Improving Student Understanding of Coulomb's Law and Gauss's Law

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Abstract. We discuss the development and evaluation of five research-based tutorials on Coulomb's law, superposition, symmetry and Gauss's Law to help students in the calculus-based introductory physics courses learn these concepts. We compare the performance of students on the pre-/post-tests given before and after the tutorials in three calculus-based introductory physics courses. We also compare the performance of students who used the tutorials and those who did not use it on a multiple-choice test which employs concepts covered in the tutorials.

INTRODUCTION

Electrostatics is an important topic in most calculus-based introductory physics courses. Although Coulomb's law, superposition principle, and Gauss's law are taught in most of these courses, investigations have shown that these concepts are challenging for students [1, 2, 3]. Despite the fact that students may have learned the superposition principle in the context of forces in introductory mechanics, this learning does not automatically transfer to the abstract context of electrostatics and students get distracted by the very different surface features of the electrostatics problems. Effective application of Gauss's law implicitly requires understanding the principle of superposition for electric fields and the symmetry that ensues from a given charge distribution. Helping students learn these concepts can improve their reasoning and meta-cognitive skills and can help them build a more coherent knowledge structure. Here, we discuss the development and evaluation of research-based tutorials and the corresponding pre-/post-tests to help students develop a functional understanding of these concepts.

TUTORIAL DEVELOPMENT AND ADMINISTRATION

Before the development of the tutorials, we conducted investigation of student difficulties with these concepts [3] by administering free-response and multiple-choice questions and by interviewing individual students. We found that many students have difficulty distinguishing between the electric charge, field and force. Students also have difficulty with the principle of superposition and in recognizing whether sufficient symmetry exists for a particular charge distribution to calculate the electric field using Gauss's law. Choosing appropriate Gaussian surfaces to calculate the electric field using Gauss's law when sufficient symmetry exists is also challenging for students. Distinguishing between electric field and flux was often difficult.

We then developed the preliminary version of five tutorials and the corresponding pre-/post-tests based upon the findings of the difficulties elicited in previous research and a theoretical task analysis of the underlying concepts. Such analysis involves making a fine-grained flow chart of the concepts involved in solving specific class of problems and can help identify stumbling blocks where students may have difficulty. The first two tutorials were developed to help students learn about Coulomb's law, superposition principle and symmetry in the context of discrete and continuous charge distributions (conceptually), the third tutorial focused on distinguishing between electric flux and field, and the fourth and fifth tutorials dealt with symmetry and Gauss's law and on revisiting superposition principle after Gauss's law. Some tutorials on related topics have been developed by the University of Washington group. Those tutorials are complementary to the ones we have developed which focus on achieving competency with symmetry ideas in Coulomb's law, superposition, and Gauss's Law. We administered each pre-test, tutorial and post-test to 5 students individually who were asked to talk aloud while working on them. After each administration, we modified the tutorials based upon the feedback obtained from student interviews. These individual administrations helped fine-tune the tutorials and improve their organization and flow. Then, the tutorials were administered to four different calculus-based introductory physics classes with four lecture hours and one recita-

TABLE 1. Average percentage scores obtained on individual questions on the pre-/post-tests (matched unless only the post-test was given) for each of the five tutorials (I-V). The pre-tests were administered after traditional instruction but before the tutorial. As shown in the table, additional questions were included either in the pre-test or post-test and the pre-tests for tutorials II and V were not administered in some of the classes. The symblo *n* refers to the matched number of students in a given class for a given pre-/post-tests and *Total* refers to the total average percentage score including all questions on a pre-test or post-test administered to a given class for a particular tutorial. For tutorial II, the relative weights for the three pre-test questions for class 2 were 30%, 30% and 40% respectively. For tutorial IV, the relative weights for the pre-test and post-test questions for classes 1 and 2 were 10%, 10%, 20%, 20%, 20%, 20% and 20%, 10%, 20%, 10%, 20%, 20% respectively while the relative weights for the pre-test and post-test questions for class 3 were 20%, 20%, 30%, 30% and 30%, 20%, 30%, 20% respectively. For tutorial V, the relative weights for both the pre-test and post-test questions for class 2 were 30%, 40% and 30% and the weights for the post-test questions for the other two groups were 10%, 20%, 20%, 20%, 10%, 20% respectively. For all other cases, the same weight is assigned to each pre-test or post-test question.

Tutorial	Class	n	PRETEST							POSTTEST						
			1	2	3	4	5	6	Pre-Total	1	2	3	4	5	6	Post-Total
I	1	82	64	57	46	45	—	—	53	92	86	93	—	—	—	90
	2	60	52	58	38	47	—	—	48	96	96	95	—	—	—	96
	3	52	44	29	45	—	—	—	39	85	77	88	—	—	—	83
II	1	84	—	—	—	—	—	—	—	68	84	68	72	90	—	76
	2	63	56	6	41	—	—	—	35	90	96	87	87	98	—	92
	3	63	—	—	—	—	—	—	—	75	84	77	77	92	—	81
III	1	78	42	—	—	—	—	—	42	77	85	72	92	81	—	81
	2	55	44	—	—	—	—	—	44	74	88	82	95	—	—	85
	3	49	40	—	—	—	—	—	40	81	78	84	96	87	—	85
IV	1	65	28	22	58	58	41	17	40	83	91	95	91	91	93	91
	2	62	39	19	51	52	42	6	36	85	84	93	96	95	84	89
	3	49	45	6	38	28	—	—	30	87	90	88	81	—	—	87
V	1	85	—	—	—	—	—	—	—	71	61	75	70	96	64	71
	2	57	21	26	35	—	—	—	27	82	76	84	—	—	—	80
	3	64	—	—	—	—	—	—	—	92	81	89	89	95	90	88

tion hour per week. Students worked on each tutorial in groups of two or three either during the lecture section of the class or in the recitation depending upon what was most convenient for an instructor. Table 1 shows the pre-/post-test data on each question from three of the classes in which the tutorials were administered. The details of each question will be discussed elsewhere. In the fourth class, the post-tests were returned without photocopying them and we only have data on student performance on the cumulative test administered after all tutorials. As shown in Table 1, for some tutorials, additional questions were included in the pre-test and/or post-test after the previous administration and analysis of data. The pre-/post-tests were not identical but focused on the same topics covered in a tutorial.

All pre-tests and tutorials were administered after traditional instruction in relevant concepts. Instructors often preferred to alternate between lectures and tutorials during the class and give an additional tutorial during the recitation. This way all of the five tutorials from Coulomb's law to Gauss's law were administered within two weeks. For the tutorials administered in lecture section of the class, pre-tests were given to students right before they worked on the tutorials in groups. Since not all students completed a tutorial during the class, they were asked to complete them as part of their homework assignment. At the beginning of the next class, students were given an opportunity to ask for clarification on any issue related to the part of the tutorial they completed at home and then they were administered the corresponding post-test before the lecture began. Each pre-/post-test counted as a quiz and students were given a full quiz grade for taking each of the pre-test regardless of students' actual performance. The pre-tests were not returned but the post-tests were returned after grading. When a tutorial was administered in the recitation (tutorials II and V which were shorter), the teaching assistant (TA) was given specific instruction on how to conduct the group work effectively during the tutorial. Moreover, since the TA had to give the post-test corresponding to the tutorial during the same recitation class in which students worked on the tutorials (unlike the lecture administration in which the post-tests were in the following class), the pre-tests were skipped for some of these tutorials due to

TABLE 2. Percentage average pre-/post-test scores (matched pairs) for each of the five tutorials (I-V), divided into three groups according to the pre-test performance. *N* denotes the total number of students who worked through a tutorial and took both the pre-/post-tests, and n_i (i=1,2,3) denote the number of students in a particular class. For tutorials II and V, only one of the classes took both the pre-/post-tests. For students in the high pre-test range, sometimes there are very few students for a meaningful statistical interpretation.

N	Tutorial	Range (%)	n1 (class 1)	pre	post	n2 (class 2)	pre	post	n3 (class 3)	pre	post
194	I	All	82	53	90	60	48	96	52	39	83
		0-33	24	19	77	21	20	92	29	22	76
		34-67	33	55	92	18	43	97	21	58	92
		68-100	25	83	99	21	80	99	2	100	100
63	II	All				63	35	92			
		0-33				30	17	89			
		34-67				32	50	94			
		68-100				1	90	90			
182	III	All	78	42	81	55	44	85	49	40	85
		0-33	31	18	76	22	20	81	22	15	85
		34-67	38	52	84	26	55	87	19	53	85
		68-100	9	84	88	7	79	88	8	78	85
176	IV	All	65	40	91	62	36	89	49	30	87
		0-33	26	15	89	31	17	85	29	16	83
		34-67	32	50	91	27	51	94	17	47	92
		68-100	7	83	96	4	78	94	3	70	93
57	V	All				57	27	80			
		0-33				42	14	77			
		34-67				10	47	90			
		68-100				5	96	92			

a lack of time. Sometimes, the instructors gave the pre-tests in the lecture section of the class for a tutorial that was administered in the recitation.

In all of the classes in which the tutorials were used, 2-2.5 weeks were sufficient to cover all topics from Coulomb's law to Gauss's law. This time line is not significantly different from what the instructors in other courses allocated to this material. The main difference between the tutorial and the non-tutorial courses is that fewer solved examples were presented in the tutorial classes (students worked on many problems themselves in the tutorials). We note that since many of the tutorials were administered during the lecture section of the class, sometimes two instructors (e.g., the instructor and the TA) were present during these "large" tutorial sessions to ensure smooth facilitation. In such cases, students working in groups of three were asked to raise their hands for questions and clarifications. Once the instructor knew that a group of students was making good progress, that group was invited to help other groups in the vicinity which had similar questions. Thus, students not only worked in small groups discussing issues with each other, some of them also got an opportunity to help those in the other groups.

DISCUSSION

Out of the five tutorials, the first two focused on Coulomb's law, superposition and symmetry. The first tutorial started with the electric field due to a single point charge in the surrounding region and then extended this discussion to two or more point charges. The second tutorial further continued the conceptual discussion that started in the first tutorial (which was mainly about discrete charges) to continuous charge distributions. The tutorials guided students to understand the vector nature of the electric field, learn the superposition principle and recognize the symmetry of the charge distribution. Students worked on examples in which the symmetry of the charge distribution (and hence the electric field) was the same but the charges were embedded on objects of different shapes (e.g., four equidistant charges on a plastic ring vs. a plastic square). Common misconceptions were explicitly elicited often by having two students discuss an issue in a particular context. Students were asked to identify the student with whom they agreed.

The third tutorial was designed to help students learn to distinguish between the electric field and flux. The tutorial tried to help students learn that the electric field

TABLE 3. The average percentage of correct responses to the "Superposition, Symmetry and Gauss's Law Test" for different groups of students. *N* refers to the total number of students for a given group. In all undergraduate courses, the test was administered after instruction on these concepts in that course except in the upper-level *E&M* course in which it was given both as a pre-test and post-test (since students had instruction in these concepts at the introductory level). The "without tutorial" group and the "Honors students" group are from the same student population (mainly physical science and engineering freshmen but some sophomores) as the tutorial group from the same institution. The second row of the table gives the *p* value for t-tests (which performed a pair-wise comparison of the performance of the tutorial group with each of the other groups before rounding off the numbers).

	Without tutorial but otherwise same type of courses (2 classes)	Honors Students (2 classes)	Upper-level Undergrads		With Tutorial (4 classes)	First Year Grads (2 classes)
			Pre-test	Post-test		
	N=135	N=182	N=33	N=28	N=278	N=33
Average	38	42	44	49	59	75
p value	1.34E-04	7.85E-04	4.33E-03	5.29E-02		1.37E-03

is a vector while the electric flux is a scalar. Also, electric field is defined at various *points* in space surrounding a charge distribution while the electric flux is always through an *area*. Students learn about Gauss's law and how to relate the flux through a closed surface to the net charge enclosed. Rather than emphasizing the symmetry considerations, this tutorial focused on helping students use Gauss's law to find the net flux through a closed surface given the net charge enclosed and vice versa.

The fourth tutorial was designed to help students learn to exploit Gauss's law to calculate the electric field at a point due to a given charge distribution if a high symmetry exists. Students were helped to draw upon the superposition and symmetry ideas they learned in the first two tutorials to evaluate whether sufficient symmetry exists to exploit Gauss's law to calculate the electric field. Then, students learn to choose the appropriate Gaussian surfaces that would aid in using Gauss's law to find the electric field. Finally, they use Gauss's law to calculate the electric field in these cases. The last tutorial revisits the superposition principle after students have learned to exploit Gauss's law to calculate the electric field. For example, students learn to find the electric field at a point due to two non-concentric uniform spheres of charge or due to a point charge and an infinitely long uniform cylinder of charge.

The pre-tests and post-tests were graded by two individuals, and the inter-rater reliability was good. The average pre-/post-test scores on matched pairs for a particular class graded by them differ only by a few percent. Table 1 shows the student performance (on each question and also overall) on the pre-test and post-test in each of the five tutorials (I-V) in percentage for each class. The classes utilizing each tutorial may differ either because additional pre-/post-test questions were added or the pre-tests for tutorial II and V were not administered to some of the classes. The differences in the performance of different classes may also be due to the differences in student samples, instructor/TA differences or the manner in which the tutorials were administered. Table 2 shows the performance of students on the pre-/post-tests for each tutorial partitioned into three separate groups based upon the pre-test performance (see the Range column). As can be seen from Table 2, tutorials generally helped all students including those who performed poorly on the pre-test. The post-test scores are unusually high and can be attributed to a variety of reasons including Hawthorne effect. Another possibility is that the tutorials were "teaching to the test" and immediately after working on a tutorial, the concepts were fresh in students' minds. We therefore administered a cumulative test which includes concepts from all of the tutorials [3]. Table 3 shows the average percentage scores from the cumulative test administered to different student populations. Although the performance of the tutorial group is not as impressive on the cumulative test as on the pre-/post-tests administered with the tutorials, Table 3 shows that students who worked through the tutorials significantly outperformed both Honors students and those in upper-level undergraduate courses, but not first year physics graduate students.

CONCLUSION

We developed and evaluated tutorials to help calculus-based introductory students learn Coulomb's law, superposition, symmetry and Gauss's law. Pre-/post-tests for each tutorial and a test that includes content on all of the tutorials show that the tutorials can be effective in improving student understanding of these concepts.

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