Homework: Newton's laws and velocity-dependent forces Name ________________________

1. Consider a ball that moves vertically under the influences of both gravity and air resistance. For the purposes of this problem, take vertically **upward** as the **positive** direction. For instance, negative velocity values ($v < 0$) correspond to downward velocities in this case, and the gravitational force on the ball would be expressed as $-mg$.

   a. Suppose that the force of air resistance on the ball were purely linear with respect to speed ($c_1 \neq 0$, $c_2 = 0$). For each equation of motion below, determine whether that equation applies to (a) a situation in which the ball moves *upward,* (b) a situation in which the ball moves *downward,* (c) *either* of these, or (d) *neither* of these. Explain your reasoning for each case.

      i. $m\,(dv/dt) = -mg + c_1 v$ ii. $m\,(dv/dt) = -mg - c_1 v$

   b. Repeat the preceding part of this problem with each of the equations of motion listed below, except now suppose that the force of air resistance on the ball were purely quadratic with respect to speed ($c_1 = 0$, $c_2 \neq 0$). Explain your reasoning for each case.

      i. $m\,(dv/dt) = -mg + c_2 v^2$ ii. $m\,(dv/dt) = -mg - c_2 v^2$

2. Follow the same reasoning that you used in section II of the tutorial by expressing the terminal velocity of a spherical object of mass m for the case in which the force of air resistance is:

   - purely *quadratic* with respect to speed ($c_1 = 0$, $c_2 > 0$)
   - expressed as a combination of *both* linear *and* quadratic terms ($c_1 > 0$, $c_2 > 0$)

NOTE: For problems 3 and 4, use the fact that the force of air resistance on a spherical object of diameter D can be approximated using coefficients $c_1 = (1.55 \times 10^{-4})D$ and $c_2 = 0.22D^2$ (all numerical values are in SI units).[1] The ratio of the quadratic and the linear terms of the force of air resistance can therefore be expressed as:

$$\left|\frac{c_2 v^2}{c_1 v}\right| = \frac{0.22 v|v|D^2}{\left(1.55\times 10^{-4}\right) vD} = (1.4\times 10^{3})|v|D$$

3. Consider a softball with diameter 10.0 cm and mass 200 g.

   a. Using the above ratio, for what range of speeds will (i) the linear term of air resistance dominate over the quadratic term? (ii) the quadratic term dominate over the linear term?

   b. Calculate the terminal speed of the softball taking into account *both* the linear *and* quadratic terms. Show all work.

   c. Reflect on your results in parts a and b above. If it were desired to approximate the effect of air resistance on a falling softball with *either* the linear term *or* the quadratic term (not both), which term would you keep? Explain your reasoning.

4. Repeat problem 3, except now consider an oil droplet from Millikan's oil-drop experiment (use $D = 10^{-4}$ cm, mass 10^{-12} g). Explain your reasoning and show all work.

[1] From *Analytical Mechanics,* 7th ed., Fowles & Cassiday (Thomson-Brooks/Cole), p. 69.