Using Bayesian Updating to Shift Epistemic Beliefs

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Introduction

A central role of physics education is to improve students' ability to evaluate proposed hypotheses and models. This ability is important not just for students' understanding of physics, but also to prepare students for future learning beyond physics. However:

Many studies have shown even reformed physics curricula often produce *losses* in students' epistemic reasoning (see e.g., [1, 2]).

To address this, new curricular materials and approaches are needed which clarify and value the act of evaluation and learning.

Evaluation of an idea involves hypothetico-deductive reasoning (HD reasoning):

HD REASONING PROCESS

IF	[the hypothesis *H* to be tested is assumed true]
AND	[a test is planned under certain assumed conditions]
THEN	[a prediction is deduced from the hypothesis]
AND	[evidence *E*, including associated uncertainties, is obtained]
THEREFORE	[confidence in the hypothesis is modified with Bayes' Thm.]

In some physics curricula, students are tacitly expected to engage in this process and then update the estimated likelihood of an assertion in a qualitative, intuitive fashion.

Here, we focus on making the act of hypothesis evaluation explicit, clear, and consistent in format. There are two types of evaluation:

Direct Evaluation: The evidence *E* is obtained from a laboratory experiment.
Indirect Evaluation: The evidence *E* is obtained from a thought experiment.

HYPOTHESIS:

Teaching students to make use of Bayes' Theorem to update their estimated likelihood of *H* can benefit their epistemic sophistication, making them more expert-like.

Bayes Theorem: The initial probability of *H*, denoted *P(H)*, is updated after evidence *E* is gathered, according to:

$$P(H|E) = \frac{P(H) * R}{(P(H) * R) + 1 - P(H)}$$

where *R* = *P(E|H) / P(E|¬H)* is a Bayes factor which encodes the inferential power of the evidence with regards to *H*. Guidelines for the meaning of *R* are provided in Table 1.

GENERAL BAYESIAN ACTIVITY DESIGN:

Students estimate *P(H)*, then after an evaluation (direct or indirect) is performed to test *H*, they select an updating coefficient *R* and apply Bayes' Theorem to update the subjective likelihood of the hypothesis.

R	Interpretation
< (1/150)	¬*H* very strongly favored
(1/150) to (1/20)	¬*H* strongly favored
(1/20) to (1/3)	¬*H* substantially favored
(1/3) to 1	¬*H* barely favored
1 to 3	*H* barely favored
3 to 20	*H* substantially favored
20 to 150	*H* strongly favored
> 150	*H* very strongly favored

Table 1. Guidelines for estimation of the Bayes factor *R* (adapted from [3]). Students are given these guidelines and use them to subjectively estimate *R* when using Bayes' Theorem to update their confidence in a hypothesis *H*. Note that ¬*H* (NOT *H*) is the hypothesis that *H* is false.

Curricular Material Designs

Although the specific values for *P(H)* and *R* used by students are subjective, as more evaluations are performed each students' posterior probability will asymptotically approach either 0 or 1 depending on whether *H* is objectively false or true.

Epistemic Corollary 1: One should never rule out any hypothesis with absolute certainty, and likewise one ought never to have absolute confidence in a hypothesis.

Epistemic Corollary 2: Knowledge can never be absolute, but it can be objective.

Bayes' Theorem in Direct Evaluation (Lab Reports):

a. *Hypothesis:* Describe the hypothesis being tested and state your initial confidence level. ←IF

b. *Experiment Design:* Describe (both verbally and with pictures) how you collected your data. Include information about calibration & setup to minimize systematic uncertainties, and any steps you took to minimize random uncertainties. ←AND

c. *Prediction:* If the hypothesis is correct, what do you predict will be true about your data? ←THEN

d. *Data:* Include your Excel workbook with the raw data, and summary tables in your report document.

e. *Analysis:* Include all analyses of the data, including calculations, graphs, and statistical work, in your Excel workbook. Describe and summarize your analysis in your report.

f. *Result:* Report your group's results, including the 95% confidence intervals. State whether your result is consistent or inconsistent with the prediction you made in section (c) above. ←AND/BUT

g. *Uncertainty Analysis:* Identify and analyze any significant sources of systematic uncertainties in your experiment. For each source of error that you list, briefly describe why you identified it as a source of error, the manner in which each error is likely to affect the results, and the magnitude of the effect.

h. *Evaluate Results:* Make a judgment whether your experiment produces a confirmatory, null, or disconfirmatory result. Justify your judgment on the basis of your prediction, results, and error analysis.

i. *Summary:* Summarize the logical structure of this experiment using the IF… AND… THEN… AND/BUT… THEREFORE… structure. Choose a value for the updating coefficient *R*, and justify that choice. Then, use that to update your confidence in the hypothesis using Bayes' Theorem. ←THEREFORE

Bayes' Theorem in Indirect Evaluation (Thought Experiments):

Strategies such as special case and limit case analysis are cast in the HD process to perform the evaluation. Bayes' Theorem is then used to update students' confidence in their proposed solution.

1. [student solves standard end-of-chapter problem]

2. a. What physical model is your solution to problem #1 based on? What do you think is the probability that your solution is consistent with the model and any prior knowledge you have about the situation (i.e., that your solution is valid)?

b. Do an indirect evaluation of your solution to #1 following the usual IF… AND… THEN… AND/BUT… THEREFORE… structure.

c. Use Bayes' Theorem to calculate your new confidence level in your solution to #1. Justify your estimate of *R*.

$$C_f = \frac{C_i * R}{C_i * R + 1 - C_i}$$

Results (EBAPS)

Algebra-Based Physics

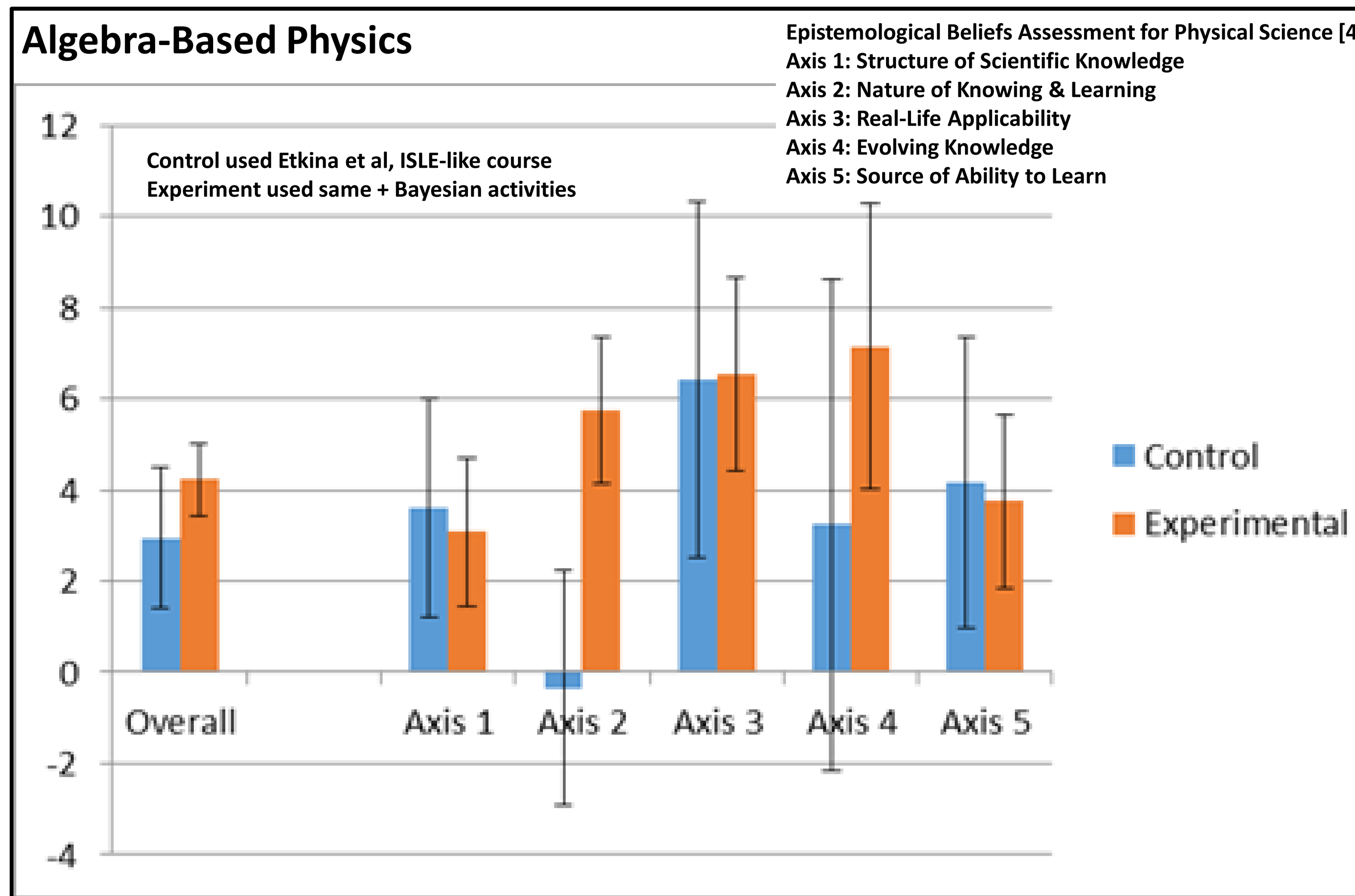

Figure 1. A comparison of the gains in mean score for each condition (C and E) on the EBAPS and each of the five subscales (N_E = 56, N_C = 18). Error bars show one standard error of the mean.

	Total	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5
Gain 95% HDI (Con) (N = 18)	$\mathbf{2.6_{-2.9}^{7.8}}$	$3.4_{-3.5}^{10.3}$	$-0.4_{-9.2}^{8.2}$	$6.2_{-5.4}^{16.0}$	$3.1_{-10.7}^{16.8}$	$4.6_{-5.8}^{14.8}$
Gain 95% HDI (Exp) (N = 56)	$4.2_{0.4}^{8.5}$	$2.9_{-2.1}^{8.2}$	$6.1_{1.6}^{11.0}$	$6.5_{-0.6}^{13.2}$	$6.8_{-3.8}^{16.6}$	$3.9_{-3.6}^{9.7}$
Difference of Gains 95% HDI	$\mathbf{1.6_{-2.1}^{5.2}}$	$-0.6_{-6.6}^{5.7}$	$\mathbf{6.4_{0.0}^{12.8}}$	$0.2_{-9.4}^{9.8}$	$\mathbf{3.6_{-9.5}^{16.7}}$	$-0.6_{-8.5}^{7.4}$
Likelihood $E_{gain} > C_{gain}$	**74.7%**	43.4%	**97.4%**	51.6%	**70.6%**	44.4%
Effect size 95% HDI	$\mathbf{0.18_{-0.34}^{0.73}}$	$-0.04_{-0.47}^{0.40}$	$\mathbf{0.51_{0.02}^{1.00}}$	$0.01_{-0.46}^{0.51}$	$\mathbf{0.14_{-0.34}^{0.61}}$	$-0.05_{-0.58}^{0.46}$

Table 2. A summary of the data. The results relevant to this study are for Axis 2 and 4 (highlighted). Bayesian Estimation Supersedes the T-test (BEST) [5] is used to estimate parameters and effect sizes.

Calculus-Based Physics

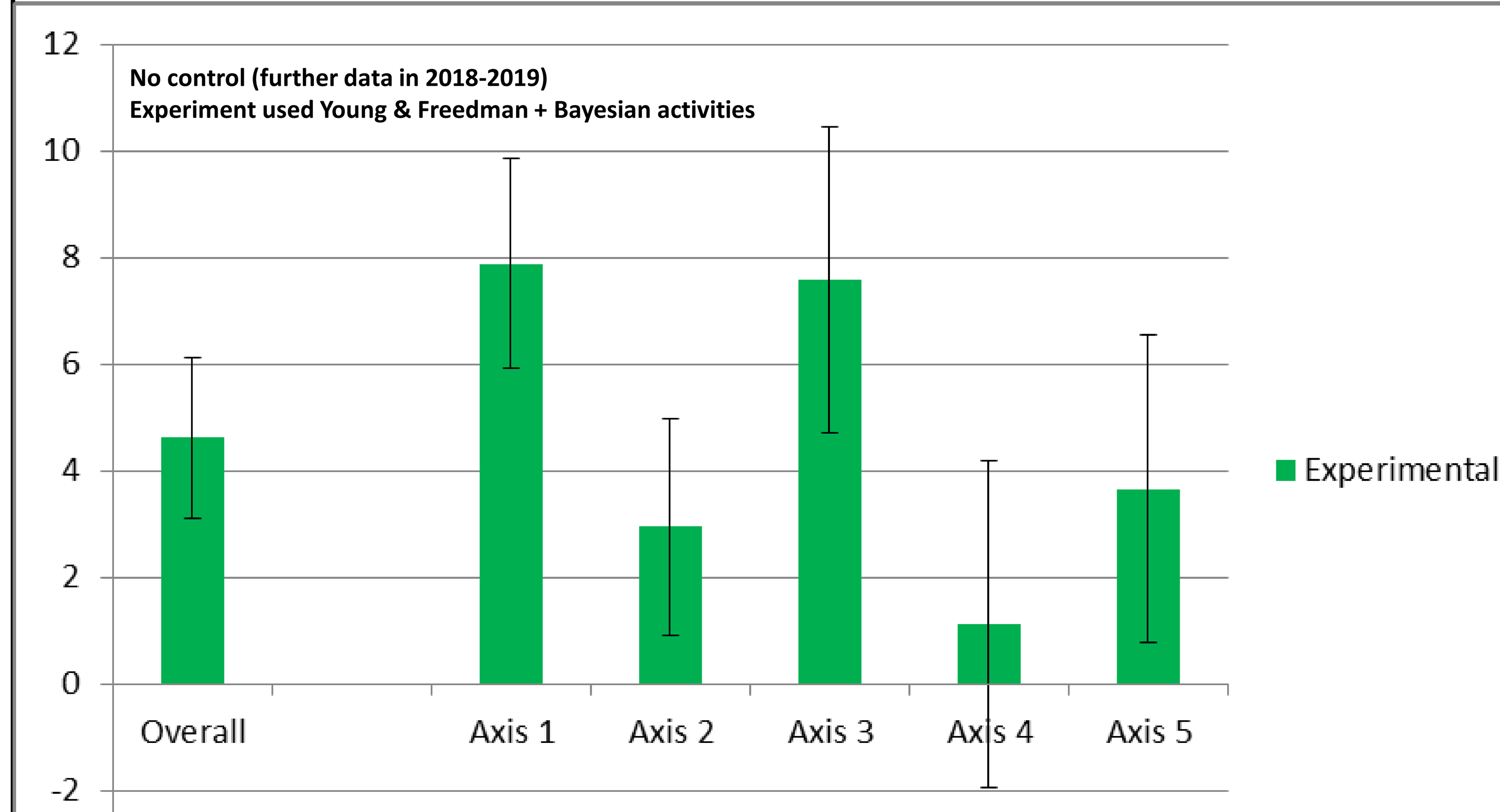

Figure 2 (above) & Table 3 (below). Gains in mean score for two experimental sections on the EBAPS and each of the five subscales (*N* = 52). BEST is used for parameter estimation.

	Total	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5
Gain 95% HDI (N = 52)	$\mathbf{4.6_{1.5}^{8.1}}$	$8.2_{3.5}^{12.9}$	$\mathbf{2.8_{-2.1}^{7.5}}$	$7.9_{1.4}^{14.0}$	$\mathbf{1.2_{-6.2}^{8.3}}$	$3.7_{-2.3}^{10.0}$
Likelihood $E_{gain} > 0$	**99.7%**	99.98%	**87.2%**	99.2%	**62.6%**	88.3%

Conclusions

Bayesian Updating Activities may enable significant epistemic gains in introductory student attitudes. Quasi-experimental data collection will take place this coming year to test effects in calculus-based physics.

References

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