Energy measurement resources in spins-first and position-first quantum mechanics

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Spins-first

- Introduces QM with spin-1/2 systems (e.g. an electron)
- Uses Stern-Gerlach experiments to explain the postulates of QM
- Focus on measurement outcomes, probabilities, superposition, and change of basis
- Heavy reliance on Dirac notation and matrix mathematics

Energy instruction in spins-first:

Energy is the eigenvalue of the Hamiltonian operator.
Context: Spin-1/2 particle in a uniform magnetic field.
Hamiltonian for this system is: $H = -\mu \cdot B$

Question 1:
An electron is initially in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

At $t = 0$, a constant magnetic field B_0 is turned on in the z-direction.

a) Measure spin in y-direction at $t = 0$. What is the probability of obtaining spin up?
b) Measuring spin in y-direction at $t > 0$ instead. What is the probability of obtaining spin up?
c) Suppose the above measurements were not made. What are the possible outcomes of a measurement of the energy and what are the probabilities of each? Explain or show how you determined your answer.

Question of interest

Example student response:

$\hat{H}|E_n\rangle = E_n|E_n\rangle$

$\omega S_z |E_n\rangle = E_n|E_n\rangle$

$|\pm\rangle_z \quad \frac{\hbar}{2} \quad |\pm\rangle_z$

$E_n = \pm\frac{\omega\hbar}{2}$

$|{}_y\langle +|\psi(t)\rangle|^2 = \frac{1}{2} - ab\sin\theta = \frac{1}{2} - i\sin\theta$

$|{}_y\langle -|\psi(t)\rangle|^2 = \frac{1}{2} + ab\sin\theta = \frac{1}{2} + i\sin\theta$

Resources coded in this response:

Energy is an eigenvalue
Operator associated with energy is the Hamiltonian
Hamiltonian for a spin-1/2 particle in a B-field is ωS_n
Eigenvalue of spin in any direction are $\pm\hbar/2$
Probability is calculated as $|\langle\ |\ \rangle|^2$

Resources	Coded Rate
Energy is an eigenvalue	0.43
Probability is calculated as $\lvert\langle\ \vert\ \rangle\rvert^2$	0.38
Operator associated with energy is the Hamiltonian	0.38
Hamiltonian for a spin-1/2 particle in a magnetic field is $H = -\mu \cdot B$ and/or $H = \omega S_n$	0.29
Eigenvalues of spin in any direction are $\pm\hbar/2$	0.19
Eigenvalues can be found by solving $\lvert H - \lambda\mathbb{1}\rvert = 0$	0.14
The probability is the coefficient squared	0.05

Quantum mechanics can be taught in very different ways. We look at two popular paradigms of instruction: spins-first and position-first. Specifically we are looking at student thinking about energy and energy measurements.

Do different instructional paradigms affect student thinking about energy measurements? and how?

We analyzed student responses to two questions that ask for possible outcomes of an energy measurement.

- Question 1 is a typical question in spins-first
- Question 2 is a typical question in position-first

A resources lens is used to analyze student responses. We are interested in the types of resources that students activate in response to each question and any differences or similarities that are seen.

Preliminary Findings: **Lots of differences!** The lists of resources are very different! In fact, only three resources overlap. Some of the differences can be explained by the different wording of the questions, but the results suggest a deeper difference that warrants further investigation.

Hamiltonian:

- On the spins question, students were more likely to be coded to activate a resource that relates the energy to the Hamiltonian.
- In position-first the energy eigenvalues and eigenfunctions are calculated once, then recalled for subsequent questions.
- In spins-first the eigenvectors need to be recalculated for each different magnetic field.
- This may create a more pronounced connection between the Hamiltonian and the energy are created.

Probability:

- In cases where the response included information about the probability, in both contexts every student correctly activated the resource that the probability is related to the absolute square of a quantity.

Imaginary numbers:

- Both questions contained imaginary coefficients.
- In the position question, students activated the resource that energy cannot be imaginary. This was given as a reason E_2 could not be measured (an incorrect answer).
- No student did this in response to the spins question, even though it is possible to rewrite the state in a similar notation: $|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$.

Mathematical vs. conceptual:

- There is data to suggest that spins-first students outperform position-first students on a common conceptual survey.
- The resources coded were more mathematical for the spin question and conceptual for the position question.
- It is possible this is due to the different methods required to solve the problem.

Next Steps: It is unclear whether the differences noted here are due to instructional paradigm, question context, or methods required to solve the problems. The next steps for this project are to create questions for each context that are more similar to their counter-part in the other context and give all four questions to students in both the spins-first and position-first instructional paradigm.

In addition, more work needs to go into classifying the resources articulated in student responses. Potential candidates for questions that mirror their counterparts are given below.

New Spins Question:
A spin-1/2 particle in a magnetic field is prepared so that the wave function at $t = 0$ is $|\psi\rangle = 0.6|+\rangle + 0.8i|-\rangle$, where $|+\rangle$ and $|-\rangle$ are the energy eigenstates with energy $+\hbar\omega/2$ and $-\hbar\omega/2$. Suppose you measured the energy at $t > 0$. What value or values would a measurement of the energy yield? Explain.

New Position Question:
A particle in an infinite square well is initially in the state $\Psi(x,0) = 0.6\Psi_1(x,0) + 0.8i\Psi_2(x,0)$ at $t = 0$. What are the possible outcomes of a measurement of the energy and what are the probabilities of each? Explain.

Position-first

- Introduces quantum mechanics using the position wave function
- Focuses on solving the time-independent Schrodinger equation for multiple simple potentials
- Uses both the position and momentum representations
- Heavy reliance on partial differential equations

Energy instruction in position-first:

The Hamiltonian is introduced as the operator form of the total energy (kinetic + potential: $H = p^2/2m + V(x)$)
Context: single particle in a one-dimensional potential

Question 2:
A particle in an infinite square well is prepared so that its wave function at time $t = 0$ is: $\Psi(x,0) = 0.6\Psi_1(x,0) + 0.8i\Psi_2(x,0)$
Where $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are the energy eigenstates corresponding to the ground state, with energy $E_1 = \varepsilon$ and the first excited state, with energy $E_2 = 4\varepsilon$
Suppose you measured the energy of this particle at time $t_1 > 0$.
What value or values would a measurement of the energy yield? Explain your reasoning.

Example student responses:

Only the $E_1 = \varepsilon$,
From the wave equation above $\Psi_2(x,0)$ is multiplied by $.8i$ and imaginary values can not be measured.

Resources: Coefficients in the state are meaningful
Energy cannot be imaginary

You can only measure one of the energies at a time and not a combination of the two
Therefore we would measure E to be either ε or 4ε depending on the probability

Resource: Multiple values are possible

$.6\varepsilon + .8(4\varepsilon) = .6\varepsilon + 3.2\varepsilon = 3.8\varepsilon$
A proportional sum of the energies... but wouldn't this allow an infinite range of energy?

Resource: Expectation value

Potential resources that should be coded

Preliminary Resources	Coded Rate
Multiple values are possible	0.33
Expectation value	0.17
The coefficients in the state are meaningful	0.15
Energy is an eigenvalue	0.13
Energy cannot be imaginary	0.10
Operator associated with energy is the Hamiltonian	0.08
Energy and time have an uncertainty relation	0.04
The probability is the coefficient squared	0.04