Characterizing covariational reasoning in physics modeling

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Background

Covariational Reasoning:

- The consideration of **how a change in one quantity is related to a change in another quantity.**
- Plays a central role in how physicists reason about quantitative models.
- Studied extensively in math contexts by mathematics education researchers (e.g., see [1]).

Mathematics education researchers Carlson et al. developed frameworks to describe levels of development of covariational reasoning, and "mental actions" that are supported by those levels [1]. The framework of mental actions describes five modes of reasoning about relationships between variables that effectively operationalize covariational reasoning in mathematics. An abbreviated version of the mental actions framework is shown in Table 1.

Mental Action	Description
MA1	Coordinating the value of one variable with changes in another
MA2	Coordinating the direction of change of one variable with changes in another variable
MA3	Coordinating the amount of change of one variable with changes in the other variable
MA4	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable
MA5	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the domain of the function

Prior work suggests that covariational reasoning "looks different" in physics than it does in mathematics [2]

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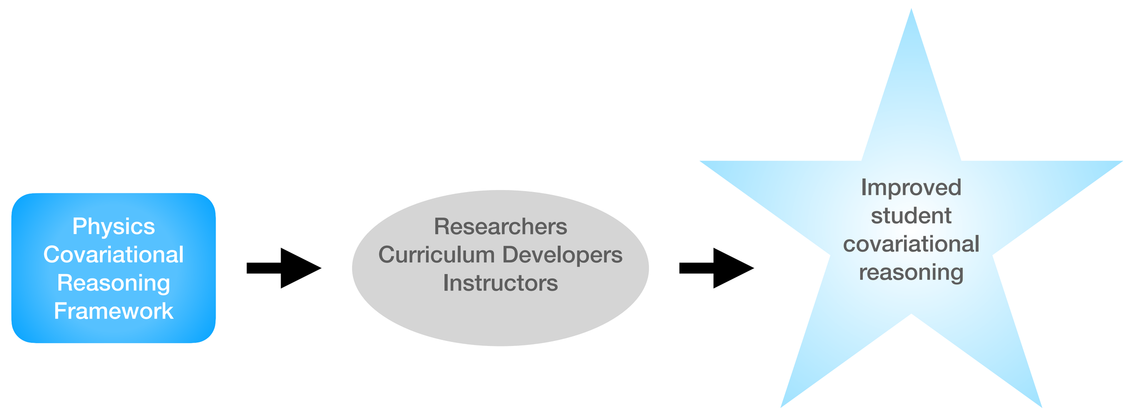
Purpose



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Framework of covariational reasoning in physics modeling

This framework characterizes how physics experts reason covariationally about quantitative models.

PROCEPTUAL UNDERSTANDING	
I. Mathematics Resources	**II. Physics Quantities**
A. Common function behavior	A. Constructing quantity
B. Common function rules	B. Mathematical structure
C. Use of common operations	C. Constraints of quantities
D. Use of common procedures	D. Symbolizing
	E. Combining quantities

PHYSICS MENTAL ACTIONS	EXPERT BEHAVIORS
I. Related Quantities	**I. Compiled Relationships**
II. Trend of Change	A. Proxy Quantities
III. Discrete Change	B. "Goes Like"
IV. Small Chunks of Change	**II. Simplification Techniques**
V. Functional Reasoning	A. Limiting Cases
	B. Physically Significant Points
	C. Symmetry

Proceptual Understanding

Covariational reasoning in physics modeling is based on *proceptual understanding* [3] of both the underlying **Mathematics Resources** and the relevant **Physics Quantities** themselves.

Physics Mental Actions

Physics mental actions (PMA) describe direct covariation of quantities; they are explicit instantiations of considering how quantities relate to each other or how changes in one quantity lead to changes in the other. The PMA are derived but are distinct from the mental actions described by Carlson et al. For all of the PMA, we stress that the variables are physics quantities, rather than just numeric values, and do not entail reasoning about continuous changes, instead focusing on small "chunks" of change. PMA V does not have an analog in the mental actions by Carlson et al., and seems to stem from knowledge of familiar physics models and relevant equations.

Expert Behaviors

Expert behaviors are used in ways that tend to limit or guide the use of PMA, though this may not be a conscious process. **Compiled Relationships** describes behaviors that use understanding of physics quantities or familiarity with physics contexts, and minimize mental effort. *Proxy quantities* is the use of a different, more familiar quantity substituted for another when covarying two quantities. *"Goes Like"* reasoning refers to ways physicists relate two quantities through a simplified function. **Use of Simplification Techniques** describes behaviors which physics experts engage in that both guide and limit the amount of novel covariational reasoning that is required in a given physics context.

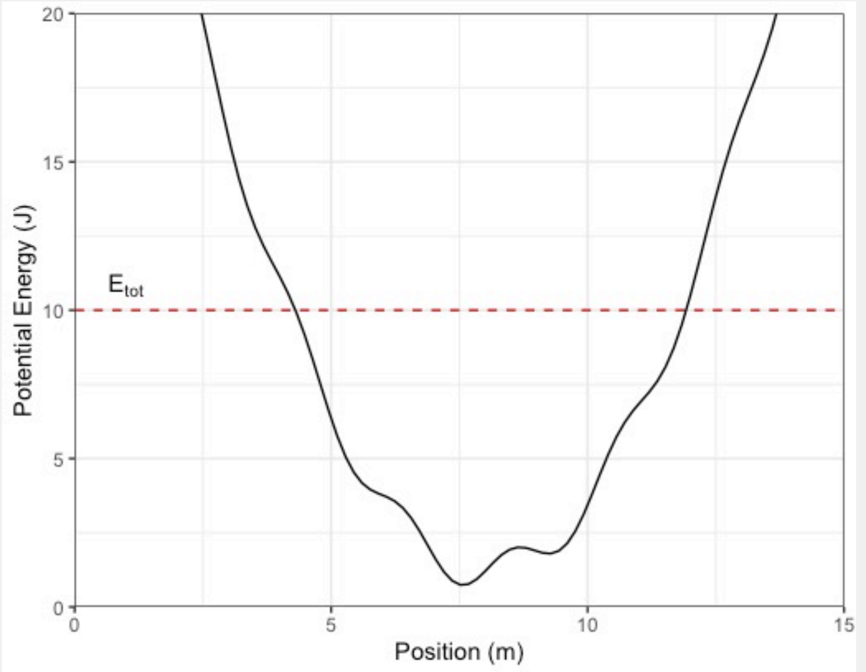
Application of the framework

Covariational reasoning "essential skills"

Physics education researchers Mikula and Heckler described a framework for improving physics essential skills (ES) [4], fundamental procedural mathematical skills that are necessary for problem-solving in STEM fields (e.g., vector superposition). The ES framework describes a structure for computer-based assignments to improve students' fluency with these skills; when students are able to do the math quickly and accurately (i.e., *fluently*), they are able to dedicate more cognitive effort to understanding physics content and reasoning. We believe that **ES assignments featuring more *conceptual* essential skills can also facilitate physics problem solving.**

Example "Graphical Features" Essential Skills item

Graphs like the one below are referred to as "potential energy diagrams" and show how potential energy varies with position (in one dimension).



How could you use the potential energy diagram above to find a quantity that has units of Newtons?

a. Find the slope of the graph
b. Find the area under the curve
c. Find the vertical coordinate of a point on the graph
d. Find the vertical intercept

Essential skills item intended to improve student fluency with interpretation of graphical features. The correct answer is **a**, and uses PU II. A, D, and E.

References

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