Student Expectations in a Group Learning Activity on Harmonic Motion

Adam Kaczynski and Michael C. Wittmann

Department of Physics and Astronomy, University of Maine, Orono, ME, 04469, USA

Abstract. Students in a sophomore-level mechanics course participated in a new group learning activity that was intended to support model-building and finding coherence between multiple representations in the context of an underdamped harmonic system. Not all of the student groups framed the activity in the same way, and many attempted tasks that existed outside of the prompts of the activity. For one group, this meant that instead of providing a rich verbal description, they framed the activity as finding a mathematical expression.

Keywords: framing, expectations, damped harmonic motion
PACS: 01.40.-d, 01.30.Cc

INTRODUCTION

Students come to any physics course with expectations about the world, science, and learning. Those expectations have an influence on the way students make observations, reason about phenomena, and draw conclusions. In certain situations, those expectations may be inconsistent with those of the physics community and may lead to results that are inconsistent with the body of knowledge in physics.

Researchers have used multiple methods in examining students' expectations. The MPEX [1] has been used to examine differences in expectations about physics, both within a population over a period of time and between populations. Results have consistently shown a gap between the expectations of introductory students and experts. Epistemological framing [2, 3] has been used to describe how students' expectations about and knowledge of a situation affect their behavior in a physics learning environment. Where Scherr [4] has suggested behavioral criteria for the kinds of epistemological framing, we are interested in the evidence to be found in the spoken elements of student group interactions.

Here we present an example of how students frame a group learning activity on underdamped harmonic motion. In doing this, we make a more explicit connection between epistemological framing in physics and framing in discourse [5]. Our results contribute to the discussion of epistemological framing in the context of problem solving during group learning activities by suggesting observable criteria for framing.

GROUP LEARNING ACTIVITY ON UNDERDAMPED HARMONIC MOTION

A new group learning activity, designed to occur over two class periods, was created for a sophomore-level mechanics course. The activity opens with a demonstration of an underdamped harmonic system. A glider is on an air track and attached to posts on either side by taut springs. It is pulled to one side, released, and allowed to oscillate until it comes to rest. A motion sensor is used to record the position of the cart, which is presented to students graphically as a function of time (Figure 1). Following the demonstration, the activity is broken into four content related steps and a final reflective step: describing the motion, determining the factors that are relevant to changing the motion, representing Newton's Second Law mathematically, interpreting the mathematics without solving the differential equation in order to describe the cart's velocity, and reflecting back on the connections between the previous steps.

DATA

The group learning activity was implemented before lecture instruction on damped harmonic motion. Student groups contained three or four members, and three out of six groups were video- and audio-recorded. One group in particular (Jackson, Kurt, Lincoln, and Matthew) engaged with the activity in an unintended but interesting way. Following is a description of this group's discussion as they addressed the first part of the activity and consideration of different perspectives on their actions.

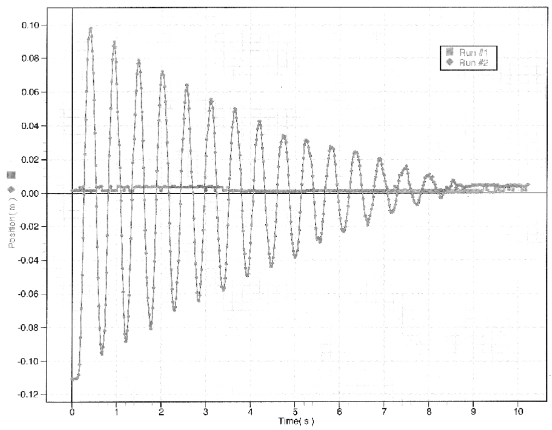

FIGURE 1. Scan of position vs time graph that was presented to students for the underdamped harmonic system

Seeking a mathematical expression

After the demonstration, the groups are prompted to provide a rich verbal description of the motion of the underdamped system and discuss any phenomena that might behave similarly.

The group quickly addressed the prompt for the verbal description by saying that the system moved back and forth and was dampened. Over the next several minutes, the students discussed several topics that did not explicitly address the prompts (an example being the role of a differential equations course on being a physicist), but they did discuss several examples of other systems that have similar behavior to the one from the demonstration.

When they were given a paper printout of the graph from the demonstration, Matthew said, "So we have to find an equation to describe this?" He then introduced a possible mathematical expression for the graph, saying, "This would be like a sine *e* to the negative *x*, right?". The rest of the group engaged with the expression, making it the focal point of the conversation.

Inventing an expression

The group proceeded with a discussion that included ideas to support and refine the mathematical expression that they introduced. In what follows, we see the group seeking consistency between the graph and the expression.

As Matthew originally introduced the expression, it read as $x(t) = sin(e^{-x})$. The group was able to take from that expression the exponential and sine components, which they matched to the graph.

> Lincoln: I was trying to think if it's going to be a sine of the *e* to the *x* function or is just sine of the *x*, then
> Kurt: It's *e* to the *x*, yup.
> Jackson: Yeah, 'cause the sine of the *x* dictates the uh, the actual motion
> Lincoln: Yeah
> Kurt: Is the- is the actual sinusoidal thing going on
> Lincoln: And then your *e* to the *x* is your um
> Kurt: Is the actual amplitude of each of that
> Lincoln: Yeah
> Kurt: which eventually the amplitude goes to zero.

The group matched the sine term to the oscillating aspect of the graph and the exponential term to the decreasing amplitude of the oscillation. Looking back to the graph in Figure 1, describing it as sinusoidal contained within an exponential envelope seems entirely plausible. They arrived at an expression consistent with that for an underdamped oscillator subject to a velocity-dependent force, $x(t) = Ae^{-\gamma t} sin(\omega t)$.

Justifying the expression

The group then compared their expression to the position expression for a simple harmonic oscillator. They did this in two ways: first by comparing the terms in the simple and underdamped harmonic motion expressions and then by transferring a strategy from free-fall problems to this situation.

Comparing terms to simple harmonic motion

The group first compared their expression for the underdamped system to the expression from simple harmonic motion, which they recalled from introductory physics.

> Lincoln: Wasn't the equa- the original spring equation *A* sine *x*?
> Kurt: Yeah, you have that amplitude, yeah, exactly.
> Lincoln: like in, like, [our introductory mechanics course], yeah
> Kurt: So that *A*- you have- so that *A* term but now you also have that *e* to the negative *x*

The group identified the leading constant of the simple harmonic expression as the component that described the amplitude. This is similar to how they described the exponential component when they matched their expression to the amplitude of the graph, so for underdamped motion, a leading exponential term is also needed. This reasoning ignores the change in frequency that also occurs when a simple harmonic oscillator is subject to a damping force.

Later in the activity, they made an argument to support this strategy, arguing for the independence of frequency and amplitude for the system.

> Lincoln: I think what you need to do is solve for each of these independent- like each- these two things independently and then once they come together they give you the- like 'cause you're gonna need a separate equation- um, completely independent of your harmonic oscillation to govern the dampening of it.
> ...
> Kurt: Yeah, because the dampening effect is- is nothing to do with your harmonic oscillation.

This is consistent with an overgeneralization from simple harmonic motion about the independence of frequency and amplitude for an oscillator [6].

Transferring a strategy from free-fall problems

The group also supported their expression by transferring a strategy from a problem previously encountered about an object in free-fall that was subject to drag. The group argued that the difference between simple harmonic motion and underdamped motion was similar to the difference between an object in free-fall and one that also includes drag. For the free-fall problem, students were assumed to be very familiar with uniformly accelerating motion. When including drag, a velocity-dependent force was needed in addition to the gravitational force in the mathematical representation of Newton's Second Law to appropriately model the system. The resulting differential equation would be solved to find the expression for $x(t)$. This strategy is similar to how we intended the groups to approach the underdamped system during a later step in the activity.

One group member described the strategy in this way:

> Lincoln: What we did with something falling is first we set up the equations for something falling completely regardless of air resistance, and then we put on, then we included your term for air resistance after we had a governed term regardless of air resistance.

We note the use of the phrase "put on" as a description of "includ[ing]" the air resistance term. The group continued:

> Lincoln: Harmonic oscillation's completely univer- like simple harmonic motion, for instance, has like governed principles that
> Kurt: For example, when we look into the air resistance things we have our mass times gravity
> Lincoln: Exactly
> Kurt: minus our cv term, we just hang on that cv term

Kurt added to the description, naming the two terms (mg and cv) and specifying the change made when including the drag force in the mathematical expression for Newton's Second Law ("we just hang on that cv term"). Their description demonstrated their recognition of this strategy and an ability to use it in the original context. The group then tried to apply the strategy to the underdamped system, but modified it.

> Lincoln: This sine x is our simple harmonic motion, like describes our simple harmonic motion, and we're- that's what we're experiencing i- like, without [the exponential] term.
> Jackson: e is just the constraints.
> Lincoln: Exactly, by putting e to the x in there, now we're- this is our dampening right here. And that's gonna govern this shrinking.

The group recognized two terms in their expression for underdamped motion, a sine term and an exponential term. They argued that the sine term described simple harmonic motion, which would be like the mg term from the free-fall situation. The group then "puts on" the exponential term, recognizing that a change was made in the free-fall problem when including drag, but not distinguishing between a change made during the setup of a problem and one made at the solution level.

DIFFERENT PERSPECTIVES ON THE DISCUSSION

Several different elements of the students' conversation can be described in terms of past research in physics education and the learning sciences. When looking at the group's conceptual understanding of the system, for example, we see that they discuss the issue of damping but do not recognize the effect that damping has on the natural frequency of a system. When looking at the group's use of mathematics, we can describe their work in terms of symbolic forms [7], in particular the idea of *base+change*. In the air drag problem, the base (mg) has a $-cv$ term added to it, while e^x is a change "put on" to the simple harmonic motion. (To be clear, we describe how the students treat the mathematics almost metaphorically, and we are less strict about the operational mathematical form of addition.)

We observe several ways in which conceptual and mathematical knowledge interact in the student discussion. We interpret these intersections as examples of meaningful epistemological activity. First, the group seeks consistency between the graphical and mathematical representations of the system. Second, the group engages in a useful problem solving heuristic of first considering (the physical and mathematical descriptions of) a simpler system. Finally, the group compares this new situation mathematically to a previously solved problem. These are all valuable habits of mind in a classroom.

FRAMING THE ACTIVITY

A way to look at the many pieces of the group's discussion is through the lens of epistemological framing. To provide evidence for how the group frames the activity, we use discourse analysis following from Tannen's work with framing during the retelling of a narrative. In this framework, a frame is a structure of expectations that is associated with a particular context, and those expectations are evident from linguistic clues [5]. In what follows, we describe the group framing the activity as "finding a mathematical expression."

Of Tannen's many ways of analyzing discourse [5], the conversational marker of *addition* provides the most visible evidence of the group's expectations during the portion of the activity we have presented. The addition of a new idea within narrative discourse implies an expectation about the appropriateness of the added information. The group spends quite some time addressing a task that they were not explicitly asked to do: finding the expression for the position of the cart as it moved. This task was introduced by Matthew when he suggested that the goal was to find an equation, and that goal was adopted by the rest of the group. Matthew's introduction of this task can be thought of as an addition to the given task, and is evidence for an expectation about its appropriateness. The group's adoption of this task, focusing the majority of their conversation around the solution during this activity, suggests that the rest of the group shared this expectation with Matthew, or at least did not find it wildly inappropriate.

We suggest *persistence* as another marker for expectations in the context of a group discussion. This bears similarity to Tannen's marker of repetition. Where repetition characterizes the repeating of an exact or nearly identical phrase, we use persistence more generally as the continued occurrence of a task or goal. We see the group's persistence in the task of finding a mathematical expression as additional evidence of their framing of the activity. Upon its introduction, the group sought to clarify the expression by comparing it to the graph of the motion. They then sought to support their expression by comparing it to simple harmonic motion. They first made a direct comparison between their expression and that for simple harmonic motion. They then compared simple and underdamped harmonic motion through an analogy to a free-fall problem. The group correctly identified a change made to the setup of the free-fall problem but used this as an argument to make a change to the solution of the underdamped problem. Each of these activities had the goal of finding a mathematical expression for the position. We note that their discussion about the mathematical expression also continued through an intervention by the facilitator (author MCW) who said, "What I would like to see is a- a kind of a rich verbal description here that complements the math stuff that you guys were just talking about." They persisted in discussing the expression following this intervention, which we believe supports our interpretation that they framed this activity as "finding a mathematical expression."

CONCLUSIONS

Individual perspectives (e.g., analysis of conceptual understanding, students' use of symbolic forms, or investigation of student habits of mind) provide insight into how students approach a group learning activity. Using Tannen's work on framing (namely, the markers of addition to a narrative and persistence in forming a particular narrative), we see that each of the individual perspectives provides evidence that students are framing this activity as "finding a mathematical expression." The task as given (and reinforced by the instructor) was explicitly designed to avoid this framing. The group's activity not only suggests preferences (about mathematics, the use of verbal descriptions in a physics class, and more) that our students bring to the classroom, but also shortcomings of the activity that was presented to them.

REFERENCES

1. E.F. Redish, J.M. Saul, and R.N. Steinberg, *American Journal of Physics*, **66**, pp. 212-224, (1998).
2. D. Hammer, A. Elby, R.E. Scherr, and E.F. Redish. In *Transfer of Learning from a Modern Multidisciplinary Perspective*, edited by J. Mestre (Information Age Publishing, Greenwich, CT, 2005), pp. 89-120.
3. E.F. Redish, in *Proceedings of the International School of Physics "Enrico Fermi," Research in Physics Education, Course CLVI*, edited by E. Redish and M. Vicentini, (IOS Press, 2004), pp. 1-60.
4. R.E. Scherr, and D. Hammer, *Cognition and Instruction*, **27**, pp. 147-174, (2009).
5. D. Tannen, *Framing In Discourse*, edited by D. Tannen, Oxford University Press, New York, (1993).
6. B.S. Ambrose, in *2006 Phys. Educ. Res. Conf.* AIP CP **883**, pp. 30-33, (2007).
7. B. Sherin. *Cognition and Instruction*, **19**(4), pp. 479-541, (2001).