"It's just a math equation": Examining resource coordination in physics students' reasoning about exponential functions and drag

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Exponential functions are foundational to modeling dynamic phenomena in physics, yet students often struggle to integrate their mathematical form with corresponding physical interpretations. This study reports on upper-division physics students' reasoning about exponential decay in the context of projectile motion with drag. Using the *knowledge in pieces* framework, we analyze how students activate and coordinate mathematical and conceptual resources during problem-solving. Case studies reveal that while participants demonstrated procedural fluency with exponential expressions, they did not construe these forms as meaningful representations of physical systems. In contrast, polynomial forms elicited stronger conceptual associations, suggesting that curricular familiarity plays a role in resource coordination. These findings underscore a persistent disconnect between symbolic manipulation and physical interpretation in students' reasoning. We argue for instructional designs that explicitly foster connections between mathematical structure (e.g., e^{-kt}) and mechanistic models (e.g., velocity-dependent drag), thereby supporting more integrated and expert-like engagement with exponential functions in physics contexts.

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I. INTRODUCTION

Exponential and logarithmic functions are a fundamental class of mathematical relationships widely used across science, technology, engineering, and mathematics (STEM) to represent and analyze dynamic systems and processes. Exponential functions describe processes where the rate of change is directly proportional to the current value, making them essential for modeling natural phenomena, such as growth, decay, and dynamic change. Due to their prevalence in real-world systems, exponentials and their inverses are foundational tools across STEM disciplines, bridging mathematical structure with physical behavior. In physics, exponential functions are central to modeling processes such as drag force, capacitor discharge, radioactive decay, and light attenuation [1, 2]. The concept has evolved historically from arithmetic roots, such as repeated multiplication and root extraction, to a powerful analytic framework formalized through the work of mathematicians like Napier, Euler, and Bernoulli for describing continuous change and growth in natural systems [3].

However, exponentials pose unique learning challenges in physics due to their symbolic abstraction and appearance in nonlinear and dissipative systems [4, 5]. Broadly, research on nonlinear change has shown that students often struggle to interpret and coordinate quantities that vary in relation to each other [6] and to connect symbolic expressions with their physical meaning in motion contexts [7]. Exponential functions in particular often emerge in contexts that are not immediately intuitive to students, which adds to their complexity [8, 9]. Exponentials can also take the form of an equation or a graph, which research has found students tend to interpret in inconsistent ways [10, 11]. Furthermore, Demkanin et al. [12] found that students often treat exponential functions as a tool for rote equation manipulation with little connection to the physical systems they are meant to model. Even when students encountered exponential behavior in mathematical settings, they rarely transferred that understanding to phenomena like velocity decay under drag.

Our study investigated upper-division physics students' reasoning about projectile motion with drag force – a canonical example of exponential decay in physics – and whether they could coordinate the mathematical form of the solution with its conceptual implications. Our goal was to understand how students activate, interpret, or connect the ideas they hold for exponential functions and for physical modeling, particularly in the context of the Classical Mechanics curriculum. By examining student responses and interview data through the lens of *knowledge in pieces* [13], we aim to identify patterns of reasoning that reveal the subtle – but critical – details in resource coordination that underlie students' successes and struggles with exponential functions and the systems they model.

II. THEORETICAL FRAMEWORK

This study is grounded in the constructivist framework *knowledge in pieces* (KiP) [13], which models knowledge as composed of diverse, context-sensitive elements that students activate and coordinate in real time; these elements – sometimes called "resources" [14] – may include intuitive ideas, personal beliefs, and fragments of formal knowledge [10, 15]. KiP-inspired work emphasizes the importance of examining how students connect and coordinate their many knowledge resources during problem-solving. For example, Sherin's [10] *symbolic forms* are used to model how students connect common structures found in equations with an accompanying intuitive conceptual schema, such as associating the symbolic representation e^{-kt} with decay, or "dying away" (Ref. [10], p.536).

Sometimes, novices activate these resources inconsistently across contexts [16]. Rather than labeling such thinking as problematic, KiP sees these naive ideas as the foundation for learning – the role of the educator is to identify the productive and counterproductive components among novices' thinking and to reinforce, refine, and support gradual change towards normative understanding. In doing so, KiP supports a more nuanced analysis of learning and reasoning beyond assessing for mere correctness or cataloging lists of misconceptions.

With this theoretical lens, we consider not just whether students can correctly use exponential functions, but what conceptual resources are activated in these moments, and how these knowledge activations can be refined to promote a deeper, holistic understanding of exponentials in physics contexts.

III. METHODS

This work is a part of the PhysMath project [17], which aims to develop digital simulations that help upper-division physics students integrate mathematical expressions with physical meaning. Effective simulation design requires understanding not only what knowledge resources students hold, but how they coordinate them (i.e., how they activate and connect conceptual and mathematical ideas in context). In this study, we set out to observe upper-division physics students' ability to interpret exponential expressions in relation to related physical processes like drag. Identifying patterns in this coordination informs how simulations can better support and refine students' mechanistic reasoning.

As part of the project, one-on-one semi-structured interviews were conducted with second-year undergraduate students in the advanced Classical Mechanics class throughout the spring 2025 semester to investigate their understanding of topics covered in the course material. One of the topics chosen for investigation was projectile motion with linear and quadratic drag forces. During these interviews, participants discussed their conceptual understanding of drag and its mathematical representation in projectile motion equations and reflected on challenges they had encountered in coursework. The participants then tried out a PhysMath digital simulation for Projectile Motion with Drag [18]. They first explored independently, then were given guided prompts to elicit their reasoning about the modeled behavior.

Each interview was video- and audio-recorded, along with screen recordings of the students' time with the simulation. Transcriptions of the interview audio were created with Otter.AI and annotated to include records of the students' actions in the simulation. The annotated transcripts were analyzed using *knowledge analysis* (KA) [19], a methodological approach developed by KiP-aligned researchers to study the structure and use of knowledge. Our KA was focused on a structural description of the distribution and connections between the knowledge resources demonstrated by the students. We did this via grounded coding [20], beginning with the identification of salient moments of interest, followed by identifying emerging patterns and iterative refinement. The first author conducted this thematic coding, and inter-rater reliability was established through independent coding of a subset of data by the first and second authors.

In total, four students participated in the spring 2025 interviews. Of these, two students exhibited the same pattern of mathematical-conceptual coordination. We present their cases as illustrative examples rather than as representations of the full range of variation in participants' reasoning.

IV. FINDINGS

To explore how students coordinate mathematical and conceptual resources when reasoning about exponential functions in physics, we present two illustrative cases from participants Vera and Reuben (pseudonyms). These cases highlight distinct ways in which two students engaged with exponential expressions.

A. Case Study 1: Vera's Procedural Activations and Negative Associations

Vera demonstrated partial fluency with the symbolic side of exponential functions; When she was asked to recall the forms that solutions to differential equations could take, she answered:

> "Yeah, I know one was like some constant, like A-e-to-the-something-x. I don't have it memorized now what would the solution be for a first-order differential, homogeneous equation."

When the interviewer asked Vera about how she conceptualized functions with e-to-the-exponent in them, she responded:

> "I just think of 'something grows really fast,' like, exponentially... I guess I don't really have a great way to conceptualize it... it's just 'something-to-the-power,' so it grows fast. I don't really think I've ever tried conceptualizing it before, to be honest... it's just a math equation, like, it's a function."

In this exchange, Vera demonstrated that she can activate resources around exponential functions in a productive manner for procedural tasks, like associating them with the solutions of differential equations. She also shows an association of the symbol template for exponents (in her words, "something-to-the-power") with the generalized concept of fast growth. However, Vera's admission of never trying to conceptualize exponentials and treating them like "just a math equation" indicates that, to her, exponential functions are not connected to or associated with any particular physical phenomena. This is especially noteworthy given that, just a few minutes prior, Vera and the interviewer had been explicitly discussing projectile motion with drag and the way her Classical Mechanics course had been covering the subject. Even with such close discursive proximity, Vera did not look to drag as a conceptual form of the exponential function. This suggests a somewhat context-dependent coordination: the knowledge resources which Vera draws upon in real time during discussions like these are constantly in flux, and incongruent (or even contradictory) ideas may exist within the same network but go unrecognized if they do not activate simultaneously.

One possible source of Vera's mathematical/conceptual disconnect with exponentials was revealed when the interviewer inquired how she compared e^x to polynomial functions (where the variable is in the base and the exponent is an integer). She responded:

> "The 'variable-to-some-power,' I feel like I'd have a better time trying to understand or conceptualize it, because it's like, 'oh, it's that variable that's being raised to the power.' The 'e,' I wouldn't really know exactly what's going on... I've seen in a few different places, like Euler's equation, that used it. I was really confused on that."

It appears that Vera has had negative experiences with exponentials in the context of Euler's equation, shown below:

$$e^{i\pi} + 1 = 0 \tag{1}$$

Here, the exponential is used within the context of imaginary numbers and the complex plane – a notoriously confusing subject [21]. Vera's statement seems to suggest that she encountered significant difficulties with exponentials in a complex-number context, and this unpleasant association sours her interactions with the function elsewhere.

Before the discussion topic moved on, Vera closed by saying "I just punch it in the calculator and call it good." This comment reinforces Vera's treatment of the exponential as a solely mathematical object – a process for a calculator rather than a concept with physical meaning. It also implies that Vera does not take additional time to understand exponentials during problem solving, preferring instead to perform the needed calculations and move on as quickly as possible.

B. Case Study 2: Reuben's Absence of Experiential Anchors for Exponential Behavior

Similar to Vera, Reuben also associated exponentials with the solution to differential equations; he listed exponentials – as well as polynomials and trigonometry – as mathematical forms which can show up, "depending on the starting equation." When the interviewer asked Reuben what ideas come to mind when he encounters the exponential function in equations, he responded:

> "I know on some level the shape of the exponential equation [traces e^x on table with finger], but I guess I usually don't necessarily think about the specific shape of the equation, more just what the equation as a whole means and how the variables relate to each other."

However, when Reuben tried to come up with an example scenario to explain this way of seeing equations, he struggled to do so. He commented:

> "I guess maybe, with the exponential, I don't really have a lot of experience with actually using it in equations and then understanding how that affects whatever property it's representing."

In this comment, Reuben identifies a key limitation in his reasoning around exponentials: a lack of experiential grounding for interpreting the physical meaning of exponential equations. In contrast, Reuben shared that other mathematical forms do cue conceptual associations:

> "With quadratics or a linear equation, if I see something squared in a physics equation, I might think, this could be, I'm working with acceleration, or some other variable changing, some sort of derivative in that way."

In this context, Reuben links a syntactic structure in an equation to a physical meaning; he associates polynomial terms (like squares) with those he has seen in the kinematic equations, and recalls how the mathematical process of differentiation in those cases maps conceptually onto rates of change. This resource chain shows that Reuben can successfully connect mathematical and conceptual ideas, and that his difficulty with exponentials is not due to a lack of coordination aptitude. As Reuben concluded, "with exponentials, I don't really have anything that comes to mind as to what that means."

Being an upper-division student, Reuben had undoubtedly encountered and solved many equations containing the exponential function, but expressed that he has no physical interpretation of the mathematical operation. Without such connections, the exponential term is little more than an inert symbol rather than a meaningful representation of dynamic, real-world behavior. As with Vera, procedural fluency with exponential expressions likely exists in Reuben's conceptual network, but accompanying interpretations of its physical meaning were not observed.

V. DISCUSSION

This study investigated how upper-division physics students coordinated mathematical and conceptual resources when reasoning about exponential functions in the context of projectile motion with drag. The knowledge resources used by Vera and Reuben were primarily mathematical in distribution, and the connections between their resources rarely (if ever) crossed into conceptual territory. Vera, for example, could recall that exponential functions often arise in differential equations and understood that their derivatives reproduce in the same functional form, indicating procedural fluency. However, she described e^x as "just a math equation" and seemed to prefer rote calculation over linking it to any real-world behavior. Similarly, Reuben reported greater conceptual understanding with polynomial forms such as x^2 – which he associated with acceleration, derivatives, and changing rates – but acknowledged that exponential functions did not evoke any intuitive interpretation or relevant physical phenomena.

These findings point to a inconsistent and likely context-dependent coordination between the students' mathematical forms and conceptual meanings for exponential functions and behavior. In both cases, students rarely connect their mathematical ideas around the exponential function to physical or conceptual resources that would support a mechanistic understanding of exponential change. While they could recognize or manipulate exponential expressions symbolically, their reasoning lacked the epistemological framing necessary to treat these forms as meaningful representations of physical processes. This aligns with previous research highlighting students' difficulties in translating mathematical knowledge into physics contexts [10, 22]. In particular, our findings confirm those found by Demkanin et al. [12], who also found that students treat exponential functions as formal structures devoid of interpretive content. Additionally, the fact that neither student mentioned drag forces as a physical application for exponential function – in spite of the conversations held only minutes prior – echoes the findings from other KiP researchers in which students' use of particular knowledge resources can shift dramatically over short timescales [23].

Furthermore, we noticed an interesting contrast emerge between the treatment of polynomial and exponential forms: both students reported having an easier time connecting the symbolic structure of polynomials to familiar physical contexts. The first point of interest with this observation is that both exponentials and polynomials have the same symbolic template [10] of a base raised to an exponent. Yet, the students reported having considerably disparate levels of conceptual connections with the two families of functions. This suggests that students' conceptual connections depend on the placement of individual symbols *within* the template (not just the template itself). Second, the students' conceptual understanding of polynomials demonstrates that they are capable of developing conceptual anchors to mathematical forms. It is possible the polynomial connections were stronger because polynomials appear more often in physics curricula, giving students a greater number of opportunities to reinforce these connections through diverse experiences. The examples given by Reuben show that he connected polynomials to tangible concepts like acceleration. We believe this indicates that students would benefit from instructional approaches around exponential functions that situate them in concrete, embodied phenomena where their meaning becomes experientially accessible.

Taken together, these findings underscore a critical challenge in upper-division physics: students may learn to manipulate exponential forms procedurally, without developing the interpretive tools necessary to model and explain physical systems governed by those forms. Future instructional design should aim to promote resource coordination by explicitly linking mathematical structure (e.g., e^{-kt}) with physical mechanisms (e.g., drag-induced velocity decay). The PhysMath Projectile Motion with Drag simulation [18] could potentially scaffold such instruction. Our prior studies with the PhysMath Bead-on-Hoop simulation [24] showed that students drew on both mathematical and conceptual resources when working with the simulation, even shifting from purely symbolic reasoning to mechanistic explanations. We believe the PhysMath Projectile Motion with Drag simulation could similarly enable learners to connect algebraic forms and graphical trends to their physical intuition in an experientially accessible way.

Our study was limited by its small sample size, as enrollment in upper-division physics courses is much smaller than introductory courses. As our analysis is limited to two case studies, our findings are not generalizable. Additionally, the semi-structured interview format may not capture how students engage with exponential functions during authentic coursework, which may have appreciable differences. Finally, the study emphasizes individual cognitive resources and, as such, does not account for broader instructional or social factors which may have influenced students' reasoning.

Without stable conceptual anchors, exponential functions remain abstract artifacts rather than meaningful models. This underscores the need for instruction that intentionally reinforces the coordination of conceptual and mathematical reasoning, making exponential functions not just solvable, but interpretable. By embedding exponential models in tangible physical systems and promoting model-based reflection, educators can help students develop a more integrated and expert-like understanding of exponential behavior in physics.

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