Upper-level students’ conceptual understanding of energy and momentum

Alexandru Maries (he/him/his)
Department of Physics, University of Cincinnati, Cincinnati, OH, 45220

Mary Jane Brundage (she/her/hers), and Chandralekha Singh (she/her/hers)
[1] *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA, 15260*

The Energy and Momentum Conceptual Survey (EMCS) is a multiple-choice survey that contains a variety of energy and momentum concepts at the level of introductory physics used to help inform instructors of student mastery of those concepts. Prior studies suggest that many concepts on the survey are challenging for introductory physics students and the average student scores after traditional instruction are low. The research presented here uses the EMCS to investigate the extent to which upper-level students have developed mastery of energy and momentum concepts both before and after an upper-level classical mechanics course. To contextualize their performance, it will be presented alongside the performance of introductory students. Additionally, a different set of upper-level students provided explanations for their answers to each question, and those explanations were useful for understanding their common difficulties with energy and momentum concepts. We discuss some of the most challenging questions on the EMCS for upper-level students and common reasons they had difficulty with those questions.

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I. INTRODUCTION AND GOAL

One goal of an introductory physics course for physical science and engineering majors is for students to develop a robust conceptual understanding of the underlying physics [1–15]. This remains one of the goals in courses for physics majors as they take advanced physics courses even though their courses use complex mathematics to solve problems [16]. In these courses, students are often expected to make the math-physics connection themselves to develop a good knowledge structure without much scaffolding support from instructors [16, 17]. In particular, the focus of upper-level undergraduate courses is often on being able to solve complex quantitative problems with course assessment focusing exclusively on such problems, which students can do without a robust understanding of the underlying concepts.

In the context of energy and momentum, students often learn about the basic concepts including the work-energy theorem, conservation of mechanical energy, impulse-momentum theorem and momentum conservation in the calculus-based introductory courses. Moreover, in the typical traditionally taught upper-level undergraduate mechanics courses, students are taught these same concepts to solve complex quantitative problems. However, these problems can often be solved algorithmically without any course assessment specifically focusing on conceptual understanding. If advanced students in these traditionally taught courses are not given grade incentives and provided scaffolding support to make the appropriate math-physics connections, they may solve problems by pattern matching without necessarily unpacking the underlying concepts for developing a deep conceptual understanding of the foundational concepts, which is important for cultivating expertise in physics [18-20].

It is therefore useful for instructors to be aware of the extent to which upper-level students have mastered foundational concepts of energy and momentum, and the extent to which students improve their conceptual understanding after a traditionally taught course in classical mechanics primarily focused on helping students develop mathematical prowess. A few conceptual surveys have been developed for energy and momentum concepts, namely, the Energy Concept Assessment [21] and the Energy and Momentum Conceptual Survey, or EMCS [7]. In this study, we use the EMCS to study the persistent difficulties of upper-level undergraduates students with concepts measured by the EMCS as follows:

RQ1: On which EMCS questions do upper-level students struggle after instruction? Are there any patterns when comparing student responses in introductory and upper-level courses for these questions?

RQ2: What are the common difficulties of advanced students based upon the alternative choices they commonly selected? Are they the same or different from the alternative choices selected commonly by the introductory students?

II. METHODOLOGY

As mentioned, we use the EMCS to investigate the performance of upper-level students. To contextualize this performance, it is reported along with the performance of introductory students at the same institution. The upper-level students were taking a required upper-level physics classical mechanics course and completed the EMCS before (first few weeks) and after traditional instruction (last few weeks) while the introductory students did the same except at the beginning and end of their introductory physics course. Two years of data were collected for the upper-level students. We note that in the data shown in Table 1, some students took the pre-test but not the post-test and vice versa. We have carried out the analysis using matched data only (all students took both the pre-test and the post-test) and found very similar results. Therefore, we elected to report the analysis for the unmatched data here.

Additionally, 26 upper-level students from a different class completed the EMCS and provided explanations for each answer. We used these explanations to identify common reasons that students struggle on some of the questions on the EMCS and we discuss in detail the most challenging six questions.

III. RESULTS

Table 1 shows the performance of introductory students and upper-level students for each question on the EMCS. Since our focus is on upper-level students, their improvement as measured by normalized gain, g, [22] defined as (post % – pre %)/(100 – pre %), is listed and the questions on which they performed well (performance above 80%) are highlighted in green and those in which they performed poorly (performance lower than two thirds) are highlighted in red.

To answer RQ1, we identified questions on the EMCS on which the performance of upper-level students was below two thirds and also compared their performance with that of introductory students to identify any patterns. We found that on roughly half the questions on the EMCS, the performance of upper level students is below two thirds, namely Q3, Q5, Q6, Q8, Q9, Q10, Q12, Q13, Q16, Q17, Q22, Q23, Q24. This suggests that the upper-level students are far from having developed a robust conceptual understanding of the concepts of energy and momentum after traditional instruction in an upper-level classical mechanics course. Comparing the post-test performance of the upper-level students with the corresponding post-test performance of introductory students on these questions, the data show that in nearly all of the questions (11 out of 13), introductory students' post-test performance after instruction was less than 50%. In fact, on all of the questions on which the performance of introductory students was below 50% on the post-test, the performance of the upper-level students after instruction was below two-thirds, indicating that the concepts covered in those questions are very challenging for even the upper-level students to grasp after traditional instruction, and instructors need to do more to help students develop a solid understanding of the underlying concepts using evidence-based approaches.

Furthermore, the data in Table 1 shows that on all the questions on the EMCS on which the post-test performance of upper-level students was high (better than 80%), their pre-test performance was greater than 70%. Also, normalized gains tell a similar story: all the highest normalized gains are on the same questions, and part of the reason for it is the high pre-test performance. For example, Q11 which showed the highest normalized gain of 0.68 was due to an increase from 85% to 95%. More importantly though, on many of the questions with low post-test performance, student performance exhibited very low or negative normalized gain. For example, Q8, with an improvement from 57% to 62%; a normalized gain of 0.11. These data point to the same interpretation: whenever student pre-test performance was below a certain level, it is very difficult for students to improve much even in an upper-level classical mechanics course.

To answer RQ2, we identified the most common alternate conceptions of students in each group by looking at the percentage of students who select each answer choice. We then identified questions in which the most common incorrect answer choices differed between introductory and upper-level students. We found that in most cases, the most common incorrect answer choices of upper-level students were the same as for the introductory students, except the percentages were lower in the upper-level student group. To illuminate some difficulties students had on the EMCS, we focus on the six questions that were most challenging for upper-level students. We note that all the questions can be found in the appendix of Ref. [7] and that for each concept, the EMCS probes student understanding in multiple contexts. Generally, student performance is context dependent, which means that they are likely to struggle in certain contexts but not in others because they are yet to have developed expertise. Our goal here is to focus on the specific contexts in which even upper-level students struggled.

TABLE 1. Performance (percent correct) of introductory (IS) and upper-level (US) along with normalized gain (g) for upper-level students on each question on the EMCS.

Q#	IS-pre (N=352)	IS-post (N=336)	US-pre (N=68)	US-post (N=42)	g
1	33%	63%	76%	88%	0.49
2	50%	81%	76%	83%	0.29
3	45%	61%	65%	48%	-
4	47%	69%	71%	86%	0.51
5	11%	27%	47%	52%	0.10
6	12%	28%	47%	38%	-
7	55%	85%	87%	90%	0.28
8	30%	45%	57%	62%	0.11
9	26%	36%	49%	62%	0.26
10	13%	35%	57%	55%	-
11	41%	79%	85%	95%	0.68
12	21%	33%	47%	60%	0.24
13	52%	53%	60%	57%	-
14	60%	70%	90%	83%	-
15	47%	58%	69%	76%	0.23
16	34%	26%	44%	62%	0.32
17	30%	46%	54%	64%	0.22
18	20%	66%	82%	90%	0.46
19	47%	70%	72%	71%	-
20	35%	58%	78%	81%	0.15
21	44%	62%	71%	83%	0.43
22	16%	29%	32%	29%	-
23	21%	25%	51%	55%	0.07
24	20%	29%	50%	62%	0.24
25	33%	59%	63%	76%	0.35

Q3 describes a situation in which a white hockey puck collides elastically with a red hockey puck (and no net external forces act on the two-puck system) and asks which statements are correct among (1) the kinetic energy of the white puck is conserved, (2) the linear momentum of the white puck is conserved, and (3) the linear momentum of the two-puck system is conserved. The only correct statement is statement (3); however, 36% of upper-level students selected statements (1) and (3), due to only focusing on the fact that kinetic energy is conserved and not recognizing that this is only true for the two-puck system in this context. For example, one student stated "No external forces so momentum of the system is conserved, and it is elastic so kinetic energy is conserved." What is interesting is that these students can clearly recognize that it's the linear momentum *of the two-puck system* that is conserved, not just the white puck, and this doesn't appear to be sufficient to help them recognize that the same must be true for kinetic energy conservation. It is possible that students have formed a strong association between "elastic collision" and "kinetic energy conservation" and in this context, have neglected to consider the applicability conditions (i.e., that it is the kinetic energy of the system is conserved), which is a common student difficulty in physics [7,19,20].

Q3 is also intriguing because it's the only question on the EMCS with a significant *decrease* in performance of 17% from 65% on the pre-test to 48% on the post-test. This is the only question which had a decrease larger than 10%. Additionally, it is the only question in which the upper-level students had a lower performance than the introductory students (48% compared to 61%), and also, more upper-level students thought that the kinetic energy of the white puck is conserved (36%) compared to introductory students (21%). This suggests that the low performance of upper-level students might be due to them not paying close attention to the fact that in a collision, it is the kinetic energy of the system that is conserved and it points to the importance of emphasizing to students whenever conservation laws are discussed that laws such as conservation of mechanical energy and conservation of momentum are always applied to a system of multiple objects and it is the total energy/momentum of the system that is conserved, not that of an individual object.

Q5 relates to the impulse momentum theorem: two bullets are shot towards blocks of equal mass, one made of wood and the other made of steel. The bullet embeds in the wood block and bounces off of the steel (elastically), and students are asked which block travels faster after the collision. Only roughly half of the upper-level students answered this question correctly and 29% of students selected that the wood block would travel faster because the bullet transfers all of its kinetic energy to it. For example, one student stated that “the wood block now has the KE of the bullet instead of splitting it between two objects,” and another stated that “all of the bullet’s energy goes into the wood, while the bullet is still moving for the steel, that implies the velocity of the wood block will be more.” These students are not recognizing that when the bullet embeds in the block, some of its kinetic energy is used to deform the block and lost to heat, so the kinetic energy of the system is not constant. In other words, students expect that both the kinetic energy and the momentum of a system is conserved in a perfectly inelastic collision. We also note how strong this alternate conception appears to be in this context: the percentage of students who have this alternate conception is the same in introductory and upper-level courses (30% for introductory students compared to 29% for upper-level students).

Additionally, analyzing students’ choices more carefully, introductory students selected that the wooden block would be moving faster after the collision for two reasons: 1) because it gains the momentum of the bullet (20%), and 2) because it gains the kinetic energy of the bullet (30%). Upper-level students who expected that the wooden block would be moving faster primarily selected the first reason, and the percentage of upper-level students who selected it (29%) was comparable to that of introductory students (30%).

Q23 is similar to Q5 in that it also relates to the impulse-momentum theorem: two balls, one made of rubber and the other made of putty, are dropped from the same height. The rubber ball bounces up after the collision with the ground while the putty ball comes to rest. The question also states that the collisions take the same amount of time Δt and asks which ball exerts a larger force on the floor. On this question, 29% of upper-level students stated that the forces are the same. For example, one upper-level student explained: “Average force applied by the surface will equal the change in momentum / dt. It is the same in both cases.” Also, students seem to answer Q5 and Q23 quite consistently: roughly equal percentages of students answer the questions correctly (52% and 55% on Q5 and Q23, respectively), and the percentages of students with the aforementioned alternate conception is nearly identical as well (29% on both questions).

Q6, the second most challenging question on the EMCS, provides a diagram showing the circular orbit of a satellite around the Earth (Fig. 1) and asks which statement is correct among (a) the gravitational potential energy of the satellite decreases as it moves from A to B, (b) the work done on the satellite by the gravitational force is negative for the motion from A to B, (c) the work done on the satellite by the gravitational force is zero for the motion from A to B, (d) the velocity of the satellite remains unchanged as it moves from A to B, and (e) none of the above. Q6 is a very difficult question for upper-level students: only 38% answered it correctly, a performance not a lot higher than that of introductory students (28%). On this question, the most common difficulties of both upper-level and introductory students were the same: 31% of upper-level and 37% of introductory students selected (d) and 21% of upper-level and 24% of introductory students selected (e). Students who selected (d) did not distinguish between velocity and speed, e.g., one student who selected this answer choice motivated their choice by saying “It must move with constant velocity to have a circular path.”

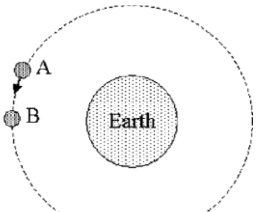

Fig. 1. Diagram for Q6.

In Q22 three balls are launched from the same height at the same speed but at different angles (ball (1) is launched vertically, ball (2) is is launched at an angle of 60° with respect to the horizontal and ball (3) is launched at an angle of 45° with respect to the horizontal). They all reach a dotted line at a certain height above their starting position and students are asked to rank the balls according to their speed when they reach the line. Q22 is the most difficult question on the EMCS: only 29% of upper-level students answered it correctly, which is identical to the percentage of introductory students who answered it correctly. On this question, roughly one third of students rank the speeds (from largest to smallest) in the same order as their *y* components and also one third rank them in the same order as their *x* components. For example, one upper-level student stated: “The parameter that matters here is the vertical velocity. for [ball 1] it is the biggest, at v_o. for [ball 2] it is the second biggest at $v_0 \sin 60 = v_0\sqrt{3}/2$. for [ball 3] it is the smallest at $v_0\sqrt{2}/2$, therefore the order is 1), 2), 3).” Also, sometimes, even if a student recognized that conservation of mechanical energy could be used in this context, they still appeared to get distracted by the fact that the velocities in the *y* direction are not the same. For example, another upper-level student who selected the same order stated: “I imagine the equation KE1=PE+KE2 at the dashed line. If this is correct, then all of the masses cancel out. The fastest ball will be whichever one has the most velocity in the vertical direction.” Other students appear to have focused on the *x* component of velocity not being affected by the force of gravity in this question, reasoning that the object with the most initial component of velocity in the *x* direction will be the one that reaches the dashed line moving fastest. For example, one upper-level student stated: “45 degrees has the highest component of horizontal velocity which gravity cannot affect and so moves more quickly when reaching the horizontal line”, and another upper-level student stated: “Vertical speeds are killed by gravity, while horizontal speeds are not.” While it is true that ball (1), which is thrown vertically, has more vertical velocity than ball (2), which is thrown at an angle of 60°, we need to consider both components of velocity because the question is asking for the total speed. It is not obvious if one does not consider conservation of mechanical energy that the horizontal velocity of ball (2) “compensates” for the lower vertical velocity such that the total speed is the same (both initially and when reaching the dotted line), so it is not surprising that not invoking conservation of mechanical energy leads to difficulties answering the question correctly. This suggests the importance of helping students consider multiple solution approaches to identify which approach is simpler, and also encouraging them to contemplate whether the use of conservation of mechanical energy is applicable whenever possible because it generally leads to more efficient solution, e.g., than using two-dimensioal kinematics.

Q10 describes an explosion in which a bomb at rest on a horizontal surface breaks up into three fragments, all of which fly off horizontally and asks which statements are true among (1) the total kinetic energy of the bomb fragments is the same as that of the bomb before the explosion, (2) the total momentum of the bomb fragments is the same as that of the bomb before explosion, and (3) the total momentum of the bomb fragments is zero. On this question, 29% of upper-level students thought that all three statements are correct, possibly because they associate explosions with both momentum and kinetic energy conservation. For example, one student motivated choosing all three by stating "by momentum and energy conservation." It is interesting that students who selected all three recognized that the initial momentum is zero (because the initial velocity is zero), and thus would presumably also be able to recognize that the initial kinetic energy must be zero as well in this particular context. But the final kinetic energy cannot be zero because all three pieces are moving. In other words, it is possible that some students have not internalized that kinetic energy is a scalar quantity. For example, one upper-level student who provided explanation reasoned that the kinetic energy stays the same by stating "the amount of total kinetic energy does not change, but just the orientations do. So even if it is represented different the overall [kinetic energy] will be the same." This type of response suggests that instructors should do more at both the introductory and advanced level to help students recognize the differences between momentum and kinetic energy in the context of collisions (and explosions).

IV. SUMMARY

We find that many upper-level students struggled with introductory mechanics concepts on the EMCS after traditional instruction in upper-level classical mechanics. The questions on the EMCS are conceptual in nature and upper-level students have difficulties with several conceptual aspects of energy and momentum covered in the EMCS. For example, on the two questions on the EMCS directly related to the impulse momentum theorem, the upper-level students exhibited very low performance of roughly 50%, indicating that they either need more support to recognize the utility of the impulse momentum theorem to understand that situation or they need more support on correctly interpreting the impulse momentum theorem. Additionally, we find that there were no questions on which the performance of upper-level students was higher than 80% *and* they showed marked improvement from the pre-test to the post-test: on all the questions on the post-test which showed a final performance of upper-level students greater than 80%, their pre-test performance was greater than 70%. While it is true that the upper-level students did improve on some questions on which they started very low in pre-test, it was unlikely for them to improve beyond a certain level. Furthermore, on roughly half the questions on the EMCS, the performance of upper-level students either decreased slightly or showed little improvement (less than 5% from the pre-test to post-test). For a few of those questions, this could be explained due to very high pre-test performance, but in most, this was not the case. We also note that on the entire EMCS, the normalized gain was a mere 14%, corresponding to an increase from a 63.2% average on the pre-test to a 68.3% average on the post-test.

We find that on all the EMCS questions on which introductory students' performance was less than 50% in the post-test (after instruction), less than two-thirds of the upper-level students provided the correct response in the post-test. It appears that on questions in which their incoming knowledge is very low, it is very difficult to improve significantly after traditional instruction in upper-level course on these concepts. We note that traditional upper-level classical mechanics teaching and assessment did not focus on the kinds of questions that are on the EMCS, although the course deals with the same concepts. Additionally, we find that the upper-level students often displayed the same types of difficulties on the EMCS questions as introductory students, usually with lower percentages, but there wasn't a significant difference between the two groups on some of the questions. These findings suggest that a traditional upper-level course (involving primarily lecture-based instruction and focus on quantitative problem solving) is not effective at helping students develop a robust knowledge structure of energy and momentum.

We note that we previously conducted interviews with physics instructors who had taught traditional upper-level undergraduate and graduate core courses. We found that some instructors incorrectly believe that learning physics concepts is easier for students at all levels than learning how to solve physics problems using "rigorous" mathematics [17]. They believe that if non-science majors can learn physics concepts, science and engineering majors and particularly physics majors can learn physics concepts on their own even if there was no conceptual assessment in the course and there is not much use in instructors wasting precious instructional time explicitly on concepts in advanced courses. Instructors with these types of beliefs often noted that they focus mainly on quantitative problem solving that will help students do complex calculations. The study discussed here suggests that this type of instruction is not effective at helping students develop a functional understanding of fundamental physics concepts. However, similar to introductory physics, instruction is likely to be more effective if upper-level classical mechanics courses use evidence-based approaches that integrate conceptual and quantitative aspects of mechanics. As Mazur noted [23], students can become very adept at regurgitating solution patterns using memorized algorithms, but not be able to answer 'simpler' questions, e.g., comparing the brightness of different light bulbs in significantly simpler circuits. If instruction does not explicitly integrate both conceptual and quantitative problem solving in teaching and assessment, students can rely on algorithmic problem-solving approaches and perform well despite lacking deep understanding of the underlying physics concepts.

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