The temperature distribution will satisfy the heat flow equation

$$\frac{\partial^2}{\partial x^2}\tau - \frac{1}{\alpha^2}\frac{\partial}{\partial t}\tau = 0.$$

Assume that the temperature in the regior is a product of two terms, $\tau(x,t) = X(x)\,T(t)$ so that

$$\frac{1}{X}\frac{\partial^2}{\partial x^2}X = \frac{1}{T}\frac{1}{\alpha^2}\frac{\partial}{\partial t}T = \kappa .$$

- What are the dimensions (units) of the constant κ?

  of the constant α?

FIG 2. A sample *Determine the Units* task, part of a longer problem on partial differential equations.

quantity differed among the three versions). The length was answered correctly by 90% of students. For the angular frequency, 75% of answers were coded as correct (1/s, radians/s, 1/time, Hz, radians/time). Answers of 'rad / s' were coded separately from answers of '1/s' as it was not clear whether students recognized that radians are dimensionless. In one version of the problem, (N = 17) students were asked about the units of the quantity b in the expression $r=R_0-b\theta;$ 7 students answered with units of meters (or distance), and 7 others answered meters / radian (or distance / radian).

Other problems involving unit identification were more difficult for students; expressions involving quantities other than distance and time seem to be more challenging for students. For example, only 4 of 15 students in one section correctly identified the units of two quantities in an expression for a potential energy. Problems with derivatives and integrals have proven to be difficult for students. When asked to find the units of the constant k in the differential equation $dV/dt = -kA$ (with V volume and A area), 58% of the students (one section, N = 17) answered correctly. In the task in Figure 2 only around 15% of students (three sections, N = 38) have correctly identified the units of the constants κ and α. Student work suggests that, for example, the units resulting from the second spatial derivative of X(x) are not clear to many students. A few students appear to be confused by notation; for example, assigning to V units of velocity rather than volume.

IV. DISCUSSION

Examining student responses to both types of problems lead to a few tentative conclusions. This portion of the project is in initial stages, and further research is needed.

First, many students entering the math methods course do not successfully reason quantitatively even with tasks designed to elicit this reasoning. The response given by some students suggest that they do not recognize that the tasks shown require them to step away from solving the problem directly or remembering its answer in order to reason whether a solution might be correct. Relatively few students spontaneously examined the expressions for special cases of the variables in the problem or related to a sense of physical mechanism.

Second, even after instruction many students struggle to identify the units in some of the expressions commonly encountered in the class. While units are a fundamental part of a physicist's toolkit, many students have difficulty with units as expressions get further from familiar quantities , as notation becomes more complicated and as mathematical operations like differentiation are included.

Finally, given that physicists value the quantitative skills described, there is a need for tasks that can be used in instruction and assessment. Redish and Kuo [9] have recently written that students "need to learn a component of physics expertise not present in math class—tying those formal mathematical tools to physical meaning….We as physics instructors must explicitly foster these components of expert physics practice to help students succeed in using math in physics." Yet the majority of problems in the course text are merely mathematical exercises that do not explicitly address these reasoning skills. Our data suggest that students need help in developing these skills.

ACKNOWLEDGEMENTS

This work is part of a collaboration with John Thompson (University of Maine), Joe Wagner (Xavier University) and Warren Christensen (North Dakota State University). The author acknowledges student researcher Marlene Vega (CSUF/NDSU REU). This work is supported by the NSF through grant PHYS-1405616; opinions, findings, and conclusions are those of the author and do not necessarily reflect the views of the NSF.

[1] M. Loverude and B. Ambrose, "Editorial: Focused Collection: PER in Upper-Division Physics Courses, *Phys Rev ST-PER* (in press).

[2] SEI: Science Education Initiative at U. Colorado, http://www.colorado.edu/sei/departments/physics.htm

[3] E. F. Redish, *Proceedings of World View on Physics Education* (2005).

[4] M.E. Loverude, AIP Conference Proceedings 1413 (2011).

[5] P. Heron, "Enrico Fermi" Course CLVI, (IOS Press, Amsterdam, 2004), pp. 341–350

[6] D. Hammer, *Am. J. Phys*. **68**, S52 (2000), Wagner *Cogn. and Instr.* **24**, 1–71 (2006).

[7] B Wilcox et al, *Phys. Rev. ST-PER* **9**, 0201191 (2013).

[8] M. Boas, *Mathematical Methods in the Physical Sciences,* Wiley (2006).

[9] E. Redish & E. Kuo, *Sci and Educ* **24**, 561-590 (2015).