



PHYSLETS FOR QUANTUM MECHANICS

By Mario Belloni and Wolfgang Christian

EDUCATORS HAVE OFTEN PINNED THEIR HOPES OF BETTER INSTRUCTION ON EMERGING TECHNOLOGIES SUCH AS TELEVISION, COMPUTERS, AND THE WORLD WIDE WEB. YET TEACHING WITH TECHNOLOGY—WITHOUT A SOUND PEDAGOGY—YIELDS NO significant educational gain.¹

In addition, students often approach typical end-of-chapter textbook problems by finding a formula that contains the variables given in the problem statement, rather than first determining a problem's conceptual foundation.^{2,3} We use the technology of Physlets combined with pedagogical techniques such as Just-in-Time Teaching (JiTT) to create alternative problems that we believe help students better develop their problem-solving ability and deepen their conceptual understanding.

The Philosophy of Physlets

Physlets—physics applets—are small, flexible Java applets usable in a wide variety of Web applications.⁴ They have attributes that make them especially valuable for science education:

- They're simple. Each Physlet deals with only a single facet of a physical phenomenon, which keeps them relatively small and eases download problems over slow networks.
- They're visual and interactive. By having students decide what measurements to make and what variables to change, and by providing them with real-time feedback, the students

are in control of the exercise (see Figure 1). A visual and interactive exercise enriches student understanding far more than if an instructor just uses a page of text to explain the physics.

- They're flexible. For example, you can use the Physlet called Animator for almost any subject in mechanics with small changes in the JavaScript (not the Java) associated with each exercise.⁵⁻⁷
- Physlet-based pedagogy is agnostic. You can use them as elements of almost any curriculum with almost any teaching style. Although we believe that interactive engagement methods⁸ such as JiTT,⁹ Peer Instruction,¹⁰ and Tutorials¹¹ can improve pedagogy, you can use Physlets as traditional lecture demonstrations and give them as end-of-chapter homework problems.
- They're Web based, which means they can run on any platform (using Physlets on a Macintosh requires OS X and the Mozilla browser) and are easily embedded in HTML pages.
- They're free for noncommercial use. You can download Physlet archives—that is, compressed archives containing compiled Java programs—from the Davidson College Web-Physics server (<http://webphysics.davidson.edu/applets/Applets.html>).

Just-in-Time Teaching

Although the rich content and interactivity that Physlets provide can be pedagogically useful, using them on their own omits the human element that is so important to effective teaching. To be truly effective, the computer's communication capabilities must be exploited to create a feedback loop between instructor and student. Researchers at Indiana University and the United States Air Force Academy have pioneered a promising approach, JiTT, which we have further developed at Davidson College to include Physlets. Although we briefly describe JiTT in this section, you can read more about it in *Just-in-Time Teaching: Blending Active Learning with Web Technology*.⁹

The JiTT pedagogy exploits an interaction between Web-based study and an active-learner classroom. Students get Web-based assignments and respond electronically with their answers before the next class period. The instructor reads the students' submissions “just in time” to adjust the lesson content and activities to suit their needs. The heart of JiTT is the feedback loop—formed by the students' and teacher's outside-of-class preparation—which fundamentally affects the teaching and learning occurring during class time.

Physlet-Based JiTT Exercises

Although you can fully implement JiTT using technically simple Web-based assignments, incorporating interactive Physlet-based exercises heightens the extent to which student

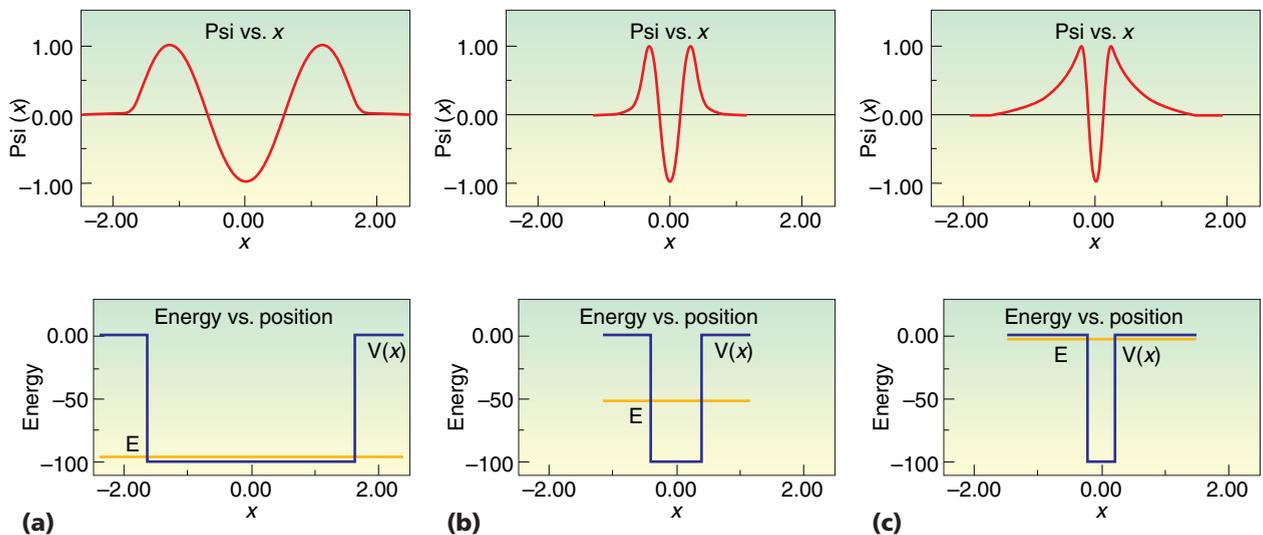


Figure 1. A Physlet-based quantum mechanics exercise. Students must change the width of the potential energy function and describe what happens to the wave function and the energy levels. As the well narrows, the number of bound states is reduced. Here we see the energy of the second excited state increase until it is almost a continuum state.

understanding can be probed and encouraged. Responding to questions that involve watching or analyzing an animation often requires different skills and a different level of understanding than responding to static questions.

Figure 2 shows part of a Physlet-based JiTT exercise from introductory physics. Students must rank six animations¹² according to an orange mass's acceleration and a rope's tension. First, students must watch the animation to determine the mass's acceleration and use this information to determine the rope's tension. Our experience with this exercise shows that students do not have trouble with the acceleration question, but they have significant trouble with the tension question. Consequently, we spend much more class time discussing how to determine (conceptually and analytically) the tension and very little class time discussing the acceleration.

Assessment of Material in Introductory Mechanics

Over the past four years, we gave the Force Concept Inventory (FCI)¹³ as a pre- and post-test to 10 of our first-se-

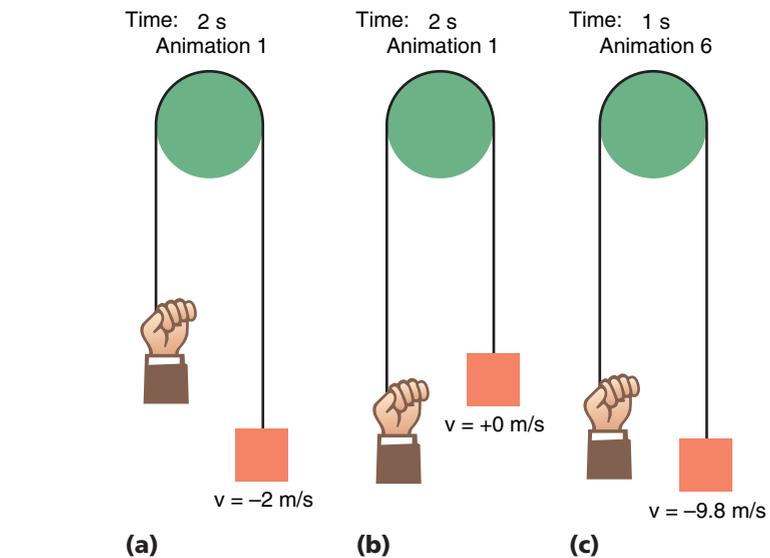


Figure 2. A Physlet-based Just-in-Time Teaching exercise about acceleration and tension. Three of the exercise's six scenarios show rankings according to an orange mass's acceleration and a rope's tension.

mester introductory (algebra-based and calculus-based) physics classes at Davidson College. (The FCI is an assessment instrument that probes student understanding of Newtonian mechanics through 30 multiple-choice

questions.) We tallied the results and calculated the course's average normalized gain (g).^{8,10} Previous results from a study of over 6,000 students⁸ show that traditionally taught students have a gain of 0.25 on the FCI. However, students

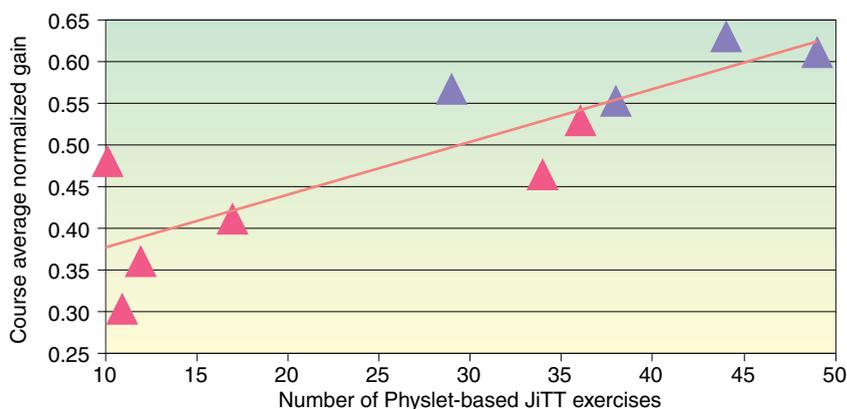


Figure 3. Data from four years and 317 students at Davidson College. The pink triangles represent the data from the algebra-based courses, and the blue triangles represent the data from the calculus-based courses. The orange line is the data’s linear fit.

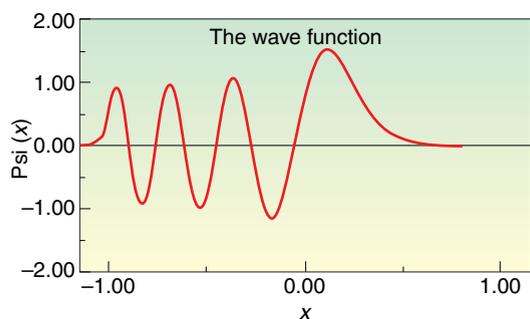


Figure 4. A sample Physlet problem. A wave function of an “unknown” potential.

Curricular Materials for Quantum Mechanics

We have completed and class tested over 50 Physlet-based JiTT exercises for a one-semester, senior-level quantum mechanics course. To support various teaching pedagogies, our curricular material comes in three formats:

- Illustrations act as interactive essays that animate what would normally be a static figure or a formula in a textbook.
- Explorations are directed interactive tutorials exploring a particular concept.
- Problems are interactive versions of traditional questions that appear at the end of a chapter in most textbooks.

This material will become *Physlet Quantum Mechanics* (to be published by Prentice Hall, 2004), but the original materials created for JiTT are and will continue to be freely available for teachers. Let’s review a few of these exercises.

Time-Independent Exercises

Several important features of wave

functions are hard for students to understand. Most student confusion lies in the fact that the standard examples presented are the infinite square well and the quantum harmonic oscillator potential. These potential energy functions are symmetric about the middle of the well and therefore do not provide a rich enough testing ground for quantum mechanical concepts regarding the wave function’s amplitude and curvature (sometimes also called the *wavelength* or *wiggleness*).

Consider the example in Figure 4. We let students observe a wave function and then ask

1. What energy state is depicted?
2. The wave function changes amplitude and wavelength as a function of position. What do these changes mean for the wave function?
3. Given your answer to number 2, what does the potential energy function look like?

In this exercise, notice that the wave function must correspond to the seventh bound state of a ramped potential energy function. This exercise is based on “one of the best quantum mechanics problems ever posed.”¹⁴ Unlike the classic problem¹⁵ in which the potential energy function is shown along with an incorrect wave function, we give students a set of open-ended questions and ask them to determine the “unknown” potential energy function.

We give this Physlet-based JiTT exercise pre-instruction and follow it with an in-class discussion of the Illustration in Figure 5, which shows both the potential energy function and the wave function. Sliders on the HTML page let students or instructors change the “ramping” of the potential energy function and immediately see the con-

taught in a more interactive way have a significantly greater gain—between 0.38 and 0.68.

Figure 3 shows the correlation between the course’s average normalized gain and the number of interactive (Physlet- and JiTT-based) exercises assigned during the course. In every class, students must complete 10 interactive JiTT pre-lab exercises. Hence, the base number of interactive exercises is 10. All courses except the ones with the lowest two gains ($g = 0.29$ and 0.35 , the more traditional courses) used either JiTT (seven courses with an average g of 0.53) or Peer Instruction (one course with $g = 0.48$). JiTT gives results that are comparable to Peer Instruction, and both methods give relatively high normalized gains.

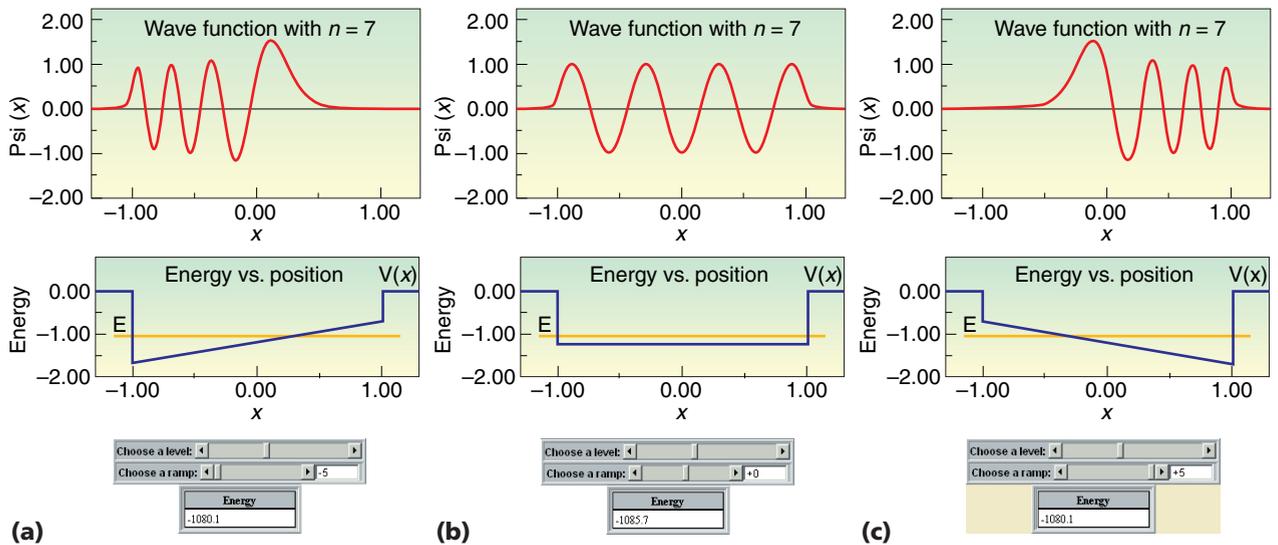


Figure 5. The ramped finite potential energy well illustration. As you look from left to right, notice how the ramping of the potential energy function affects the wave function. In (a), the ramp is deeper on the left half of the well; in (b) the well is symmetric; and in (c), the ramp is deeper on the right half of the well.

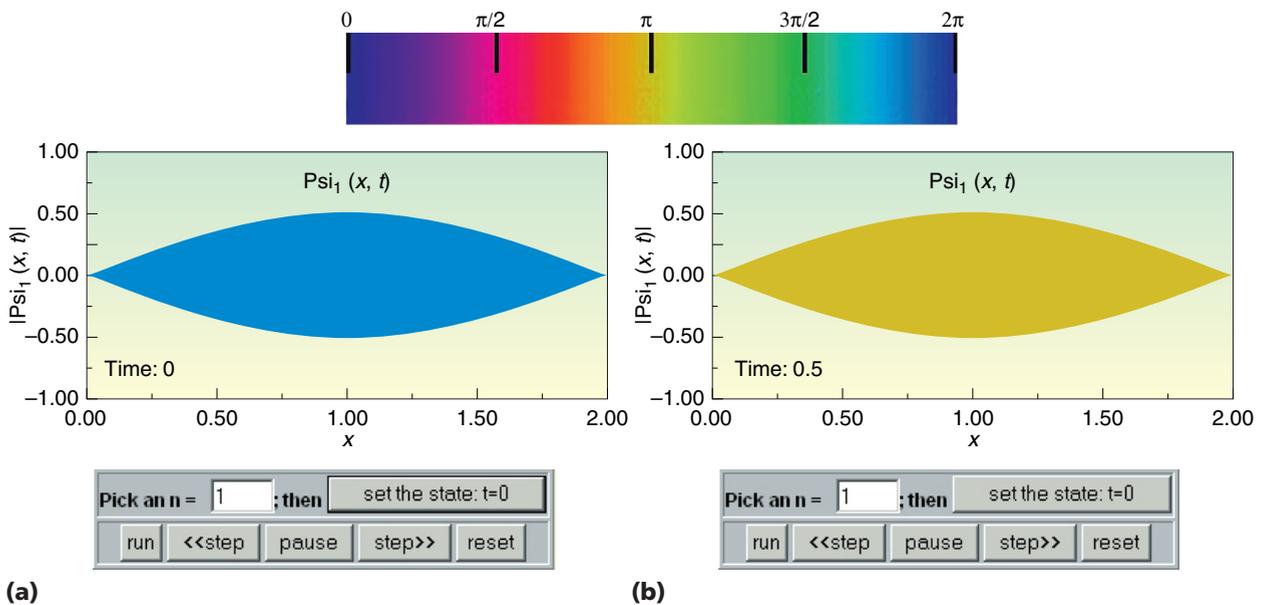


Figure 6. The ground state wave function of the time-dependent infinite square well shown at (a) $t = 0$ and (b) $t = 0.5$, where $t = 1$ is the ground state revival time. Note the correspondence between the wave function's color and phase.

sequences on the wave function.

One important misconception this Illustration elicits is that the particle's energy is changing across the well. Because these are energy eigenstates, the energy is, of course, the same across the well. This misconception

probably comes from the infinite square well problem. Because the potential is constant (and zero) for that problem, changing the wave function's curvature means that the total energy changes (it goes from one energy eigenstate to another). However,

with the ramped potential energy function, this is not the case. The potential energy function changes as a function of position and therefore the wave function's curvature must also change to maintain the total energy across the well.

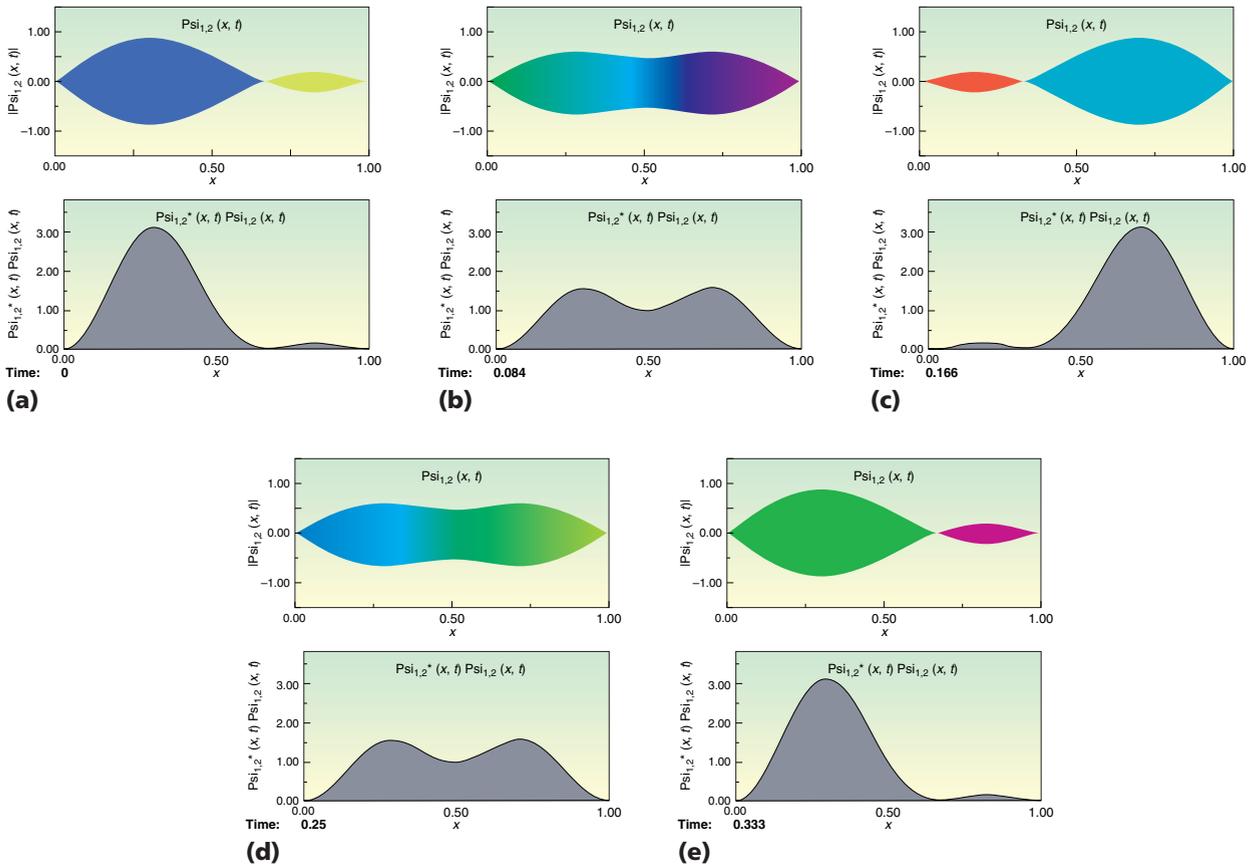


Figure 7. The time evolution of an equal-mix superposition of ground state and first excited state wave functions for the infinite square well. In the figure, time evolves from (a) to (e) from $t = 0$ to $t = 0.33$. A time of 0.33 corresponds to the time it takes for the probability density to return to the distribution it had at $t = 0$. The wave function, however, has not yet returned to its original value by $t = 0.33$ and won't until $t = 1$.

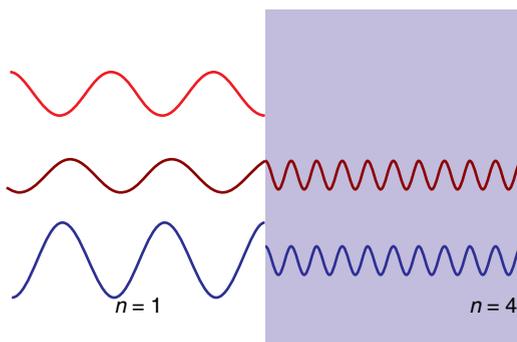


Figure 8. A classical electromagnetic wave (in blue) traveling to the right through two media of different indices of refraction. Shown in red is the left-moving (the reflected) part of the EM wave. In dark red is the superposition of the right- and left-moving waves.

Time-Dependent Exercises

One of the most intriguing problems (from a computer animation or simulation viewpoint) is quantum mechanical time development. How to portray complex wave functions in time evolution is an important consideration, and various authors have chosen slightly different conventions.^{16,17} We show either separate real and imaginary components of the wave function, or we show the wave function's amplitude with its phase superimposed on it as color, much like other authors have done.

Figure 6 shows our first time-evolution exercise, which represents the wave function's phase as color. We chose infi-

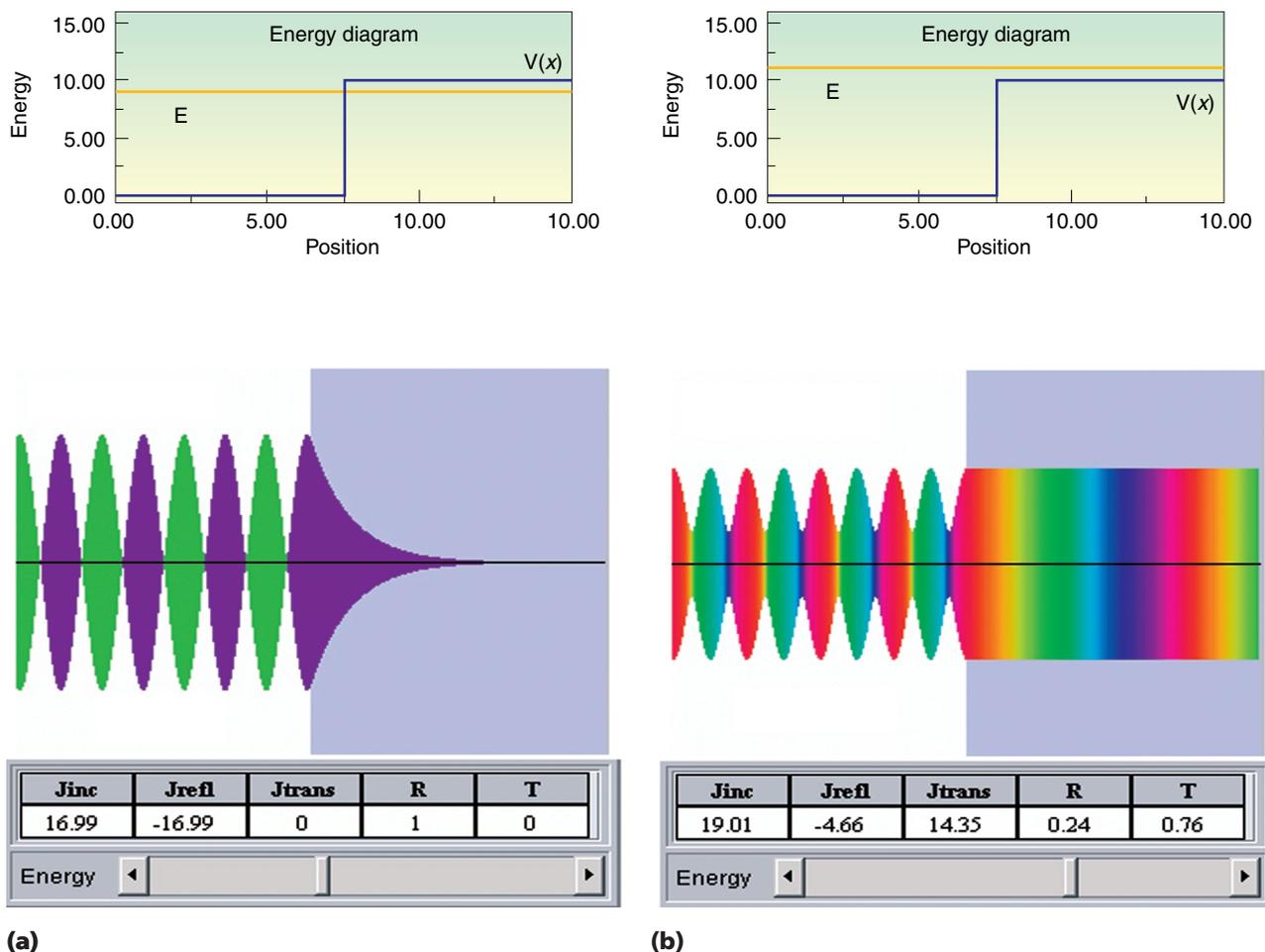


Figure 9. A quantum mechanical plane wave traveling from (a) to (b) through a change in potential energy. The graphs at the top show the energy diagram.

nite square well states because of students' familiarity with these states. Students must play the animation and describe the wave function's features. At $t = 0$, the wave function vanishes at $x = 0$ and $x = 2$, which correspond to the well's sides. However, a normalized wave function should also have a maximum at the center of the well with a value of 1. To match this fact with the representation shown, the wave function's total height must be the amplitude. (There are two standard representations for the complex wave function. Figure 6 shows one; in the other representation, the amplitude is always shown as positive. We prefer the representation described in this article because it seems to better represent scattering problems.)

Because we can represent any com-

plex number as $Ae^{i\theta}$, where A is the amplitude and θ is the phase angle from the real axis (e is 2.718 and i = square root of -1), the wave function's color corresponds to its phase. Quantum mechanical time evolution has a minus sign in the exponential, so the time-dependent wave function must rotate clockwise in the complex plane. In the animation shown in Figure 6, $t = 1$ corresponds to the ground state wave function's revival time. A real wave function at $t = 0$ evolves to become negative and imaginary at $t = 0.25$, then negative and real at $t = 0.5$, and then positive and imaginary at $t = 0.75$ before becoming real and positive again at $t = 1$. We can determine the color progression by reading the color strip in Figure 6 from right to left.

Because students do not often "see" the

wave function's time development, they have no reason to believe that a difference could exist between the time evolution of stationary states and a superposition of states. In fact, because the time-evolution operator is a unitary operator, it usually drops out of calculations for the probability density. A superposition of states is different, however. Although the total probability is still 1, independent of time, the probability density does vary with time, as Figure 7 shows.

The probability density's oscillation frequency is proportional to the difference in energy between the two states in the superposition. However, the wave function's oscillation frequency is the time it takes for the phases of the underlying wave functions to return to zero simultaneously. This is the ground

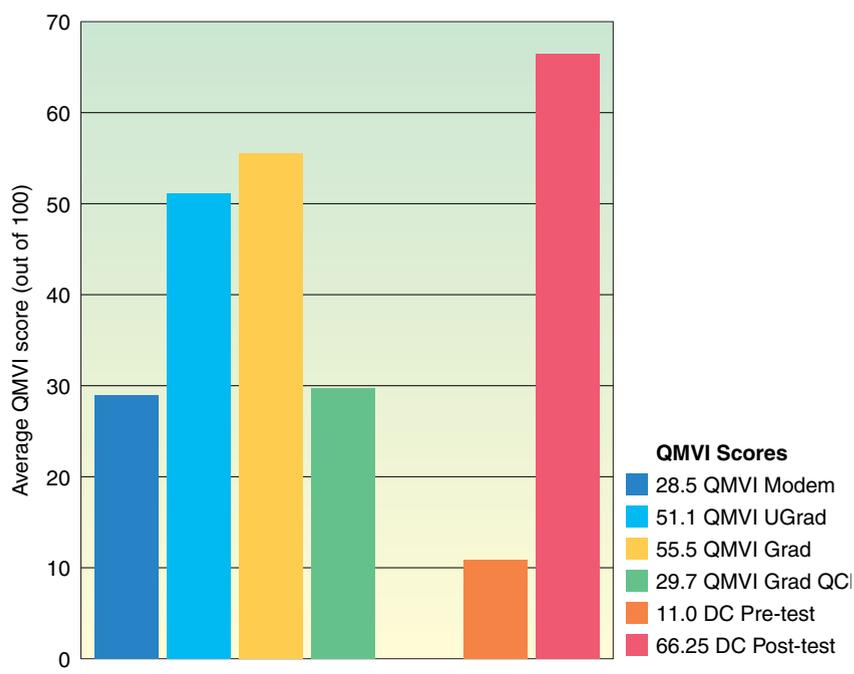


Figure 10. Quantum Mechanics Visualization Instrument results that appear elsewhere¹⁹ compared with the Fall 2001 Davidson College QMVI results.

state revival time,¹⁸ which is 1 for the animation shown in Figure 7.

Barrier Problems

Let's now look at the similarities and differences between a classical electromagnetic wave incident on a change in index of refraction and a quantum mechanical plane wave incident on a change in potential energy.

Figure 8 shows the classical electromagnetic wave animation. It also shows an important feature of the wave in the left medium: the reflected wave travels in the opposite direction of the original wave and is 180 degrees out of phase with the original wave. Therefore, when you add up both the left-moving (reflected) wave and the right-moving (original) wave, the resulting central dark red wave resembles a standing wave when the change of index of refraction is large enough. We show students an animation without the superposition first, ask them what the superposition of the right- and left-moving waves will look like, and then show them the superposition shown in Figure 8.

Students then see a quantum mechanical plane wave traveling to the

right through a change in potential energy, as Figure 9 shows. The figure shows the energy diagram along with the wave function. Showing the energy diagram is important because students often think that the plane wave loses energy when it is in the potential energy barrier.

Assessment of Material in Quantum Mechanics

During our Fall 2001 quantum mechanics course, we assigned 16 Physlet-based JiTT exercises to four senior physics majors. They took the Quantum Mechanics Visualization Instrument¹⁹ during the first week of the semester (the pre-test) and then at the end of the semester (the post-test). The QMVI probes conceptual and visual understanding of quantum mechanics through 25 multiple-choice questions.

Figure 10 shows results that appear elsewhere¹⁹ for modern physics (28.5 out of 100), undergraduate quantum mechanics (51.1), graduate quantum mechanics (55.5) and graduate quantum chemistry (29.7), next to the results from the Davidson College un-

dergraduate quantum mechanics course (66.25). The gain for this class was 0.62. Students completed our quantum mechanics course with a conceptual understanding of quantum mechanics at or above the level of graduate students after they took a graduate course in quantum mechanics. Because the pre-test scores are so low (11, where a score of 10 would be expected from random guessing), we see this class's gain as evidence that our students' conceptual understanding is due to the interactive nature of the exercises we created for the quantum mechanics course.

We have created over 50 Physlet-based exercises for the teaching and learning of quantum mechanics. These materials provide a new, exciting, and effective way to deliver interactive curricular material to students in advanced physics courses. These materials, and the examples shown in this article, are available for browsing and download at http://web-physics.davidson.edu/cise_qm. Their effectiveness is supported by our preliminary assessments of the use of interactive curricular material in both introductory physics and quantum mechanics courses. In addition to the forthcoming *Physlet Quantum Mechanics*, we are working on *Physlet Physics: Interactive Illustrations, Explorations, and Problems for Introductory Physics*, which Prentice Hall will publish in 2003.

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