Clarifying Quantum Mechanics: The Importance of Formalization in Education

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The degree to which curricula can be understood depends on how they are defined and presented. If features of the content itself are not well-defined, or if the content is presented in a vague or misleading manner, then this places limitations on conceptual understanding. Such curricula lack a degree of formalization. Formalization is the process of translating an unclear or incomplete idea into a precise, well-defined concept or mathematical structure. This has applications in physics and physics education. Quantum mechanics is notoriously difficult to understand, especially with its departure from classical intuition, but we argue that a lack of formalization in quantum models and curricula is a major source of that confusion. Formalization in this context is compared with other conceptual difficulties such as quantum indeterminacy and philosophical interpretations of quantum mechanics. Recommendations are offered for physics educators and future research.

2025 PERC Proceedings edited by Pawl, Zwolak, and Leak; Peer-reviewed, doi.org/10.1119/perc.2025.pr.Bass

I. INTRODUCTION

Richard Feynman once remarked “I think it’s safe to say that no one understands quantum mechanics" [1]. Quantum mechanics (QM) as a field of physics seems fairly unique in the volume and intensity of such comments. The very meaning of QM has been in dispute since its founding, as N. David Mermin states “Quantum mechanics is the most useful and powerful theory physicists have ever devised. Yet today, nearly 90 years after its formulation, disagreement about the meaning of the theory is stronger than ever. New interpretations appear every year. None ever disappear" [2].

Why is QM so difficult to understand? Relativity also departs from classical intuition but seemingly generates less confusion in comparison. Can we fully comprehend QM, or is there something which prevents this in principle? This has strong implications for scientists, educators, and students who seek a complete understanding of QM. We might approach a subject which is difficult to grasp in practice (but still possible in principle) quite differently than topics which cannot be understood in principle.

The numerous interpretations of QM suggest that the foundations of QM could be ambiguous. Indeed, a statement is ambiguous (by definition) if there are multiple valid interpretations. Consequently, we explore this topic using the concept of formalization. Specific research questions include: 1) to what extent has QM been formalized? and 2) which implications does this have for QM education?

II. FORMALIZATION

A. Defining Formalization

Formalization is the process of translating an unclear or incomplete idea into a precise, well-defined concept or mathematical structure [3]. A well-known example is the epsilon-delta definition of limits. Informally, if a function approaches some value b as the input approaches some value a, then $\lim_{x\to a} f(x) = b$. But what does that mean, *precisely*? As a formal definition, $\lim_{x\to a} f(x) = b$ if for all $\epsilon \in \mathbb{R}^+$, there exists some $\delta \in \mathbb{R}^+$ where for all $x \in \mathbb{R}$, $0 < |x - a| < \delta \Rightarrow |f(x) - b| < \epsilon$. This assigns a formal definition to the previous informal notion. There are degrees of formalization (i.e., some ideas are more precise or complete than others), and one might formalize only some features of a system.

Formalization is sometimes conflated with other concepts like standardization or rigor. Standardization is a collective adoption of ideas or practices (standards) within some discipline. Standards exist in physics (e.g., measurement standards such as SI units) and education (e.g., Common Core). While formalization would assist in standardization, they are independent activities. For instance, one could have a standard but informal definition or a formal but non-standard definition.

The term rigor appears in many STEM contexts and typically refers to a strict adherence to methodological rules, often for argumentation (e.g., mathematical rigor for mathematical proofs). The role of rigor in physics continues to be discussed [4]. Rigor has a different use in education, where its meaning and role is heavily debated but tends to be associated with workload, level of difficulty, and performance or grading standards [5][6]. Since rigorous arguments generally require greater clarity, rigor can be extended to include formalization [7]. Indeed, some use “rigorous" and “formal" interchangeably. Regardless of definition preferences, note that increasing precision can take many forms, and here we are more concerned with precise communication than rigorous justification.

B. Formalization in STEM

Formalization has applications across STEM practices. This is apparent when considering that mathematical modeling itself is an example of formalization. One could say that “light bends as it enters a new medium", but Snell’s law is a formal description of this statement. “Objects with mass attract each other" was made formal by Newtonian gravity. There are many other empirical relationships which could also be described as “light bending" or “objects attracting", but these models resolve that vagueness. Imagine trying to solve problems or communicate results without this. Formalization offers benefits for nearly every activity in scientific practice.

Furthermore, formalization tends to strengthen the foundations of a discipline by revealing implicit premises and challenging longstanding assumptions. Once these are developed into formal axioms, each axiomatic system provides a different theoretical framework of results. For instance, the independence of space and time and the parallel postulate of Euclidean geometry can appear self-evident, but dropping these assumptions facilitated special and general relativity, respectively.

Is formalization always necessary or helpful? Formal definitions of limits (and a broader emphasis of rigor in mathematics) became more emphasized in the 19th century [8]. Calculus and limits were defined and utilized long before this, even over advanced topics in multivariate calculus, differential equations, and calculus of variations. So a formalism is not required to do or understand calculus.

How were scholars using limits without a formal definition? To illustrate, consider that most of us do not have a formal definition of a cat, but we can reliably identify cats and distinguish a cat from a horse. Such results can be obtained through intuition (i.e., unconscious inference). Students have an intuitive understanding for the meaning and result of many limit problems. An informal approach can be sufficient (and more efficient) for tasks where intuition is accurate and rigorous justification is unnecessary. To support that intuition is sound without engaging in formal analyses requires some

means of evaluating its accuracy. This can be readily checked in some cases (e.g., checking mental math with a calculator).

Without a means to directly measure accuracy, signs that intuition might be correct include reliability (in a sense of repeatability and consensus) and conviction (i.e., intuition accompanied with confidence). A failure in any of these conditions is grounds to doubt intuition, but passing such criteria is not a guarantee since intuition and consensus are fallible even among experts. Famous examples are the Monty Hall problem in probability and Bell's spaceship paradox in relativity [9][10][11]. In contexts where intuition falls short, formalization might be the *only* means to effectively define, discuss, or solve a problem.

C. Formalization of STEM Curricula

The term "curriculum" has numerous definitions [12], but here we refer to the learning goals and their lesson materials. To illustrate the progression of formalization in curricula, consider descriptions of a hydrogen atom in Table I.

Difficulty Level	**Hydrogen Atom**
Elementary	An electron circles a proton.
Middle	An electron orbits a proton at different energy levels.
High	$E_n = -\frac{13.6\text{ eV}}{n^2}$
University	$\psi_{nlm}(r,\theta,\phi) = \cdots$

TABLE I. Descriptions of hydrogen across education levels.

The target level of formalization depends on the audience and learning goals. As we see in Table I, the way we describe the configuration of a hydrogen atom can greatly differ across K-16, and we typically add features and increase precision as levels advance. Survey and introductory courses are less formal, and students learning content for practical applications tend to be less concerned with underlying details, but those seeking a complete understanding will benefit most from greater formalization.

For audiences ultimately seeking professional competency, it might seem tempting to start with formal definitions. This does offer some advantages; we can state the definition once without continually modifying previous content, but a serious cost is that students may find it incomprehensible at first. Historically, most concepts begin as more intuitive, informal notions and later become formalized. Without proper prerequisites and motivation, a formalism can feel like a foreign language. We should build familiarity with more intuitive exercises. This culminates with a statement of the formal definition and subsequent practice to reinforce the concept.

In cases where student intuition continually falls short, a formalism might be the best means to facilitate learning (just as we noted previously for STEM practices). The degree to which curricula can be understood depends on how they are defined and presented. If the content itself is unclear, incomplete, or misleading, then this places limitations on conceptual understanding.

Furthermore, curricula are limited by the extent to which that content has been formalized in that STEM discipline. For example, calculus students lacked the epsilon-delta definition of limits before its development. Having introduced formalization more generally, we will explore this concept within QM and quantum education.

III. FORMALIZATION OF QUANTUM MECHANICS

A. Postulates of Quantum Mechanics

A model is made more formal by adopting an axiomatic system (also called a formal system) to cover the base assumptions or premises of that framework. The "standard" or "canonical" postulates of QM ultimately trace to the work of Paul Dirac and John von Neumann [13][14][15]. The notation and number of postulates vary but tend to express the same content. Some QM textbooks present these as an explicit list of statements [16][17]. Others cover these rules throughout the text [18].

The postulates stated below are for non-relativistic systems and observables with discrete, non-degenerate eigenvalues. Postulates can be extended to degenerate eigenvalues, continuous eigenvalues, and composite systems (e.g., multiple particles). Extending the postulates to composite systems essentially involves changing the ket to a density operator over a tensor product of Hilbert spaces. Treatments also differ on terminology such as "Hermitian" vs. "self-adjoint" and qualifications like whether Hilbert spaces are separable. These generalizations and details do not alter the challenges presented here. Quantum field theory is beyond the scope of this paper, and although it has some of the most accurate predictions in science, some believe it lacks a "canonical" axiomatization [19].

These postulates of QM are synthesized from [16][17][20]:

P1. State – the state of an isolated quantum system at time t is given by a unit ket $|\psi(t)\rangle$ in some Hilbert space $\mathcal{H}$ called the state space.

P2. Physical Quantities – every measurable physical quantity A corresponds to a Hermitian operator $\hat{A}$ acting on the state space $\mathcal{H}$.

P3. Quantum Measurement:
   (a) The result of measuring a physical quantity A is an eigenvalue of the corresponding operator $\hat{A}$.
   (b) When a measurement occurs, the probability of obtaining an eigenvalue a_n is given by $\mathbb{P}(a_n) = |\langle a_n|\psi\rangle|^2$ where $|a_n\rangle$ is the corresponding eigenstate (unit eigenvector).

(c) If measurement of physical quantity A gives the result a_n, then the resulting state upon measurement is proportional to $\hat{P}_n |\psi\rangle$ where $\hat{P}_n = |a_n\rangle \langle a_n|$ is the projection operator for the eigenstate $|a_n\rangle$.

P4. Time Evolution – the time evolution of a quantum system (between measurements) is given by the time-dependent Schrödinger equation $i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle$ where $\hat{H}(t)$ is the Hamiltonian or total energy operator.

Adding the condition "between measurements" to P4 is essential, as the time-dependent Schrödinger equation (TDSE) does not provide the future state for all time if a quantum measurement occurs. Many sources omit this caveat and some explicitly assert that the TDSE is sufficient to describe time evolution of the system. For example, when introducing the TDSE in *Modern Quantum Mechanics* by Sakurai (a common textbook for graduate QM), they state "This is the Schrödinger equation for the time-evolution operator. Everything that has to do with time development follows from this fundamental equation" [21]. This is misleading at best and arguably false. Dirac notes that the TDSE applies provided that an observation does not occur, and von Neumann also distinguishes two types of time evolution: the unitary, reversible evolution governed by the TDSE, and the non-unitary, irreversible process of measurement [13] [14].

These postulates lack a mathematical relation describing how or when quantum measurement occurs. Consequently, the mathematical formalism under these postulates is underspecified. To illustrate, imagine some model expressed as a differential equation with initial conditions that allow us to generate a solution $x(t)$. Suppose that another process sometimes happens in the model, called "jumping", where the state x_1 can instantly jump to another state x_2, but we don't specify the conditions for how or when jumping occurs. If jumping occurs, then regardless of our interpretation of x and other quantities in the model, the model is *mathematically* underspecified. Likewise, QM under these standard postulates has gaps independent of their interpretation. We explore the ramifications of this in the next section.

B. Interpretations of Quantum Mechanics

After we formally define the mathematical structure of a model, we still need to interpret its meaning. There are numerous interpretations of QM, including Copenhagen (sometimes called orthodox), many-worlds interpretations, and pilot wave theory (also known as Bohmian mechanics). Debates over quantum models and interpretations have existed since their conception. Virtually every physicist commonly mentioned in a QM textbook (e.g., Einstein, Bohr, Heisenberg, Schrödinger, de Broglie, Pauli, Born, Dirac, von Neumann, Wigner, Bell, etc.) had views on QM which differed in some degree. Recent informal polls of physicists reveal that while Copenhagen-like interpretations are the most common, there are a wide variety of views and strong disagreements remain over fundamental questions in QM [22][23][24] .

To make matters more difficult, there are differing views within these broader interpretations. There is no definitive "Copenhagen interpretation", and historical evidence suggests there never was [25][26]. This view is often attributed to Bohr and Heisenberg, but they disagreed over core concepts within QM [27]. Copenhagen-like interpretations could be described as a family of views which share common features, such as inherent indeterminism, the Born probability rule, and complementarity [28]. There are also different many-worlds interpretations and disagreement regarding whether these satisfy the Born probability rule [29][30]. Some have attempted to compile notable interpretations into a table, but such an exercise inevitably invites controversy [31].

Interpretations exist at varying levels of precision. Without formalization, we cannot fully evaluate or compare these views. Despite being named "interpretations", some views have different sets of axioms (e.g., many-worlds interpretations drop postulate 3c of wave function collapse). The standard postulates of QM best match Copenhagen-like views. If it happens that two axiomatic systems are not logically equivalent, then they are distinct *models*, not merely different interpretations of the same mathematical formalism. Again, this depends on the exact versions of each broader view that one is comparing.

There are several interpretive challenges in QM, such as the meaning of quantum states, superposition, etc. However, perhaps the greatest difficulty lies in understanding measurement. Some denote this as the measurement problem, but again, "the measurement problem" is better understood as several related but distinct problems [32][33]. Questions include: what exactly is measurement? What governs if, how, or when measurement occurs? How does a superposition of states yield a definite outcome? What is the role of the measurement apparatus or an observer? Can measurement happen spontaneously? Does it involve consciousness? These questions do not have clear answers, but they are essential to *completely* describe and understand QM.

That said, there are quantum models which reduce the mathematical and interpretive issues with measurement, such as the various "objective collapse models" [34–37]. These are nonlinear stochastic QM models which offer novel, testable predictions, but experiments have yet to support or falsify such hypotheses [38][39].

In closing this section, note the multiple signs that intuition is insufficient for key problems in QM (e.g., lack of consensus). Thus, formalization would be helpful (and perhaps necessary) to resolve these challenges. This has implications for quantum education.

IV. IMPLICATIONS FOR QUANTUM EDUCATION

A. Formalizing QM Curricula

With disagreements over the meaning of QM, which QM model(s) and interpretation(s) should instructors address?

Such diverse views naturally extend to the role of quantum interpretations in curricula. Regardless of interpretative preferences, evidence suggests that explicitly discussing quantum interpretations improves various student outcomes [40]. Some meaningful interpretation is necessary for conceptualizing a model beyond pure mathematics. However, without careful planning, we could easily invest too much time here or overwhelm students.

Copenhagen-like views became the historical "default" and were maintained by continued support and traditional inertia, so textbooks tend to adopt such positions. If we strictly prioritize precision, we might teach objective collapse models, but these require tenuous assumptions as such models currently have unverified experimental predictions. While Copenhagen views are less complete, these provide a stronger introduction to QM due to a large availability of resources and the broader familiarity within the physics community. So for instructors introducing a formalism, we recommend the standard postulates because they best match Copenhagen views.

Ideally, we would properly motivate and develop the postulates of QM through intuitive exercises. However, quantum education literature consistently regards QM as unintuitive or counterintuitive [41]. It's often not obvious how to even translate classical physical systems and quantities into their quantum counterparts (i.e., Hilbert spaces and Hermitian operators). It would help to have a rule like "given a classical system S with some quantity A, the Hilbert space is $\mathcal{H}$ and the quantum operator is $\hat{A}$". Dirac assumed the relation $\{A, B\} \rightarrow \frac{1}{i\hbar}[\hat{A}, \hat{B}]$ where $\{\}$ denotes the classical Poisson bracket [13]. This was shown to be false by Groenewold's theorem [20]. Moreover, some quantum systems seemingly lack a classical counterpart (e.g., spin-1/2 systems). Thus, QM can be difficult to make intuitive.

We previously argued that increasing formalization helps when intuition fails. Without intuition or a formalism, it can be difficult for a student to find any bearing. Bridging this gap is difficult in introductory QM courses, especially under typical prerequisite and time constraints. Perhaps this supports why quantum curricula can devolve into mathematical exercises without being grounded in meaningful interpretations, a culture often expressed as "shut up and calculate" [42]. It might appear as if formalization mirrors this sentiment (with its more mathematical nature), but formalization seeks to provide and clarify meaning. Ultimately, we're acknowledging that this situation is no easy task for QM instructors.

B. Recognizing Inherent Limitations

Even if students comprehend the standard postulates, recall that QM is not *fully* formalized (mathematically or interpretively) under this framework. This places inherent limitations on curricula. In other words, unlike limits in calculus, if a student asks "What is QM, precisely?" then we cannot fully address this question. As John Bell states, "It remains that the theory is ambiguous in principle, about exactly when and exactly how the collapse occurs, about what is microscopic and what is macroscopic, what quantum and what classical" [43]. That said, students can receive confident yet diverging answers (some mutually contradictory) from faculty or other reliable sources.

Anecdotal evidence suggests that while most faculty recognize that quantum foundations are a complex and difficult subject, many are less familiar with the boundaries or full extent of these challenges. This is understandable; the high predictive accuracy of QM can imply that the main problems have either been addressed or are inconsequential in practice (perhaps belonging more to philosophy than physics). Also, quantum curricula and textbooks tend to cover what we do know without emphasizing the boundaries of our knowledge, but this frontier is much closer than many realize.

It helps for students and educators to be aware of this issue. First, note that there are common misconceptions in QM we *can* address. For example, work has been done to study student difficulties with understanding quantum measurement [44–49]. There are also concept inventories to assess formal, foundational content, such as the Quantum Mechanics Formalism and Postulate Survey [50]. However, what if a student struggles with an unresolved question? If the instructor is unaware, they might provide unsatisfying or even misleading guidance. If the student is unaware, they could expend significant time and effort with little success and ultimately blame themselves for this failure.

To be clear, instructors should not give the impression that these limitations undermine the achievements of QM, as this could have adverse effects on student attitudes. Students at all levels of QM can sometimes feel lost, overwhelmed, frustrated, etc. A tactful presentation of these challenges provides guideposts and can foster a sense of comfort and belonging by revealing that quantum physicists throughout history wrestled with the same ideas. Lastly, since quantum foundations is an active area of research [51], a brief introduction will better orient students to the overall discipline of QM and possibly capture the interest of prospective students.

V. CONCLUSION

Examining the degree to which QM is formalized under the standard postulates, we find that measurement is mathematically and interpretively underspecified. This ambiguity places limitations on QM understanding and curricula. We promote increased efforts towards formalizing quantum theory and recommend that educators be explicit but tactful with students about the boundaries of quantum knowledge, noting any underlying assumptions. Future goals and research include: 1) further investigate the role of formalization in physics, physics education, and quantum education, and 2) obtain empirical evidence which could accompany these arguments and explore the impact of misconceptions we can address vs. the inherent difficulties in modern QM.

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