Multiple-Choice Question Sequence (MQS): Measurement Uncertainty for 2-state systems

Development, validation, and comparison of implementations | P. Hu, Y. Li, & C. Singh

Learning objectives - students are able to do the following:

- Identify which observables are **well-defined** for a particular measurement.
- **Calculate the uncertainty** of a measurement in a given state.
- Describe the **results when successive measurements are made** of observables whose operators do or do not commute

Development and validation

For the sequence, we created new questions and leveraged existing ones for which cognitive task analysis from student and expert perspectives had been conducted. We iterated the questions with faculty and students to reduce ambiguity. The final sequence prioritizes conceptual knowledge and contains checkpoints to foster class discussion.

Virtual and In-person Implementation

Implementation took place after traditional lecture instruction on time-development of two-state systems.
Pre-test and post-test questions that were multiple-choice are labeled as [MC].

1. Pre-test → **2. MQS** → **3. Post-test**

Virtual (N = 27)	Pre-test mean
Q1	78%
Q2	94%
Q4	95%
Q5a	78%
Q5b	52%
Q6a	87%
Q6b	69%
Q6c	70%
Q6d	37%
Q6e	31%

In-person (N = 23)	Pre-test mean
Q1	80%
Q2	91%
Q4	83%
Q5a	65%
Q5b	35%
Q6a	63%
Q6b	50%
Q6c	46%
Q6d	43%
Q6e	48%

Student difficulties that were addressed

- Identifying observables that are well-defined (Q1-3) *[MC]*
- Identifying and calculating uncertainty (Q4a-b)
- Using measurement collapse to determine uncertainty of a subsequent measurement (Q5a-e)
- Conflating eigenstates and eigenvalues (Q5a-e)

Example question

Uncertainty 1.1

Consider a system with a Hamiltonian $\hat{H} = C\hat{S}_x$, where C is an appropriate constant. (Note: The Hamiltonian is the operator that corresponds to energy.)

Choose all of the following statements that are correct for the system if it is in an eigenstate of $\hat{S}_z$, i.e., $|z\rangle$ or $|-z\rangle$.

I. The observable S_x is well-defined.
II. The observable S_z is well-defined.
III. Energy is well-defined.

A. I only B. II only C. II and III only D. I and III only E. All of the above

Post-test mean	Normalized gain	Effect size (Cohen's *d*)
94%	0.72	0.93
98%	0.60	0.33
93%	-0.50	-0.10
85%	0.33	0.22
76%	0.50	0.64
93%	0.43	0.21
81%	0.41	0.39
83%	0.44	0.38
76%	0.62	0.91
48%	0.24	0.40

Post-test mean	Normalized gain	Effect size (Cohen's *d*)
99%	0.93	1.06
94%	0.33	0.14
94%	0.67	0.39
83%	0.73	1.24
78%	0.38	0.38
96%	0.88	1.07
83%	0.65	0.86
78%	0.60	0.77
63%	0.35	0.48
80%	0.63	0.88

Typical normalized gains	
Traditional lecture	~0.23
Active learning	~0.45

Cohen's *d* effect sizes	
Small	~0.20
Medium	~0.50
Large	~0.80

Developing an MQS to address common student difficulties is effective!