Presentation of Integrals in Introductory Physics Textbooks

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How are integrals presented in introductory physics textbooks?

Students are most likely to come out of a calculus course with a perimeter and area or antiderivative conceptualizations of integrals which are neither common nor productive [1,2]. In contrast, the most common representations in physics texts are adding up pieces and procedural.

Motivation

Setting up an integral based on a given physical system can be a significant barrier to success for students [3]. Prior research has shown that the way students think about integration (area under the curve, the antiderivative, or adding up pieces) before they start a problem has an effect on their success when setting up the necessary integrals [1]. These studies informed our research question.

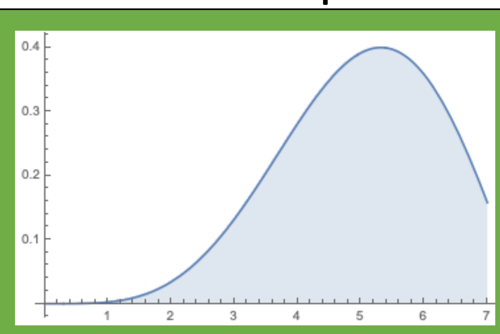
Methods

Five introductory physics textbooks were selected for analysis.

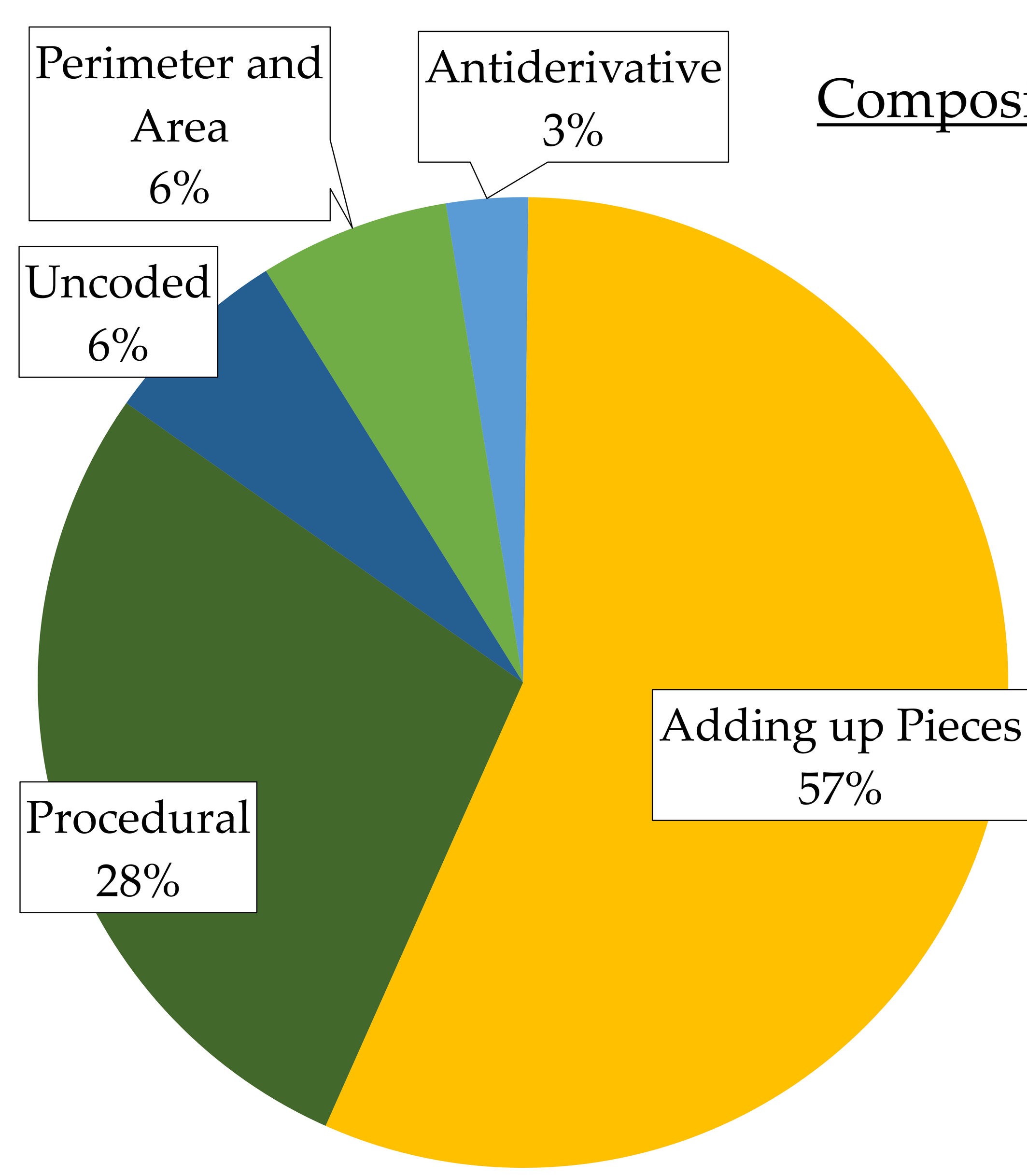
A. Halliday Resnick and Walker, Fundamentals of Physics 9th ed.
B. Young and Freedman, University Physics 11th ed.
C. Giancoli, Physics for Scientists and Engineers 4th ed.
D. Knight, Physics for Scientists and Engineers 3rd ed.
E. Serway and Jewett, Physics for Scientists and Engineers 8th ed.

Each integral presented in the text was coded according to the scheme summarized in the table to the right.

Data & Analysis

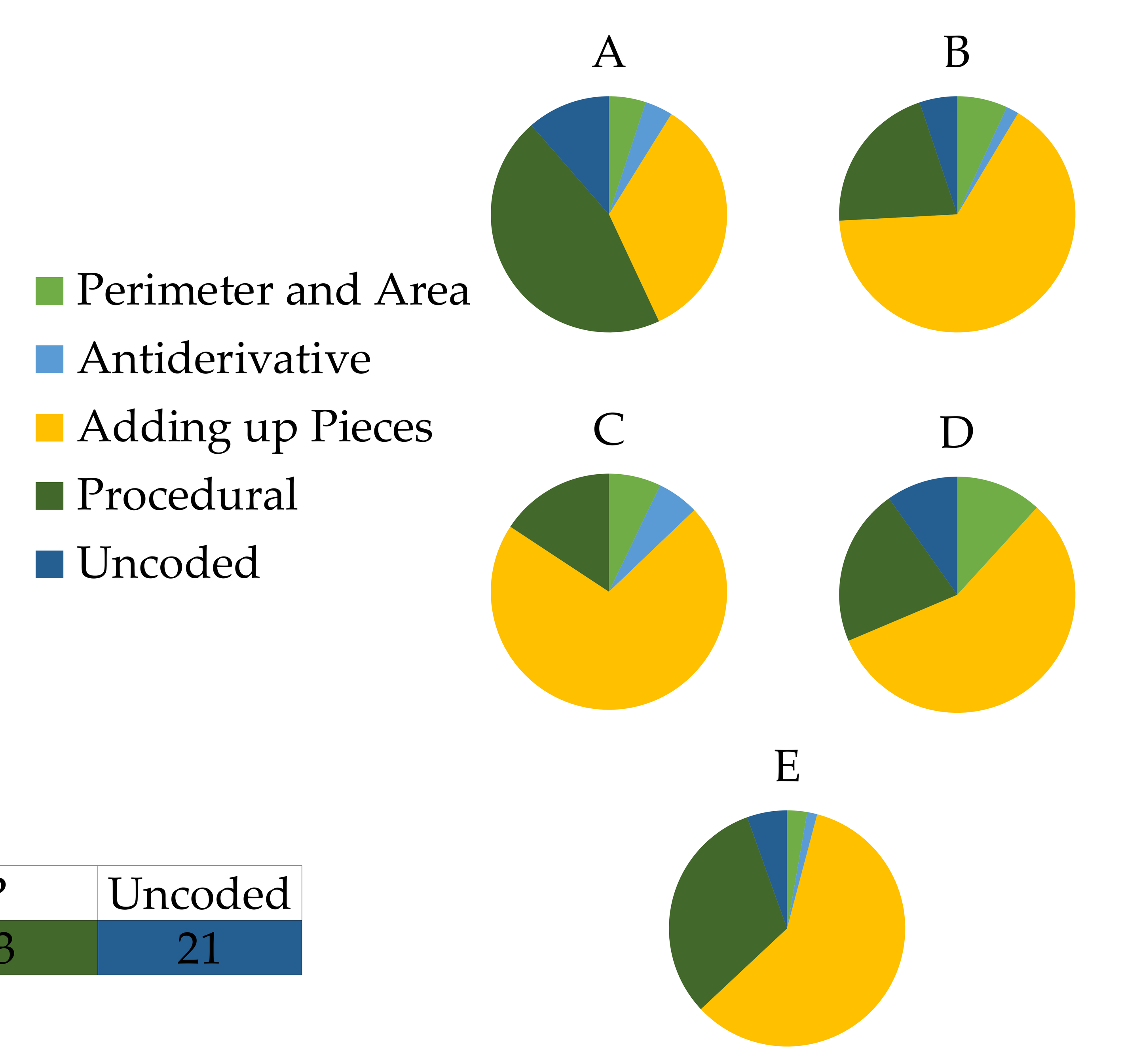
Categorizations	Keywords and Ideas	Example
Perimeter and Area (P&A)	- Area referenced in the text - Shaded region on graph	
Antiderivative (AD)	- Indefinite Integral - Start and end with a function - Use of constant of integration	$U = \int \mu B \sin\theta \, d\theta$ $= -\mu B cos\theta + C$
Adding up Pieces (AUP)	- Summation Notation - Infinitesimal divisions - Summation referenced in text	$\lim_{N\to\infty} \sum_{i=1}^{N} \frac{Q}{4\pi\varepsilon_0 L} \frac{r\Delta y}{(y_i^2+r^2)^{1/2}}$ $\frac{Q}{4\pi\varepsilon_0 L} \int_{-L/2}^{L/2} \frac{r dy}{(y_i^2+r^2)^{1/2}}$
Procedural (P)	- Adapted from analytical derivation epistemic game [4] - Integration used as a tool - Deriving an algebraic expression - Solving differential equation	$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC} dt$ $Q = Q_0 e^{-t/RC}$

Composite Data



P&A	AD	AUP	P	Uncoded
21	9	187	93	21

Individual Texts



Discussion

By a significant margin the most common presentation of an integral was categorized as adding up pieces.

- 187/331 (57%) in AUP
- 280/331 (85%) coded as either P or AUP
- 30 (9%) integrals between P&A and AD

The number of integrals between the textbooks varied depending on the number of examples and other factors but the contexts in which integration was utilized varied very little.

Acknowledgements

This work is supported by the NSF through grant PHYS-1405616 and by the Dan Black Family Foundation.

References

[1] Jones, S. R. (2015a). Areas, anti-derivatives, and adding up pieces: Definite integrals in pure mathematics and applied science contexts. *Journal of Mathematical Behavior*, *38*, 9–28. https://doi.org/10.1016/j.jmathb.2015.01.001

[2] Jones, S. R. (2015b). The prevalence of area-under-a-curve and anti-derivative conceptions over Riemann sum-based conceptions in students' explanations of definite integrals. *International Journal of Mathematical Education in Science and Technology*, *46*(5), 721–736.

[3] Cui, L., Sanjay, R., & Fletcher, P. (2006). Transfer of Learning from college calculus to physics. *Proceedings of the National …*, (2004), 1–10.

[4] Bajracharya, R. R., & Thompson, J. R. (2016). Analytical derivation: An epistemic game for solving mathematically basedphysics problems. *Physical Review Physics Education Research*, *12*(1), 1–21. https://doi.org/10.1103/PhysRevPhysEducRes.12.010124