Tools for Understanding the Microscopic World of Quantum Mechanics: Analogies in Textbooks

Diana Ryder (she/her)
School of Physics and Astronomy, Rochester Institute of Technology, Rochester, New York 14623 and
Department of Astrophysical and Planetary Sciences, University of Colorado Boulder, Boulder, Colorado 80309

Mike Verostek (he/him)
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627 and
School of Physics and Astronomy, Rochester Institute of Technology, Rochester, New York 14623

Benjamin M. Zwickl (he/him)
School of Physics and Astronomy, Rochester Institute of Technology, Rochester, New York 14623

Analogies are known to be powerful tools for making sense of unfamiliar ideas in terms of already understood concepts. Science students regularly encounter unfamiliar ideas, such as microscopic objects that are invisible to our everyday experience and behaviors dictated by quantum mechanics. An understanding of basic concepts of quantum mechanics is useful in many disciplines, especially with the growing field of quantum information sciences and technologies. Physics researchers often use analogies in their own research and science communicators use them to make quantum ideas accessible to K-12 students and across STEM disciplines, but analogy use in upper-division teaching has been less researched. Our research goal is to understand how analogies are used to teach quantum mechanics, and specifically, what prior knowledge is used as a basis for analogies within two widely used quantum mechanics textbooks. This textbook analysis shows the most common bases for analogies include: mathematical structures from linear algebra, which are applied to model quantum systems; everyday life examples, which are used to make quantum systems more familiar and understandable; and macroscopic classical phenomena, which are used to highlight differences between classical and quantum mechanics. We also find authors use different conventions, based on the various cue words that authors use to indicate analogy-based reasoning. In the STEM classroom, this research has implications for enhancing student learning about abstract topics in science.

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I. INTRODUCTION

Finding ways to enhance education in physics, particularly in quantum mechanics, is important in light of the growing multidisciplinary field of quantum information science [1–3]. As a result, efforts are being made to make quantum ideas more accessible to the general public, K-12 students, and across STEM disciplines [4–7]. In this paper, we investigate the ways that analogies may be used to enhance teaching of quantum physics. Analogies come in a variety of different forms and can be used in many contexts. In physics education, analogies are widely used as a way to make the unfamiliar world of physics more understandable. Indeed, sophisticated scientific ideas are often explained in pop science books in a way that makes them more accessible and understandable to the general public.

Quantum mechanics is viewed as a more advanced physics subject, traditionally taught after a student has completed several physics classes, such as classical mechanics and electricity and magnetism. This is potentially because students can benefit from learning these subjects beforehand and use that prior knowledge as a learning resource. Still, quantum mechanics can be unintuitive to students, since the rules that govern quantum systems (e.g. atoms, quantum bits) are very different from the rules that govern objects in everyday life and vary from the rules from previous physics classes. Hence, analogies are especially suited to teach quantum mechanics because they are specifically made to bridge prior knowledge, intuition, and experiences to new knowledge. As such, this study analyzes analogy use in two common quantum mechanics textbooks. The following are the research questions we want to address with this analysis:

- What, if any, are common words and phrases authors use that indicate the presence of an analogy?
- What bases of analogies are commonly used in quantum mechanics textbooks?

II. BACKGROUND

Students learn in a variety of different ways, including but not limited to analogies. Indeed, the authors of *ABCs of How We Learn* note that "Analogies help people learn principles and apply those principles in new situations" through recognizing a common underlying structure, despite surface differences [8]. Thus, analogies are tools for learning through facilitating conceptual understanding, and several specific methods have been formed to help create and understand analogies [9–11]. Further, analogies are generally agreed to be mappings from a *base* of familiar knowledge to a *target* of unfamiliar knowledge [9, 10, 12]; for the purpose of this analysis, this is the definition of analogy we use. The usefulness of analogies is demonstrated through the following research, which cover a wide range of scientific contexts and usages.

In an analysis of analogies in physics textbooks, Körhasan and Hıdır indicated that analogies are "suitable for teaching scientific concepts by comparing an unknown with a known" [13]. Similarly, Podolefsky and Finkelstein demonstrated that analogical scaffolding substantially increases student comprehension and corresponding test scores in upper-division electricity and magnetism. Overall, student learning was seen to increase with analogy use compared to lecture-style teaching without analogy use [11]. Further, Clement investigated how analogies may be used in introductory mechanics as a tool to gauge the level of current student knowledge and understanding and ease the transition into new content [14].

Specifically related to quantum mechanics, Wittmann and Morgan focused on the integration of analogies within lecture, emphasizing student experiences in a non-majors quantum physics course. Through emphasis of everyday experiences and situations, analogies aided student sense-making about quantum mechanics [15]. Schermerhorn *et al.* investigated the use of analogy-based tutorials to teach students upper-division quantum mechanics based on students' prior classical mechanical knowledge [16]. Similarly, Hoehn and Finkelstein investigated the circumstances of when modern physics students used analogies and other ontologies to connect classical and quantum ideas [17]. These papers demonstrate the versatility of analogies towards aiding student learning. Although Wittmann and Morgan focuses on using everyday experiences while Schermerhorn *et al.* and Hoehn and Finkelstein focus on classical mechanics, all frame those experiences and knowledge in a way to help students learn quantum mechanics.

Beyond the classroom, professional scientists use analogies to help overcome conceptual challenges in their research [18, 19]. The fact that analogies are a tool used in authentic research environments demonstrates the usefulness and importance of integrating them into upper-division curricula to benefit problem-solving skills [18].

However, research has also shown that caution is necessary when using analogies, as appropriate usage (context and phrasing) and sufficient explanation are necessary for the meaning and importance of analogies to be understood by students [13, 14, 20]. Otherwise, students may map unintended features from the base to the target, or even extend the analogy beyond the intend scope, which leads to an inaccurate understanding.

Our study complements prior research by focusing solely on analogy use in textbooks, whereas previous research has predominantly focused on classroom implementation. Focusing on textbooks will allow us to potentially learn what bases of knowledge authors assume readers have, what target knowledge readers may learn from analogies, and how analogy use in textbooks may be improved.

III. METHODS

To explore the ways analogies are used in quantum mechanics instruction, we conducted a textbook analysis [21, 22]. The textbooks analyzed include *Quantum Mechanics*

by McIntyre [23] and *Introduction to Quantum Mechanics, Third Edition* by Griffiths [24]. These textbooks were selected based on wide usage in physics classrooms and varying writing styles and approaches to teaching.

McIntyre was analyzed first, because the chapters were shorter and contained fewer topics. The chapters in Griffiths covering similar topics were then analyzed. Chapter selection was based on topics typically covered in a first semester quantum mechanics course: operators and measurement, the wave function and time evolution, angular momentum, and identical particles.

We iteratively coded chapters in these textbooks in order to develop themes regarding their analogy usage. During the first pass, the textbook chapters were read and we applied a single a priori code of *analogies*. Each identified analogy was also coded with a short descriptive initial code (following grounded theory) summarizing the base, target, and context/topic [25]. For this analysis, we use the following definition of analogy: an analogy is a mapping of features from a *base* of familiar knowledge to features of a *target* of unfamiliar knowledge [9, 10, 12]. To be coded as an analogy, a base and target domain, along with at least one mapped feature, must be identified; the analogy could be a mapping to show either similarity or dissimilarity of features between the base and target. Identifying analogies was not a trivial process, given that some analogies have implied components. As such, it became helpful to identify key words that indicated a relationship between a base and target. Given the usefulness of these key words, they became a part of the methods used for identifying select analogies. We examine trends in key word use within Sec. IV A.

During a second pass through the data, codes were further categorized by corresponding base. The main categories of base knowledge were *Comparison to Classical Mechanics*, *Mathematical Comparisons*, and *Everyday examples*. An analogy was coded with *Comparison to Classical Mechanics* when the base of the analogy incited knowledge from classical mechanical classes; *Mathematical Comparisons* was used when the base of the analogy referenced knowledge from mathematics classes (frequently linear algebra); lastly, an analogy was coded with *Everyday examples* when the base involved common everyday experiences (for clarification of this term, see Sec. IV B). When applying these main codes, it did not matter whether the analogy was meant to demonstrate similarity or dissimilarity to quantum mechanics.

For example, "The quantum oscillator is strikingly different from its classical counterpart - not only are the energies quantized, but the position distributions have some bizarre features" (Griffiths, Ch. 2 pg. 71) was given the following initial code: *CM oscillator to QM oscillator, probabilities*. Identification of the base knowledge as a classical oscillator led to further categorization, which we labeled as *Comparison to Classical Mechanics*. Throughout the coding process and writing of this paper, regular meetings were held between all three authors to establish reliability of the codebook and the emergent categories.

IV. RESULTS

There are four main results from the analysis. The first result summarizes the key words that are used to indicate the presence of an analogy, while the remaining three results deal with specific ways that analogies are used: making connections between everyday experiences and the quantum world, making mathematical comparisons between classical and quantum systems; and understanding the differences between classical and quantum mechanics.

A. Analogies Indicated by Key Words

The discovery of keywords and phrases came about during the analysis, and so was not initially coded for. Instead, key words became tools to identify select analogies and became their own unanticipated research topic.

Determining what is and what is not an analogy is not a trivial process. Thus, it is helpful to pick up on certain key words that indicate a relationship between a base and target. The following is a list of common words found in the analysis that indicate an analogy: is/are, analogue/analogous, just as/like, like/same/similar, satisfy/satisfies, different/difference, etc.

The key words "is/are" are most frequently used to set an equivalence or satisfying mathematical properties. For example, "$A^\dagger$ is the hermitian conjugate of A" (Griffiths, Ch. 2 pg. 63) uses a base of linear algebra and matrices to explain the hermitian conjugate of a quantum mechanical operator. The words "analogue/analogous" are mainly used to indicate a mathematical relationship or to identify a similar way of solving certain mathematical systems, such as,"The Schrödinger equation plays a role logically analogous to Newton's second law" (Griffiths, Ch. 1 pg. 16). "Just as/just like" is used to identify similarity between mathematical properties or solving a mathematical system; "The Schrödinger equation determines $\psi(x,t)$ for all future time, just as, in classical mechanics, Newton's law determines $x(t)$ for all future time" (Griffiths, Ch. 1 pg. 16). Additionally, "like/same/similar" are used most frequently to note similarities between solving techniques of quantum systems and comparing quantum mathematics to linear algebra concepts, such as "You may not yet know how to solve [the Schrödinger equation], but you do know how to solve a very similar one - Newton's second law" (McIntyre, Ch.5 pg. 151). Although these keywords and phrases have inherent similar meaning and function, they have different presentations based on the author.

Further keywords include "satisfy/satisfies" and "different/difference":"satisfy/satisfies" is solely used to identify shared mathematical properties, and "different/difference" is most frequently used to identify specific differences between fundamental systems, especially if there are known similarities between the systems as well.

Still, keywords and phrases are not always helpful in identifying analogies, as the relationship between a base and tar-

get is often implied. Take the following quote for example: "Classical mechanics relies on Newton's second law $F = ma$ to predict the future of a particle's motion. The ability to predict the quantum future started with Erwin Schrödinger and bears his name" (McIntyre, Ch. 3 pg. 68). In this quote, McIntyre uses similar wording ("predict the future" and "predict the quantum future") to establish a connection between the base of Newton's second law and the target of the Schrödinger Equation, but there is no cue word or phrase that indicates the analogy. Rather, it is the parallel sentence structure. In a similar manner, Griffiths uses a semi-colon to indicate a contrasting analogy between the previously covered bases of the infinite square well and harmonic oscillator and the target free particle: "Because the infinite square well and harmonic oscillator potentials go to infinity as $x \to \pm\infty$, they admit bound states only; because the free particle potential is zero everywhere, it only allows scattering states" (Griffiths, Ch. 2 pg. 83).

B. Connecting everyday experiences to the quantum world

Both McIntyre and Griffiths use analogies in order to make connections between everyday experiences and the quantum world. In this case, everyday experiences refer to occurrences and scenarios that most people (not only scientists) encounter and would recognize. Analogies connecting everyday experiences to quantum mechanics were coded as *Everyday experiences*. It was found that Griffiths uses this type of analogy more than McIntyre.

Examples in McIntyre of this include the following: "From these plots, it is now clear why we call $\psi(x)$ the wave function. These energy eigenstates have a 'wavy' spatial dependence, much like the modes on a guitar string" (McIntyre, Ch. 5 pg. 124). To emphasize this point, McIntyre continues the analogy: "First, the energy levels can be adjusted, or 'tuned,' by changing the thickness of the quantum well layer" (McIntyre, Ch. 5 pg. 147). Through the everyday base of a guitar, McIntyre demonstrates how the mathematics of a quantum potential well work.

In comparison, Griffiths relates the No-Clone Theorem to a Xerox machine: "Indeed, if you could build a cloning device (a 'quantum Xerox machine') quantum mechanics would be out the window," and continues the analogy by saying "schematically, we want the machine to take as input a particle in state $|\psi\rangle$ (the one to be copied), plus a second particular in state $|X\rangle$ (the 'blank sheet of paper'), and spit out two particles in the state $|\psi\rangle$ (original plus copy)" (Griffiths, Ch. 12 pg. 583). Thus, Griffiths compares the functions of a Xerox machine to that of a hypothetical quantum cloning device as a means to contrast classical and quantum behaviors.

Similarly, McIntyre leverages hypothetically fluctuating sock properties as a means to demonstrate the unintuitive nature of quantum mechanics: "Quantum particles behave as mysteriously as Erwin's socks - sometimes forgetting what we have already measured" (McIntyre, Ch. 1 pg. 1). This example serves to demonstrate how intuitive ideas about the properties of socks are insufficient for understanding the spin properties of quantum systems.

C. Leveraging Previous Mathematics Towards Quantum Systems

Another common base of knowledge that emerged from the textbook analysis is mathematical knowledge, as might be seen in other mathematics or non-quantum physics courses. Analogies that used mathematics and mathematical processes as the base knowledge were coded with *Mathematical Comparison*. This can take one of two forms: using similar problem-solving techniques or using known mathematics to build a basis for concepts, notation, and associated formalism.

Concerning similar problem-solving techniques, McIntyre compares how to solve the differential form of the Schrödinger equation to solving Newton's second law: "You may not yet know how to solve [the Schrödinger equation], but you do know how to solve a very similar one - Newton's second law" (McIntyre, Ch. 5 pg. 151). Similarly, Griffiths has the following quote comparing the time evolution between the two: "The Schrödinger equation plays a role logically analogous to Newton's second law: given suitable initial conditions (typically, $\psi(x, 0)$), the Schrödinger equation determines $\psi(x, t)$ for all future time, just as, in classical mechanics, Newton's law determines $x(t)$ for all future time" (Griffiths, Ch. 1 pg. 16).

Often, both textbooks make analogies relating linear algebra concepts (base) to quantum mechanical concepts (target) in order to build the mathematical formalism of quantum mechanics. Thus, both authors assume that readers are knowledgeable of linear algebra, and that readers should be able to understand the analogical connections that the authors make. When forming this relationship between linear algebra and quantum mathematics, both McIntyre and Griffiths use key words, such as "satisfy/satisfies" and "is/are" among others, which may be seen in the following examples.

Griffiths compares abstract vectors (base) with wave functions (target) and linear transformations (base) with operators (target): "Mathematically, wave functions satisfy the defining conditions for abstract vectors, and operators act on them as linear transformations. So the natural language of quantum mechanics is linear algebra" (Griffiths, Ch. 3 pg. 119).

Similarly, McIntyre compares geometric vectors (base) and basis vectors (target): "Continuing the mathematical analogy between spatial vectors and abstract vectors, we require that these same properties (at least conceptually) apply to quantum mechanical basis vectors" (McIntyre, Ch. 1, pg. 11). McIntyre establishes a connection between the linear algebra (base) and quantum (target) versions of the adjoint: "Equation (2.50) tells us that the matrix representing the Hermitian adjoint $A^\dagger$ is found by transposing and complex conjugating the matrix representing A. This is consistent with the defini-

tion of Hermitian adjoint used in matrix algebra" (McIntyre, Ch. 2 pg. 44).

D. Using classical knowledge to contrast with quantum mechanics

Analogies are frequently used to demonstrate similarities, but analogies can also be used to emphasize differences. For instance, while previous examples demonstrated similarities between classical mechanics and linear algebra in connection to understanding quantum mechanics, analogies can also be used to demonstrate the differences between them. Analogies that contrasted classical mechanics and linear algebra to quantum mechanics were coded under *Comparison to Classical Mechanics*.

In McIntyre and Griffiths, this difference most often comes in the form of stating a quantum mechanical phenomena or mathematics and discussing how it is different from classical mechanics, corresponding classical intuition, or linear algebra. These classical-quantum analogies frequently discuss experimental results and mathematical expressions.

Experimental results are discussed in both textbooks, and the following are examples from each textbook in relation to the phenomenon of quantum tunneling or barrier penetration. From Griffiths, "Classically, of course, a particle cannot make it over an infinitely high barrier, regardless of its energy [...] Quantum scattering problems are much richer: The particle has some non-zero probability of passing through the potential [...] We call this phenomenon tunneling" (Griffiths, Ch. 2 pg. 114). From McIntyre, "Quantum mechanical particles have a finite probability of being found where classical particles may not exist! This is a purely quantum mechanical effect and is commonly referred to as barrier penetration" (McIntyre, Ch. 5 pg. 133). Both quotes demonstrate how a classical particle cannot pass through an infinite barrier, but a quantum particle can.

The authors also demonstrate differences between mathematical expressions using analogical reasoning. The following are examples from each textbook of the following base-target pairing: comparing the classical use of a complex-valued function (base) to that of quantum mechanics (target). From Griffiths, "Incidentally, in electrodynamics we would write the azimuthal function in terms of sines and cosines, instead of exponentials [...] But there is no such constraint on the wave function" (Griffiths, Ch. 4 pg. 176). From McIntyre, "Note that the imaginary components of these kets are required. They are not merely a mathematical convenience as one sees in classical mechanics" (McIntyre, Ch. 1 pg. 25). Both quotes demonstrate how the wave function in quantum mechanics utilizes imaginary components, whereas in classical mechanics, they are not.

V. CONCLUSION AND DISCUSSION

Our analysis of analogy use in quantum mechanics textbooks reveals two main ideas, in connection to our research questions and results. One, key words or phrases can be helpful in identifying select analogies, and serve to emphasize different types of analogical mappings. And two, analogies have many different roles in quantum mechanics textbooks; analogies can help (1) connect everyday experiences to the quantum world, (2) solve new quantum systems through using prior knowledge from mathematics and classical physics, and (3) understand the differences between classical and quantum mechanics.

Although our analysis focused on textbooks, it suggests areas to investigate around teaching and learning. We observed that each textbook author will make assumptions as to the current level of student knowledge, and will base analogies off that. If the reader's knowledge is not aligned with the analogy's base knowledge, the intended target will probably not be understood.

When it comes to using analogies as learning tools, we hypothesize that students may have difficulty recognizing analogies or the extent of the analogical mappings used. We observed that, besides normal variation in writing style (wording, sentence structure, etc.), variation within textbooks may additionally appear within key word usage, specificity in topic coverage, and as a result, frequency and appearance of analogies. For example, Griffiths heavily uses analogies based on everyday experiences, while McIntyre prefers to use analogies for comparing quantum to classical mechanics. Thus, it may be beneficial to give students practice in identifying analogies, and particularly bases, targets, and corresponding mappings. In general, when analogies are designed at an appropriate level, are readily identifiable, and students are given time to understand the analogical mappings, they can be useful tools in aiding student understanding and learning.

Reflecting on the relationship between analogies and models, we viewed mathematical models as a particular type, or subset, of analogies. Specifically, all mathematical models are analogies but not all analogies are mathematical models; the distinction is in the directness and completeness of the mapping [26]. For instance, some examples above may be viewed as mathematical statements or definitions; these are analogies, but depending on the directness of the statement, could also be viewed as mathematical models.

Potential limitations of the current analysis include only focusing on particular topics in the textbooks and not examining key words more closely. These limitations may be addressed in future research. Potential next steps include investigating how students recognize and interpret analogies within texts and in the classroom. More broadly, this research has implications for educating the general public and students.

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[1] J. C. Meyer, G. Passante, S. J. Pollock, and B. R. Wilcox, Today's interdisciplinary quantum information classroom: Themes from a survey of quantum information science instructors, Physical Review Physics Education Research **18**, 010150 (2022).

[2] M. F. Fox, B. M. Zwickl, and H. Lewandowski, Preparing for the quantum revolution: What is the role of higher education?, Physical Review Physics Education Research **16**, 020131 (2020).

[3] Subcommittee on Quantum Information Science, *Quantum Information Science and Technology Workforce Development National Strategic Plan*, Tech. Rep. (2022).

[4] National Q-12 Education Partnership, QIS Key Concepts for Early Learners: K-12 Framework High School Chemistry, Available at https://q12education.org/learning-materials-framework/chemistry (2023).

[5] National Q-12 Education Partnership, QIS Key Concepts for Early Learners: K-12 Framework High School Computer Science, Available at https://q12education.org/learning-materials-framework/cs (2023).

[6] National Q-12 Education Partnership, QIS Key Concepts for Early Learners: K-12 Framework High School Physics, Available at https://q12education.org/learning-materials-framework/physics (2023).

[7] The Quantum Atlas, The Quantum Atlas - Get Started, Available at https://quantumatlas.umd.edu/start/ (2023).

[8] D. L. Schwartz, J. M. Tsang, and K. P. Blair, *The ABCs of how we learn: 26 scientifically proven approaches, how they work, and when to use them* (WW Norton & Company, 2016).

[9] D. Gentner, Structure-mapping: A theoretical framework for analogy, Cognitive science **7**, 155 (1983).

[10] D. Gentner and A. B. Markman, Structure mapping in analogy and similarity., American Psychologist **52**, 45 (1997).

[11] N. S. Podolefsky and N. D. Finkelstein, Analogical scaffolding and the learning of abstract ideas in physics: An example from electromagnetic waves, Physical Review Special Topics - Physics Education Research **3**, 010109 (2007).

[12] K. J. Holyoak and P. Thagard, Analogical mapping by constraint satisfaction, Cognitive science **13**, 295 (1989).

[13] N. D. Körhasan and M. Hıdır, How should textbook analogies be used in teaching physics?, Physical Review Physics Education Research **15**, 010109 (2019).

[14] J. Clement, Using bridging analogies and anchoring intuitions to deal with students' preconceptions in physics, Journal of Research in Science Teaching **30**, 1241 (1993).

[15] M. C. Wittmann and J. T. Morgan, Foregrounding epistemology and everyday intuitions in a quantum physics course for nonscience majors, Physical Review Physics Education Research **16**, 020159 (2020).

[16] B. P. Schermerhorn, G. Corsiglia, H. Sadaghiani, G. Passante, and S. Pollock, From Cartesian coordinates to Hilbert space: Supporting student understanding of basis in quantum mechanics, Physical Review Physics Education Research **18**, 010145 (2022).

[17] J. R. Hoehn and N. D. Finkelstein, Students' flexible use of ontologies and the value of tentative reasoning: Examples of conceptual understanding in three canonical topics of quantum mechanics, Physical Review Physics Education Research **14**, 010122 (2018).

[18] M. Verostek, M. Griston, J. Botello, and B. Zwickl, Making expert processes visible: How and why theorists use assumptions and analogies in their research, Physical Review Physics Education Research **18**, 020143 (2022).

[19] K. Dunbar, How scientists think: On-line creativity and conceptual change in science, in *Conceptual structures and processes: Emergence, discovery, and change* (American Psychological Association, 1997) pp. 461–493.

[20] D. T. Brookes and E. Etkina, Using conceptual metaphor and functional grammar to explore how language used in physics affects student learning, Physical Review Special Topics - Physics Education Research **3**, 010105 (2007).

[21] M. Kapanadze, G. Jonas-Ahrend, A. Mazzolini, and F. Joubran, Evaluation of Physics Textbooks, in *The International Handbook of Physics Education Research: Special Topics* (AIP Publishing LLC, Melville, New York, 2023) pp. 17–1.

[22] K. S. Serbin, M. Wawro, and R. Storms, Characterizations of student, instructor, and textbook discourse related to basis and change of basis in quantum mechanics, Physical Review Physics Education Research **17**, 010140 (2021).

[23] D. H. McIntyre, *Quantum mechanics* (Cambridge University Press, 2022).

[24] D. J. Griffiths and D. F. Schroeter, *Introduction to quantum mechanics* (Cambridge University Press, 2018).

[25] J. Saldaña, *The coding manual for qualitative researchers* (sage, 2021).

[26] E. Etkina, A. Warren, and M. Gentile, The role of models in physics instruction, The physics teacher **44**, 34 (2006).