

Comparing Physics and Math Problems

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Abstract. Presented is a subsection of a larger project to understand and facilitate students' use of mathematics when solving physics problems. Specifically, this study is an examination of how students group/pair problems in math and physics. The results show that whether students examine surface or structure is in part dependent on context.

Keywords: physics education, math, similarity

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INTRODUCTION

Few physicists would argue against the necessity of mathematics for understanding physics principles; aside from a traditional calculus-based introductory course, even a basic conceptual physics course requires an amount of proportional reasoning skills.

The intersection of math and physics is a topic which has been examined with increasing frequency in the recent past. In the 2011 PERC proceedings alone, there were 11 papers on different aspects of this topic, from vectors to partial derivatives, and with a heavy focus on integrals [1]. Still, it is easy to see that much work remains to understand more fully how students incorporate techniques from these two subjects and from that data create possible interventions to assist in the melding of disciplines.

The work presented here is a continuation of a larger study designed to examine how students identify which math concepts are needed to solve a particular problem and how they are implemented as they work toward a solution.

LITERATURE REVIEW

While the topic of problem solving has pervaded the physics education research literature, there has until recently been very few studies focusing on the utilization of mathematics in problem solving [2]. Classic work by Bassok and Holyoak studied the ordering effects of learning either math or physics problems first [3] and was built upon by Rebello *et al.* to examine the type of transfer as a function of the level of structure to show that students may not have improved problem-solving performance despite their ability to recognize similar problems [4].

More recent work has specifically focused on different representation issues between the

disciplines in attempt of explaining the effect of the different representations [5, 6]. Presumably because of their vast use in physics, specific attention has been paid to integration. The representations of integrals is one area of recent study [7, 8], as well as the 'area under the curve' interpretation [1, 9] and broader examination of the application of integrals to specific physics topics, such as electricity [10].

THIS STUDY

The previous research provides an excellent foundation from which to begin a research study encompassing a more holistic view of student use of math in solving physics problems. In previous phases of this project, students were provided a written sample of physics and mathematics problems and were asked to pair the problems according to similarity [11]. Those results showed that students had difficulty with the integrals, as expected from the literature review, and that their difficulties seemed to focus on the difference between definite and indefinite integrals. Still, the small sample size and written-response nature of the data collection indicated that more examination was needed.

As a continuation of those results, this phase sought to more deeply probe how students compare math and physics problems, and the features they look for when determining similarity. In this phase, group intervening techniques were used to obtain a richer set of data and allow for probing by the interviewer as necessary.

Methodology

This research was conducted with a phenomenological approach to ensure coherence with the idea of a student-centered learning experience [12].

Table 1. Problem sets provided to students

MATH PROBLEMS	PHYSICS PROBLEMS
<p>A. What is 572 divided by thirteen?</p> <p>B. What is the value of x as given by: $H(x, y, z) = xy + xz + yz^2$ if $H = 5.4$, $y = 2.3$ and $z = 0.80$?</p> <p>C. Evaluate the integral $\int(\sqrt{x} + 4x^2) dx$.</p> <p>D. Evaluate the integral $\int_1^4(3x^2 - 2x + 5)dx$.</p> <p>E. Calculate the area under the curve given by $y(x) = x\sqrt{9 - x^2}$ between $x=0s$ and $x=8s$.</p> <p>F. Perform the following calculation: 2423×157.</p> <p>G. What is the minimum value of a curve given by $f(x) = x + \frac{32}{x^2}$ over the interval $[0,10]$?</p> <p>H. A certain function is given by the equation $f(x) = 3x^2 - 4x^3 = 6$. At what points are there relative extrema?</p> <p>J. The equation of a parabola is $(x + 3)^2 = 4p(y - 2)$, where p is the focus of the parabola. Find the focus if a single point on the parabola is given by the coordinate $(-7,-4)$.</p> <p>K. What is the integral of the following function: $g(t) = t^4 + 3t^2 \sin(t) - 8t + 2?$</p>	<p>1. Two airplanes are flying in the sky – the displacement of plane A is $d = \sqrt{(5.8t^2 - 2)^2 + (6 - t^3)^2}$ relative to plane B. What is the closest distance that the two planes come to each other?</p> <p>2. A force $\vec{F} = (3x^2)N\hat{i} + (4)N\hat{j}$ with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates $(2m, 3m)$ to $(3m, 0m)$?</p> <p>3. A disk is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by $\theta(t) = -3.4 + 2.3t - 0.43t^2$ with t is seconds, θ in radians, and the initial angular position at zero. At what time does $\theta(t)$ reach its minimum value?</p> <p>4. A time-varying force of $F(t) = 3(t^5 - 5t + 1)N$ pushes a block along a horizontal surface. How much work does the force exert on the block in the first 8m of the motion?</p> <p>5. A car travels at a velocity of 89m for 6.5s. Find how much distance the car traveled.</p> <p>6. If the acceleration of a jet ski is given by $\vec{a} = (4t^2 - 8)\frac{m}{s}\hat{i} + (6 - 7t^2)\frac{m}{s}\hat{j}$, what is the equation for the jet ski's velocity?</p> <p>7. The electric potential of a charged disk is given by $V = \frac{\sigma}{2\epsilon_0}(\sqrt{z^2 + R^2} - z)$. Write an expression for the electric field at any point from this disk.</p> <p>8. The average rate at which energy is conducted outward through the ground surface in North America is $54.0\frac{mW}{m^2}$, and the average thermal conductivity of the rocks near the surface is $2.50\frac{W}{m\cdot K}$. Assuming a surface temperature of $100^\circ C$, find the temperature at a depths of 35.0km below the surface.</p> <p>9. The mass of an iron atom is $9.27 \times 10^{-27}kg$, and its volume is $1.178 \times 10^{-30}m^3$. What is the density of an iron atom? Remember that density is defined as mass/volume.</p> <p>10. A batter hits a ball when its center is 1.22m above the ground. The ball leaves the bat at an angle of 45° with respect to the ground. With that launch angle, the ball should have a horizontal range (as measured at the 1.22m level) of 107m. Does the ball clear a 7.32m-high fence that is 97.5m from the launch site?</p>
Solution-based Pairings:	A-F / 5-9 B-J / 8-10 G-H / 1-3 D-E / 2-4 C-K / 6-7

A set of group interviews was conducted at Mercyhurst University during the Winter 2011-2012 term. All students were enrolled in the second term of calculus-based introductory physics; they participated in one of 10 self-selected groups of 3-4 students per group. In previous phases, students were given a set of math problems and a set of physics problems, and were asked to pair them. However, to more deeply probe the ways in which students ‘compare problems’, the protocol for this study was lengthened. In this study, students were first given a set of 10 purely mathematics problems and were asked to group the problems in any way they could think of. Then, they were prompted to search for pairs of problems if they had created larger groups. Next, students were given a set of 10 physics problems along with the same directions to group the problems in any way they felt appropriate. Based on the cues from the first task, however, most groups immediately attempted to pair the physics

problems. Each of those sets of pairs was retained by the interviewer. Finally, the students were asked to match math problem pairs to physics problem pairs. The group interview format was chosen for several reasons: previous iterations of this study showed that the cognitive load was at times overwhelming for student without peer support, the participant population was more open to discussion in groups than as individuals, and because the task was so lengthy that individual interviews were time and cost prohibitive. The problems were printed and laminated onto large index cards for easy manipulation. Throughout the entire interview, students were encouraged to think aloud, to interact heavily with their group members, and were asked numerous questions by the interviewer about their choice of pairings and the features they were comparing. The math and physics problem sets can be seen in Table 1. The interviewers only intervened to clarify/understand the students explanations and to

encourage them to continue working. If needed, the students were prompted to think about the solutions to the problems, but no indication was ever given as to quality of the students explanations or pairings.

This study revolves around studying how students use mathematics when learning physics. As such, it was decided that pairings would be created based on the mathematics involved in obtaining a solution. Based on the results of previous phases, the problems were designed so that the following general ideas were represented: simple math, algebra, indefinite integration, definite integration, and relative extrema (derivatives). Further, certain features were purposefully embedded to boost the level of the problems and the potential for student distracters: for example, some integration limits are given as a data points while some are listed outright, there are 2D definite integrals, and some are embedded within a mathematical context (ie the equation of a parabola).

All interviews were fully transcribed so that data could be obtained for not only what pairs students created, but also their explanations and discussions. Due to the short nature of this paper, the results below contain only a subsection of the findings and focus on the largest-grained features of the analysis.

FINDINGS

Math Problems

As the students were handed the set of 10 math problems, it took relatively little time for them to begin sorting the problems into piles. Many students indicated that they were looking only for key words or symbols, such as an integral symbol or the word “extrema.” In many cases, the students began with only 2 or 3 piles of problems; for example, one group made piles of ‘calculus’ and ‘not calculus’ problems.

When prompted to create pairs, 9 out of the 10 groups were able to do so fairly quickly; one group struggled with the task and ultimately created only 2 pairs of problems before giving up on the task entirely. Of the nine groups to successfully create problem pairs, 6 groups did so as intended. One group mixed pairs of definite (D-E) and indefinite (C-K) integrals, and the two remaining groups mixed all the integral and extrema/derivative pairs (G-H). Even the group who avoided the task matched the simple arithmetic pair (A-F).

Nevertheless, the students seemed to promptly sort the problems into what they viewed as the most logical pairs. When asked to explain how they sorted the problems, students in the nine groups reported that they sorted by solution type. As one student put it: “you read the problem, and you just know what math it wants you to do to get the answer.” When

specifically prompted about the context dependence of the parabola problem (B-J pair), the students indicated that it was irrelevant because both problems were still just ‘plug-and-chug’ regardless of what the equation represented.

Physics Problems

The physics problem pairings were much more challenging to all of the groups. The initial difference that the interviewer noticed was how much longer the students spent reading the problems. Whereas they mentioned looking for key words or symbols in the math problems, they said they needed to read each physics problem in its entirety to determine what type of problem it was. It should also be noted that the students immediately sought to create pairs of problems when sorting and did not attempt to group them first, presumably because they had just been asked to pair the math problems.

Only two groups paired all of the problems as intended on the first try. Four other groups created some mismatched pairs, but while they were explaining their pairings they realized they were not satisfied with their original matches and ultimately rearranged the problems into the intended pairings. The remaining four groups had a variety of different pairings, with no noticeable trends in their other matches. The group who had difficulty with the math problems did complete this task, and used a very topics-based approach to the pairings.

For the physics problems, the groups had many varied explanations for their method of pairings. Many of the groups indicated that they should be paired based on the topic/content they covered. For example, one group paired problems 3 and 10 because they both were asking you to find a minimum distance or displacement. This was not uncommon: each of the four groups with alternative pairings had used a mix of “topic” and “solution type” pairs. One group in particular was very explicit about the importance of the physics topic/content, and insisted on creating two independent sets of pairings (ultimately, they did not even coherently pair all problems in either category).

One other interesting trend is that the same ambiguity between types of integrals existed in the physics problems. Students often commented on the problems requiring calculus or integration to solve, but did not always make the distinction between definite (2-4) and indefinite (6-7) integrals. As was seen in the previous phase, students had little difficulty recognizing the extrema/derivative problems in the physics context, with 9 out of 10 groups pairing those problems (1-3) as intended. The simpler arithmetic problems (5-9) and ‘plug-

and-chug' problems (8-10) were paired by 8 and 9 groups, respectively.

Combining Math and Physics

Regardless of the accuracy of their initial pairings, the students were then asked to combine the math and physics problems together and find similarities and create combinations there in. Naturally, this process was easier for those groups of students who had paired at least a majority of the problems in the individual sets as intended by their mathematical solution. Still, it provided an opportunity to discuss many aspects of physics problems and the ways in which you can classify and group problems together.

In the end, three groups successfully matched their math and physics pairs and provided an argument for their pairings based upon the mathematical solution process. The integral confusion continued to pervade in this task, though it did spark a discussion of the meaning of limits in a number of the groups.

Interestingly, all three of these groups were those who had to reconsider their initial physics pairings and make changes to their first matches. The group who declined to finish matching the math pairs also declined to attempt this part of the task – in general, they indicated that the idea of even comparing problems to begin with was “pointless” and “not helpful for learning how to solve them.”

CONCLUSIONS AND FUTURE WORK

This is a small representation of the data obtained from group interviews which were conducted with students enrolled in a calculus-based physics course. And with a purpose of studying how they compared a variety of physics and math problems. Initial findings show that students categorize math and physics problems in very different ways. When looking at math problems, students indicate looking for key words or symbols and categorizing problems based on the mathematical technique required to obtain a solution. When categorizing physics problems, however, students seem to use a mix of contextual, topical, and mathematical features. The group dynamic highlighted the different ways students think about and approach problems, which in turn provided a more dynamic and challenging task for all involved.

This work is a continuation of a larger research project aimed at examining how students use mathematics in the physics problem solving process. To that end, future work will include and be based on a more fine-grained analysis of the student

discussions surrounding their pairings. While much of the current literature focuses specifically on the topic of integration, this study seeks to take a broader approach and focus on how students decide which solution path to take and which mathematical techniques to incorporate as they work toward a solution and build their problem solving repertoire.

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