Peer interaction facilitates co-construction of knowledge related to quantum mechanics formalism and postulates

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Collaborative learning with peers can lead to students learning from each other and solving physics problems correctly not only in situations in which one student knows how to solve the problems but also when none of the students can solve the problems alone. In the latter situation, students are co-constructing knowledge that helps them solve the problems, while in the former, one student helps the other construct knowledge. In this study, we investigated student learning measured by student performance on a validated quantum mechanics survey and frequencies of construction and co-construction of knowledge when students first worked individually after lecture-based instruction in relevant concepts and then worked with peers during class without receiving any feedback from the course instructor. We find that construction of knowledge consistently occurred at a high rate during peer collaboration. However, rates of co-construction were more varied. High rates of co-construction were generally achieved when approximately half of the students knew the correct answers initially. We also conducted an analysis of some of the survey questions with high rates of co-construction to gain insight into what students converged on after peer interaction and what types of difficulties were reduced. Our findings can be valuable for physics instructors who want to provide in-class and out-of-class opportunities for peer collaboration, e.g., in their quantum mechanics courses.

I. INTRODUCTION AND FRAMEWORK

Classroom time is often one of the most constraining factors for any instructor looking to improve their instruction and student outcomes. In quantum mechanics (QM) courses, instructors may not have as many students as they do in introductory courses, but students must learn an entire new paradigm of physics in a limited time. Thus, in addition to other support provided by the instructor, incentivizing and providing opportunities for students to learn from their peers both inside and outside of the classrooms may be an effective tool for helping them develop a good grasp of QM concepts.

Students in physics courses may benefit from peer collaboration for a variety of reasons. For example, if they all share the prior knowledge useful for solving a problem, students may benefit from peer interaction and collaborative work if some of the group members cue relevant prior knowledge that all students did not individually identify as relevant to the problem solution. Students are also more likely to remember what they discussed and learned together than alone [1-3]. Alternatively, if none of the students initially identified the relevant information or approach needed to solve a problem, they may have adequate combined knowledge to be able to do it correctly consistent with the framework of distributed cognition and Zone of Proximal Facilitation Model (ZPFM) since the capacity of their working memory is increased when students work collaboratively to solve problems [4]. Not only are students likely to benefit from checking the logic and rationale of each other's solutions while collaborating with their peers [5], they are also likely to think more deeply about the logic of their own solution when discussing and explaining their approach to peers. These deeper reflections may reveal inaccuracies in their initial reasoning and may help students co-construct knowledge. If classroom instructional time is limited, out-of-class peer collaboration can enhance and complement classroom instruction [6].

Peer collaboration has been shown to be effective in the context of college physics instruction [7-12]. Mazur's peer instruction method has been widely used successfully to help students develop a solid grasp of concepts and involves short but frequent instances of collaboration during class with the help of multiple-choice questions [7, 12]. This formative assessment approach has been effective in improving student learning and can help keep students engaged in the learning process [7, 12]. Also, quantitative problem solving using context-rich problems has been effective in introductory physics instruction [13, 14].

Inspired by the distributed cognition and ZPFM frameworks supporting effectiveness of peer collaboration, even without any intervention from the instructor, we investigated the impact of peer collaboration on construction and co-construction of knowledge as measured by student performance in the first semester of their quantum mechanics course. We used the (redacted for review) Survey (QMFPS), a validated survey, to investigate the effectiveness of peer collaboration for upper-level undergraduate and first-year graduate physics students after traditional lecture-based instruction in relevant concepts. Although prior research on collaborative work involving three or more students [13-16] has found it beneficial to assign roles to students collaborating with each other (e.g., group leader, scribe, skeptic, time-keeper etc.), prior research also suggests that there are significant benefits to letting students select their partners if students are asked to collaborate in pairs [17, 18]. Since our investigation involved mainly pairs of students (except in two cases when students were in a group of three), we let students select who they wanted to collaborate with while brainstorming how to solve the QMFPS questions correctly.

It is not clear a priori how much students will benefit from unguided peer discussions during their collaboration particularly because the paradigm of QM differs from introductory physics and has the potential to affect peer interaction. However, even though QM is abstract and nonintuitive, students often have common difficulties in learning this new paradigm and research shows that they sometimes transfer ideas from classical to quantum contexts or from one quantum context to another quantum context in which those ideas are not applicable [19]. In other words, even in the QM contexts, students' reasonings in different contexts show patterns, which can impact the manner in which students are likely to construct and co-construct knowledge during unguided peer collaboration.

This study investigates the extent to which construction and co-construction of knowledge occurs when students collaborate with their peers mainly in groups of two in the context of QM courses using a validated survey that focuses on the formalism and postulates of QM after having worked on it individually after traditional instruction in relevant concepts. Here, we define construction as the instance in which only one student answered a question correctly individually before working with a peer, and then as a group they were able to converge on the correct answer. We also investigate specific features of a survey item that may have led to high rates of construction or co-construction. Specifically, we aim to answer the following research questions regarding the QMFPS when physics upper-level undergraduates and graduate students in advanced quantum mechanics courses worked mainly in pairs after working individually after traditional instruction:

RQ1. How often do students in groups choose a correct answer if one student (but not both) chose the correct answer individually?

RQ2. How often do students in groups choose a correct answer if no student in that group chose the correct answer individually?

RQ3. What item features correlate with high rates of construction and co-construction of knowledge for some of the survey items?

II. METHODS

All participants attended the same large, public, and urban research institution in the US. All students were either enrolled in the first mandatory QM course in a two-semester undergraduate sequence for physics juniors/seniors or in a mandatory first-year first semester graduate introduction to teaching course in the physics department. All students had received instruction on all topics covered by the survey in their undergraduate courses before taking the QMFPS. In the undergraduate course, the QMFPS was administered at the end of the first-semester QM course, first individually and then in groups in the following class period. In the first-year first semester graduate teaching of physics course that helps graduate teaching assistants reflect upon evidence-based approaches to teaching and learning, the survey was administered in the first few weeks of the semester to help graduate students (most of whom were also teaching assistants for various introductory physics courses) reflect upon the value of collaborative learning. Although most of these graduate students were simultaneously enrolled in the first core graduate QM course for Ph.D. students in physics, since the QMFPS was administered in the first few weeks of the semester for the graduate students, their main learning of the QM content was in their undergraduate courses similar to the undergraduates who participated in the study. Student QMFPS performance, construction, and co-construction from the graduate and undergraduate groups were very similar, so here we provide statistics on combined data. Students (N=78) first took the QMFPS individually and were given a full 50-minute class period to complete the survey. Then, in the next scheduled class, students took the survey a second time in groups of two (N=36) or three (N=2) without any assistance from the instructor. Students were given a full 50-minute class period to collaborate on the surveys. They were not given feedback on their performance between the first and second survey attempts, so they did not know if their initial responses were correct.

The QMFPS is a validated survey which includes topics that were not focused on in other QM surveys [19], such as Dirac notation, Hilbert space, state vectors, physical observables and their corresponding Hermitian operators, compatible and incompatible observables, projection operators and writing operators in terms of their eigenstates and eigenvalues, and spin angular momentum. The final version of the survey contains 34 multiple-choice items and is designed primarily for junior/senior-level undergraduates and first-year graduate students. It has been validated [19] using initial faculty and student interviews, learning from previous open-ended and multiple-choice questions given to QM students, and additional faculty and student interviews for each iteration of the survey. Student interviews served multiple purposes, including assuring that students interpreted questions as intended and development of alternative incorrect choices for multiple-choice questions aligned with common student difficulties when the questions were presented as open-ended questions [19].

First, for each question, we calculated the percent of students who selected the correct answer individually, as well as the percent of student groups that answered the question correctly. We also analyzed the rates of "construction" and co-construction for each question. We define construction as the ratio of groups that chose the correct answer to all groups when individually one student answered correctly and one incorrectly. When looking at the combined individual and group scores, we write the binary scores (where 0 is incorrect and 1 is correct) of each student and then the group as a whole. For example, a group that constructed knowledge can be written as 011 or 101. The rate, or percentage, of construction on each question can be found as follows:

$$\frac{N(101 + 011)}{N(100 + 010) + N(101 + 011)} \; x \; 100\%$$

We define co-construction on each QMFPS question as the number of groups that chose the correct answer if no students in the group chose the correct answer individually, over the total number of groups in which no students chose the correct answer initially. The rate or percentage of co-construction on each question is:

$$\frac{N(001)}{N(000 + 001)} \; x \; 100\%$$

Analyzing each QMFPS question individually allows us to find patterns in data such as correlations between the percentage of individuals that answered the questions correctly individually and the rate of co-construction. We also analyzed the average rate of construction and co-construction for questions in content-based groups, such as Quantum states and Dirac notation. This allowed us to find any content-based patterns; for example, do students have particularly high rates of co-construction for questions regarding commutation relations? Finally, we analyze the content of some of the questions with particularly high rates of construction or co-construction individually. This allows us to reflect upon potential features of questions that may have facilitated productive peer interaction.

III. RESULTS AND DISCUSSION

How often do students in groups choose a correct answer if some students chose the correct answer individually?

If one student in a group answered a question correctly on the individual survey and the other student in that group answered the question incorrectly, but the group answered the question correctly, we call this situation peer construction of knowledge ("construction"). The average rate of construction for each individual question was varied and ranged from 57% (Question 23) to 100% (Questions 3

and 11). However, the average rate of construction is quite high (80%, standard deviation=11%), and the correlation between construction and individual survey scores was non-significant *(r = 0.02, p = 0.90)*. Consistently high rates of construction among survey questions add to the existing evidence that unguided group work is a valuable tool for student learning in a variety of circumstances [20-23]. All topics on the survey had between 74% and 85% construction. Thus, students who do not understand the concepts well can learn from peers even when the instructor does not have the time or resources to interact with them individually.

It is also important to address the cases in which one student in a group answered a question correctly on the individual survey, the other student answered the question incorrectly, and the group answered the question incorrectly. This may happen for a variety of reasons. For example, a student with a dominant personality might convince others that their incorrect logic is correct. However, even though the group work was unguided by an instructor, the benefits of construction likely outweigh the risks of one student convincing the other of an incorrect answer. We did find that in some cases, if both students chose the correct answer individually, the group answered the question incorrectly. In these cases, group work had a negative effect on student performance. For twenty-six of the thirty-four items, there was no instance of this happening. For seven items, this occurred in 3% of groups, and for one item this happened in 8% of groups. It was rare for peer collaboration to result in decreased student performance, this type of unguided peer collaboration activity has very small risk for students.

How often do students in groups choose a correct answer if no students chose the correct answer individually?

If neither student in a group answered a question correctly on the individual survey, and the group answers the question correctly, we call this peer co-construction of knowledge ("co-construction"). The average rate of co-construction for each individual question was quite varied, ranging from 0% (Questions 2, 10, 11, 14, and 15) to 71% (Question 27). Moreover, we find that the highest rates of co-construction were for questions that approximately 50-60% of students answered correctly on the individual survey. This is consistent with Mazur's established Peer Instruction method guidelines which suggest that most effective peer instruction occurs when approximately half of students can answer a question correctly individually [12]. Similarly, the lowest rates of co-construction tended to be for clusters of items that had individual scores furthest from the 50-60% range. Mazur's observation aligns with the Zone of Proximal Facilitation Model. For questions that few students were able to answer correctly (for example, Question 29, which 18% of students answered correctly individually) they may not know the necessary concepts to answer the question correctly in a group, even if their cognitive resources and skills are increased by working with a partner [2, 24].

Alternatively, for questions that most students were able to answer correctly individually, since a high percentage of students already understand the material, there is little room for improvement via peer collaboration [2, 24].

We also investigated rates of construction and co-construction for topics, rather than individual questions. The rates of co-construction were highest for commutators and compatibility (33%) and eigenstates of operators corresponding to physical observables (31%). The rates of co-construction were lowest for dimensionality of Hilbert space (8%), and expectation values of observables (15%).

What item features correlate to high rates of construction/ co-construction of knowledge on some of the survey items?

Due to space constraints, we only analyze features of question 20 on the QMFPS that had a high rate of construction and co-construction. Item 20 below investigates understanding of eigenstates and eigenvalues of operators in the context of spin as well as concepts related to simultaneous eigenstates of two commuting operators. The correct answer (option B) is bold, underlined, and italicized:

Question 20. For a spin-1/2 particle, suppose $|s, m_s\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ is a simultaneous eigenstate of $\hat{S}^2$ and $\hat{S}_z$ with quantum numbers $s = \frac{1}{2}$, and $m_s = -\frac{1}{2}$. Choose all of the following statements that are correct.

(1) $\hat{S}_+ \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ is an eigenstate of both $\hat{S}^2$ and $\hat{S}_z$.

(2) If $\hat{S}^2 \left|\frac{1}{2}, -\frac{1}{2}\right\rangle = \frac{3}{4}\hbar^2 \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$, then $\hat{S}_+ \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ is an eigenstate of $\hat{S}^2$ with eigenvalue $\frac{3}{4}\hbar^2$.

(3) If $\hat{S}_z \left|\frac{1}{2}, -\frac{1}{2}\right\rangle = -\frac{\hbar}{2} \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$, then $\hat{S}_+ \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ is an eigenstate of $\hat{S}_z$ with eigenvalue $-\frac{\hbar}{2}$.

A. 1 only   ***B. 1 and 2 only***   C. 1 and 3 only   D. 2 and 3 only   E. all of the above

The percentages of students and groups that chose each answer option for question 20 as well as percentages of construction and co-construction (out of all eligible cases) can be found in Table I. Table I shows that the most common

TABLE I. On question 20, the percentage of students or groups that chose each answer option. The correct answer is bold, underlined, and italicized. Here, N is the number of students in each category, individual (Ind) or group. The construction and co-construction rates are abbreviated as Con and Co-Con, respectively.

Item #		N	A	B	C	D	E	Con	Co-Con
20	Ind.	78	22	***47***	4	4	22	77%	44%
	Group	38	8	***74***	0	3	16		

individual answer was option B, the correct answer. However, options A and E were also common choices. Thus, most students knew statement 1 to be true, though some students did not realize that statement 2 was true or that statement 3 was false when answering individually. These difficulties were reduced after peer collaboration, and this item had a co-construction rate of 44%. Our prior interviews with some students during the validation of the QMFPS suggest that some students struggled with raising and lowering operators and did not realize that $\hat{S}_+ \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ yields an eigenstate of of $\hat{S}^2$ with eigenvalue $\frac{3}{4}\hbar^2$ (statement 2) but it yields an eigenstate of $\hat{S}_z$ with eigenvalue $\hbar/2$. We also note that $\hat{S}_z \left| \frac{1}{2}, -\frac{1}{2} \right\rangle = -\frac{\hbar}{2} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ was provided to students earlier in the survey in the instructions but some students may have overlooked the information provided.

Analyzing item 20, we note that it had two incorrect answer choices selected by 22% of the students individually. It requires knowledge of foundational concepts such as eigenstates and eigenvalues of spin operators that commute with each other. With regard to taxonomies of cognitive achievement such as Bloom's or Marzano's taxonomies, this question is lower than other questions on the QMFPS [25, 26]. The rate of co-construction may be high because students may either be able to cue each other about relevant knowledge, or they may combine their knowledge bases to construct and converge on the correct answer [24].

Another category consists of questions that require complex, multistep solutions which are higher on Bloom's or Marzano's taxonomies. In the Zone of Proximal Facilitation model, these would be labeled complex problems. In these questions, students must combine their knowledge of formalisms and postulates in ways that require multiple steps or combine multiple concepts simultaneously. For example, although we do not discuss it in detail here, an item we categorize in this group is QMFPS item 33, in which students needed to know how to find the probability of measuring energy in the position representation and the probability density for measuring position for a given quantum state. We hypothesize that in complex questions like these, the high rate of co-construction may at least partly be due to distribution of cognition between peers, which may have allowed students to process more information during problem solving using their joint cognitive resources as an external aid to converge on the correct reasoning [27-29].

IV. CONCLUSIONS

In this study, we found that advanced physics students in quantum mechanics courses who initially worked individually on the QMFPS improved their performance when working in groups even without any instructor feedback between individual and group attempts. We also found that construction (i.e., a group choosing a correct answer when only one or some of the students in the group chose the correct answer individually) occurred frequently on the survey. Because construction rates were high across all questions and the risk of decreased learning outcomes from group work overall were low, our findings suggest that unguided peer collaboration in which some students know the correct solutions is a useful tool for instructors to use and incentivize in their courses both inside and outside of the classroom.

We found that co-construction (i.e., a group choosing a correct answer when no students in the group chose the correct answer individually) occurred for most questions, but the rate varied more than for construction. The highest rates of co-construction were found for questions that approximately 50-60% of students answered correctly on the individual administration. Thus, co-construction may work best for concepts that are not extremely difficult or easy for most students. In the former case, two students may not have the combined knowledge to solve the problem and in the latter, the concepts are so easy that there may not be significant opportunities for co-construction.

Finally, we conducted analysis of some of the items individually with the highest rates of co-construction, e.g., question 20 discussed here. We found co-construction may work well for items that focus on fundamentals of quantum mechanics that may be challenging for students. These include Dirac notation, eigenstates and eigenvalues of spin operators that mutually commute, compatible observables and time evolution of quantum states. These concepts are new for many students, and we find that students benefited from peer collaboration. Also, co-construction may work well for QMFPS items that require many steps or many related concepts to answer correctly, e.g., those related to the probability of measuring energy in position representation or probability density of measuring position. Even in these instances, we found that students benefited from peer collaboration which may at least partly be due to students being able to reduce their cognitive load during problem solving via distributed cognition and expanding the capacity of the individual working memory [27, 29].

Importantly, peer collaboration provided students an opportunity to articulate their thoughts and understand their peer's thought processes. These types of opportunities can help students develop ability to communicate physics concepts in a low anxiety environment. Unfacilitated peer collaboration does not require extensive effort from instructors and carries minimal risk for students. Thus, this is an effective tool that instructors can implement even outside of their classes by providing students appropriate incentives, e.g., grade incentives. In future studies, students can be given an individual assessment after group work to analyze the effect of group work on individual learning.

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