Assessing Students' Ability to Solve Introductory Physics Problems Using Integrals in Symbolic and Graphical Representations

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Abstract. Integration is widely used in physics in electricity and magnetism (E&M), as well as in mechanics, to calculate physical quantities from other non-constant quantities. We designed a survey to assess students' ability to apply integration to physics problems in introductory physics. Each student was given a set of eight problems, and each set of problems had two different versions; one consisted of symbolic problems and the other graphical problems. The purpose of this study was to investigate students' strategies for solving physics problems that use integrals in first and second-semester calculus-based physics. Our results indicate that most students had difficulty even recognizing that an integral is needed to solve the problem.

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INTRODUCTION

Unlike mathematical problems, where students are required to compute integrals that are already provided, most physics problems do not provide students with pre-determined integrals. Rather, students are required to derive the integrals depending on the physical situation in the problem.

Previous studies investigated students' conceptual understanding of integration and the relationship between a definite integral and the area under the curve [1-4]. McDermott, et al. [1] identified difficulties that students encounter while connecting graphs to physical concepts for example in kinematics. Meridith [5] investigated the mathematical resources students invoke to guide their work as they integrate in the context of electrostatistics. Manogue et al. [6] and Wallace et al. [7] investigated students' difficulties in interpreting and calculating the integral in Ampere's law equation. In recent studies, Nguyen et al. [2, 3] investigated how students relate a definite integral with area under the curve when solving problems in mechanics and electricity and magnetism (E&M). Studies in mathematics education have also addressed students' use of integration. Both Sealy [8] and Thompson et al. [9] emphasized that using the concept of area under the curve is helpful only if students relate it to the structure of the Riemann sum.

The studies in physics education have focused on students' difficulties with using integration in physics problem solving in a specific topical area while the studies in mathematics education have focused on the mechanics of solving an integral. However, no detailed study has yet been completed that investigates the strategies that students use to solve physics problems requiring the use of integration over a broad range of topical areas.

We designed a survey to assess the strategies that students apply to problems requiring integration, encompassing a wide range of topics in mechanics and E&M. The problems were presented in graphical and symbolic representations. The survey was administered to students after they had completed mechanics and again after the same cohort of students had completed E&M in their second semester of calculus-based physics. Our research questions are:

- What strategies do students use while solving physics problems requiring integrals in the symbolic and graphical representations?
- How do students' strategies change after completing second semester calculus-based physics?

METHODOLOGY

Participants in our study were N = 222 students enrolled in second semester calculus-based physics at Kansas State University (KSU). Test 1 was administered at the beginning of second semester calculus-based physics covering E&M. Students had already completed first semester calculus-based physics covering mechanics. There was a hiatus of three months between these classes. The prerequisites are Calculus I for the first semester and Calculus II for the second semester. Test 1 consisted of 16 questions, -- eight in graphical and eight in symbolic representations. The topics covered in test 1 were linear and rotational kinematics, impulse, work done by force, rolling frictional force and gas laws. Test 2 was given at the end of second semester calculus-based physics. Test 2 required similar integration strategies as Test 1, but covered the topics of electric and magnetic flux, electric energy, work done by electric forces, change in current through an inductor, voltage change in a capacitor and potential difference.

In the symbolic problems, students had to set up the integral and compute it using the algebraic expression provided. In the graphical problems students had to set up the integral and compute it using area under the graph. On each test, students were provided an equation sheet containing the non-integral version of the physics formulae and an integral table

Figures 1 and 2 show the graphical problem on test 1 and the corresponding problem on test 2.

The velocity of a block moving along the x-axis with respect to time is given as per the following graph. Find the displacement of the block on the x-axis from t = 1.0 s to t = 3.0 s.

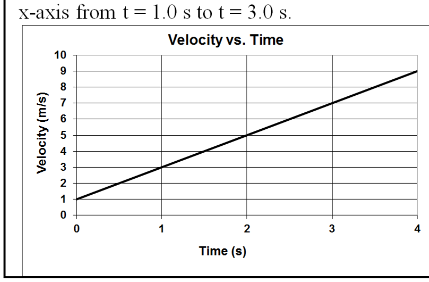


FIGURE 1. Graphical problem on test 1.

The direct electric current *i(t)* through a wire is changing with time *t* as per the following graph. Find the total charge passing through a cross-section of the wire during the time from t = 1.0 s to t = 3.0 s.

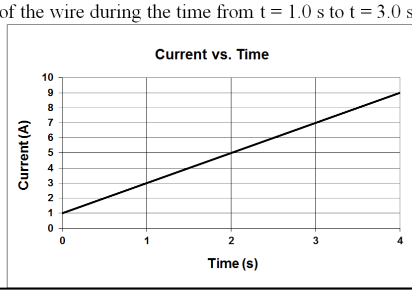
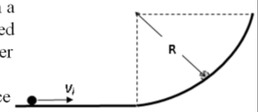


FIGURE 2. Graphical problem on test 2 corresponding to the test 1 problem in Fig. 1.

A symbolic problem on test 1 and a corresponding problem on test 2 are in Figs. 3 and 4 respectively.

A small particle moves on a track as shown. The curved part of the track is a quarter of a circle with radius, R=10cm. The friction force on this circular part of the track is given as per the equation:



$F(\theta) = -1.5\,\theta + 3$

where θ is angle in radian, F is the magnitude of friction force in Newton. Find the magnitude of the work done by friction on this circular part of the track.

FIGURE 3. Symbolic problem on test 1.

The electric field *E(r)* in a region of space varies with respect to distance *r* is given as per the equation:

$$E(r) = \frac{5}{r}$$

where *r* is position in meters and *E(r)* is the magnitude of the electric field in kilo-Volts per meter. Find the work done by the electric force when a charge Q = –2.0 μC moves from r = 0.3 m to r = 0.1 m.

FIGURE 4. Symbolic problem on test 2 corresponding to the test 1 problem in Fig. 3.

We coded the written solutions for the strategies they used to solve the problem [10]. An inter-rater reliability of 100% between two independent raters was established after discussion.

RESULTS AND DISCUSSION

Students' Problem Solving Strategies

We first describe in general the strategies that students used while computing graphical and symbolic problems on both test 1 and test 2. The strategies used for the graphical problems were:

S-1 (area): Finding the area under the curve S-2 (averaging): Averaging values from the end points of the curve and multiplying this value with the change in dependent variable.

S-2 (averaging): Averaging values from the end points of the curve and multiplying this value with the change in dependent variable.

S-3 (conversion): Using values from the graph, plugging them into a calculator that provides the algebraic representation equivalent to the graph and then symbolically integrating the algebraic representation.

In principle, any of the strategies above can be used to successfully solve the problem. Some of the students were able to use one of the aforementioned strategies, but did not succeed in getting the correct answer. They incorrectly calculated the area under the curve, incorrectly computed the integral (writing the incorrect equation of the curve) or made some other calculation mistake.

Strategies used by students who did not recognize the need for an integral were categorized as S-4 (no integration). These students computed quantities assuming that no integration was needed

In addition to S2 (averaging) that was used for the graphical problems, solutions for the symbolic problems also included two other strategies:

S-5 (computing): Directly integrating the symbolic expression.

S-6 (plotting): Plotting a graph from the given symbolic equation and then finding the area under the curve of the graph.

Similar to the graphical problems some of the students did not recognize the need for an integral. These strategies were coded as S-4 (no integration)

Table 1 summarizes the strategies used for the two kinds of problems – graphical and symbolic.

TABLE 1. Description of problem solving strategies used by students on graphical and symbolic problems

Graphical Problem Solving Strategies	Symbolic Problem Solving Strategies
S-1 (**area**): Find area under curve.	**S-5** (**computing**): Integrate the symbolic expression.
S-2 (**averaging**): Average end point values and multiply with change in dependent variable.	**S-2** (**averaging**): Same as for graphical problem, but compute values from expression.
S-3 (**conversion**): Find symbolic expression corresponding to the curve and integrate it.	**S-6** (**plotting**): Plot the graph from the symbolic expression and then find area under curve of graph.
S-4 (no integration): Do not recognize the need for an integral and finding the quantity using constant functions.	

Graphical Problems Results

Figure 5 shows the percentage of responses that were coded under each of the strategies used for graphical problems on both test 1 and test 2. Students used the same set of strategies as described above while solving graphical problems on test 1 and test 2 although the percentage was different. A majority (about 65%) of the responses were categorized as S-4 (none) on test 1, whereas they were about 50% on test 2. The students, who successfully used S-1 (area) on test 1 was about 21%, almost comparable to 22% who successfully used S-1 (area) on test 2. Twenty-six percent of students used S-3 (conversion) on test 1 as compared to about 7% on test 2. The percentage of students using S-2 (averaging) on test 1 was 6%, slightly higher than 2% on test 2.

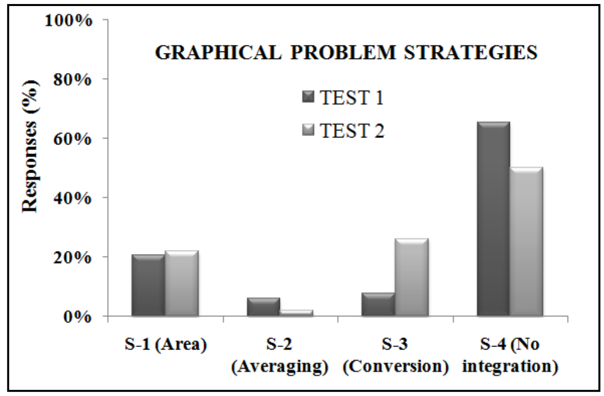


FIGURE 5. Test 1 versus Test 2 comparison for graphical problems.

Symbolic Problem Results

Figure 6 shows the percentage of responses that were coded under each of the strategies used for symbolic problems on both test 1 and test 2. Students used the same set of strategies as described above while solving symbolic problems on both test 1 and test 2, although the percentage was different. The only exception was that no responses using S-6 (plotting) were found on test 2 although they accounted for about 2% on test 1. On the symbolic problems on test 1, similar to the graphical problems, a vast majority of responses - about 69% - were categorized as S-4 (no integration), whereas it was about 43% on test 2. About 56% of the responses used the S-5 (computing) strategy successfully on test 2 as compared to 22% on test 1. However, about 8% and 2% of the responses were categorized as S-2 (averaging) and S-6 (plotting) on test 1 as compared to 1% and 0% on test 2, respectively.

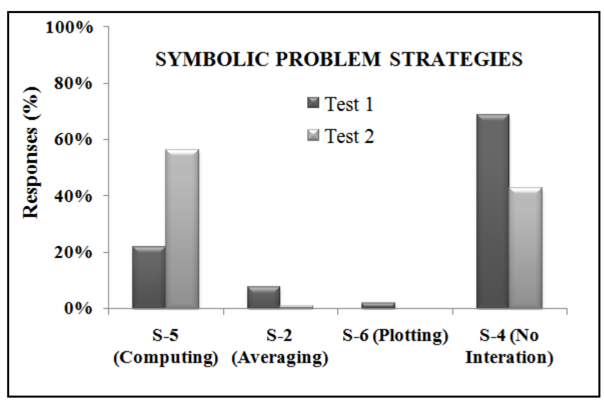


FIGURE 6. Test 1 versus Test 2 comparison for symbolic problems.

Test 1 versus Test 2 Comparisons

A chi-square test was used to determine if there was a statistically significant difference between the strategies on test 1 and test 2. Fisher's exact test was used when expected cell counts were less than five. Adjusted residuals were examined to reveal which cells contributed to the significance when a statistically significant result was found [10].

We found a statistically significant difference between the types of strategies on test 1 and test 2 for graphical problems, $\chi2$ (3, N = 1676) = 119.51, $p < 0.001$, V = 0.267. S-4 (no integration) and S-2 (averaging) occurred more often on test 1 followed by S-1 (area), whereas S-3 (conversion) followed by S-1 (area) occurred more often on test 2.

On the symbolic problems we again found a statistically significant difference between the strategies on test 1 and test 2, $\chi2$ (3, N = 1676) = 119.51, $p < 0.001$, V = 0.267. Here again S-4 (no integration) occurred more often on test 1 followed by S-2 (averaging) and S-6 (plotting), while S-5 (computing) occurred more often on test 2

In general, this comparison suggests a statistically significant improvement on test 1 to test 2 as evinced by migration away from the S-4 (no integration) category. This trend is a bit more prominent in the symbolic problems than the graphical problems. There is significant increase in the number of responses using computational strategy (S-5) for symbolic problems but no such improvement in the graphical problems.

CONCLUSIONS

We found that students used a variety of strategies to solve problems requiring integration. However, the percentages of responses demonstrating various strategies differed from test 1 to test 2 and between graphical and symbolic problems

For both kinds of problems, a vast majority of responses on test 1 indicate that students did not even seem to recognize the need for integration on test 1. Our results show a number of responses indicate that students who did not even demonstrate a recognition of the need for an integral was prominently higher on test 1 as compared to test 2 for both symbolic and graphical problems.

Our results indicate a marginally greater improvement on the symbolic problems compared to the graphical problems. This result could be because most problems in second semester calculus-based physics require symbolic integration while few, if any, require the use of finding the area under a curve.

LIMITATIONS & IMPLICATIONS

The most important limitation of this study is that due to the paper-and-pencil nature of the task, we were unable to gain any insights into students' thought processes as they solved these problems. For instance, a large number of responses indicated the inability to recognize the need for an integral. We would need to interview these students to find out how and why they believed no integral was necessary to solve these problems. These findings suggest the need for interventions that focus on enabling students to appropriately recognize the need for integration in physics problems.

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