Student reasoning about whether a solution is “sensible”

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ABSTRACT: Practicing physicists value a variety of answer-checking behaviors such as reviewing units, limiting case analysis and numerical estimations, which we refer to as "the three usual ways" of answer-checking. Students, however, often do not adopt these behaviors even when the instruction includes explicit efforts to encourage it. In previous work we have documented settings in which all students demonstrate the ability to perform solution checks, finding that checking units is most readily adopted, limiting case analysis, less so, and numerical estimation, least of all. Here, undergraduates in two upper-level courses---electrodynamics and intermediate lab--- were asked to check whether a given solution is "sensible" and their responses were studied with an eye toward better understanding the "simple" mathematical and physical reasoning that was preferentially invoked. We report on stark differences between the two groups, largely related to previous instruction, we believe.

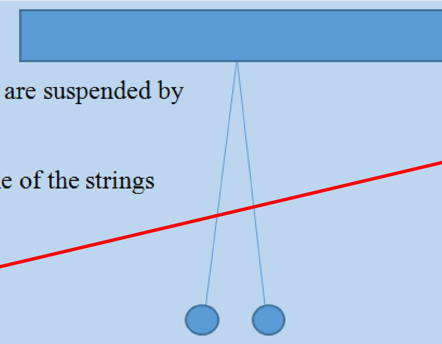
Scenario

The first author asked his “new” and “returning” physics students in intermediate lab and E&M to examine a potential solution to an electrostatics problem:

4) Two small identical charged balls of mass m and excess charge q are suspended by insulating strings of length L as shown.

Someone proposes the following formula for the static tension in one of the strings

$$T = mg\sqrt{1+\left(\frac{kq^2}{4mgL^2}\right)^{2/3}}$$



(Here $k = 9\times10^9$ Nm^2/C^2 is the Coulomb constant and g is the usual gravitational for the earth’s surface.)

Check to see if this formula is sensible in as many ways as you can think of, explaining your thinking clearly.

TRY IT: Check to see if this formula is sensible in as many ways as you can think of.

Instructional Rationale & PER Interests

Teaching students to **check solutions for sensibility** is hypothesized to help them catch errors in their or others’ work, improve future problem-solving performance, and generate new physics [1]. Problem-solving protocols do not seem to encourage students to check solutions. Prior work [2,3] suggests that with sufficient time and encouragement, most students will at least check units and attempt some sort of limiting case analysis for their own or others’ solutions.

Limiting case checks have piqued our interest recently because it provides especially rich detail from the students when it is implemented, and because the “new” students did not seem to have this check on their radar initially. Meanwhile, other researchers have shown that intentional instruction strategies that include exercises that specifically address limiting or special case analysis improve problem-solving performance in areas that are conceptually related to the exercises [4]. Thus, we are led to look closer at limiting case analysis.

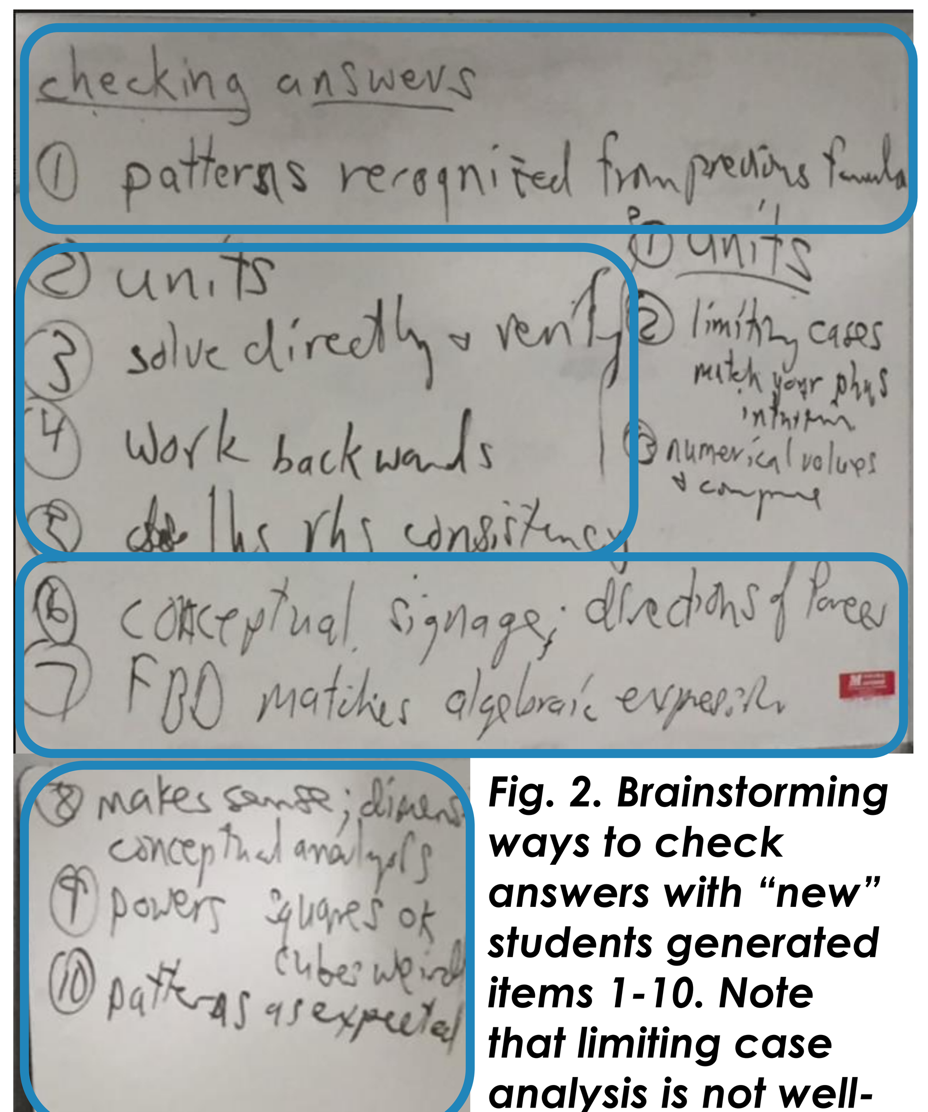
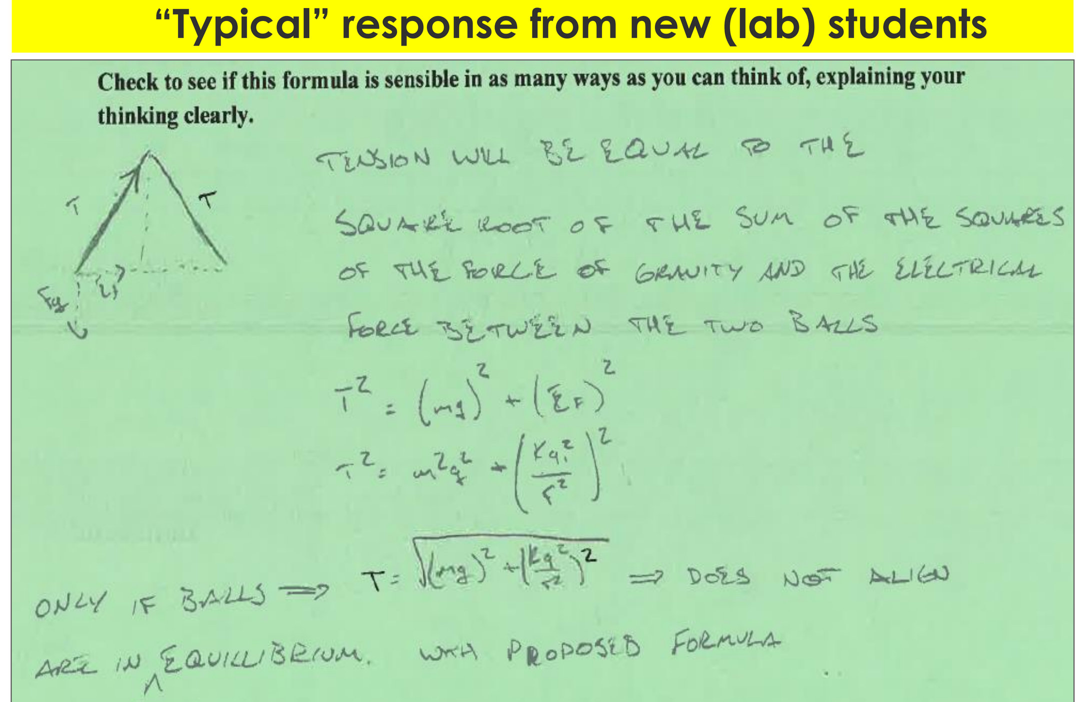


Fig. 2. Brainstorming ways to check answers with “new” students generated items 1-10. Note that limiting case analysis is not well-represented.

“New” vs. “Returning” Student Responses

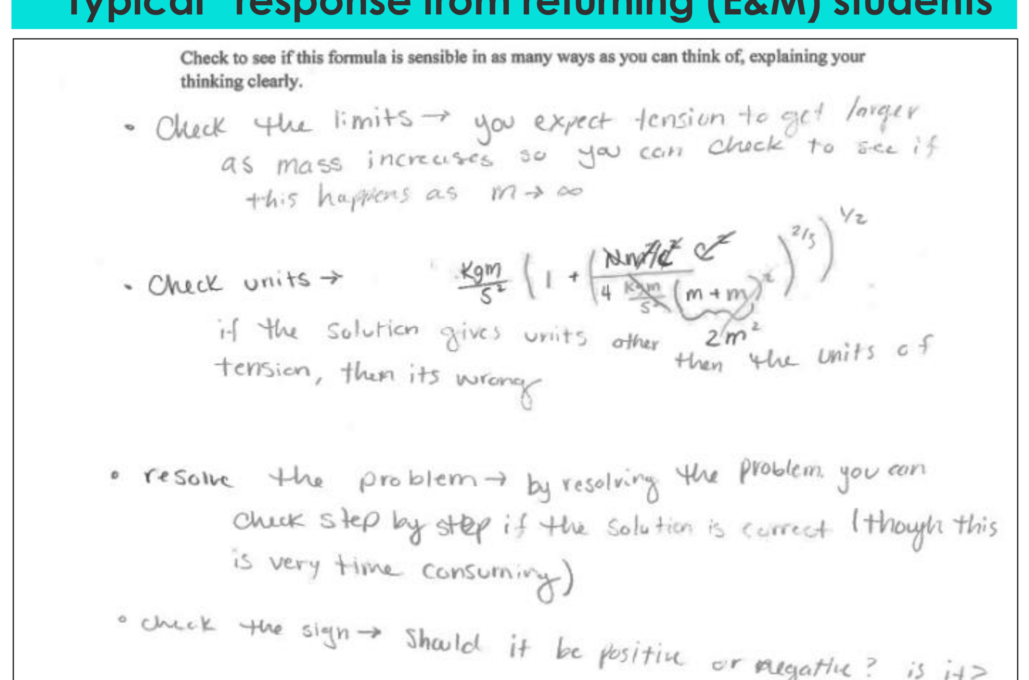
NOTE: *The “returning” E&M students were mostly those who had taken an upper level course with the professor (who emphasized the three usual checks) in a previous spring (7 of 10), while none of the “new” lab students had taken a course with this professor, or had upper level physics courses.*

“Typical” response from new (lab) students



- **Most (10 of 13) draw FBD and attempt to derive this formula.**
- Some (4 of 13) check units. Several mention the “weirdness” of the exponent, but no one checked limiting cases.
- In brainstorming session, limiting cases did not come up as a student idea (see photo above).

“Typical” response from returning (E&M) students



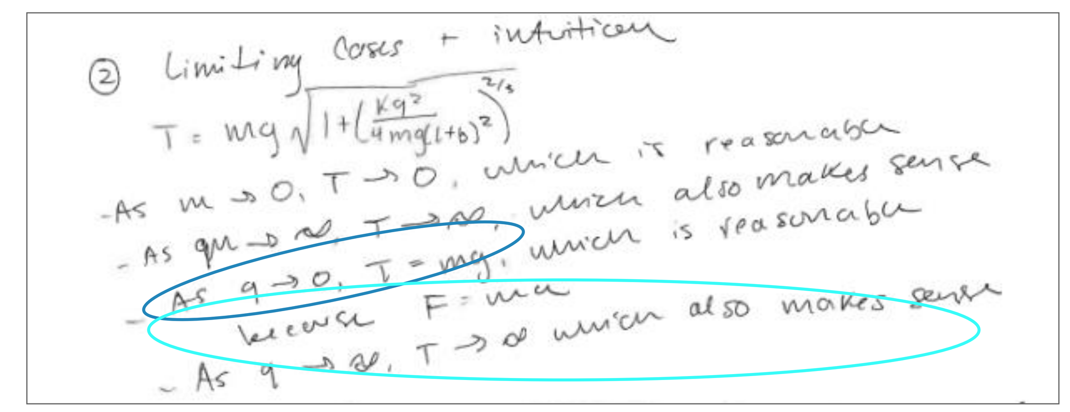
- **Most (5 of 7) make bullet point list of things to check, including units.**
- Qualitatively, while appearing more rote, the responses seemed more generic, applicable to a broader array of problems
- A few (3 of 7) specifically mention checking limiting cases of the expression above.

Closer look at students’ limiting case analysis

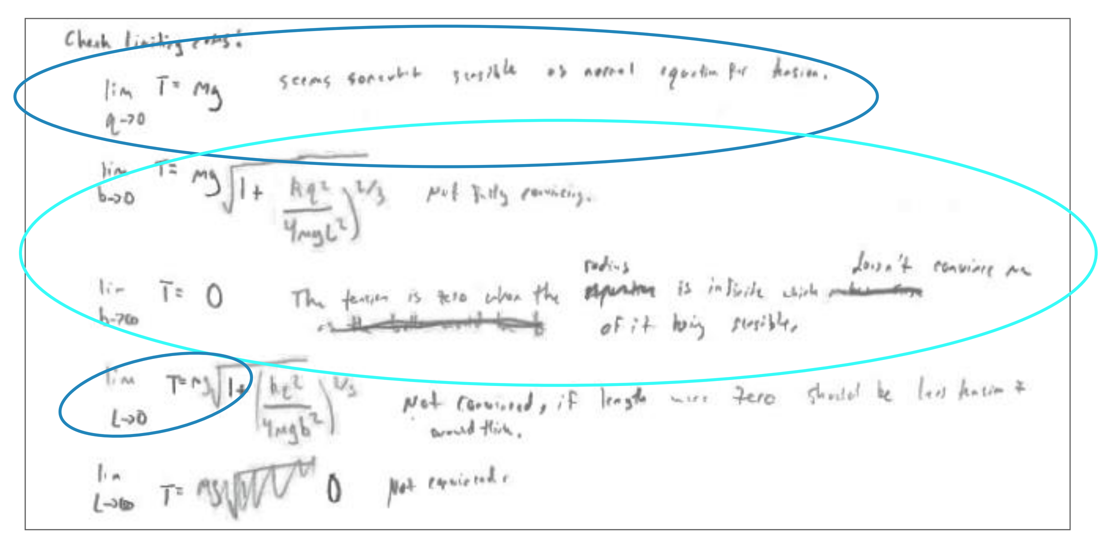
What limiting cases do students analyze?

Students choose m, q, and L, but not g or k.

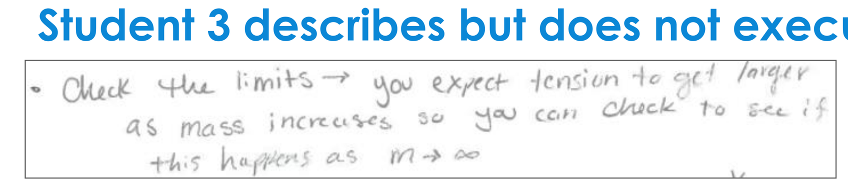
Student 1 determines the formula is sensible



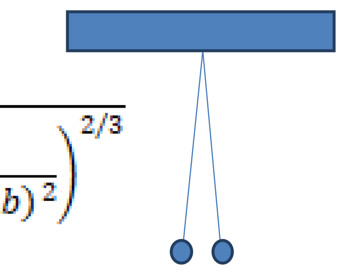
Student 2 is “not convinced”



Student 3 describes but does not execute check



$$T = mg\sqrt{1+\left(\frac{kq^2}{4mg(L+b)^2}\right)^{2/3}}$$



“Returning” students 1 and 2 at left note that when the charge is zero that the tension goes to mg, as they expect, citing F=ma or the “normal” equation for tension.

IF students conduct limiting case analysis at all, then they usually check more than one variable (m, q, L) and/or more than one limit (0, ∞).

Compared to m, limiting behavior for L and b were difficult for students to interpret.

Physical intuition informs the selection AND evaluation of limiting case checks.

How do students decide what limits to take?

From an interview with Student 3, a “returning” student, ~1 year after producing data at left:

S: Some of it is kinda just a guess…I think in some problems, it's a little more clear, like in the wording …I checked the mass because the mass was a fairly easy one to check, although I forgot that there was a mass right here as well (revisits mass check and notes a problem) that I didn't think about when I was initially doing @this…also at least in my mind, **I already kind of had an idea of what the // should happen if the mass gets larger…but** I think if I had been doing this on a real test, I probably would have also checked q um, which I didn't because the charge also could've been // would be a variable, that if that changed, **could have a big impact.**

Later, S3, on another aspect of choosing:

…I think that if you check [L and]um, the radius [b], that then they're basically in some sense the same, that if I make b bigger or if I make L bigger if the other one's held constant they're still gonna kinda do the same thing in the problem **so I guess you don't really need to check both of them…**

Evidence of considering what quantities to hold constant and which to vary [5].

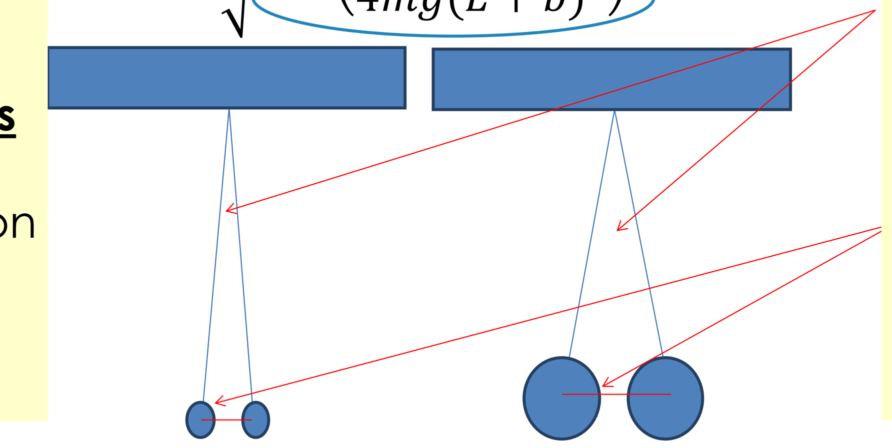
How do students decide if the limiting case results “make sense”?

Student 3, initially considering <u>incremental change</u> in b:

“..**when the radius approaches, um, infinity** what happens to this that'll make the bottom of this equation get smaller um of this fraction, um because of this number gets larger, um, **<u>it creates a smaller and smaller fraction</u>** which means that the total tension will also get larger, um, **so the tension will also be tending towards infinity** I believe.”

Note: S3 was asked to evaluate a different formula for *T* in the interview:

$$T = \frac{mg}{\sqrt{1-\left(\frac{kq^2}{4mg(L+b)^2}\right)^{2/3}}}$$



Student 3, later in the interview, invoking <u>physical intuition</u>:

S: …as the radius approaches infinity the tension increase // er approaches infinity and I believe this is what we would expect um, because um **<u>a larger radius is going to increase that angle between the two strings they're gonna be pushed farther apart (gestures) um which should cause more tension</u>…even if this little distance here** (surface to surface distance) **didn't change the center to the center if they have a bigger radius, then you also have to account for that radius** to get to the center, so I guess to me it seems like they'd have to be farther, they'd have to be pushed farther apart…

Conclusions and Next Steps

As discussed in the literature, we see evidence that while conducting limiting case analysis, students invoke and coordinate symbolic and pictorial representations, mathematical knowledge, and physical intuition [4, 5]. Limiting case checks prompt students to consider which quantities to treat as variable, and which quantities to treat as constant; interestingly, no students in our study treated k or g as variables. We find evidence of students implementing limiting case analysis by imaging how incremental change to a variable would impact the formula, as well as checking behavior at extremes. The dependency on b and L was difficult for students to interpret. Physical intuition seemed to be invoked both in selecting checks and in evaluating the results. Interviews provided more detailed evidence of students' physical intuitions, compared to written prompts. We are curious in future work to better understand how students respond when limiting case checks generate results that students think are not sensible (i.e., Student 2 above), and whether students learn new physics from limiting case analysis [6].

References

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