TABLE 1. Categorization of responses from students in Courses A-C who did not answer Q4(b) correctly [N = 79]. Roman numerals (i)-(v) correspond to the illustrative student responses quoted in the text.

TYPE OF REASONING	%
Magnitude of **J** increasing to the right (i)	47
Field lines becoming more dense (ii)	13
Charge density as source of divergence (iii)	8
Non-zero net flux of **J** through a surface (iv)	6
Source/sink exists somewhere (v)	5
Other	9
Blank / no reasoning provided	12

quoted above [(i) – (v)] corresponds to one of the first five categories listed in the table, as indicated.

During the development of the CURrENT, validation interviews were conducted with 7 students to establish that they were interpreting these questions as intended, and also that their written responses were consistent with the reasoning they expressed verbally. This process was recently repeated with 4 students using the latest version (V.5), with the same results [6]. We would therefore argue that the categorization of written responses in Table 1 is an adequate reflection of student thinking regarding the divergence in this specific context. Throughout its history, this question has remained essentially unchanged, so we were also able to determine the percentage of correct responses for a larger and more diverse population. Across eight different institutions, only 146 from a total of 376 students (=39%) correctly stated that $\nabla \cdot \mathbf{J} = 0$ in this situation.

DISCUSSION AND CONCLUSIONS

The data presented here show that students may rely on a number of ideas when reasoning about the divergence of a vector field, some of which are productive in specific contexts, but not universally. Most students at St Andrews recognized during the midterm exam that the divergence of a field is only non-zero at the location of a source (when asked about an electric dipole field), but many did not access this same resource when later considering the current density inside a wire, even though they had all seen a similar homework problem (albeit one that did not contain the distraction of "converging" field lines). Instead, they appealed to other resources that are not necessarily wrong in and of themselves, but lead to erroneous conclusions when applied in the wrong situations. In other words, there were many students who understood the meaning of divergence in the context of Gauss' law, but in other contexts did not employ an epistemological framing [10] that led them to access the relevant knowledge (e.g., not recognizing the significance of the word "steady" in the problem statement of Q4).

Despite instruction to the contrary, a significant number of students expressed incorrect reasoning, primarily having to do with the divergence being non-zero when at least one of the components of a field is changing with distance (non-zero partial derivatives), or wherever field lines in a diagram are becoming more or less closely spaced. Students were easily distracted by the *semblance* of a divergence in a field; and some believed the existence of a source/sink at one point in space implies the divergence is everywhere non-zero. Our experience suggests that students face analogous difficulties with the curl of a vector field, which requires further investigation.

Although we have insufficient evidence to demonstrate a direct link between specific instructional choices and the student thinking described herein, there are good reasons to believe that when students are introduced to the concept of divergence without reference to field sources, and without explicit attention paid to common difficulties, they can develop robust misconceptions that will be resistant to later instruction. We suggest that instructors be conscious of this when developing the tools of vector calculus in the physical context of electromagnetism; and should explicitly address these specific student difficulties within a variety of topics, in order for students to develop proficiency in determining when a particular conceptual resource will be productive.

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