Students' sensemaking using pictorial analysis game in upper-level electrostatics

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Instructors and researchers who focus on student sense-making try to understand 'how' students navigate the problem-solving process. Understanding these mechanisms plays an important role in helping students become good problem solvers. In particular, investigating student sense-making in the context of physics problem-solving can be useful for developing curricula and pedagogies to help students learn. We used individual interviews to investigate student sense-making in upper-level E&M in the context of problems that can be efficiently solved using the method of images as part of the development and validation of a research-based tutorial. Some of the valuable findings using the pictorial analysis epistemic games proposed by Tuminaro and Redish are presented.

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I. INTRODUCTION AND FRAMEWORK

Learning upper-level Electricity and Magnetism (E&M) is challenging. Not only does it require students to have a solid understanding of relevant mathematics including vector calculus but also the ability to integrate physics and mathematics [1-8] seamlessly during problem-solving [9-14]. In E&M, the Method of Images (MoI) is a powerful technique that can help to efficiently solve certain kinds of boundary value problems, e.g., those involving a charge distribution near conductors [15]. To provide learning support, we developed and validated a tutorial and the corresponding pretest and posttest on MoI. As a part of the development and validation of the tutorial (including pretests and posttest), we conducted interviews with graduate (grad) students at a university in the US. During the interviews, we observed students' sensemaking while they solved problems.

Sensemaking [7, 16-22] is the process of figuring out how to solve a problem and can lead to building new knowledge or creating connections within preexisting knowledge [23]. Observation of the sensemaking of students can help us to understand how students think while solving a problem. It can help us to understand what resources get activated in a particular situation and what additional resources could potentially be activated if students are provided with some guidance and scaffolding.

We investigated the sensemaking of students through the lens of the epistemic [17-19, 24, 25] game framework adapted by Tuminaro and Redish [26] in the physics context from the framework originally proposed by Collins and Ferguson [27]. An epistemic game involves the activation of a collection of resources. The structural components of an epistemic game are entry conditions, moves, and ending conditions [26]. Out of the six epistemic games proposed by Tuminaro and Redish, we find the Pictorial Analysis game to be the most prominent in the MoI problems as expected because it involves a deep analysis involving figures. This game includes the determination of the target concept, selection of an external representation (picture), a narration of a conceptual story about the physical situation based on the spatial relation among the objects and finally filling up of the "slots" in the representation [26]. We observed that students play other games embedded within the Pictorial Analysis game or switch between different games but those are not explicitly discussed here. In this paper, we discuss how two grad students played the Pictorial Analysis game on problems that can be solved efficiently using MoI.

II. METHODOLOGY

This investigation was conducted in the context of collecting qualitative data for the development and validation of a tutorial (including pretest and posttest) on MoI. We conducted interviews with six different grad students from a large public university in the US which were recorded online via Zoom, while the students shared their screens. The students were asked to turn the video off and replace their names with dummy names to keep their identities anonymous. Students shared the PDF files of the completed tutorial and pre- and post-tests at the end of each interview session. The interview was divided into three different sessions involving unscaffolded and scaffolded pretests, tutorial, and then unscaffolded and scaffolded posttests on MoI. An unscaffolded version contains direct question(s) similar to the textbook problems without guidance, whereas a scaffolded version has questions broken down into multiple parts since some scaffolding is provided to proceed. In this paper, we discuss three questions, labeled Q1 – Q3 from the pre/post tests and the tutorial as two grad students, G1 and G2, do sensemaking.

Q1. [Unscaffolded pretest] **Consider the surface of a grounded conductor as the x-y plane. The region z > 0 is vacuum. Two-point charges are placed on the z-axis: charges q at z = a and –2q at z = b. Write down the expression for the potential in the region of interest**. [correct number of image charges: two, -q (0,0, -a) and +2q (0,0, -b)]

Q2. [Tutorial] **Consider two semi-infinite, perpendicular, grounded conducting planes (x = 0 plane and y = 0 plane). A point charge +q is placed at the coordinates x = +a, y = +b. z = 0**. **a) Draw a sketch of this situation. b) To exploit the MoI for the potential in the quadrant of interest, how many image charges are needed? Explain. c) Draw a sketch of where any image charge(s) should be placed to solve the potential using the MoI. Include the values and signs of the image charges. d) Check the boundary conditions explicitly for the new problem to see if they match the boundary conditions for the original problem. e) Use the MoI to write down the potential in the quadrant where the original point charge is located.** [correct number of image charges: three, -q (-a, b,0), +q (-a, -b,0) and -q(a, -b,0)]

Q3. [Unscaffolded posttest] **Consider two grounded conducting planes, both perpendicular to the x-y plane, intersecting at an angle of 60°as shown in the figure** [Fig. 1.]**. One plane is y = 0 and the other makes a 60° angle with respect to it. A point charge 'q' is placed at the point specified by the coordinates $\mathbf{x = \sqrt{3}a}$, $\mathbf{y = a}$, $\mathbf{z = 0}$ (along the line bisecting the region at 30°) in the sextant of interest. Write down the expression for potential in the region of interest.** [correct number of image charges: five, $-q(0,2a,0), +q(-\sqrt{3}a,a,0), -q(-\sqrt{3}a,-a,0)$, $+q(0,2a,0), -q(\sqrt{3}a,-a,0)$].

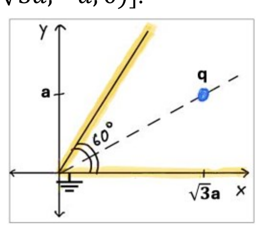

FIG. 1. Figure provided with Q3.

Students were asked to think aloud while working on the problems during the interview. The interviewer acted as an observer who did not interrupt or help except when any clarification was needed or to ask students to try more if they were giving up easily on a problem. Two researchers discussed and agreed with the analysis of students' sensemaking primarily using the pictorial analysis game. We included excerpts from interviews with students in this paper. Redundant words such as 'like' and "umm" are removed from the transcript for better comprehension.

III. RESULTS AND DISCUSSION

In this section, we discuss three different scenarios of sensemaking of two grad students G1 and G2 through the lens of pictorial analysis epistemic game.

A. Mixing up of methods and impact of "nudge"

We first discuss the sensemaking of G1 for Q1 in which the statement of the question prompted them to enter the Pictorial Analysis game. They determined the target concept as the potential (V) and started drawing the picture in Fig. 2 saying,*'...So, I like to have a visual in my mind first...'*. To arrive at Fig. 2, G1 first drew the axes and the x-y plane and placed the point charge 'q' at a distance 'a' on the positive z-axis. They placed the charge '-2q' at a distance 'b' on the negative z-axis. At this point, G1 misinterpreted the question and placed -2q on the negative z-axis instead of the positive. They read the question again to make the next move and said, '*…I'm…confused over what we mean by the region of interest but what I remember is that…this* [wrote down Poisson's equation (PE), $\nabla^2 \mathrm{V} = -\frac{\rho}{\epsilon_0}$] *is what we had for solving for potential…and so over here, we're using method of images…I'm not sure but maybe…we want to add… another point charge, so that it… balances the system* [net charge becomes zero including all the regions]*…So that we will turn this equation* [Poisson's equation] *into…zero* [wrote down Laplace's equation (LE), $\nabla^2 \mathrm{V} = 0$]*…and what we are trying to do with the method of images...we could try adding a point charge somewhere over here on the z-axis…*'. G1 incorrectly thought that the purpose of MoI is to find the image charges such that all charges '*balance*' each other and the net charge becomes zero. They also incorrectly thought that the local charge density ρ in PE is the net charge including all regions. G1 mixed up MOI and PE. They tried to find the image charges such that all charges including the original charge sum up to zero so that they could set ρ=0 in PE, $\nabla^2 V = -\frac{\rho}{\epsilon_o}$, and it goes to LE ($\nabla^2 V = 0$).

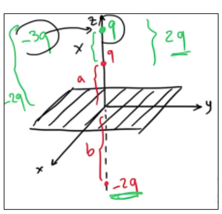

FIG. 2. G1's drawing for Q1

After this they tried to think about the magnitude, position, and nature of the image charges using intuition and said, *'…maybe we could try adding…a point charge somewhere over here on the z-axis* ['q' at the top as shown in Fig. 2]… *so that we're going to end up like with 2q* [after adding original and image charge] *on this* side [above x-y plane], *and then we have -2q on the other side* [referring to the original charge present below x-y plane] *but I don't know like how I'm supposed to find this distance* [of the image charge]...'. G1 was not sure about the magnitude of the charge and said, '*... also…instead of positive q…maybe I should have added negative 3q* [above x-y plane] *so…on both sides, I'm going to end up with –2q... But once we add that then I think the equation that expresses the potential would be…the Laplace equation...*'. G1 incorrectly thought that after placing the image charges at different points to make the net charge zero, solving LE would provide the expression for the potential but they did not proceed to solve the LE explicitly. They decided to end the game here.

At this point, the interviewer asked G1 to think about the expression for the potential, '*Okay, so do you want to try writing the potential in the region? Want to think about it?*' This prompt made G1 realize that an expression for V can be written without solving the LE explicitly and they said, *'…I'm not sure but… we would write a potential based on… the charges that we had. So, we have…this – 2q over the distance from this surface* [x-y plane] *over here* [to the charge -2q] *... over b plus depending on either, if I choose like this* [q] *or this* ['-3q'], *which I don't know. Let's go with this one* [-3q] *plus… -3q over this distance* [x-y plane to the image charge], *let's call it x + a plus q/a. I don't know maybe something like this...*'. They wrote the form of the potential as $V = -\frac{2q}{a} + \left(-\frac{3q}{x+a}\right) + \frac{q}{a}$.

In summary, the student was confused about the region of interest and therefore did not think about the negative consequences of the incorrect placement of the charge. We also find that G1 got confused and had difficulty in differentiating quantities such as the local charge density and the net charge (including the image charges they placed) and converted PE to LE. G1's sensemaking about the potential in this context shows that when students learn different concepts and methods in a course with a small-time gap, they may get mixed up. The difficulty in distinguishing between the local charge density ρ and the net charge in a region is also reminiscent of challenges in understanding the variables, e.g., in the differential and integral forms of Gauss's law in electrostatics. G1 had learned both MoI and LE but combined them while solving the problem even though they are two different methods to solve for V. Depending upon the constraints of the problem, one of these methods might be an easier approach to find V, e.g., MoI was easier for the given problem.

We also find that prompting by the interviewer to write an expression for the potential made the student abandon their earlier view that LE must be solved to find the potential

(V) in the given situation. What is useful to note here is that when the interviewer asked G1 to write down the expression for the potential, the student came out of one game and re-entered with a new entry condition. In particular, G1 realized that potential can be written without solving the LE. This shows that even small nudging can completely change students' entry/exit conditions or moves in the epistemic game they are playing to solve the problem. This could be of great advantage if we could figure out the right spots where this nudging could provide scaffolding.

Moreover, we note that G1 misinterpreted the distance in the expression for the potential as a distance between the origin and the charges instead of the distance from a charge to the point where the potential is to be found. Furthermore, a similar pattern was observed in other interviews (not included here) as well which shows that students may choose distances that appear reasonable to them but they may not be the distances from each point charge to the point where V is to be calculated.

B. Identification of the correct number of image charges at the right positions

We now discuss the sensemaking of G2 for Q2 which asks students to draw a sketch and thus makes them enter the Pictorial Analysis game. Here, G2 started by drawing a picture as shown in Fig. 3(a) in response to Q2(a).

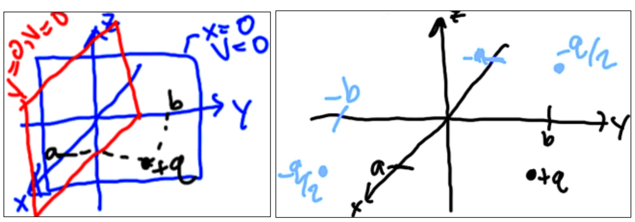

FIG. 3. (a) (Left) G2's drawing for Q2 (a). (b) (right) G2's drawing for Q2 (c).

For Q2(b), G2 used their intuition to figure out the number of image charges by saying, '*... we have a charge, and it is somewhere, not on an axis and I want to say we would need two* [image charges]. *And now I'm gonna try to justify that...*'. They tried to look at the boundary conditions at the origin (V = 0) to justify that the correct number of image charges is two -q (a, -b,0) and -q (-a, b,0) (the correct number of image charges is three). When G2 could not get the desired result, they decided to change the magnitudes of the two charges to (-q/2 and -q/2). An interesting thing we noticed in this case as well as in other interviews (not included here) is that students typically preferred to draw the 3D picture in their first attempt. However, this 3D figure often seemed to be blocking the view of one of the quadrants instead of helping them in visualization. To answer Q2(c), G2 redrew the picture without planes as shown in Fig. 3(b). G2 said, '*... I guess I needed two image charges, and these were -q/2, -q/2 ...*'. For Q2(e), G2 wrote down the expression for the potential as

$$V = \frac{kq}{\sqrt{(x-a)^2+(y-b)^2+z^2}} - \frac{kq}{2\sqrt{(x-a)^2+(y+b)^2+z^2}} - \frac{kq}{2\sqrt{(x+a)^2+(y-b)^2+z^2}}.$$

They obtained V=0 by substituting x = 0, y = 0, and x = ∞, y = ∞. Thus, in this case, the approach of G2 was to use intuition to determine the number of image charges first and then check if it satisfies the boundary conditions. They made a good attempt to obtain the expression for V. The twist it took was when the charges did not seem to satisfy the boundary conditions, G2 decided to change the magnitude of the image charges instead of the number of image charges. G2 checked the boundary condition at the origin which seemed to be satisfied with q/2 and thought that the chosen image charges are correct. They did not check the boundary condition for each conducting plane separately. This entire problem-solving process gave G2 the confidence that the answer they got was correct. This suggests that sometimes students put a lot of effort into solving a problem, but they may incorrectly think that whatever approach they used is correct and then they may carry those incorrect approaches with them to other problems as was found in the interviews.

C. Exploring symmetry and impact of scaffolding

We now discuss the sensemaking of G1 for Q3, which requires careful consideration of the geometry. After reading the question, G1 entered the **Pictorial Analysis game** by writing the angle and potential (V=0) on the grounded conducting planes in the figure provided in the question. They realized that MoI can help to solve the given problem and image charges should be placed at various points in the figure. They decided to draw a completely new picture as shown in Fig. 4 (a) to place the image charges. G1 drew the axes and the original charge as given in the question and then tried to place image charges. Placing an image charge in the fourth quadrant G1 said, '*…So, if I add one charge same distance over here* [puts an image charge -q below the x-axis as shown in Fig. 4 (a)]. *so, from here it's gonna be '-a' and this is* [distance on x axis] $\sqrt{3}a$ *...*'. Then, G1 used geometry and said later, '*... so that there are similar triangles, and this is also going to be 30 degrees over here* [along y axis]. *So, if I continue this line...* [drew a similar triangle with y axis].'

They labeled the lengths of the sides of the triangles as shown in Fig. 4 (a). Then G1 immediately decided to redraw the figure saying, '*... hmm. I don't know if I'm visualizing this right or not let me look at it over here again…*'. They redrew the picture with the axes and original charge and then placed both the negative image charges using the geometry. While drawing these image charges G1 said, *'... So, if I want to put the mirror image of this* [original charge] *on the other side* [on the positive y-axis] *I put a -q and I have the other one* [image charge] *to be over here* [in the fourth quadrant] *-q. Now, the question is whether this is going to satisfy the boundary conditions of the original problem or not…*'.

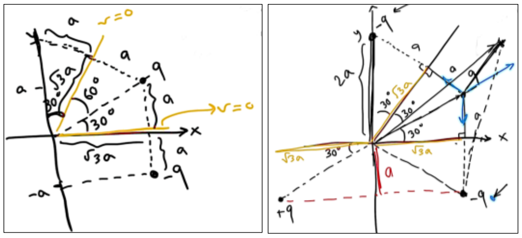

FIG. 4. (a) (Left) G1's drawing to find the positions of the image charges using geometry for Q3. (b) (right) picture redrawn by G1, to find three image charges for Q3.

G1 thought that the image charges should be placed such that the net force on the original charge is zero. They tried to relate it to another problem (Q2) solved before saying, *'... we looked at the addition of all of the fields or the forces on a point charge from this point charge to this one* [-q placed in fourth quadrant] *from this point charge* [original charge 'q'] *to this one* [-q placed of positive y axis] *and from this one to this one, we're gonna have this. So I'm gonna end up needing something that cancels out these vectors with this* [pointing away from the point charge] ...'. G1 drew another image charge in the third quadrant opposite to the original point charge as shown in Fig. 4 (b) and thought that this complete set-up of charges would make the net force zero on the original point charge 'q' given and hence would satisfy the boundary conditions automatically. At this point, G1 did not check the boundary conditions explicitly and ended the Pictorial Analysis game by writing an expression for the potential using the original point charge and the image charges they placed.

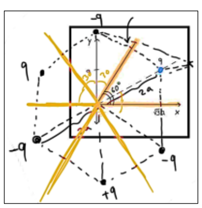

FIG. 5. G1's drawing shows five image charges for scaffolded version of Q3.

When G1 got this question again in the scaffolded version, their approach was different. They started playing the Pictorial Analysis game with different entry conditions. In the scaffolded version of Q3 the options for number of image charges were provided which helped G1 to look at different options before correctly deciding the final number of image charges to be five by saying, '... *Well, I used 3 but let's think, maybe I was wrong, so I might have wanted to use more…'*. G1 realized that perhaps they did not place the charges correctly in the unscaffolded version and the vectors were not drawn correctly. They played the Pictorial Analysis game to calculate the positions of the image charges by exploring the symmetry saying, '... *So based on the symmetry that I want to kind of preserve; I would have added negative q here* [adjacent to original charge] *and then I would have again added...* [next to it] *q over here....*'. They eventually figured out the correct number of image charges, '*5 image charges. Yeah. But there might also be a way that I could like control, for the amount of the mirror charge that I am putting or maybe the distance and then only use 3 image charges. So I'm also doubting on this one. It's definitely not 4 or 6....*'. And then G1 decided to go with the 5 image charges as shown in Fig. 5.

Thus, we find that G1 entered the Pictorial Analysis game with different entry conditions and improved the moves over time. G1 was provided with the unscaffolded version of the test first and then got the scaffolded version immediately after that. Initially, instead of looking at the potential at the boundary, G1 incorrectly focused on obtaining the vector sum of all the forces on the original point charge to be zero. They thought that when the net force from the image charges they placed sum up to zero on the original point charge, it would automatically satisfy the boundary conditions but did not check V at the boundaries. When G1 was asked on the scaffolded posttest to choose the correct number of image charges and was provided with the options of three, four, five, and six, this small scaffolding of providing options helped them to think about the symmetry as they had to find the reasons to discard 4 and 6 image charges. The question with options about the number of image charges may have helped and prompted them to think about the correct choice involving 5 image charges even if it did not help them in completely understanding the concept of potential. Thus, sometimes intermediate questions or small scaffolding can help in bringing out the appropriate resources to steer them towards a useful direction

IV. SUMMARY

We discussed episodes of student problem-solving involving the MoI by mapping them onto pictorial analysis epistemic games [26]. Observing the sensemaking of students can help make instructors aware of how different students think. As we saw in the first example, students knew the methods but mixed them up. Depending upon the organization of knowledge, different knowledge resources got activated in different situations. Also, we find that minor nudging can change the frame of the student completely and can be utilized positively by knowing and providing the right scaffolding at the right time. In the second example, we saw that the initial picture drawn by the student added hindrance in visualization. Also, the student did not check the boundary conditions on the plane except at the origin and infinity and assumed that their image charges were placed correctly and considered their incorrect approach to be the correct one. In the last case, we observed how the student gradually improved in their exploration of the symmetry of the problem and how a minor scaffolding helped activate the right resources and helped them solve a highly complex problem correctly. All these examples are useful as they show us the importance of observing the sensemaking process during problem-solving so that we as researchers or instructors can provide appropriate scaffolding for improving students' sensemaking while solving problems.

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