

Analyzing students' assumptions to varying degree of prompting during problem solving

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Reports on pedagogical transformations have called for promoting authentic knowledge-building practices in science classrooms. Making assumptions is one such practice that is integral to “doing” physics. In this study, we analyze the nature and characteristics of students’ assumptions when they are (i) not prompted, (ii) prompted explicitly at the beginning, and (iii) prompted at the end of physics problems. Preliminary observations indicate that students seldom generate assumptions unless prompted. When explicitly asked at the beginning of problem solving, students perceive making assumptions as a separate task dissociated from the problem-solving process. However, when asked to reflect on the validity of their solutions in light of their assumptions, not only do students make assumptions that are closely “tied” to their solutions, but go an extra mile by articulating the implications of the violations of their assumptions. Implications of these findings for instruction and assessment design are discussed.

I. INTRODUCTION

Reports on pedagogical reforms in higher education have called for shifting the focus of classroom learning from rote-memorization of concepts to engaging students in authentic knowledge-building practices [1]. Consequently, researchers have promoted the use of multiple representations [2], argumentation [3], evidence-based [4] and analogical reasoning [5] in physics learning environments. Making effective assumptions is another such component which is integral to “doing” physics. Assumptions dictate the underlying conditions in which a model or principle holds true, and play a key role in idealization and approximations in physics.

Given this significance, researchers have explored how students and professional physicists reason about assumptions. These studies have noted students’ difficulties in converting an “ill-defined” problem into a “well-defined” one through appropriate assumptions [6] and conflating assumptions with algebraic denotation of physical quantities [7, 8]. Recent studies have also noted the centrality of assumptions in theorists practices especially in checking the mathematical behavior of models and their role in troubleshooting [9–12]. Despite its centrality in physics, studies towards reliably eliciting assumptions from students tend to be relatively scarce.

The current manuscript seeks to bridge this gap by investigating how introductory students generate assumptions to varying degrees of prompting in physics problems. We analyze the nature and characteristics of students’ assumptions when they are (i) not prompted, (ii) prompted explicitly at the beginning, and (iii) at the end of physics problems. Preliminary observations indicate that students seldom generate assumptions unless prompted. When explicitly asked at the beginning of problem solving, students perceive making assumptions as a separate task dissociated from the problem solving process. However, when asked to reflect on the validity of their solutions in light of their assumptions, not only do students make assumptions that are closely “tied” to their solutions, but go an extra mile by articulating the implications of the violations of their assumptions. These results suggest the need to explicitly emphasize (i) assumptions in physics pedagogy, and (ii) the “reflection of results” in light of the underlying conditions during problem solving.

This manuscript is organized as follows: In the next section, we discuss theory of assumptions before discussing the physics problems and methods of data collection and analysis in Section III. We then present the results in Section IV before discussing their implications in Section V.

II. ASSUMPTIONS

Researchers across multiples disciplines including business studies and linguistics have contemplated the characteristics and significance of making assumptions. In science education literature, discussions on assumptions have revolved around two broad themes: (i) what counts as an “assumption”

and (ii) the purposes served by assumptions. According to Scates [13]: “An assumption is a mental datum which is not fully established, but which is used as a basis for continuing the thought or study.”

Scates extends this definition in arguing that assumptions cover the terrain of facts, principles, or other concepts, “the truth of which is taken for granted for particular purposes without insistence upon specific proof”.

Assumptions have also been described in terms of the two purposes they serve: in making the problem tractable [6, 14] (which we refer as “*constraining assumptions*”), and in relation to the things that the person making assumptions believes to be true in a problem (“*given assumptions*”) [9]. One of the ways *constraining assumptions* work is by making an “ill-defined” problem “well-defined”. Ill-defined problems are the tasks with no specific starting/ending points and with no established process of solving them [15]. Tasks such as modeling an abstract system and designing experiments to validate a theoretical idea represent examples of ill-defined problems. Well-defined problems (e.g., end-of-the-chapter physics problems) on the other hand, are the problems with a specific beginning/ending points and an established laid out process between the two. Typically transitioning from an ill-defined problem space to a well-defined one is facilitated through approximations and idealizations in physics. On the other hand, *given assumptions* represent the features of a system or model that practitioners consider to be true. Examples include: the absence of friction and the constant magnitude of the acceleration due to gravity during free-fall of an object. We refer to these forms of assumptions which are often taken to be true in a problem as “*given assumptions*”.

In the current study, we adopt the above-mentioned Scates’ definition of an assumption. Since the problems discussed in this study are well-defined, the purpose that assumptions serve is to mainly reflect what students consider to be true in their approach. In the rest of this manuscript, we address the research question: How does prompting in tasks affect students’ articulation of assumptions during problem solving?

III. METHODS

The objective of the current study is to investigate the characteristics of the students’ assumptions in response to the varying degree of prompting in tasks. The tasks involved real-world contexts based on two different amusement ride contexts: Gravitron and Roller coaster. These contexts were chosen since real-world scenarios provide opportunities to make subjective assumptions during problem solving [6]. The “Gravitron task” consists of a cylindrical rotating ride in which riders lean against a wall. The task asks students to determine whether a rider would fall off the Gravitron’s walls under the specified parameters. Figures 1 and 2 represent the problem statements of the task. Due to the space constraints, we refrain from providing detailed solutions and refer readers to our earlier work [16] for the same.

You are asked to design a Gravitron for the county fair, an amusement park ride where the enters a hollow cylinder, radius of 4.6 m, the rider leans against the wall and the room spins until it reaches angular velocity, at which point the floor lowers. The coefficient of static friction is 0.2. You need this ride to sustain mass between 25-160 kg to be able to ride safely and not slide off the wall. If the minimum ω is 3 rad/s, will anyone slide down and off the wall at these masses? Explain your reasoning using diagrams, equations and words.

FIG. 1. Statement of the open-ended Gravitron problem

The second problem, the “Roller coaster task”, entails testing of an amusement train ride along a specified path. Students are asked to determine whether the applied force would bring the train to a halt at the end of its track. Figure 3 represents its problem statement along with the given diagram of the track. One of the ways to approach this problem is by determining the difference in potential energies of the train between the points A and D (assuming that the train’s mechanical energy is conserved). This difference in the potential energies will be equal to the train’s kinetic energy at point D. Extracting the value of the train’s velocity from the kinetic energy term, and using the kinematic equation $v^2 - u^2 = 2as$ (where $s = 113\text{ m}$), one can calculate the value of acceleration and thus force required to stop this train. Comparing the calculated force with the given value, one observes that the applied force to be inadequate in stopping the train.

The Gravitron task had two versions: an open-ended and a scaffolded version. The open-ended version (Figure 1) contained no explicit prompt asking students to make assumptions. The scaffolded version however, contained three sub-parts with the first sub-part explicitly asking “What assumptions do you need to make to be able to solve this?”. The Roller coaster problem, on the other hand, was an open-ended task but with an explicit prompt at the end asking “Under what conditions do you think your conclusion is valid?” In this way, we had three tasks with varying degree of prompting towards making assumptions: first task having no explicit prompt, second task with an initial explicit prompt to articulate assumptions before solving the problem, and the third asking students to specify the assumptions in which their conclusions would be valid.

On a side note, all three problems were part of a study aimed at analyzing students’ engagement on the scientific practices [1] and were designed using the Three-Dimensional Learning Assessment Protocol (3D-LAP) [17]. While the open-ended Gravitron task was designed to engage students in the practice of “Developing and Using Models”, the scaffolded Gravitron and Roller coaster tasks were designed to elicit the practice of “Using Mathematics”.

Data were collected for the three tasks from introductory students through a series of think-aloud interviews during Spring-2018 and Spring-2019. Data for the first task included

You are asked to design a Gravitron for the county fair, an amusement park ride where the enters a hollow cylinder, radius of 4.6 m, the rider leans against the wall and the room spins until it reaches a specified angular velocity (ω), at which point the floor lowers. The coefficient of static friction is 0.2. You need this ride to sustain mass between 25-160 kg (i.e., they should be able to ride safely and not slide off the wall).

(A.) What assumptions do you need to make to be able to solve this?

(B.) Create a free body diagram for the rider when the room is spinning. Note all applicable forces and label them.

(C.) If the floor drops out when ω is 3 rad/s, will anyone slide off the wall in the given mass range? Explain your reasoning.

FIG. 2. Statement of the scaffolded Gravitron problem. Part of the problem statement asking to generate assumptions has been highlighted in bold. However the bold font was not presented to students.

responses from 10 introductory students collected in Spring 2018. Of the ten interviews, 2 had audio/video issues and are not part of this study. Data for the scaffolded version of the Gravitron and Roller coaster tasks involved another ten introductory students and the responses were collected in Spring of 2019. We highlight that the data set across the second and the third problem is the same, i.e., the same students attempted the two problems (Figures 2 and 3) in a single think-aloud interview. The interview protocol involved asking students to consider the problem-solving exercise as an untimed exam and to articulate their thoughts out loud. During moments of prolonged silences, the interviewer interjected with questions such as “what are you thinking” to nudge students to continue articulating their thoughts. Students were compensated with \$20 for their participation.

We analyzed the interviews by taking into account students’ verbal arguments and their written solutions. For the current study, we focus on the explicit assumptions articulated by students either in their verbal arguments and/or in their written solutions. We particularly focused on the frequency, and the circumstances in which explicit assumptions were invoked or reiterated. The “explicit” assumptions in students’ verbal and written solutions were identified by noting phrases such as “assuming”, “given that”, etc. While the number of assumptions invoked by a participant (frequency) reflected the influence of prompting, the accompanying circumstances reflected on how students employed assumptions during problem solving.

IV. RESULTS

In what follows, we describe how students generated assumptions to the varying degree of prompting across three problems represented in Figures 1, 2 and 3.

Engineers are testing a new roller coaster-like ride before it starts functioning. The sandbags are strapped into the train to simulate passengers and the total mass of the train with sandbags is 1000 kg . It is supposed to start from rest at point A and stop at point E. The train starts braking at point D so that it will come to a stop at point E. If the brake system applies an average force of 6749 N , will it be enough to stop the train at point E? **Under what conditions do you think your conclusion is valid?** The heights from the ground to points A, B, C, D and E are 173 , 145 , 124 , 95 and 95 (in m), respectively. The distance from D to E is 113 m .

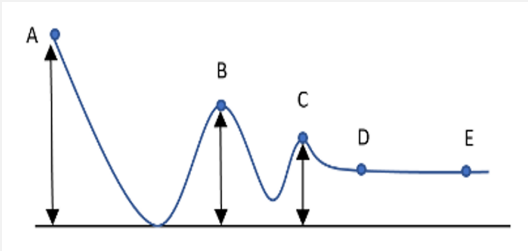


FIG. 3. Statement of the open-ended Roller coaster problem. Part of the problem statement asking to generate assumptions has been highlighted in bold. However the bold font was not presented to students.

A. Students' assumptions on open-ended Gravitron task

The open-ended version of the Gravitron task had no explicit prompt asking students to make assumptions. Out of eight students, we found only one participant articulating their assumptions while solving the problem. At the outset, this finding implies that when not explicitly prompted, articulating assumptions is often not the primary focus of students during problem solving. Our finding is in agreement with existing observation that students' prior familiarity plays a crucial role in engaging with this practice [6].

B. Students' assumptions on scaffolded Gravitron task

Unlike the open-ended version, the scaffolded version of the Gravitron task consisted of an explicit prompt asking students to first articulate their assumptions before constructing free body diagrams and solving the problem (Figure 2). All ten participants articulated assumptions which mainly spanned across the orthogonality between the ground and the walls of the Gravitron, riders' weight, and the coefficient of friction offered by the wall. The generated assumptions have been summarized in the second column of Table I.

Furthermore, approximately half of the total participants generated more than one assumption explicitly and interestingly, none referred to the assumptions while solving the remaining parts of the problem. Based on these observations,

we infer that explicit prompting in the initial phases of problem solving can nudge students to generate assumptions but also can make the participants treat the exercise as a separate task dissociated from the problem. This finding substantiates existing observations that explicit prompting in tasks often gets treated as a separate task and leads to the "algorithmic approach" during problem solving [18].

C. Students' assumptions on open-ended Roller coaster task

The third task – Roller coaster problem – was open-ended and asked students to reflect on the conditions in which their solutions were valid (Figure 3). Of the ten participants, eight articulated explicit assumptions in response to the prompting. Of the remaining two, one participant did not complete the problem and the other did not articulate assumptions despite completing the task (summarized in the third column of the Table I). These observations reflect a couple of limitations of this form of prompting. Firstly, asking students to reflect on the conditions that determine the validity of their solutions would force students to employ assumptions *after* instead of *during* problem solving. Secondly, successful completion of the task becomes the prerequisite to generate assumptions.

Furthermore, all the eight participants who articulated assumptions did so only after completing the problem (in response to the prompt) but not *during problem solving*. This observation reinforces our observation made in the open-ended Gravitron task (Section IV A) that generating assumptions is often not a primary focus of students during problem solving. Also, a key characteristic feature of the student generated assumptions in this task is that few of the participants articulated additional arguments discussing the implications of their assumptions' violations. For example, one of the participants (Participant 1 in Table I) wrote in their solution:

"Assuming there is a friction offered by the track itself and is not an ideal system. The remaining [energy] will be dissipated".

The participant further goes on to mention verbally

"If you don't take out that [friction], its gonna create problem."

This reflection on the violation of the assumptions despite the problem statement not asking for it is an interesting feature in our observations. It is interesting because the same participants did not ponder on the violation of their assumptions while engaging with the scaffolded version of the Gravitron task (Section IV B). These arguments indicate that asking students to introspect the validity of their results in light of the accompanying conditions to be a *relatively* effective approach in eliciting assumptions during problem solving.

TABLE I. Summary of the assumptions made by students to the scaffolded version of the Gravitron (Figure 2) and the open-ended Roller coaster problem (Figure 3). The assumptions from participants have been rephrased and briefly summarized due to space constraints.

Participant	Scaffolded Gravitron task	Open-ended Roller coaster task
1	Magnitude of the acceleration due to gravity remains constant, and clothing material of every rider has same the coefficient of static friction.	The track offers friction and train-track system is not an ideal system. Consequently excess excess energy gets dissipated.
2	Every rider weighs 160 <i>kg</i> and stays put on the wall.	-
3	The coefficient of static friction is same for all riders; their angular velocity remains constant.	Tracks are not icy and thus offer friction.
4	Every rider weighs 160 <i>kg</i> or more.	Train's motion is similar to "sliding motion" and brakes offer uniform and continuous force.
5	The weight of 55 <i>lb</i> is too small for the Gravitron ride.	-
6	The Gravitron's walls are flat and are perpendicular to the floor.	The track offers friction to train's motion.
7	The Gravitron's walls are flat and are perpendicular to the floor.	There is no friction between the track and the train.
8	Magnitude of the acceleration due to gravity remains constant.	There is no friction between the track and the train.
9	The Gravitron's walls are flat.	There is no friction between the track and the train, and thus no energy gets dissipated between the points A and D.
10	Clothing material of every rider has same the coefficient of static friction.	There is no friction between the track and the train, and thus no energy gets dissipated.

V. DISCUSSION, CONCLUSION, AND FUTURE WORK

We analyzed how students generated assumptions to varying degree of prompting in physics problems. The first problem was open-ended with no explicit prompting (Figure 1). The second problem was the scaffolded version of the first task with an initial explicit prompt to generate assumptions (Figure 2). And lastly the third task was open-ended asking students to reflect on the conditions in which their solutions were valid (Figure 3). We observe that students rarely generated assumptions in the first task. However, in the second task, all participants generated assumptions, although they did not effectively integrate them into their problem-solving process. In the third task, not only were students' assumptions closely linked to their solutions, but they also considered the implications of violating those assumptions.

Based on these observations, we claim that (i) students typically do not prioritize making assumptions explicit when engaged in problem-solving, (ii) explicit prompting to generate assumptions at the beginning of problem-solving tasks may not yield productive results, as students tend to perceive it as a separate task detached from the problem-solving, and (iii) encouraging students to reflect on the conditions under which their solutions are valid proves to be a *relatively effective* strategy for guiding students in making assumptions.

For instructors, these results indicate the need to emphasize the role of assumptions in classroom instruction, especially by reflecting on the validity of results in light of the underlying conditions. Our findings also provide insights on developing assignments and examinations that effectively nudge students in productively engaging with assumptions. For researchers, our findings call for expanding research on students' generation of assumptions to various degrees of

prompting by focusing on "ill-defined" problems. Explorations probing characterizations of students' explicit vs implicit assumptions also is a potential avenue.

However, claims made in this study accompany few limitations. Ideally we expect students to blend assumptions while making sense of physics problems. None of the prompting strategies achieved this objective. In response to this shortcoming, we rephrased our third claim that nudging students to reflect their solutions in light of the accompanying conditions was a *relatively effective* strategy in eliciting assumptions. Secondly, our observations are drawn from responses of only ten introductory students. Observations from larger data-set by taking into account students' demographics would undoubtedly enrich the results. Three, by the very design, the problems were relatively well-defined. Consequently, our analysis captured the "given assumptions" without exploring students' "constraining" ones (Refer Section II). Analyzing data from ill-defined problems with varying degree of prompting would capture the spectrum of reasoning around students assumptions in physics.

Future work would involve expanding our analysis to more well-defined problems based on real-life from our data set. We further seek to explore the patterns (and potentially hierarchies) in the students' generated assumptions in response to varying degree of prompting.

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- [1] N. R. Council *et al.*, *A framework for K-12 science education: Practices, crosscutting concepts, and core ideas* (National Academies Press, 2012).
- [2] E. Brewe, Modeling theory applied: Modeling instruction in introductory physics, *American Journal of physics* **76**, 1155 (2008).
- [3] M. J. Ford, A dialogic account of sense-making in scientific argumentation and reasoning, *Cognition and Instruction* **30**, 207 (2012).
- [4] R. S. Russ and T. O. B. Odden, Intertwining evidence- and model-based reasoning in physics sensemaking: An example from electrostatics, *Physical Review Physics Education Research* **13**, 020105 (2017).
- [5] N. S. Podolefsky and N. D. Finkelstein, Use of analogy in learning physics: The role of representations, *Physical Review Special Topics-Physics Education Research* **2**, 020101 (2006).
- [6] D. Fortus, The importance of learning to make assumptions, *Science Education* **93**, 86 (2009).
- [7] A. Sirnoorkar, A. Mazumdar, and A. Kumar, Students' epistemic understanding of mathematical derivations in physics, *European Journal of Physics* **38**, 015703 (2016).
- [8] A. Sirnoorkar, A. Mazumdar, and A. Kumar, Towards a content-based epistemic measure in physics, *Physical Review Physics Education Research* **16**, 010103 (2020).
- [9] M. Verostek, M. Griston, J. Botello, and B. Zwickl, Making expert processes visible: How and why theorists use assumptions and analogies in their research, *Physical Review Physics Education Research* **18**, 020143 (2022).
- [10] J. Park, K.-A. Jang, and I. Kim, An analysis of the actual processes of physicists' research and the implications for teaching scientific inquiry in school, *Research in Science Education* **39**, 111 (2009).
- [11] M. Wells, D. Hestenes, and G. Swackhamer, A modeling method for high school physics instruction, *American journal of physics* **63**, 606 (1995).
- [12] D. R. Dounas-Frazer and H. Lewandowski, The modelling framework for experimental physics: Description, development, and applications, *European Journal of Physics* **39**, 064005 (2018).
- [13] D. E. Scates, Types of assumptions in educational research., *Journal of Educational Psychology* **26**, 350 (1935).
- [14] C.-H. Ho, Some phenomena of problem decomposition strategy for design thinking: differences between novices and experts, *Design Studies* **22**, 27 (2001).
- [15] S. K. Reed, The structure of ill-structured (and well-structured) problems revisited, *Educational Psychology Review* **28**, 691 (2016).
- [16] A. Sirnoorkar, P. D. Bergeron, and J. T. Laverty, Sensemaking and scientific modeling: Intertwined processes analyzed in the context of physics problem solving, *Physical Review Physics Education Research* **19**, 010118 (2023).
- [17] J. T. Laverty, S. M. Underwood, R. L. Matz, L. A. Posey, J. H. Carmel, M. D. Caballero, C. L. Fata-Hartley, D. Ebert-May, S. E. Jardeleza, and M. M. Cooper, Characterizing college science assessments: The three-dimensional learning assessment protocol, *PloS one* **11**, e0162333 (2016).
- [18] A. F. Heckler, Some consequences of prompting novice physics students to construct force diagrams, *International Journal of Science Education* **32**, 1829 (2010).