Student Sensemaking about Equipotential Graphs

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The electric potential is often represented graphically using equipotential lines. Representing a multivariable function like the electric potential in this way can be challenging to interpret, and it is often helpful to make sense of a contour graph by making connections to other physical and mathematical ideas. We describe how upper-level physics students make sense of two-dimensional equipotential graphs representing the electric potential. Our data are the students' responses to a matched pair of open-ended questions given at the beginning and end of a junior-level electrostatics course. Students predominantly discussed the sign, shape, and location of the charged objects that give rise to the potential. We also find that, while some students discuss how the potential changes in space, they rarely connect the potential explicitly to an electric field, even after junior-level instruction on electrostatics.

I. INTRODUCTION

A crucial aspect of learning physics is developing the ability to move fluidly between representations. In electrostatics, students are taught about representations such as field lines, equipotential curves, charge distributions, graphs, equations, and vector field maps. Typically, students are explicitly instructed to translate from one representation to another specific representation, but we also expect students to learn what other representations they should invoke to deepen their understanding in any particular context. This coordination between representations is one instance of what we and others term *physics sensemaking*. Our working definition for sensemaking, influenced by several aspects of the physics education research literature [1–6], is the seeking of meaning or coherence between different representations of knowledge or information.

We are beginning to explore how students seek this coherence in the context of upper-division electromagnetism. Our broad research question is: How do students coordinate symbolic, graphical, and physical representations of the various scalar and vector fields used in electromagnetism? We have chosen to focus initially on the electrostatic potential, and to center our investigation on equipotential graphs, which are an extremely common and powerful tool for describing the potential. Choosing this focus has allowed us to articulate a more detailed research question: What connections do students make between a given equipotential graph and other aspects of physics knowledge? We note that there has been some prior research identifying student resources and difficulties with equipotential graphs [7, 8], but that our goal is instead to describe the representations of knowledge that students are connecting to a given equipotential graph, without focusing on whether or not such knowledge is correct.

II. INSTRUCTIONAL CONTEXT

The subjects of this study are students who were enrolled in a junior-level electrostatics course at Oregon State University (OSU) called *Static Fields*. *Static Fields* is an intensive 5-week course that is part of the Paradigms in Physics program, the reformed upper-division physics curriculum at OSU. The course includes seven in-class hours and two homework assignments each week, and was taught using a diverse array of interactive engagement strategies. The first and second authors served as the instructor and TA for the course, respectively. A total of 25 students completed the course.

The *Static Fields* course has a particular emphasis on students building geometric knowledge of the various scalar and vector fields associated with electromagnetism (and to a lesser extent, Newtonian gravitation). This focus extends to the use of (partial) derivatives to interrelate these fields, especially the gradient (*e.g.,* $\vec{E} = -\vec{\nabla}V$), the divergence (*e.g.,* $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$), and the curl (*e.g.,* $\vec{B} = \vec{\nabla} \times \vec{A}$). Instruction on gradient, divergence, and curl is both geometric and symbolic [9], and it is also primarily student-driven.

Virtually all of the students in the course were physics majors or minors and had completed an introductory physics sequence, including introductory electromagnetism. Additionally, all students had either completed or were co-enrolled in *Techniques of Theoretical Mechanics*, a sophomore-level course that included a particular focus on physics sensemaking. In *Techniques of Theoretical Mechanics*, students learned not only physics content but also a variety of strategies for physics problem solving and for evaluating physics problems and solutions, including but not limited to: dimensional analysis, special-case analysis, and visualizing functions using Mathematica [10, 11].

Partially due to the influence of this sophomore-level course, the instructor for *Static Fields* elected to include an explicit sensemaking focus in the course. Every homework assignment included the following instruction:

"Remember that you should do some sense-making about every problem and result (e.g., describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points."

In grading and providing feedback on students' work, the points given for sensemaking were roughly equal to the points given for completing the homework problem itself. Sensemaking was also discussed explicitly in class as part of both lectures and small-group activities.

Shown at right is a graph of an electric potential V.

Make sense of what the graph tells you in as many different ways as you can.

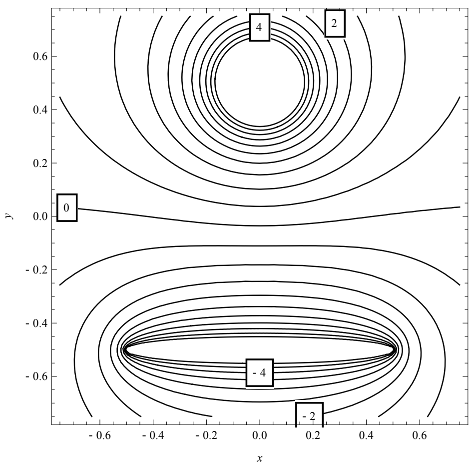

FIG. 1. A task given as an in-class assignment and as an exam question. The exam figure was rotated by 90°, had charges with reversed signs, and labeled all five equipotentials between $+1$ and -1.

III. METHODS AND FRAMEWORK

To investigate how students make sense of the electrostatic potential, we administered the open-ended prompt shown in Fig. 1. The prompt was initially given at the beginning of the course as a low-stakes in-class assignment (N=23). Students were given about 10 minutes and worked independently.

The prompt was readministered as part of the final exam (N=24). The exam prompt was kept the same, but the figure was modified in two ways. First, the graph was rotated and the signs of the charges were reversed (to reduce retest effect without substantially changing the underlying charge distribution). Second, the labels were modified because the exact location of the labels made it difficult to determine the corresponding equipotential lines. The exam figure labeled the five contours between $+1$ and -1 at unambiguous locations.

The responses to both assessments were analyzed by the first two authors (independently) using an open-coding approach [12]. A phenomenographic approach was taken, aimed at identifying the broadest possible set of connections made by students [?]. Attention was paid to all connections regardless of whether or not an expert would consider them correct or relevant. The researchers developed codes by first reading the student responses and identifying all statements made by students, then comparing lists qualitatively. After discussion, each researcher independently developed codes and applied them to the data.

Once the researchers had independent codes, they increased the reliability of the coding by comparing and resolving differences until consensus was reached and a single set of consistent codes was accepted. The researchers then worked together to group the individual codes into broad categories. Due to the interconnected nature of sensemaking, there were many sensible ways to categorize the individual codes. The final categories discussed below are those that were found to be most helpful in interpreting the student work.

An additional source of data that we considered for this study were several homework problems in which students are given a charge distribution and asked to find the electric potential. However, we found that very few students used equipotential graphs to answer these questions or to make sense of their symbolic answers, and so we do not include this data in our analysis.

IV. RESULTS

We identified four broad categories of sensemaking carried out by students. The specific kinds of sensemaking varied widely from student to student. Since our research perspective is phenomenographic, we present as broad a set of results as possible. We also do not focus on reporting the specific number of students whose responses aligned with all categories, though we do try to give some general sense for how common each type of response proved to be.

A. What kind of graph is this?

Most students described the kind of graph or how the graph displays information. Some specifically called it a "contour" or "topographical" map. Many used the term "equipotential," a word that is common in introductory physics. Some students explicitly stated that the potential is constant along each line, as in the following example:

"The lines on the graph represent paths with the same value for potential on every point of the path."

A few students were less specific, as below:

"Each curve (continuous solid lines) represents an electric potential ranging from -5 to $5 = V$."

The difference between these two quotes is that the first clearly explains that the potential has the same value along each path, whereas the second only vaguely indicates a relationship between the lines and the potential. Three students mentioned that the difference in potential between adjacent contours is always equal. Many students used language consistent with more than one of the above descriptions when interpreting the graph.

In addition to describing the kind of graph that was given, some students described alternative graphs. The most common was invoking a third dimension to describe the potential (see the left of Fig. 2). Verbal descriptions of perspectives like those in the figure, as well as the use of language like "hill" and "valley," was also indicative of this line of thinking. One student drew a cross-section (see the right of Fig. 2).

A few students observed that the given graph is a two-dimensional projection of what should be a function of three dimensions. For example, one student wrote on the exam:

"It appears as if the graph shown is a contour plot of a 2-dimensional potential function, or a contour plot of a 3-dimensional potential function where z is set to some constant or zero."

Another referred to the graph as "some slice of the potential."

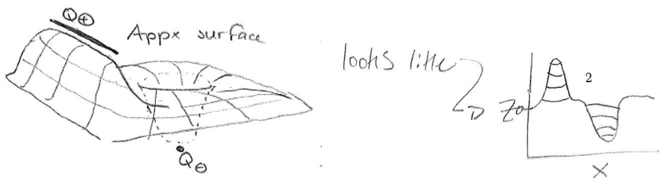

FIG. 2. Two sketches showing alternative graphical representations of the electric potential.

Some students were confused about the meaning of the boxed numbers on the graph. The implicit meaning is that each number is the value of the potential along that line—an interpretation given by three students on the exam. On the beginning-of-class survey, two students wrote that they did not understand the meaning of the numbers. A few students appeared to interpret the numbers as referring to the strength of the charge at that location.

B. What charges are making this *V*?

The most common sensemaking was to discuss the physical charges that might give rise to the potential. Students wrote about the location, sign, shape, and magnitude of the charges. The example below includes the first three of these:

"Appears that there is a positive (?) point charge located at approximately $(0, 0.5)$ *and a negative rod of charge/line charge (?) spanning* $(-0.5, -0.5)$ *to* $(0.5, -0.5)$*."*

As in this example, some students gave precise coordinates for the locations of the charged objects. This student appeared to be uncertain about whether the charges should be positive or negative, but most students conveyed more certainty, and only one student gave an incorrect sign for the charged objects. A fair number of students gave multiple options for the shape of one or both charges, all of which tended to be reasonable. A few students identified that the charges should have opposite sign; others identified the two objects as having total charges that are equal in magnitude.

Some students justified their claims about the charge: this was rare on the initial survey but became more common (and more thoughtful) on the exam. Most commonly, students justified proposed shapes for charged objects using the shapes of the equipotential lines, as follows:

"The way the shapes are structured suggests a line of positive charge and a sphere of negative charge. I say this because as you get closer to the sphere the potential becomes increasingly negative."

This student justified the sign of the charged objects using the sign of the equipotentials, which was also common.

Another student makes an argument about why the objects should have charges of equal magnitude:

"We can also make an assumption that the two charges are of equal magnitude as the same number of equipotential lines exist from the $V = \pm 1$ *to the charge."*

Only one other student gave an argument of this type. However, several students did argue that the objects have opposite charge using reasoning like the following:

"They have similar total charge because 0-potential is roughly halfway between."

One student identified the direction of the electric field using the sign of the equipotentials (see section IV D) and used this to justify the sign of the charged objects.

C. How does *V* behave?

Many students identified specific functional behavior of the given potential. Several called attention to the zero equipotential, some as justification for a claim about charges (see section IV B). Others explained that the locations of the charges account for the location or shape of the zero equipotential. Two students identified the sign of the potential in different regions and three students described how the potential changes at different points and in different directions.

Several students gave an interpretation of the circular and elliptical regions in the graph. A few interpret these regions as being "flat" (having constant potential). However, because the map is of electric potential, which approaches infinity near point charges, there is a limit to what an equipotential graph can show. The given graph deals with this by not showing any equipotential lines after some threshold is reached, even though the potential continues to rise (this is standard for contour maps generated by Mathematica). Four students (all on the exam) interpret the circular and elliptical regions as corresponding to $V \to \infty$.

Half the students mentioned the spacing between (or density of) the equipotential lines in relation to some property of the potential. Most commonly, students claimed that the spacing represents how fast the potential changes, as below:

"Ideally the lines change values linearly, so we can tell where the potential changes rapidly (where lines are close together) or gradually (where lines are far apart)."

Use of the term "rate of change" was not uncommon among students who gave similar answers. A few students related the spacing of the lines to slope, as in the following example:

"The closer the lines are, the 'steeper the slope' of the curves, meaning there is a greater change in potential."

Although ideas about change and slope have been shown to be common ways of thinking about the derivative for physics students [14], relatively few students made sense of the density of the equipotential lines in terms of derivatives or the electric field (see section IV D).

A handful of students (inappropriately) related the density of the lines to the value of the potential itself, as below:

"You can see that the contours are closer together towards the shapes, indicating that there is a larger potential [there]."

Confusion between a function's value and its change is known to be common in both math and physics contexts [15, 16]. Also, a few students identified regions on the graph where V is changing fastest (*i.e.,* by circling where the lines are closely spaced), but who did not mention the line spacing in words.

D. What about the *E*-field?

Some students explicitly discussed the electric field corresponding to the given graph, especially on the exam. For our purposes, we included any discussion of derivatives or the gradient to be related to the electric field because $\vec{E} = -\vec{\nabla}V$. Five students wrote this symbolic relationship, often as the only connection to the electric field. (A different student claimed that the potential is the gradient of an electric field.) A few students associated the density of the equipotentials with the magnitude of either the electric field or the gradient, rather than to how the function is changing (see section IV C).

Some students discussed the direction of the electric field, almost always on the exam. The following example shows multiple ways of thinking about the direction of $\vec{E}$:

"E-field points from high to low potential [...] always perpendicular to the contours."

Four students noted that $\vec{E}$ should be perpendicular to the equipotentials, while two claimed it points from high V to low V. Some related $\vec{E}$ to the locations of charges, as below:

"E-field would be strongest near charges; $(\frac{\partial V}{\partial x})$ *&* $(\frac{\partial V}{\partial y})$ *greatest where contours are most clustered."*

Very few students drew electric field lines or vectors on the given graph as part of their sensemaking, even if they discussed these vectors. (One student claimed it would be possible to find such a field.) In general, students gave less detail about the electric field than about any other connection.

We observed several different claims about the electric field that were each made by only a single student. For example, one student noted that the curl of $\vec{E}$ is zero but that the divergence is not. Another related the location of charge to the divergence of $\vec{E}$. One student asserted that $\vec{E}$ is path independent, and another commented on the work required to move a test charge. One student referred to the given graph as an electric field.

V. DISCUSSION AND IMPLICATIONS

The results indicate that students can make sense of an equipotential graph using a wide variety of ideas. Students most commonly made connections between the graph and the charged objects that might give rise to it. Many students justified claims about the shape and sign of charges, especially on the exam. These justifications strike us as particularly strong instances of physics sensemaking because the students sought coherence between at least three representations: the graph, the charges, and conceptual physics knowledge (*e.g.,* superposition or electric field). That more students justified claims about charged objects after the course than before is particularly encouraging because it suggests that the electrostatics course, which emphasized sensemaking, is helping students become more sophisticated sensemakers.

For the most part, students seemed to understand what kind of graph they were given and how it displays information. The diverse language that students used—contour, equipotential, and topographical—suggests a broad base of knowledge present in the classroom as a whole. Not only were students able to identify where the value of V is high and low, but many were also able to use the density of the equipotential lines to describe how V is changing, which is particularly important for interpreting the electric potential in physics.

However, we observe that few students made detailed connections between the given graph and the electric field (or gradient). Connecting these two quantities was an explicit focus of the course, and yet the sensemaking related to the electric field on the exam was not as complete as we expected. We do not believe this implies the students are incapable of relating $\vec{E}$ and V. Rather, it appears that students may not view this connection as sensemaking in the same way that they view connecting the potential and the charge distribution. This has implications for future instruction: if we want or expect students to go back and forth between V and $\vec{E}$ spontaneously, especially graphically, then explicit language and support is necessary even at the upper-division level.

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