Representational Format, Student Choice, and Problem Solving in Physics

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Abstract. Student problem-solving ability appears to be tied to the representational format of the problem (mathematical, pictorial, graphical, verbal).[1] In a study of a 367-student algebra-based physics class, we examine student problem solving ability on homework problems given in four different representational formats, with problems as close to isomorphic as possible. In addition, we examine students' capacity for representational self-assessment by giving follow-up quizzes in which they can choose between various problem formats, and look for factors that may influence their ability or choices. As a control, part of the class was assigned a random-format follow-up quiz. We find that there are statistically significant performance differences between isomorphic problems. We also find that allowing students to choose which representational form they use improves student performance under some circumstances and degrades it in others.

INTRODUCTION

Student competence with different representational formats is a popular topic in modern science and mathematics education. By 'representational formats', we refer to the many ways in which a particular concept or problem can be expressed. Scientists have to be able to interpret all of these formats effectively and are able to integrate and translate among them. As a result, a possible instructional goal is to instill this representational facility in science students. In physics education research, there have been several studies in which students are explicitly taught to handle multiple representations of the same topic.[2,3,4] Other studies have addressed students "meta-representational competence",[5] in particular student generation and critique of science and math representations.[6,7]

In this study, we directly compare student performance on different representational formats in the style of Meltzer.[1] We also attempt to broaden the examination by investigating whether students can assess their own representational competence, what motives they have for handling a problem in a particular representational format given a choice of formats, and whether providing this choice affects their performance.

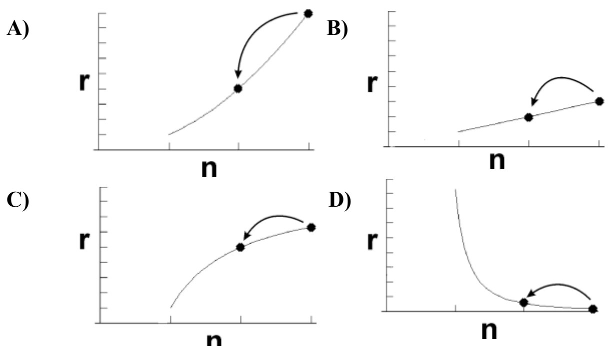
One can categorize representations according to whether they are formal or informal, abstract or concrete, or text-based versus graphics-based, to name just a few. Studies involving representational formats have taken many approaches to this division, including comparisons of mathematical problems couched in words to those stated primarily in equations,[8] comparisons of learning environments that are virtual to those that are concrete,[9] and comparisons between verbal, mathematical, graphical, and diagrammatic formats.[1] Here we take the approach of the last of these, and divide our study problems into verbal, mathematical, graphical, and pictorial formats.

METHODS

We administered our study in recitation to a large (367 student) traditionally taught second-semester algebra-based physics class at the University of Colorado at Boulder. We performed the study in two different subject areas: wave optics and atomic physics. In each subject area, the students were assigned four multiple-choice homework questions that covered the same concept in four different representational formats,

Question 3 – Graphical

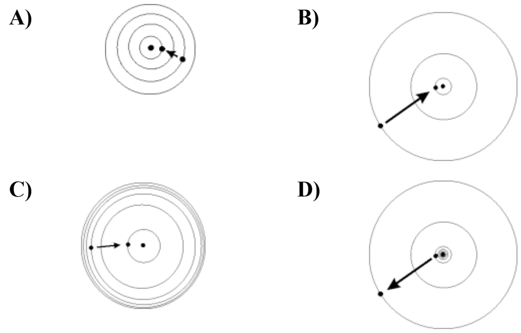
An electron in a Bohr hydrogen atom jumps from the n=3 orbit to the n=2 orbit. The following graphs show the orbit radius r as a function of the orbit number n. Choose the graph that best represents the relative locations of the electron orbits



E) None of these.

Question 4 – Pictorial

An electron in a Bohr hydrogen atom jumps from the n=3 orbit to the n=1 orbit. Choose the picture that best represents the relative locations of the electron orbits.



E) None of these.

FIGURE 1. Isomorphic homework problems (in graphical and pictorial/diagrammatic formats) regarding Bohr-model electron orbit radii.

as well as a one question multiple-choice quiz. We attempted to make the problems as isomorphic as possible, though the mapping between formats cannot be perfect or the problems would be identical. It is the case that for some representation pairs the mapping is more complete than for others. It is also worth noting that we use the word 'isomorphic' to mean isomorphic from the point of view of a physicist. A student may have a different view of the similarity (or lack thereof) between these problems.[10]

DATA

The above trials provided a wealth of data, primarily in the form of student success rates on the different problems. Various examinations are possible, including but not limited to comparisons of student performances on isomorphic problems in different formats and comparisons of student performance in choice and random-assignment (control) recitation sections.

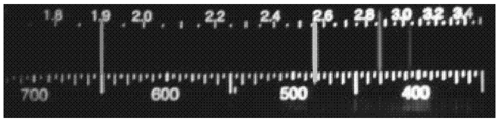
Performance Across Representational Format

We shall first look at examples of varying student performance on problems of different representational format that are otherwise very similar. Consider the pair of questions shown in Figure 1. Both require knowledge of how the electron orbit radius varies with the principal quantum number in the Bohr model. The questions differ only in which specific transition is being presented and in whether the problem and solutions are expressed in graphs or pictures/diagrams. Of the 218 students who answered both problems, 77% answered the graphical problem correctly and 62% answered the pictorial problem correctly. This difference is statistically significant ($p = 0.006$, 2-tailed binomial test) and is particularly interesting in that the graphical representation is a rather non-standard one. Students had not yet seen any graphs of orbital radius versus quantum number, but the pictorial representation of electron orbits should be somewhat familiar since it is featured in both the textbook and the lectures. Further examination of the individual student answers on these two questions indicates that this performance difference can be attributed almost entirely to the 36 students who answered the graphical problem correctly and missed the pictorial problem by choosing the distractor C. This distractor bears a strong resemblance to the energy-level diagrams seen in the Bohr model section of the text and lectures. It appears that in this case representational variations may be traceable to a very topic-dependent cueing on visual features of one of the problems.

Another example of performance variation across isomorphic problems exists in the second quiz given, which deals with the emission spectrum of a Bohr-model hydrogen atom. The students were prompted to recall the spectrum of hydrogen, and were asked how that spectrum would change if the binding of the electron to the nucleus were weaker. Figure 2 shows the problem setups and one distractor for the verbal and pictorial formats. Note that one week previous to

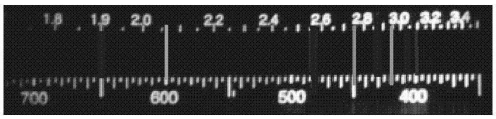
Spectroscopy Problem -- Pictorial Format

The Balmer series of spectral lines is shown below, as seen through a spectrometer:



Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus and the ionization energy is 13 eV instead of 13.6 eV. Choose the picture that best represents what the new spectrum would look like.

B)



Spectroscopy Problem -- Verbal Format

Consider the Balmer series of spectral lines from hydrogen gas. Now suppose we are in a world where electric charges are weaker, so the electron is not held as tightly by the nucleus. This means that the ionization energy for the electron will be smaller. What will happen to the Balmer lines that we see?

B) The spectral lines will all shift to shorter wavelengths (toward the bluer colors).

FIGURE 2. Setup and second answer choice for the verbal and pictorial format quizzes given in the second trial. The other distractors match up between the formats as well.

the quiz, students had a lab covering emission spectroscopy, and the quiz images match what students would have seen through their equipment. Nineteen students in the control group were randomly assigned a verbal format quiz, and 18 were assigned a pictorial format quiz. 32% of the verbal group answered the question correctly, while 83% of the pictorial group answered correctly. This difference is significant at the $p = 0.0014$ level. Answer breakdowns indicate that the students in the verbal group that missed the question almost always chose the distractor corresponding to the spectral lines moving in the wrong direction (pictured in Figure 2), whereas only one student from the pictorial group made this error.

Effects of Student Choice on Representational Performance

We next examine how providing students a choice of representational format affected their quiz performance, as compared to students who received a randomly assigned quiz format. There were a total of eight choice/control comparisons available (two trials with four formats each). Of these eight, six showed a statistically significant performance difference. These data are summarized in Table 1.

These results are notable in that the effects are in some cases quite strong. For instance, 90% of the 'choice' group answered the math format question correctly for the second topic, while 13% of the control group answered the same problem correctly. In addition, the direction of the effect can vary. In four of the six cases, giving students a choice of formats significantly increased performance, while in two of the six cases it resulted in a significant decrease. Furthermore, when comparing across content areas we see reversals in the direction of the effect. On the wave optics quiz, students in the choice group do better than the control group on the pictorial representational format and worse on the graphical representational format, while on the atomic physics quiz the students in the choice group do worse on the pictorial representation and better on the graphical representation.

DISCUSSION AND CONCLUSION

Explaining the performance differences between choice and control groups is non-trivial at this point. In breaking down the answers problem by problem, we see that in some cases the performance differences can be attributed a particular distractor, as in the case of the atomic physics homework questions. In other cases, the group that performed more poorly does not show a preference for a particular distractor as compared to the higher performing group. Examining the student comments, we do see that the students who chose the pictorial format were much more likely to cite a recent lab in making their decision, despite the fact that the labs included all four of the representations from each quiz. However, this also varied by subject, as students taking the pictorial quiz over diffraction were more likely to reference the lab than students taking the pictorial quiz over atomic physics.

TABLE 1. Fraction of students answering a quiz problem correct, broken down by representational format and choice/random assignment section. P-values compare success rate of choice and control groups (binomial test, 2-tailed).

	Wave Optics			**Atomic Physics**		
	Graphical	Pictorial	Verbal	Math	Graphical	Pictorial
Choice	0.04 (N=26)	0.82 (N=72)	0.81 (N=21)	0.90 (N=42)	0.96 (N=28)	0.39 (N=58)
Control	0.25 (N=16) p = 0.04	0.53 (N=19) p = 0.03	0.32 (N=19) p = 0.002	0.13 (N=15) p < 0.0001	0.53 (N=17) p = 0.0004	0.83 (N=18) p = 0.0012

Our results provide examples of performance differences on isomorphic problems. They also indicate that representational performance can vary strongly with subject area and with other factors such as student choice. Judging from these data, it is possible that people have underestimated the severity of the effect of representational format on student performance. Also, while the performance variations we observe are difficult to explain, their arrangement suggests that the effect is more complicated than a simple alignment of student choices with some individual learning style. When we compare the wave optics and atomic physics quiz performances of the same students choosing the same representational formats, we see substantial performance differences. For example, many of the students who chose a pictorial format quiz for the first subject area chose it again for the second area. Despite this overlap, the choice group substantially outperformed the control group on the wave optics class but was then outperformed by this same control group on the atomic physics quiz. It appears that student representational performance may be heavily influenced by the context of the problems and of the course in which they are presented.

Future work will include student interviews to probe students' problem solving strategies on these quizzes as well as to further elucidate their reasons for choosing the formats that they do. We hope that this and other work will contribute to a more complete characterization of physics students' representational skills and 'meta-skills', perhaps with implications for building this competence through explicit instruction.

ACKNOWLEDGEMENTS

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