized as the following four types: 1) The super ball was a rubber one; 2) it had a solid, compact structure; 3) it made an elastic collision with the table; and 4) its initial kinetic energy still stayed in the form of kinetic energy right after the collision. However, student reasoning why the sandbag would not bounce back was interesting. Three students explained that since the sandbag would get deformed or its bottom would get flat after the collision, it stopped. Two students reasoned that since the sandbag had a loose internal structure or less compact structure, it could not bounce back. Another two students mentioned that the sandbag would stay on the table afterward since its initial kinetic energy was transferred to moving around sand particles or molecules inside and to reshaping them. For the rest of three students, one said that because the sandbag made an inelastic collision with the table, its kinetic energy was not conserved. So it did not have kinetic energy to bounce back after the collision. Another one indicated that the kinetic energy of the sandbag before the collision was transferred to another type of energy and not stayed in the form of kinetic energy after the collision. So it stayed on

the table afterward. The last student was not asked to make a prediction about the first demonstration task.

After the demonstration was actually shown, that is, after the students actually saw that the sandbag stayed on the table and its bottom flattened after hitting the table, their responses to a question "where the kinetic energy of the sandbag goes" were very interesting. As summarized briefly in Table 5.11, all the students realized that the initial kinetic energy of the sandbag could be converted to the energy associated with the deformation of the bag or a change of its shape. But it was apparent that they did not know what term in physics should be used to describe this type of energy. One student explained:

> Student: "It [sandbag] stayed on the table afterward. So it had no kinetic energy since it's not moving. It had no potential energy either since it stayed on the ground [table]. Of course, it had no spring-type energy because of no spring here. But the [kinetic] energy had to go somewhere. It may go to, ΔU, called internal thermal energy, because it's [sandbag] flattened out."

Task of the collision between a super ball and a box: It was also very interesting to look at student responses to the question asking for a comparison of the kinetic energy of the super ball right before the collision with that of the box right after the collision. Of ten students, three explained that the kinetic energy of the super ball before the collision was equal to the kinetic energy of the box after the collision. Here was a typical response by one of these four students:

> Student: "Kinetic energy is conserved, since the super ball from here has potential energy. It goes down here and has kinetic energy. Afterwards, it stops, the kinetic energy has to go somewhere—go to move the box. If it bounced back, the kinetic energy would have gone to the super ball there."

One student first answered that the kinetic energy of the super ball should be the same as that of the box, since no external force exerted on the super ball and also it

Individual student demonstration interview, Spring 1999 Calculus-based students, N = 10
Student explanations about "where the kinetic energy of the sandbag goes"
Student 1: The kinetic energy of the sandbag goes to deformation. Sand particles inside move from the bottom to the top, and there is friction against each other. Student 2: The kinetic energy of the sandbag goes to sound, heat, and to a change of the shape. Student 3: The kinetic energy of the sandbag goes to deforming the bag. Student 4: The kinetic energy of the sandbag is transferred to the *force* to change its shape. It's called changing shape energy, maybe. I don't know the term. Student 5: The energy of the sandbag goes into the sand. You see the bag is smashed in, deformed. So the energy goes to that internally. And maybe some goes into the table too. Student 6: The kinetic energy of the sandbag is transferred to a type of potential energy to make the bag flattened out and stretched out. Student 7: I don't know where the kinetic energy of the sandbag goes. But the sandbag's deformed, so it just stopped. Student 8: It (sandbag) stayed on the table afterward. The energy had to go somewhere. It may go to, ΔU, called internal thermal energy, because it's flattened out. Student 9: The energy of the sandbag goes to change its shape. It is transferred to moving molecules inside to spread out or to reshape them. Student 10: The energy of the sandbag is transferred to another type of energy afterward. Its shape is changed, so the energy cannot stay in kinetic energy.

Table 5.11: Student explanations about "where the kinetic energy of the sandbag goes" during the collision between a sandbag and a table.

stopped after the collision. Then he added that some kinetic energy of the super ball may be transferred to internal energy, sound or compression. So his conclusion was that he was not sure whether or not the kinetic energy of the super ball should be equal to the kinetic energy of the box after the collision.

Another student was also not sure whether or not the kinetic energy of the super ball before the collision should be equal to or greater than the kinetic energy of the box after the collision. But she believed that the kinetic energy of the box afterward should not be greater than the initial kinetic energy of the super ball since the energy could not be gained from nowhere.

Among the last five students, three answered that the kinetic energy of the super ball before the collision could be slightly greater than the kinetic energy of the box since a tiny bit of energy could be lost to friction, sound, or heat. But they believed that the kinetic energy of the super ball should be close to that of the box after the collision. Only two students said that the kinetic energy of the super ball before was greater than that of the box after. One of them explained that because some initial kinetic energy could be transferred to compress or stretch the ball and the cardboard box. But the other one gave the following explanation:

> Student: "The kinetic energy of the ball would be greater than the kinetic energy of the box after because it's not a completely elastic collision. This is what I remembered from my lab (Pendulum-Box Bash lab using a super ball as the pendulum bob). First, I did think the kinetic energy will be equal because I just figured it's an elastic. From the lab we found that it wasn't. Some energy was lost to the sound, maybe. And…I don't know."

The result of the student responses is summarized in Table 5.12.

Individual student demonstration interview Calculus-based students, N = 10			
Student responses to the question asking if the kinetic energy of the super ball before the collision is greater than, equal to, or less than that of the box after the collision.			
Greater than		**Equal to**	**Greater than or equal to**
Slightly greater than	Greater than	N = 3	N = 2
N = 3	N = 2		

Table 5.12: Student responses to the question asking for a comparison between the kinetic energy of the super ball before the collision with that of the box after the collision.

Regarding student responses to the question asking for a comparison of the magnitude of the momentum of the super ball right before the collision with that of the box right after the collision, all ten students answered that they should be equal to each other. But their reasoning for this answer differed. Two students mentioned that because there were no external forces exerted on the ball and the box, the momentum should be same before and after the collision. This was the proper answer. But the interviewer did not ask the students to analyze the forces exerting on the ball and the box in this particular situation. So we did not know whether they just recited what they just learned from their laboratory instructors or if they really understand why there were no outside forces on the ball-box system during the collision.

Six students said that momentum was always conserved. But they did not know why that was. Below are explanations from three students:

> Student 1: "Momentum is conserved. And mass of the ball times its velocity should be same as the velocity of the box times its mass. I don't know why the momentum should be conserved. It may be something related to energy. It's conserved also. (Pause) I don't know why. I cannot think of any reasons now."

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Student 2: "Momentum should be equal. The teacher told us that momentum is always conserved. I never understand why or why not."

Student 3: " Momentum should be equal, because momentum is always conserved no matter in elastic or inelastic collisions...This a an inelastic collision because a little bit of energy is lost. But momentum is conserved since it is always conserved."

One student answered that the smaller mass of the ball times the bigger velocity equaled to the bigger mass of the box times the smaller velocity of the box. Last, one student said that the momentum was conserved since the kinetic energy was conserved during the collision between the super ball and the box. She further explained that if the kinetic energy was not conserved, the momentum was not conserved either.

Task of the collision between a sand ball and a box: It was even more interesting to analyze students' responses to the question asking for a comparison between the kinetic energy of the sandbag just before the collision with that of the box right after the collision. All ten students predicted that the kinetic energy of the sandbag before the collision would be greater than the kinetic energy of the box just after the collision. They all mentioned that this was due to the deformation of the sandbag. Again, they were unable to name appropriately the type of energy associated with a change of sandbag's shape. Here are two examples of the students' reasoning:

Student 1: "Sandbag would flatten out. The kinetic energy goes to that. I don't know the physics term about this type of energy, but I can physically see this."

Student 2: "Kinetic energy of the bag does not all transfer to the box. Because the box's heavy enough, the sandbag would deform again, and the energy will give into the bag. It (energy) loses when this (sandbag) starts deforming. Some work has been done during the collision. This work is to smash in the bag and to move the particles."

For the question asking for a comparison between the magnitude of the momentum of the sandbag just before the collision with that of the box just after the collision, student responses and reasoning were very similar to their answers to the

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question asking for a comparison between the magnitude of the momentum of the super ball just before the collision with that of the box just after the collision. Still two students used a "no outside forces" idea to reason that the momentum should be conserved during the collision between the sandbag and the box. Six students said that momentum was always conserved. Again they did not know why that should be. One student still answered that the smaller mass of the bag times the bigger velocity equaled the bigger mass of the box times the smaller velocity of the box. Last, one student said that the momentum was not conserved since the kinetic energy was not conserved during the collision between the sandbag and the box. Below was the last student's explanation:

> Student: "Momentum would not be equal, since some energy would be lost to change the shape of the bag. For the super ball case, since a little bit of energy is lost to friction, kinetic energy after is just slightly less. So momentum is conserved. But for this situation [sandbag case], since the energy would be lost to change its shape, it is different. So I guess the momentum would not be equal."

5.3.3.3 Discussion of Interview Results

Of the ten students interviewed, six received A and B final grades for the course while the other four received C, D, and E. Thus, to some extent the data discussed above reflected some detailed ideas of the understandings of different ability-level students.

First, it was apparent that the super ball version laboratory was not effective to help students visualize "where the kinetic energy goes" during the partially inelastic collision. There could be several reasons for this. During the PBB laboratory using a super ball, students hardly saw any deformation of the ball or the cardboard box during the collision. They were just told by their laboratory instructors that some initial kinetic energy was transferred to the internal potential energy associated with a change of the ball's shape and the box's shape. The use of a super ball could mislead some students to

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reason that its collision with the box was elastic. In the light of students' explanations why the super ball would bounce back, apparently, some students believed that the super ball was a rubber one and it made elastic collisions. Because the ball was moving and the box at rest before the collision and the reverse after, students might recognize this collision as elastic. Then they memorized that the kinetic energy was conserved during an elastic collision.

Second, it appeared that deformation of a sandbag helped students visualize "to what the kinetic energy was lost". Visualizing a change of the sandbag's shape helped students understood why the kinetic energy was not conserved during the partially inelastic collision.

Third, students did not know what quantity or concept in physics to use to describe energy associated with the deformation. This was not surprising since they never learned it. During the interview, the students did not receive any instruction about the concept of internal potential energy.

Fourth, it was apparent that most students had difficulties understanding why the conservation of momentum could be apply to the inelastic collision. As a result, some students thought that if, during the collision, the kinetic energy was conserved, the momentum was conserved. Otherwise, both were not conserved.

5.3.4 Discussion of Evaluation results

Overall, based on the written test data from a large number of students and the detailed interview results from a small number of students, the deformation of a sandbag is found helpful in letting students visualize "where the kinetic energy goes". Visualizing this aided students in understanding the energy transformation and in recognizing a

collision as partially inelastic, but did not help address student understanding of why the conservation of momentum could be applied to the collision.

5.4   Discussion and Summary

Collisions, as fundamental physical phenomena, occur every day from the microscopic scale of subatomic particles to the astronomic scale of colliding stars. In an introductory mechanics course at the university level, students learn the content of collisions as an important application of the conservation of momentum and energy principles (including the conservation of kinetic energy). Student difficulties applying these conservation laws in problem solving are known to anyone who teaches the introductory course. The studies reported in this chapter have not only identified some student difficulties which are documented by other researchers, but also have found some undocumented difficulties that introductory physics students have in understanding inelastic collisions.

Introductory physics students after standard instruction lack a conceptual understanding of "where the kinetic energy goes" or "to what the kinetic energy is lost" during inelastic collisions. They are unable to link a change of a solid's shape with a gain of its internal potential energy at an atomic level.

Most students also do not develop a meaningful understanding of the fundamental condition that is necessary in order for momentum to be conserved during a collision. They believe that momentum is always conserved during collisions.

The visual aid of a deformation of a sandbag is first used to help students physically see "where the kinetic energy goes" and to understand the energy transformation during inelastic collisions. Assessment results have shown that

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deformation of a collision object, which models internal potential energy change, helps students (especially weak students) visualize and better understand energy transformation during an inelastic collision and help them develop a mental image about the abstract concept of internal (potential) energy.

But it is apparent that visualizing the internal energy does not naturally help students address the difficulty understanding the conservation of momentum. Research-based learning materials are needed to address this student difficulty.

ENDNOTES OF CHAPTER 5

1. Chabay, R. W., and Sherwood, B. A., *Matter & Interactions* (Preliminary Edition, John Wiley & Sons, New York, 1999), pp. 202-203.

2. Chabay, R. W., and Sherwood, B. A., *Matter & Interactions* (Preliminary Edition, John Wiley & Sons, New York, 1999), pp. 272.

3. Van Demelen, D., *The development of problem decomposition diagnostic*, Ph.D. dissertation, The Ohio State University, 2000.

4. Grimellini-Tomasini, N., Pecori-Balandi, B., Pacca, J. L. A., and Villani, A., "Understanding conservation laws in mechanics: Students' conceptual change in learning about collisions," Science Education **77**, 169 (1993).

5. The regular calculus-based introductory physics classes at the OSU use the text— *Fundamentals of Physics* (5th Ed.) by D. Halliday, R. Resnick, and J. Walker. For the problems at the end of Chapter 10—Collisions, many are either about elastic or completely inelastic collisions. In addition, the problem statements explicitly inform students the types of collisions involved in the problems. Students do not need to make decisions themselves about what types of collisions occur, and they could just find associated equations to use and plug into numbers.

6. McDermott, L. C., "Guest comments: How we teach and how student learn—A mismatch?" Am. J. Phys. **61**, 295-298 (1993).

7. A small amount of internal thermal energy and wave energy was produced during the collision between the pendulum bob and the box as well. But a major amount of the initial kinetic energy of the bob could be converted to the internal potential energy of the bob-box system.

8. Van Heuvelen, A., "Experiment Problems for Mechanics," Phys. Teach. **33**, 176-180 (1995).

9. Van Heuvelen, Allen, L., and Mihas, P. "Experiment Problems for Electricity and Magnetism," Phys. Teach. **37**, 482-485 (1999).

10. A set of these experiment problems are available on the OSU Physics Education Research web site: http://www.physics.ohio-state.edu/~physedu/index2.html.

11. Some students' explanations, for instance, "the kinetic energy of the fist was 'lost' to deformation of the boxing bag, heat, or sound," were also included.

12. Van Heuvelen, A., Bulletin of the American Physical Society, 36, 1359 (April, 1991).

CHAPTER 6

CONCLUSION

In the preceding chapters, studies concerning the use of the multiple representations and visualizations in student learning of physics have been illustrated with specific examples from work and energy. As a body of formal, complex knowledge, physics is a difficult subject for many introductory students. To help them develop a meaningful understanding and acquire expertise in problem solving, it is very important to explore how to develop and use pedagogical representations and visualizations in student learning.

As part of ongoing research by the Physics Education Research Group at The Ohio State University (OSU), this dissertation project has been conducted to probe student conceptual understanding of the abstract concept of internal energy and their problem solving in the context of work and energy and to explore the use of visualizations and multiple representations in enhancing student understanding and in developing expert-like problem-solving skills. The investigations took place at OSU primarily with engineering students in calculus-based introductory physics classes and used the common methods of physics education research.

Individual student demonstration and think-aloud interviews were conducted to gain an in-depth understanding of student reasoning and problem solving. We found that

the individual student demonstration interviews, in particular, were useful in exploring student conceptual understanding of an individual topic. During the interview, demonstrations using real equipment serve the basis of communication between the researcher and the students. Students' reasoning could be probed thoroughly because the researcher was able to ask the students follow-up questions and to let them provide as detailed reasoning as possible. Individual student think-aloud interviews were suitable for probing student problem solving. In this setting, students verbalized aloud their thinking while solving the problem. During the interview, to avoid interrupting student thinking during the middle of the task, communication between the researcher and the students was minimized. But it was found that after the students completes the task, it was useful to ask them to report their thinking sequence experienced while solving the problem. This could serve three purposes. First, retrospective data could provide the researcher useful information on understanding students' think-aloud data. Second, the students often reported important, major thinking steps while being asked to recall what they remembered during the problem solving. This was very helpful in getting insight into higher-level problem solving cues that the student used. Third, the researcher had an opportunity to ask follow-up questions if needed. We also found that it was important to give the students warming-up problems, which could help them understand what they were expected to do during the "think-aloud" interviews and to get comfortable with *verbalizing* their *thinking* at the same time.

These interviews are useful tools to probe in-depth thinking from a small number of students. We also used written tests to access to a large number of students' ideas and to assess their performance in classroom settings. Detailed interview data help develop

written test questions and help understand students' reasoning that appeared in written tests, which consist of students' responses or solutions and often with less detailed information about the students' thinking. But written tests could be administered to a large number of students in naturalistic classroom settings, and test data could be easily analyzed. We used both research techniques in gaining an understanding of students' reasoning and problem solving skills.

The model used in the studies consists of three components: the initial state of student knowing, the desired state of student knowing, and the learning system transformer that can help fulfill the transformation from the former to the latter. To achieve insight into the student initial state (Ψ_0) of understanding, systematic investigations, including testing and interviewing, were conducted either before or after students experienced a particular topic in standard instruction. Understanding student's initial states of knowing, in particular, students' difficulties, was essential for developing the effective learning system transformer ($\hat{e}$). The desired state (Ψ) of student learning was carefully constructed so the learning system transformer was able to help students transform from the initial state to the desired state. After students had been exposed to the learning system, student understanding, as a final state, was investigated. The effectiveness of the learning system was assessed by determining how students' final state of understanding compared to the desired state. On the basis of the assessment results, combined with results of the research into the student initial state, the learning system transformer was modified and revised. Then the new learning system transformer interacted with students and a new cycle of this model began.

The concepts of work and energy consist of a fundamental and important part of introductory physics. Empirical data in this dissertation have shown that beginning introductory physics students solve work-energy problems using a numerical-representation-centered technique: represent related energy concepts in numbers first, assemble these numbers together based on the conservation of energy principle, and calculate the values for the unknown variables. We have found that, using this naïve approach, it is difficult for students to develop a conceptual understanding of the work-energy processes.

A multiple-representation approach developed by Alan Van Heuvelen views a work-energy problem as a physical process, which can be represented in verbal, pictorial, work-energy bar chart, and mathematical representations. This strategy attempts to help students develop expertise in problem solving—the use of physical representations to understand and reason about the problem before any mathematical equations are applied. Extensive assessment activities were conducted to evaluate the effectiveness of this technique for student work-energy problem solutions. Empirical evidence has shown that students do apply this strategy in their own problem solving. In particular, students use the work-energy bar charts 1) to understand conceptually and reason qualitatively about work-energy problems first, 2) to help set up the generalized work-energy equation correctly, and 3) to make inferences and evaluate their solutions.

Apparently, the work-energy bar charts, as a physical representation to represent work-energy processes, play a central role in student problem solving. Its function is like a bridge, leading students to move easily with smaller steps from an abstract verbal representation or a pictorial representation with surface features to a scientific,

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mathematical representation. By connecting pictorial and mathematical representations together, the work-energy bar charts also help students produce mental images for the different energy quantities and to make mathematical symbols meaningful.

As part of the research effort to develop and assess this multiple-representation strategy, student understanding of internal energy has been investigated. This is a very abstract and difficult concept for introductory students. Without a meaningful understanding of this concept, students use either surface features or common-sense ideas to understand the energy transformation during inelastic collisions.

We have found that introductory physics students have difficulty in conceptualizing "where the kinetic energy goes" during inelastic collisions. They consider "heat" as "internal thermal energy," but they do not understand either of them. They are unable to link the change in a solid's shape with a gain of its internal potential energy at an atomic level. In a word, students lack meaningful mental pictures of the concept of internal energy.

The use of visualizations is introduced to enhance student understanding of this abstract concept. The guided-inquiry learning activity was developed to help students visualize "invisible" internal thermal energy produced during inelastic collisions and the hands-on interactive laboratory activity was developed to help students physically see "the initial kinetic energy goes to internal potential energy" during a partially inelastic collision.

The guided-inquiry learning activity consists of two parts: student worksheets and demonstrations using a pair of rubber balls (happy and unhappy balls) and a pair of model carts. The student worksheets were developed under a guide for inquiry learning—*elicit,*

confront, resolve—used by the Physics Education Group at the University of Washington. The model carts were modified and constructed on the basis of Ganiel's work and empirical data collected in this dissertation. Assessment activities, including written tests and individual student demonstration interviews, have shown that this learning activity is helpful in students visualizing "internal thermal energy produced on a microscopic scale" during inelastic collisions and in developing a mental picture of this concept. It has been discovered that students' visualization of this concept helps them conceptually understand the conservation of energy during inelastic collisions.

In the hands-on interactive laboratory activity, the visual aid of a deformation of a sandbag is used to help students physically see "where the kinetic energy goes" and to understand the energy transformation during a partially inelastic collision. Assessment results have shown that the deformation of the colliding object, which models internal potential energy change, helps students (especially weak students) visualize and better understand the energy transformation during a partially inelastic collision and helps them develop a mental image about the abstract concept of internal (potential) energy.

In addition to curriculum development, classroom implementation is also an important component of the systematic learning system transformer. "Learning by passively listening" is not effective for most students in achieving a meaningful understanding in physics classes. Active learning is an essential base of the learning system. Also, students' difficulties must be explicitly addressed. Moreover, students learn to learn better if they understand the reasons for various pedagogical strategies.

In summary, this dissertation has demonstrated that it is possible to use multiple representations and visualizations to help students develop a meaningful understanding of

abstract concepts and acquire expertise in problem solving. To understand naïve student problem solving from a perspective of the multiple representations, two types of mathematical representations—algebraic and numerical representations—are distinguished for the first time in this dissertation. It has been discovered that naïve students use a numerical-representation-centered approach to solve work-energy problems. Empirical evidence has shown that the systematic strategy of representing work-energy processes in the multiple representations can help students abandon this naïve problem solving method and develop expertise in problem solving. This dissertation also presents a way to use visual aids in helping students gain a meaningful understanding of abstract concepts and in developing mental pictures. The visual aid of a deformation of a sandbag is first used to help students physically see "where the kinetic energy goes" and to understand the energy transformation during partially inelastic collisions. A pair of model carts, constructed and modified based on previous research and investigations done in this dissertation, is used to help students visualize the concept of internal thermal energy on a molecular scale.

APPENDIX A

WARM-UP QUESTIONS AND A WORK-ENERGY EQUATION SHEET USED FOR
THINK-ALOUD INDIVIDUAL STUDENT INTERVIEWS (AUTUMN 1999)

1. Warm-up questions

Problem 1: Estimate how far this room is above the ground in meters.

Problem 2: Calculate 1234 time 56.

Problem 3: If you were a skydiver and jumped out of a plane at 4000 meters, how long
would it take you to fall 1000 meters before you opened your parachute.

2. A work-energy equation sheet

Kinetic Energy	$$K = \tfrac{1}{2}\,mv^2$$ m = object mass v = object speed
Gravitational Potential Energy (near the surface of Earth)	$$U_g = mgy$$ m = object mass g = 9.8 m/s^2 y = the height of the object above or under the origin of the vertical direction
Elastic Potential Energy	$$U_s = \tfrac{1}{2}\,kx^2$$ k = force constant x = the compressed or stretched distances
Work	$$W = Fd$$ F = force exerted on the object d = distance of the object moving in the force direction
Conservation of energy	$$E_i + W = E_f$$ Initial energy + Work = Final energy

APPENDIX B

THREE THINK-ALOUD INDIVIDUAL STUDENT PRE-INTERVIEW PROBLEMS
(WITH WORK-ENERGY BAR CHARTS, AUTUMN 1999)

Problem 1

Problem Statement

A woman skier of mass 60 kg descends along the hill indicated in Fig. 1. She starts from rest at the point A at a height 40.0 m above the bottom point B of the hill, and jumps off at the point B. (The effects of friction are neglected.)
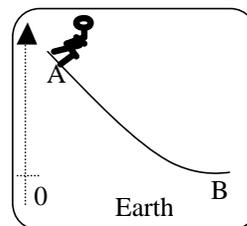
Fig. 1

Question 1.0: Complete the following work-energy bar chart.

Initial Energy + Work = Final Energy

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int}(\text{friction})$$

Question 1: What is the woman skier's speed at B?

Question 2: A male skier of mass 100 kg descends along the same hill indicated in Fig. 1. He starts from rest at point A as well, and jumps off at the bottom point B. Would his speed at point B be greater than, equal to, or less than the woman skier's speed at point B? Explain your reasoning.

Question 3: If the woman skier jumped off at point C that is above point B (see Fig. 2 below), would her speed at point C be greater than, equal to, or less than her speed at point B? Explain your reasoning.

Fig. 2

Problem 2

Problem Statement

A golf ball of mass 0.05 kg is thrown up vertically (see Fig. 1 below). The initial speed of the ball is 30 m/s. (The effects of friction from the air are neglected.)

Initial Energy + Work = Final Energy

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int}(\text{friction})$$

30 m/s

Earth

Fig. 1

Question 1: What is the maximum vertical height that the ball achieves?

Question 2: If the initial speed is doubled, how does the maximum vertical height change? Explain your reasoning.

Question 3: If a lighter ball is thrown up vertically with initial speed of 30 m/s. Would the maximum vertical height that this lighter ball attains be greater than, equal to, or less than that of the ball of 0.05 kg? Explain your reasoning.

Problem 3

Problem Statement

A HotWheel^{TM} toy car of mass 0.05 kg is launched by a spring to slide up along the smooth ramp which is 2.0 m long and 0.8 m high. The spring has its force constant of 100 N/m, and it is compressed by 0.2 m. The spring is initially held at rest. The ramp is fixed and 1.0 m above the floor (see Fig. 1). After the spring is released, the toy car moves up the ramp, jumps off, and eventually lands on the floor. (The effects of friction are neglected.)

Initial Energy +Work = Final Energy

$$K_o + U_{go} + U_{so} + W = K + U_g + U_s + \Delta U_{int}(\text{friction})$$

0.8

1.0

Fig. 1

Question 1: What is the speed of the toy car just before it hits the floor?

Question 2: If the height of the ramp is lowered, how would the speed of the toy car just before landing on the floor change? Explain your reasoning.

Question 3: If the spring is initially compressed more, how would the speed of the toy car just before landing on the floor change?

APPENDIX C

THREE THINK-ALOUD INDIVIDUAL STUDENT PRE-INTERVIEW PROBLEMS (WITHOUT THE WORK-ENERGY BAR CHARTS, AUTUMN 1999)

Problem 1

Problem Statement

A woman skier of mass 60 kg descends along the hill indicated in Fig. 1. She starts from rest at the point A at a height 40.0 m above the bottom point B of the hill, and jumps off at the point B. (The effects of friction are neglected.)

Fig. 1

Question 1: What is the woman skier's speed at B?

Question 2: A male skier of mass 100 kg descends along the same hill indicated in Fig. 1. He starts from rest at point A as well, and jumps off at the bottom point B. Would his speed at point B be greater than, equal to, or less than the woman skier's speed at point B? Explain your reasoning.

Question 3: If the woman skier jumped off at point C that is above point B (see Fig. 2 below), would her speed at point C be greater than, equal to, or less than her speed at point B? Explain your reasoning.

Fig. 2

Problem 2

Problem Statement

A golf ball of mass 0.05 kg is thrown up vertically (see Fig. 1 below). The initial speed of the ball is 30 m/s. (The effects of friction from the air are neglected.)

Fig. 1

Question 1: What is the maximum vertical height that the ball achieves?

Question 2: If the initial speed is doubled, how does the maximum vertical height change? Explain your reasoning.

Question 3: If a lighter ball is thrown up vertically with initial speed of 30 m/s. Would the maximum vertical height that this lighter ball attains be greater than, equal to, or less than that of the ball of 0.05 kg? Explain your reasoning.

Problem 3

Problem Statement

A HotWheel^{TM} toy car of mass 0.05 kg is launched by a spring to slide up along the smooth ramp which is 2.0 m long and 0.8 m high. The spring has its force constant of 100 N/m, and it is compressed by 0.2 m. The spring is initially held at rest. The ramp is fixed and 1.0 m above the floor (see Fig. 1). After the spring is released, the toy car moves up the ramp, jumps off, and eventually lands on the floor. (The effects of friction are neglected.)

0.8

1.0

Fig. 1

Question 1: What is the speed of the toy car just before it hits the floor?

Question 2: If the height of the ramp is lowered, how would the speed of the toy car just before landing on the floor change? Explain your reasoning.

Question 3: If the spring is initially compressed more, how would the speed of the toy car just before landing on the floor change?

APPENDIX D

DETAILED SOLUTIONS PRODUCED BY TWO STUDENTS FOR THE THINK-ALOUD INDIVIDUAL STUDENT PRE-INTERVIEW PROBLEMS
(WITH WORK-ENERGY BAR CHARTS, AUTUMN 1999)

| Problem 1
Problem statement and questions | Student detailed reasoning
& performance | Representations that the student
used to answer each question |

| Problem statement | → | Read the statement, and looked at the sketch.
(Not make his own sketch) | → | Verbal Representation

Pictorial Representation |

Q1.0: Qualitative Question → Understood each term and the structure of the work-energy bar charts and completed the bar charts correctly. → Physical Representation

Q1: Quantitative Question → Tried to recall and to use kinematics equations.

Asked by the interviewer to use the concepts of work and energy

Used work-energy formula in an equation sheet. Wrote down U = mgh, plugged in numbers, and calculated U.
Then wrote down K = 1/2mv^2 , plugged in numbers, and calculated v. → Numerical Representation

Q2: Qualitative Question → Looked at his numerical solution for the initial potential energy, compared the woman's mass with the man's mass, and compared their initial potential energy. → Numerical Representation

Found his own calculation error made in his solution for question 1.

Used U = mgh, plugged in numbers, calculated both woman's and man's initial potential energy, and concluded that man's kinetic energy at B was greater. So his speed was greater.
(Incorrect answer.) → Numerical Representation

Found mass included in his numerical solution of the kinetic energy from question 1.
Then used k=1/2mv^2, plugged in numbers, calculated man's & woman's speed, and concluded that they were the same. → Numerical Representation

Q3: Qualitative Question → (Looked at the sketch given in the statement) Used kinematics ideas, broke v into x & y components, reasoned that gravitational force pulled down in y, decided y component of v at C less, and concluded that v at C was less → Pictorial Representation

Problem 2 **Problem statement and questions**	**Student detailed reasoning & performance**	**Representations that the student used to answer each question**

| Problem statement | Read the statement, and looked at the sketch
(Not make his own sketch and not use the work-energy bar chart) | Verbal Representation
Pictorial Representation
Physical Representation |

| Q1: Quantitative Question | Used work-energy formula in worksheet.
Wrote down K = 1/2mv^2, plugged in numbers, and calculated K (K = 22.5 J).
Then wrote down 22.5 J = mgy , plugged in numbers, and calculated y. | Numerical Representation |

| Q2: Qualitative Question | Wrote down K = 1/2(0.05 kg)(60 m/s)^2, and calculated out K (K = 90 J).
Then wrote down 90 J = mgy , plugged in numbers, and calculated y. | Numerical Representation |

Q3: Qualitative Question	"I guess what I am thinking qualitatively that I throw up a golf ball and baseball as height as possible. I can throw the golf ball higher. The lighter ball goes higher."	(Internal) Pictorial Representation
	Then he had a long pause "Thinking about formula in my head, since I know acceleration of gravity is the same on all objects . They could possibly have the same height. " **(He commented that "this is the concept learned from school, but it is not something natural .)**	(Internal) Mathematical Representation
	"It is the best & easy way to calculate it mathematically." (Given a number for lighter ball's mass = 0.02 kg) Wrote down K = 1/2mv^2 = 1/2(0.02 kg)(30)^2, and calculated K [K = 9 J]. Then wrote down 9 J = mgy , plugged in numbers, and calculated y.	Numerical Representation
	[After this, he realized that mass did not matter] He wrote down 1/2mv^2 = mgy, and cancelled out m. "Mass does not matter. They all go the same height.	Algebraic Representation

Problem 3 Problem statement and questions	Student detailed reasoning & performance	Representations that the student used to answer each question

Problem statement

Read the problem and used the given sketch to understand the problem (Not make his own sketch])

Read the given work-energy bar chart s and matched bars with the concepts of energy in the initial and final states .

- Verbal Representation
- Pictorial Representation
- Physical Representation

Q1: Quantitative Question

"I'm thinking what equations to be used." (He looked at the given equation sheet.)

Then looked at the sketch to try to understand what types of energy that car had initially. [He understood spring potential energy, but was confused with gravitational potential energy 1) if it should be zero since origin at initial position or 2)if it should be NOT zero since it is ABOVE floor.]

Then found that in the given work-energy bar charts the final potential energy bar was negative. So he decided the potential energy must be zero initially.

Looked at the equation sheet, wrote down U = 1/2kx^2, and plugged in numbers.

Looked at given work-energy bar charts, and realized that the energy was conserved.

Then wrote down E = 1/2mv^2 - mgy ,and plugged in numbers.

- Mathematical Representation
- Pictorial Representation
- Physical Representation
- Numerical Representation
- Physical Representation
- Numerical Representation

Q2: Qualitative Question

Attempted to use his solution of question 1, but figured out he did not plug in the height of ramp in his solution.

 Looked at the work-energy bar chart. "So it (height of ramp) does not matter, but I doubt."

"But thinking qualitatively, v might be the same. Since I visualize if the ramp height is lowered, its speed at top of ramp would be greater, and the car can go farther horizontally, but slower in vertically. So they trade off. "

- Mathematical Representation
- Physical Representation
- (Internal) Pictorial Representation

Q3: Qualitative Question

Used equations in his solution for question 1, reasoned that , if elastic potential energy was increased, the speed of the car was increased.

 Looked and used the work-energy bar chart to check his answer.

- Algebraic Representation
- Physical Representation

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| Problem 1 Problem statement and questions | Student detailed reasoning & performance | Representations that the student used to answer each question |

Problem statement → Read the statement, and looked at the sketch (Not make his own sketch) → Verbal Representation / Pictorial Representation

Q1.0: Qualitative Question → Understood each term in the work-energy bar chart.

Calculated initial potential energy as:
60(10)40 = 24000 J.

"Each bar [unit] represents 6000J, so 4 bars for potential energy. It's all converted to kinetic energy, and no friction, so energy is conserved. "
→ Numerical Representation / Physical Representation

Q1: Quantitative Question → Wrote down Ug = 60(40)(10) = 24000 J.
Then 24000 J = 1/2(60)v^2 , and calculated out v. → Numerical Representation

Q2: Qualitative Question → Looked at his numerical solution for the initial potential energy.

"His initial potential energy is greater since it depends on mass and his mass is greater. Since energy is conserved, his final kinetic energy is greater. So his speed is greater." → Numerical Representation

Q3: Qualitative Question → (Used the sketch given in the statement)

"Her initial potential same, 24000J. All turns to kinetic energy at B. Then she's back up a little bit, and the kinetic energy turns to potential energy. So her speed is less at C. → Pictorial Representation

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Problem 2 **Problem statement and questions**	**Student detailed reasoning & performance**	**Representations that the student used to answer each question**

Problem statement

Read the statement, and looked at the sketch
(Not make his own sketch and not use the work-energy bar charts)

Verbal Representation

Pictorial Representation

Physical Representation

Q1: Quantitative Question

Looked at work-energy formula in the given equation sheet.

Wrote down 1/2(0.05)(30)^2 = 22.5 J,
            (0.05)(10)(y) = 22.5 ,
and calculated out y

Numerical Representation

Q2: Qualitative Question

Wrote down K = 1/2(0.05 kg)(60 m/s)^2 = 90
Then wrote down 90 = 0.05(10)(y),   and calculated y.

Numerical Representation

Q3: Qualitative Question

(Given a number for mass = 0.025 kg--half mass of golf ball)

Wrote down 1/2(0.025)(30)^2 = 11.25,
           11.25 = (0.025)(10)(y),
and calculated y.

Numerical Representation

| **Problem 3**
Problem statement and questions | **Student detailed reasoning**
& performance | **Representations that the stude**
used to answer each question |

Problem statement → Read the problem and used the given sketch to understand the problem [Not make his own sketch].

[He did not look at the given work-energy bar chart]

→ Verbal Representation

Pictorial Representation

Physical Representation

Q1: Quantitative Question → [He looked at the equation sheet]
Wrote down 1/2(100)(0.2)^2 = 2 J for the spring potential energy. [Then he got a pause.]

[Then he set up this spring potential energy equal to a gravitational potential energy]. Wrote down as: 2 J = (0.05)(10)(h), and calculated out h = 4m.

Back to read the problem statement. Then realized h was the maximum height to shoot straight up.

[A long pause. He looked at the sketch. Then he realized that at the top there was some kinetic energy in horizontal direction and the car fell anther meter below]

Then wrote down
1/2(0.05)v^2 = 2 + (0.05)(9.8)(1),   and calculated out v.

→ Numerical Representation

→ Numerical Representation

→ Verbal Representation

→ Pictorial Representation

→ Numerical Representation

Q2: Qualitative Question → (He used his solution of question 1 to reason.]
Since final kinetic energy is equal to the initial elastic potential energy plus the gravitational potential energy, the height of the ramp makes on difference in the equation. It only changes...it will go further in x, but slower in y direction. Velocity would not change."

← Mathematical Representation

Q3: Qualitative Question → [He used his solution of question 1 to reason.]

"Since the spring is compressed more, it has more elastic potential energy. Since initial energy is greater, kinetic energy will be greater because of conservation of energy. Kinetic energy is greater, v will be greater."

→ Mathematical Representation

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APPENDIX E

DETAILED SOLUTIONS PRODUCED BY ONE STUDENT FOR THE THINK-
ALOUD INDIVIDUAL STUDENT PRE-INTERVIEW PROBLEMS
(WITHOUT THE WORK-ENERGY BAR CHARTS, AUTUMN 1999)

Problem 1 **Problem statement and questions**	**Student detailed reasoning & performance**	**Representations that the student used to answer each question**

Problem statement → Read the statement, and looked at the sketch (Not make his own sketch) →
- Verbal Representation
- Pictorial Representation

Q1: Quantitative Question → Wrote down U = mgh, plugged in numbers, and calculated out U. Then wrote down K = 1/2mv^2, plugged in numbers, and calculated out v. → Numerical Representation

Q2: Qualitative Question → "If I set up these two equations equal to each other, I got mgh = 1/2mv^2 . The mass is dropped out. The speed is equal." → Algebraic Representation

Q3: Qualitative Question → (Looked at the sketch given in the statement) "Her speed at C is going to be less than that at B because a point B she got her maximum amount of kinetic energy, which is her maximum speed. When she got to C, she's going to lose some of that kinetic energy, and return back to potential energy. And that's going to cause decreasing velocity." → Pictorial Representation

Problem 2 Problem statement and questions	Student detailed reasoning & performance	Representations that the student used to answer each question

Problem statement

Read the statement, and looked at the sketch (Not make his own sketch)

Verbal Representation

Pictorial Representation

Q1: Quantitative Question

Wrote down K = 1/2mv^2, plugged in numbers, and calculated out K.
Then wrote down K = mgh, plugged in numbers, and calculated out h.

Numerical Representation

Q2: Qualitative Question

"If we double the initial speed, it's going to have a double... (looking at his solution) it's going to be four times of initial kinetic energy. Kinetic energy is a squared function, and potential energy is a linear function. So it is going to have... (a pause). The maximum height is going to a lot higher. I am not going to say double, but 4 times possible, but I am not sure".

Wrote down K = 1/2(0.05)(60)^2, and calculated out K.
Then wrote down 90 = (0.05)(10)h, and calculated out h.

Numerical Representation

Numerical Representation

Q3: Qualitative Question

"The same situation. The initial energy is going to be 1/2mv^2. That's all translated to potential energy mgh. The mass is cancelled out. It is going to have the same height."

Algebraic Representation

Problem 3 Problem statement and questions	Student detailed reasoning & performance	Representations that the student used to answer each question

Problem statement → Read the statement, and looked at the sketch .
Then made his own sketch & labeled it. →

- Verbal Representation
- Pictorial Representation

→ Pictorial Representation

Q1: Quantitative Question →

Calculated initial potential energy: U1 = (0.05)(10)(1) = 0.5
Calculated potential energy at the top of the ramp: U2 = (0.05)(1.8)(10) = 0.9.
Calculated spring potential energy: Us = 1/2kx^2= 1/2(100)(0.2) ^2 = 2.

"At the top of the hill, potential energy is going to be...[pause] I am trying to think If the energy is going to be zero. it has potential energy up to here. it also has kinetic energy. This kinetic energy is going to equal to the difference of these two potential energy, plus the work done by spring. "
Then he wrote down : 2 - 0.4 = 1.6,   1.6 = 1/2(0.05)V^2 to calculate v.

Then he started to use kinematics equations to solve the projectile motion.

→ Numerical Representation
→ Numerical Representation
→ Algebraic Representation

Q2: Qualitative Question →

"If we lower the height, [looked at his sketch], you lower the potential energy. If you move down like that [drew it on the sketch], the ramp is lowered. It is going to have... the spring is going to do less work to overcome this change of potential energy. Then it is going to have more available for kinetic energy. [Pause].

"This height is 0.8 before, now assume 0.4. Then the potential energy at here [new ramp top] is going to be less. Overall the system is going to have less energy. So I think the final velocity is going to be lower. "

→ Pictorial Representation
→ Pictorial Representation

Q3: Qualitative Question →

"If the spring is compressed more, more work done on the system. And more energy in the system. Automatically entire energy of the system eventually will convert to kinetic energy. So it has more kinetic energy, then greater speed."

→ Pictorial Representation

APPENDIX F

THREE THINK-ALOUD INDIVIDUAL STUDENT FOLLOW-UP INTERVIEW
PROBLEMS (AUTUMN 1999)

Problem 1

Problem Statement

Two snow sleds, A and B, are placed at rest on the starting line on a very icy lake. Sled A is four times as massive as sled B. Two identical engines fixed on the sleds supply equal forces to each sled, continuously pushing each sled towards the finish line.

Question 1: Which sled has the greater kinetic energy when it reaches the finish line? Explain your reasoning.

Question 2: Which sled has the greater speed when it reaches the finish line? Explain your reasoning.

Question 3: Now a new finish line for sled A is set four times farther. Sled A, starting at the same starting line, is pushed continuously by the same force towards the new finish line. When sled A reaches its new finish line, would the kinetic energy of sled A be same, greater, or less than the kinetic energy of sled B when it reaches the original finish line? Explain your reasoning.

Problem 2

A spring of force constant 100 N/m is horizontally fixed to a smooth table. The top of the table is 1.0 m above the floor. The spring is initially compressed 0.2 m. After the spring is released, a HotWheel^{TM} toy car of mass 0.05 kg is launched by the spring. After the toy car moves 0.5 m away from the spring, the car goes up a smooth ramp fixed to the same table top. The ramp horizontally makes an angle of 30^0 with the table top, and the ramp top is 0.8 m above the table top. After the toy car moves up the ramp, it flies off the ramp top, and eventually lands on the floor. **(The effects of friction are neglected. g = 10 m/s^2)**

Question 1: What is the speed of the toy car just before it hits the floor?

Question 2: If the angle that the ramp makes with the table top is decreased, will the speed of the toy car, just before it lands on the floor, be the same, greater, or less than the speed in Question 1? Explain your reasoning.

Question 3: If the height of the table is lowered, will the speed of the toy car, just before it lands on the floor, be the same, greater, or less than the speed in Question 1? Explain your reasoning.

Problem 3

Problem Statement
A company hires you to design a new concept in roller coasters. They first ask you to test out the design in a lab setting. To do this, you are given the following information, equipment, and models of the design.

A spring of force constant 80 N/m is fixed to a horizontal model plateau that is 1.0 m above the floor. The model track goes down from the plateau, and then forms two additional hills and a horizontal end part on the floor. Initially, the spring is compressed 0.2 m. After the compressed spring is released, a model cart of mass 0.1 kg is launched to move off the plateau and to ride on the track.

Question 1: The first hill top is 0.5 m tall above the floor. What maximum height could the second hill have so that the cart just makes it over the second hill? (The effects of friction can be neglected for this question. $g = 10$ m/s^2)

Question 2: Now, let the second hill be 1.6 m tall. The first hill is still 0.5 m tall. The end part of the track is given 8.0 m long, laying flat on the floor. What is the minimum frictional force needed to make the cart come to a stop at the very end? (The effects of friction can be neglected, except for the end part of the track. $g = 10$ m/s^2)

Question 3: If the height of the second hill in Question 2 is greater, will the minimum frictional force needed to stop the cart at the very end be the same, greater, or less than the minimum frictional force needed in Question 2? Explain you reasoning.

APPENDIX G

DETAILED SOLUTIONS PRODUCED BY ONE STUDENT FOR THE THINK-ALOUD INDIVIDUAL STUDENT FOLLOW-UP INTERVIEW PROBLEMS (THIS STUDENT LEARNED THE CONCEPTS OF WORK AND ENERGY USING THE MULTIPLE-REPRESENTATION STRATEGY, AUTUMN 1999)

Problem 1 (Problem statement and questions	Student detailed reasoning & performance	Representations that the student used to answer each question

Problem statement → Read the problem. Constructed a sketch and labeled related variables → Verbal Representation

Pictorial Representation

Q1: Qualitative Question → Started with the definition of kinetic energy, 1/2mv^2.

Applied Newton's 2nd law and kinematics equations to derive speeds of Sleds A and B at the finish line. → Plus the speeds into the kinetic energy equations and then compared the kinetic energies. (Correctly answered the question.) → Mathematical representation

Q2: Qualitative Question → Used the mathematical representation from his answer of question 1 to reason this question. . (Correctly answered the question.)

Q3: Qualitative Question → Used the mathematical representation from his answer of question 1 to reason this question. (Correctly answered the question.)

Problem 2 **(Problem statement and questions**	**Student detailed reasoning & performance**	**Representations that the student used to answer each question**

Problem statement

Read the problem.

Constructed a sketch and labeled related variables

Verbal Representation

Pictorial Representation

Q1: Quantitative Question

Construct the work-energy bar chart.

Followed the bars in the chart to set up the generalized the work-energy equation. Plugged in numbers and solved for the unknown quantity.

(He correctly answered the question)

Physical Representation

Algebraic Representation

Numerical Representation

Q2: Qualitative Question

Used the generalized work-energy equation from question 1to answer this question.
(He correctly answered this question)

Q3: Qualitative Question

Used the work-energy bar chart to reason about this question.
(he correctly answer this question)

Problem 3 (Problem statement and questions	Student detailed reasoning & performance	Representations that the student used to answer each question

Problem statement

Read the problem.

Constructed a sketch and labeled related variables

Verbal Representation

Pictorial Representation

Q1: Quantitative Question

Construct the work-energy bar chart.

Followed the bars in the chart to set up the generalized the work-energy equation. Plugged in numbers and solved for the unknown quantity.

(He correctly answered the question)

Physical Representation

Algebraic Representation

↓

Numerical Representation

Q2: Quantitative Question

Construct the work-energy bar chart.

Followed the bars in the chart to set up the generalized the work-energy equation. Plugged in numbers and solved for the unknown quantity.

(He correctly answered the question)

Physical Representation

Algebraic Representation

↓

Numerical Representation

Q3: Qualitative Question

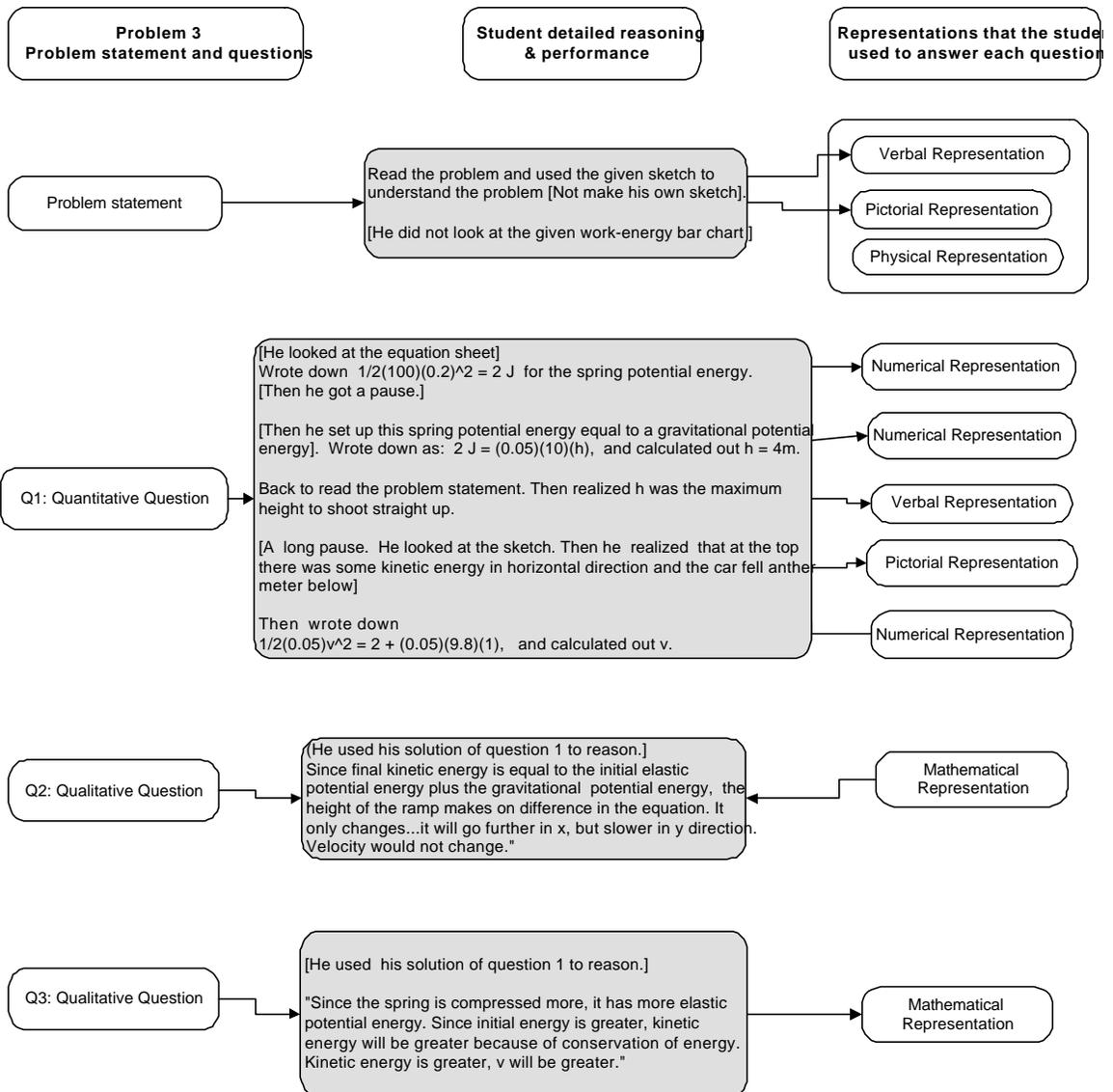Used the generalized work-energy equation for question 2 to reason about this question. (he correctly answered this question)

APPENDIX H

DETAILED SOLUTIONS PRODUCED BY ONE STUDENT FOR THE THINK-
ALOUD INDIVIDUAL STUDENT FOLLOW-UP INTERVIEW PROBLEMS
(THIS STUDENT LEARNED THE CONCEPTS OF WORK AND ENERGY USING
OTHER APPROACHES, AUTUMN 1999)

Problem 1 **Problem statement and questions**	**Student detailed reasoning & performance**	**Representations that the student used to answer each question**

Problem statement → Read the problem. (No sketch) → **Verbal Representation**

Q1: Qualitative Question → Looked at the equation sheet. Wrote down W = Fd. "They travel the same distances, and the forces supplied are the same. So the work done is the same. But it's not kinetic energy. (Pause) But the work done equals to the change of kinetic energy." → **Mathematical representation**

"Now I' thinking about the force. Force equals ma. (wrote down F=ma.) They have different masses. So their accelerations will be different. The more massive one slows down."

Wrote down K=1/2mv^2 for both sleds A and B. Plus mass into the kinetic equations. Then she commented:"if speeds for sleds A and B are the same, then they have the same kinetic energy at the finish line. But I don't know if their speeds are same or not. "

Used F =ma and kinematics equations to derive velocities of Sled A and B at the finish line. Her final results for speed were Va = (F/4Mb)t, Vb = (F/Mb)t.

(She subconsciously assumed that time t is the same for both sleds A and B.)

Plus the velocities into the kinetic energy equations.

(She gave an incorrect answer that sled B have greater kinetic energy at the finish line.)

Q2: Qualitative Question → Start with speed equations from Question 1. Va = (F/4Mb)t, Vb = (F/Mb)t. (incorrectly answered that the sled B's speed is less.) → **Mathematical representation**

Q3: Qualitative Question → Thought about distance, then used a work idea. Wrote down Wa =4Wb

Then looked at the equation sheet, and linked the work done with a change in kinetic energy. She wrote down W = dK/dt.

Then used the final kinetic energy expression from Question 1, plus into the work expression.

(Incorrectly answered that the kinetic energy of sled A was the same as that of sled B.) → **Mathematical representation**

| **Problem 2**
Problem statement and questions | **Student detailed reasoning**
& performance | **Representations that the student**
used to answer each question |

Problem statement

Read the problem.

Constructed a sketch and labeled related variables

She said:"I would have a hard time without drawing a picture."

Verbal Representation

Pictorial Representation

Q1: Quantitative Question

Started with thinking about the goal: toy car's kinetic energy just before hitting the floor, then looked the ramp.
"Using energy, at the top of the ramp there is only potential energy. At the floor level, only kinetic energy."

Wrote down P = mgy = (0.05 kg) (10 m/s^2) (1.8 m) = 0.9 J.
Then wrote down K = 0.9 J, 1/2mV^2 = 0.9 J , and solved for v.

(Incorrect answered this question)

Numerical Representation

Q2: Qualitative Question

Used her sketch to answer the question qualitatively.

(Incorrectly answered this question.)

Q3: Qualitative Question

Used her sketch to answer the question qualitatively.

(Incorrectly answered this question.)

Problem 3 Problem statement and questions	Student detailed reasoning & performance	Representations that the student used to answer each question

Problem statement → Read the problem.

Constructed a sketch and labeled related variables

→ Verbal Representation

→ Pictorial Representation

Q1: Quantitative Question → Started with a work-energy idea. Tried to identified what types of energy that the model cart had at the point leaving the plateau, at the top of the first hill, and at the top of the second hill.

P1 = mgh
= (0.1 kg)(10 m/s^2)(1m)
= 1 kg m/s^2 (1 m)
= 1 J.

Us = 1/2 (80 N/m)(0.2 m)^2
= 40 N/m (0.04 m^2)
= 1.6 J
K = 1.6 J

P = 1 J + 1.6 J = 2.6 J
mgh = 2.6 J
h = 2.6 m
(Correctly answered the question)

→ Numerical Representation

Q2: Quantitative Question → Construct a sketch and labeled related information.
Used U = 2.6 J from her solution for Question 1.
Calculated the potential energy for the second hill with the new height: P = (1.6 m)(10 m/s^2)(0.1 kg) = 1.6 J.
Then wrote down K = 2.6 J for the kinetic energy at the beginning of the track.

Wrote down
1/2mv^2 = 2.6 J
mv^2 = 5.2 J
v^2 = 52 m^2/s^2
Then realized that v was not the desired variable.

Calculated the change in kinetic energy, and used the work-energy theorem to solve the question.
Change of kinetic energy = 2.6 J
W = 2.6 J, d = 8.0 m,
W = Fd, F = W/d = 0.325 N.
(Correctly answered the question)

→ Numerical Representation

Q3: Qualitative Question → Used the numerical representation from Question 2 to reason about this question.
(Correctly answered this question)

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APPENDIX I

INDIVIDUAL STUDENT DEMONSTRATION INTERVIEW PROTOCOLS
(SPRING 1998)

Part 1. Introduction

Part 2. Demonstrations and questions

Demonstration 1: Drop a happy ball and an unhappy ball onto a table from the same height

- These are two "identical-looking" black balls. If I drop them from this height simultaneously, what would happen to them? Explain your reasoning.

- Please provide reasons why you think one of the balls did not bounce back, but the other did.

- What kind(s) of energy did each ball have just before they hit the table?

- What do you think happened to the energy of the non-bouncing ball just after the collision?

Demonstration 2: Drop an unhappy and a modeling clay ball from the same height

- If I drop this unhappy ball and this modeling clay ball from this height simultaneously, what would happen to them? Explain your reasoning.

- What happened to the shape of each ball after the collision?

- What kind(s) of energy did each ball have just before it hit the table?

- What do you think happened to the energy of each ball just after the collision?

Demonstration 3. Show the two model carts. They have identical mass, bumpers, and wheels

- If these two carts are pushed toward that wall with the same initial speed, what would happen to them? Explain your reasoning.

- Could you please provide reasons why you think one cart bounced back a significantly shorter distance than the other one?

- What do you guess about inside of the carts?

Demonstration 4. Take off the two covers from the model carts

- Again, if these two carts are pushed toward that wall with the same initial speed, which one would you predict bounce back a significantly shorter distance? Explain your reasoning.

- Could you please provide reasons why you think this rubber-band cart bounced back a significantly shorter distance than this rigid-rod cart did?

- What kinds of energy did each cart have just before they collided with the wall?

- Did you observe the small weights on the rubber bands randomly vibrate much faster after the collision than before? Where did the small weights get kinetic energy to vibrate?

- Now think about the carts modeling the molecular structure of a solid. Could you match the black balls with these two carts? Explain your reasoning.

- Consider the two carts as a simple analogy of the two black balls. Now explain again why the "unhappy ball" died on the table after the collision.

APPENDIX J

STUDENT WORKSHEETS DEVELOPED IN THE AUTUMN QUARTER OF 1998

1. Two "identical" black balls are dropped from some height onto a table surface. Write down what you see before and after the collisions.

2. Provide one or more reasons why you think one of the balls did NOT bounce back, but the other did.

3. What kind(s) of energy did each ball have just before they collided with the surface?

4. What do you think happened to the energy of the "unhappy" ball just after the collision (the "unhappy" ball is the one that did not bounce)?

5. Now we have **two carts A and B,** and each of them has the same total mass. They both have the same spring bumpers in the front of their base frames. Make a prediction about **what** will happen to each cart if they roll on a smooth table with the same initial speed and then collide with very heavy blocks in the way? Explain why this will happen.

6. Now watch the two carts hit the heavy blocks with the same initial speeds. Write down what you observed, and provide one or more reasons why that happened.

7. Now the instructor takes off the covers on the carts. Match the carts with your observation, that is, which cart bounced back a long distance, and which cart bounced back a very short distance? Provide your reasoning.

8. Now again watch the two carts roll on the smooth surface with the same initial speed and then collide with the heavy blocks. This time observe and write down what happens to the small **brass disks** on each cart before and after the collisions.

9. Did you observe the brass disks on the rubber bands vibrate much faster just after the collision than before? Where did the brass disks get kinetic energy to vibrate? Did the brass disks on the other cart vibrate and have kinetic energy, too?

10.The microscopic molecular structure inside solids such as the balls can be modeled as molecules connected by bonds that are similar to springs (see the figure). For example, the bonds of some solids are so soft that individual molecules can move and rotate around easily. In contrast, the bonds inside some other solids are very stiff, and individual molecules can NOT easily move and rotate around. So now using this model, try to explain what happened to the initial energy of the rubber-band cart after the collision.
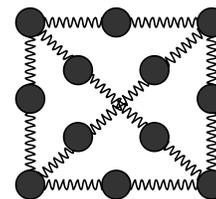
11. Now think about the "carts-model" for the molecular structure of the black balls above. Please match the carts with the balls, that is, which cart models which ball. Why?

12. Kinetic energy that individual molecules have is one example of **internal energy**. The faster molecules inside a solid move and rotate, the larger their internal energy, and the "hotter" the solid is. So now, using the concept of internal energy, explain why the "unhappy" ball did not bounce back, but the "happy" ball did.

APPENDIX K

STUDENT WORKSHEETS DEVELOPED IN THE AUTUMN QUARTER OF 1999

1. Two "identical" black balls are dropped from some height onto a table surface. Observe and write down what happens to the balls after the collisions.

2. Provide one or more reasons why you think one of the balls did NOT bounce back, but the other did.

3. What kind(s) of energy did each ball have right before they collided with the surface?

4. What do you think happened to the energy of the "unhappy" ball just after the collision (the "unhappy" ball is the one that did not bounce)?

|	
5. Now we have two carts A and B. Each of them has the same total mass, and the same spring bumper in the front of its base frame. Now watch the two carts roll on a smooth surface with the same initial speed, and collide with a fixed block. Observe and write down what happens to cart A and B after the collisions.	6. Provide one or more reasons why you think one of the carts bounced back significantly less than the other cart.
7. Now the covers on cart A and cart B are taken off. We can see that each of the carts has a similar structure. But one has small red disks tied to rubber bands, and other has small red disks fixed to very stiff light rods. Which cart do you think is the one that bounced back significantly less after the collision? Explain your reasoning.	8. Now again watch the two carts roll on the smooth surface with the same initial speed and then collide with the block. This time observe and write down what happens to the carts and to the small red disks after the collisions.
9. Did you observe the red disks on the rubber bands randomly vibrate much faster after the collision than before? Where did the red disks get kinetic energy to vibrate? Did the red disks on the other cart randomly vibrate, too?	10. Please explain why the cart with rubber bands bounced back significantly less than the other cart.

11. The molecular structure inside a solid can be simply modeled as molecules connected by bonds that are similar to springs (see the figure). The average random kinetic energy that individual atoms have is one example of internal energy. The faster the molecules inside a solid randomly move or vibrate, the greater its internal energy, and the "hotter" the solid is (i.e., the higher temperature the solid has). But if the molecules inside a solid lack freedom to vibrate, energy transfer into this type of internal energy cannot occur during a collision.

Now think about the carts modeling the molecular structure of a solid. Which cart gained the internal energy during the collision? Explain your reasoning.
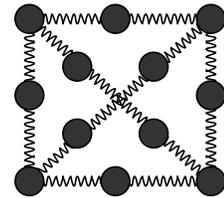
12. We can consider the two carts as a simple analogy of the two black balls. Now explain why the "unhappy" ball died on the surface after the collision.
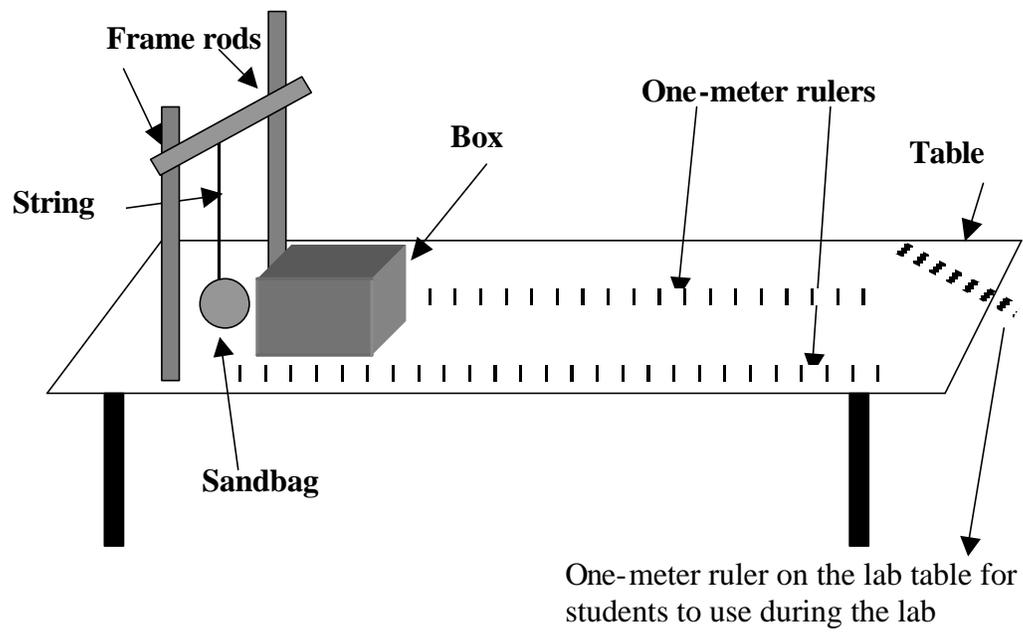
13. Now using the concept of this type of the internal energy, explain which ball and cart have a more **elastic** collision and which have a more **inelastic** collision? Explain your reasoning.

APPENDIX L

PENDULUM-BOX BASH EXPERIMENT PROBLEM SETUP

1. A sketch of the setup

One-meter ruler on the lab table for
students to use during the lab

2. A picture of the actual setup in a laboratory setting

3. Detailed apparatus information

Sandbag: Mass = 155 ~ 165 grams. Commercially-available "stress reliever" squeeze toys work well for this.

Box: Mass = 290 ~ 310 grams. Use any convenient cardboard box and fill with masses until the correct mass is reached. The sandbag should stop after the collision, and not rebound or keep moving forward.

String: Length = about 80 cm, measured from the pivot point to the center of mass of the sandbag. Tie one end of the sandbag to the pivot point (a rod or dowel), and have an alligator clip at the other end of the string. The clip holds the sandbag, and allows the sandbag to be removed for massing.

Rulers: Three metersticks are needed for this lab. Two should be taped down parallel to each other on either side of the box, to create a track for the box. They should not be too close to the sides of the box, however. The zero end of these metersticks should be directly under the pivot point or the pendulum. The third meterstick is provided for measuring other quantities.

APPENDIX M

131 LAB MANUAL USED IN THE SPRING QUARTER OF 1999

PENDULUM-BOX BASH FRICTION MEASURER

A company wants to hire a group to design experiments to measure friction between different surfaces. As part of the interview process, they give your group a specific task. You are to determine the coefficient of kinetic friction between a box and a horizontal table across which it will slide. The unusual part is that you have to determine it using the following technique. A pendulum bob swings down and hits the box, which is resting on the table. The box slides across the table and stops. Use any measurements you wish with the provided apparatus and the theories of physics to determine the coefficient of kinetic friction between the table and the box. (The determination has to be based on this experiment and not on some other experiment of your own design.) Be sure to provide a clear solution to this fun problem. **After you have completed the solution, check your work with the lab instructor. Then try to answer the questions on the next two pages, and check your answers with your partners first and then with the lab instructor.**

Name: _____

Answer the following questions based on your lab work. Please clearly give your explanations.

(a) Is the kinetic energy of the pendulum bob right before the pendulum-box collision *greater than, equal to or less than* the kinetic energy of the box right after the pendulum-box collision? Show all of your work and the reasoning to support your answer.

(b) If the kinetic energy of the pendulum bob right before the pendulum-box collision *is NOT equal to* the kinetic energy of the box right after the pendulum-box collision, by how much did it change? Explain where it goes if getting less or explain where it comes from if gaining more.

(c) Is the magnitude of the momentum of the pendulum bob right before the pendulum-box collision **greater than, equal to or less than** to the magnitude of the momentum of the box right after the pendulum-box collision? Please clearly explain your reasoning.

(d) Is the direction of the momentum of the pendulum bob right before the pendulum-box collision **the same as or opposite** the direction of the momentum of the box right after the pendulum-box collision? Please clearly explain your reasoning.

(e) If the magnitude of the momentum of the pendulum bob right before the pendulum-box collision *is NOT equal to* the magnitude of the momentum of the box right after the pendulum-box collision, by how much did it change? Explain where it goes if getting less or explain where it comes from if gaining more.

(f) What physics term describes the kind of the pendulum-box collision in this case? Please briefly explain your choice.

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