Graduate student understanding of quantum mechanical spin

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RESEARCH QUESTIONS:

(1) What are the important yet currently unidentified graduate student difficulties with understanding essential concepts related to quantum spin?

(2) To what extent does student understanding of these topics change over the course of typical graduate instruction, as observed with several instructors and institutions?

METHODS:

- Classroom observation with field notes
- Draft assessment items based on field notes
- Revision of assessment items based on trial use and student feedback
- Used assessments on multiple cohorts over four years, at two major Midwestern universities
- Student interviews

1. Suppose you are working on spin-1/2 particles, and you measure a certain spin to be in the state $|+\rangle_y$. Which of the following states is/are orthogonal to this? Circle all that apply.
(a) $|-\rangle_y$ (b) $|+\rangle_x$ (c) $|-\rangle_x$ (d) $|+\rangle_z$ (e) $|-\rangle_z$

2. Suppose a spin-1/2 particle is sent through a Stern Gerlach device (inhomogeneous magnetic field) and a measurement is made of the particle's deflection that shows the state must be the eigenstate of $\hat{S}_x$: $|\psi\rangle = |+\rangle_x$, giving it an eigenvalue of$+\hbar/2$. Immediately afterwards, what is the ***magnitude*** of the projection of the state $|\psi\rangle$ onto each of the following states?
$|+\rangle_z$ _____ $|-\rangle_z$ _____ $|-\rangle_x$ _____ $|+\rangle_y$ _____ $|-\rangle_y$ _____
(Answers: **1/√2 for all but $|-\rangle_x$, which has zero projection**)

3. Consider two spin-1/2 fermions (A and B). At a particular time, in the uncoupled (z) basis, their spin states are $|m_A = 1/2\rangle$ and $|m_B = -1/2\rangle$. If a measurement is made (immediately, with negligible passage of time) of the total spin of the system s_{Tot}, what possible values might be obtained?

4. Two spin-1 bosons are together in a harmonic oscillator potential. Your colleague has prepared them in a particular state, but he/she has not told you what that state is. Below are several statements about what the states could possibly be. Circle all statements that are FALSE.

(a) The total spin angular momentum of the system could be $s_{total}=1$, but this would require the two bosons to be in different spatial energy states.
(b) If they are both in the spatial HO ground state, their spin state can either be symmetric or antisymmetric under exchange, since there is no Pauli Exclusion Principle for bosons.
(c) Both particles must be in the ground state and have the same spin state because they are bosons.
(d) The bosons cannot occupy the first excited state, because that state is spatially antisymmetric.
(e) One boson could be in the ground state, and the other could be in the second excited state, but this requires that their spin states be antisymmetric under exchange.
(f) Bosons must always be in a spin state that is symmetric under exchange.

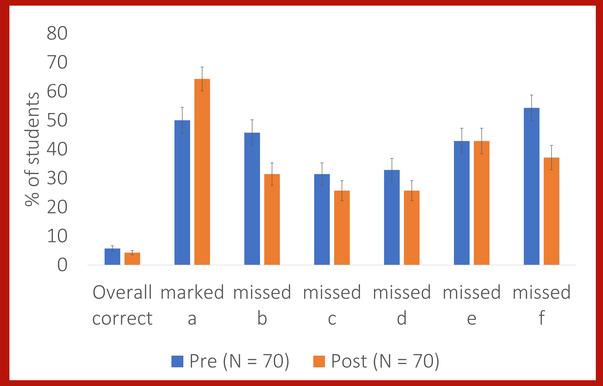
Findings

- Generally, student posttest performance was not statistically different from pretest performance.
- Pretest and posttest scores generally fell between 30% and 70%, far from floor or ceiling effects.
- Instructors (who validated items) were often surprised by student difficulties.
- There may be a confusion of parity and exchange symmetry, as in "spatially symmetric" vs. "a spatial part that is symmetric under particle exchange".

Item	Institution	Cohort	*N*	Correct Pre (%)	Correct Post (%)	Pre/post ϕ (all)	Pre/post McNemar p (all)	Cohort Fisher Ex. p on post
1	1	AU16	26	65	65	**0.53**	0.33	0.87
	1	AU17	35	57	71			
	2	AU17	17	65	65			
2	1	AU17	31	29	42	**0.60**	1.0	-
3	1	SP17	30	30	53	0.11	0.11	0.65
	1	SP18	30	43	50			
	2	SP18	10	60	70			

Item	Institution	Cohort	*N*	Score Pre (%)	Score Post (%)	95% CI gains (all)	Time p (all)	Cohort × time p
4	1	SP17	28	58	60	**[1.7, 16]**	**0.016**	**0.014**
	1	SP18	29	61	59			
	2	SP18	13	47	73			

Summary and comparisons of pre and posttest performance for all items. Here "AU##" and "SP##" refer to autumn and spring semesters, respectively, of the year 20##. Items 1-3 were scored binary, such that the quantity of interest is the percentage of answers that were correct. Pre/post comparisons are made using a McNemar test; comparisons between cohorts were made using a Fisher exact test. Item 4 was scored closer to a continuum; pre and posttest mean scores are shown. A two-way ANOVA was applied to these, with the main effects being time and cohort. Statistics with p<0.05 appear in bold.



Response proportions for Item 4 (on exchange symmetry). Note that errors are not mutually exclusive. Error bars indicate binomial standard error; note that within-student error on the pre and posttest is correlated.

Note that items were not asked of cohorts that did not cover the relevant material. Instructors were consulted and items were only used if the instructor deemed them fair and relevant to the content of the course.

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COMMON ERRORS:

- Some students (esp. Item 1) seemed to conflate orthogonality in a Hilbert space with orthogonality of Cartesian axes.
- Many students say that a spin "up" and a spin "down" particle can only form a system with a net spin of zero, ignoring possible coherent addition of spin along some axis other than *z*.
- Students displayed a variety of misunderstandings in the context of exchange symmetry. These include possible confusion of reflection symmetry (parity) with symmetry under exchange, and lack of consideration of both spatial and spin wave function parts to determine overall exchange symmetry.

FUTURE WORK / FRAMEWORKS:

The student answers to the questions investigated in this study indicate that the student difficulties in these topics may not be most productively described in terms of stable, coherent misconceptions, but rather in terms of other models of understanding or student answering, such as a resources model [1] or a dual process model [2,3]. A simple example of this is the observed inconsistency between student answers to Items 1 and 2, both of which require the concept of orthogonality but in slightly different contexts and representations. Item 1 explicitly uses the term "orthogonal" which may, in terms of a resources model, naturally evoke a graduate student's substantial resources of Cartesian coordinate systems. Or in terms of a dual process model, rapid, highly accessible associations cued by the term "orthogonal" dominate the decision process.

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