Multiple Modes of Reasoning in Physics Problem Solving, with Implications for Instruction

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Abstract. Problem-solving is an important part of physics teaching, learning and assessment. It is widely assumed that the way that experts solve problems, and students should, is by systematic application of basic physics principles. Model solutions are laid out this way, and teaching of problem-solving usually consists of 'going over' such solutions step by step. However, while this does represent the physics structure of the final solution, it does not adequately reflect how people actually think when tackling problems. Real cognition is complex. This study was prompted by students trying to 'map across' result features recalled from previous cases instead of working from basics. Since our instruction emphasizes the power and generality of basic principles, our first response was to re-emphasize principles, but we found that experts in fact draw extensively and effectively on rich compiled case knowledge. We investigated cognition in detail for geometrical optics. Research methods included analysis of written solutions, reflections on thinking, and interviews. Cognitive modes emerged from the initial research stages, and were then used to code individuals' problem-solving pathways. Learners and experts alike used multiple modes of cognition, significantly principle-based reasoning, case-based reasoning and experiential-intuitive reasoning. Case-based reasoning using pre-compiled knowledge played a pervasive role in conjunction with, and sometimes in conflict with, principle-based reasoning. The implications for instruction are that it should reflect what we know about cognition and expertise, and hence include teaching case-based as well as principle-based reasoning. We are doing this in optics, by using cases and variations, identifying topic knowledge schema 'sub-assemblies', and modeling their use in problems.

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INTRODUCTION

A goal and characteristic of science is the use of relatively few fundamental concepts and principles to explain a variety of situations, and accordingly our inquiry-based physics course for prospective teachers emphasizes a 'powerful ideas' theme throughout. We hope that students come to appreciate this perspective in both learning the physics fundamentals and applying them to solve problems.

It would appear to follow that we should teach problem solving by modeling how solutions are obtained by systematic application of concepts and principles. Thus teaching physics problem solving, if done explicitly at all, usually consists of 'going over' worked-out solutions as a stepwise application of principles. While this does represent the physics structure of the final solution, it turns out that it by no means reflects the richness of how people actually think when tackling problems. Hsu et. al.[1] recently provided a useful review of work on problem solving in physics, and Bodner [2] notes how typical problem instruction in chemistry does not reflect actual thought processes.

Observations during teaching of geometrical optics led us to believe that the cognitive processes and compiled knowledge involved in physics problem-solving might be far more complex and multifaceted than is generally realized, and not reflected in solutions conventionally presented in teaching and textbooks. Subsequently, we found that both learners and experts use multiple modes of reasoning in physics problem solving, three significant modes being principle-based reasoning, case-based reasoning and experiential-intuitive reasoning. By case-based reasoning we mean drawing quickly by association on outcome features of recalled similar cases, or retrieving previously compiled knowledge parts (schemata or subassemblies), and adapting to the new case, rather than constructing a principled solution from scratch. Kolodner [3] discusses case-based reasoning in several fields.

Elucidating the nature of cognition and expertise should have important implications for more effective instruction [4, 5]. Toward that end, we undertook a study of cognition in problem solving in optics, consisting of an observational exploratory stage and a designed experimental stage.

OBSERVATIONAL STUDY

The impetus for this study arose from observing unexpected student attempts to solve geometrical optics questions in our physics course for prospective teachers. It is clear that many entering students are not used to thinking and working via fundamentals. However, our emphasis on powerful ideas does have an effect during the course, and most students learn to work in principled fashion applying those ideas. Nevertheless we observed a mode of operating where students seemed to try 'mapping across' specific results recalled from similar-looking problems encountered earlier, instead of proceeding from principles.

This tendency first became apparent in the section on reflection and images. Students had investigated reflection, formulated the law, learned about image formation, and applied these ideas to locate virtual and real images by ray construction for various plane, convex and concave mirror cases. Given a new variant of a concave mirror problem, students produced all sorts of diagrams in their attempts.

Fig. 1 illustrates two concave mirror cases. The left-hand diagram, for a source close to the mirror, is a correct construction to find the (virtual) image.

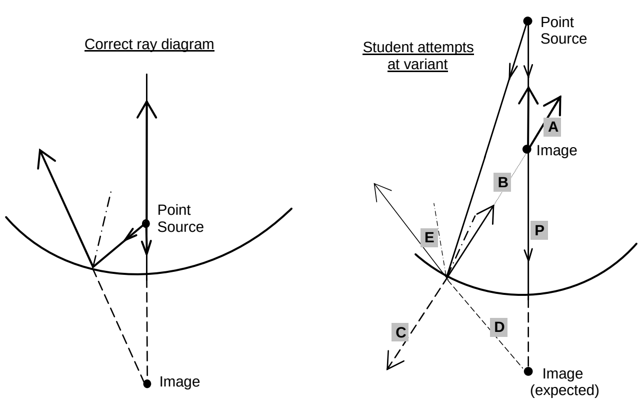

FIGURE 1. Ray constructions and image location for two curved mirror cases.

The right-hand diagram, for the case of a source further away, is a composite diagram to illustrate the main kinds of things that students did. These are labeled A–E. Most students reflected the perpendicular ray P correctly, but did all sorts of things for the other ray. Students who reflected it correctly using the basic law constructed rays ending at (A) and marked the image at the ray intersection. Some who initially reflected rays correctly erased them later (B), then did other things. Others made dotted ray extensions, diverging behind the mirror (C) and then gave up. Some drew dotted 'lines' converging (D) independently of rays. Some drew 'reflected' rays (E) actually violating the law, with dotted extensions converging behind the mirror to an 'image' point.

Mystifying attempts like this had first been interpreted as displaying near complete lack of understanding. However, an incident cued us as to what might be happening. A distraught student approached an instructor in an exam with her attempt saying: "*I don't know what I'm doing wrong… I did it like we did before… and my rays sort of reflect up here somewhere… I know the image is supposed to be here, behind… but I just can't make the rays go there..*."

Despite her confusion, she actually had most of the declarative and procedural knowledge needed to solve image problems. It seemed that she was trying to balance her knowledge of principles and procedures with a conflicting expectation of where the image should be, arising from the 'mapping across' of result features from a previous case onto a new case – a simplistic form of case–based reasoning.

Learner modes of operating

Intrigued by this mode of operating we undertook a study of learners' problem solving in geometrical optics. We focused on reflection and refraction, each of which involved both a fundamental law and its geometric application to image formation.

Our simple 'mapping' hypothesis was subject to immediate test – if the tendency was not idiosyncratic but more widespread, further examples should be found in other students' existing responses. We revisited all previous responses to the concave mirror problem. Encouraging for the predictive power of the hypothesis was the fact that supporting examples were now recognized in other responses to the same question, once we knew what we were looking for and better able to interpret students' diagrams.

Furthermore, we made a second testable prediction: if the tendency was not specific to this problem, but more general, it should be found in solutions to other problems on the same exam. This second hypothesis was also borne out, in a quite different problem on plane mirrors.

In several problems subsequently used in the course it became clear that most students were at least able to reflect rays correctly and seek ray intersections. Nevertheless, some students tended not to go that way or not to persist in it, but tried to work backwards from the kind of result they thought they ought to get. The feature-mapping mode of operating was often strong enough to override working from scientific principles when results conflicted, so that students would even resort to distortions to resolve this and get the features they had induced from other cases.

Our main focus was case-based and principle-based thinking, but we also noticed that in situations related to everyday experience, students' responses could sometimes be interpreted as arising from experiential resources such as phenomenological primitives (p-prims) [6]. An example is believing that no image is possible if an object is off to the side of a mirror, based on personal experience with being able to see (or not see) one's *own* image in a mirror. We have called this mode experiential-intuitive reasoning. The tendency seems considerably less for optics than mechanics.

EXPERT MODES OF OPERATING

Our first response to the result-mapping tendency had been to discourage it and re-emphasize working from basic principles. However, we also undertook a preliminary study of *expert* cognition, in areas of mechanics and optics, in which it became clear that experts too used case-based reasoning in problem solving, extensively and effectively! From experience, they drew on a rich case knowledge, in conjunction with basic principles. However, experts were also aware of case applicability conditions and limitations. Reif and Allen [7] discuss the use of case-specific knowledge for the concept of acceleration.

EXPERIMENTAL STUDY OF LEARNER REASONING MODES

Recognition of the major role of case-based reasoning for both novices and experts indicated that instead of discouraging associative thinking in learners, we should investigate it more closely, and learn how best to include it in instruction. Therefore, we extended the study to a designed experimental phase. We produced an instrument consisting of five sets of reflection and refraction problem tasks. Each set contains three different case variants, for use in examples, homework, and tests. The variants have different result features but the same underlying principles, and are thus of known relation to each other. For each problem students had to describe and explain their reasoning, thinking paths and levels of confidence.

Qualitative and quantitative data sources were: • Students' written problem solutions. • Students' written (metacognitive) accounts and reflections on what they did and why. • Explanations of confidence level. • Individual interviews. • Group observations.

Cognitive coding scheme

Significant modes of reasoning were first identified in the observational stage and extended and refined in the experimental stage. Usually more than one mode occurred and a subject's solution process was analyzed into episodes. Useful mode subdivisions emerged for classifying and coding episodes along thinking paths. Case knowledge items invoked in episodes were also noted. An individual's thinking path could thus be represented by a code sequence. Table 1 presents the codes and subdivisions devised:

Table 1. Cognitive coding scheme
Reasoning modes:
PBR– Principle-Based Reasoning:
F–Forward from principles.
B–Backward to principles.
PEC–Principles/Procedures Extracted from Case
CBR– Case-Based Reasoning:
RM–Result Mapping.
FM–Feature Mapping (partial or complete).
PPT–Principles & Procedures Transfer,
FMA–Feature Mapping Adjusted for case differences
EIR – Experiential-Intuitive Reasoning:
P-Prim–Phenomenological Primitive
Comparison and resolution:
C&R: Comparison & Resolution:
TP–Trusting Principles
BB–Bending the Basics (to fit result)
NA–No Answer reached, knows something is wrong
Correctness: Each episode coded, as well as overall solution
√ – used correctly X– used incorrectly
Confidence level:
VC-very confident, SC-somewhat confident, NC- not confident
Case Knowledge items accessed.
Specified for each episode.

Illustrative results

Qualitative and quantitative data analysis involved written, observational and interview data, coded as described. To illustrate briefly the nature of some quantitative results, Table 2 presents data from the Summer 2007 physics section (20 students).

Table 2. Results for the concave mirror image problem Figures are number of students out of 20 in the section.
Reasoning modes:
4/20 Reported only PBR
13/20 Used combination PRB and CBR
3/20 Confused; not enough information to classify
No apparent EIR evoked by this particular problem.
Comparison and resolution:
11/20 Compared PBR and CBR, of which:
3/11 PBR overrode CBR
2/11 CBR overrode PBR; student 'bent basics'
6/11 Unresolved by student, no solution.
Correctness of aspects:
8/20 Correct solution overall, of which:
4/8 Correct by PBR,
3/8 Correct by combination PBR & CBR
1/8 Case as a starting point to extract Procedure.
12/20 Incorrect overall, of which:
8/12 Potential to succeed if conflict resolved well
4/12 Unable to do it at all.

This is consistent with both the exploratory studies and our findings for other problems, though mode mix naturally depends on problem and context. The work is being extended to larger numbers in regular semesters.

FINDINGS AND DISCUSSION

We find that learners and experts alike invoke multiple modes of cognition in physics problem solving. Three important modes of reasoning emerging from this study in optics are principle-based reasoning, case-based reasoning and experiential-intuitive reasoning. Case-based reasoning, drawing extensively on pre-compiled knowledge, is pervasive during the process, though this may not be evident in final constructed solutions, and there is much interplay between case- and principle-based thinking. Experiential-intuitive thinking was little evident for these particular optics problems. Significantly, working linearly and systematically from basic principles is not usually the primary mode of initial thinking, of either learners or experts, although it is the way that we conventionally present model solutions, post-hoc, in teaching and textbooks.

Thinking by association seems to be a natural human tendency. It has pros and cons, in both everyday and scientific contexts. Working by association instead of via principles, although it did not serve some students well in these physics problems, is not broadly undesirable. It works well in everyday situations and is quick. Used correctly it can be very productive scientifically too. Tapping into previously compiled case knowledge is very efficient, compared to working everything out anew from scratch every time a new situation is encountered. Experts do this as part of their repertoire! They have a rich store of compiled knowledge and particular cases to draw upon. In fact this is one of the features of expertise, and indeed of structural understanding of a topic, and it is hard to imagine operating without it – everything would be cumbersome. But when drawing on case knowledge, experts are aware of what they are doing, know the status, applicability conditions and limitations of that knowledge, and are able to crosscheck against basics. They also recognize that various cases are just particular instances arising from the same underlying principles or mechanisms.

Returning to the mirror example, note that once one has solved one or two cases it is hard to imagine solving a variant without drawing on the mental picture already in place, which represents not just the result but also the method, and encapsulates one's previous construction of understanding of such situations. However this kind of compiled case knowledge, where the essence is understood as well, is different from the type of simplistic mapping of results features which we observe in some students, who do not perceive both similarities and differences in cases.

Note that in discussing how case features compiled from previous examples are used in tackling new problems we are not talking about the conventional use of example problems to teach principle-based reasoning.

Work on transfer is relevant to this study, recently that of Marton [8] on recognizing both sameness and difference in transfer. Of course if transfer between cases is to occur, instructors would prefer students to carry across deep features (underlying principles and procedures), rather than surface features of results. Students are also known to *categorize* problems by surface features [9], though this is different from solving by mapping *result* features.

Associative thinking and previous examples play a pervasive role in problem solving, but novices do not use them very well without instruction. Students need to be aware of how science works, and also that scientific thinking is different from everyday thinking in certain ways [10]. They need to appreciate the status of both general principles and particular examples, and know that applicability and generalizability need to be kept in mind. Such epistemological knowledge, though often tacit, is as important as declarative and procedural knowledge of the subject matter.

IMPLICATIONS FOR INSTRUCTION

Implications for more effective instruction are that it should reflect what we know about real cognition and the nature of expertise. This includes teaching case-based as well as principle-based reasoning and making such thinking 'visible.' Simply re-teaching the physics without the cognition will not work effectively.

We are proceeding in optics in a number of ways: identifying knowledge 'sub-assemblies' for each topic: presenting case variations to assist discernment of both commonalities and differences [8], as formative assessment for learning [11]; promoting explicit reflection and metacognition; and modeling these aspects in teaching and the design of learning tasks.

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